

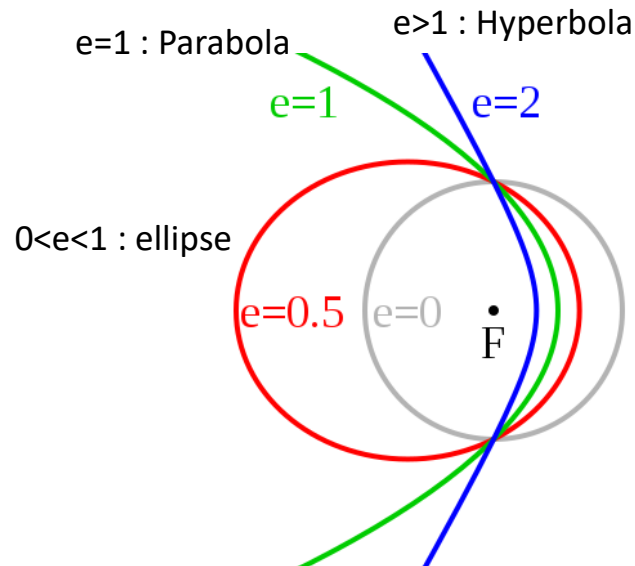
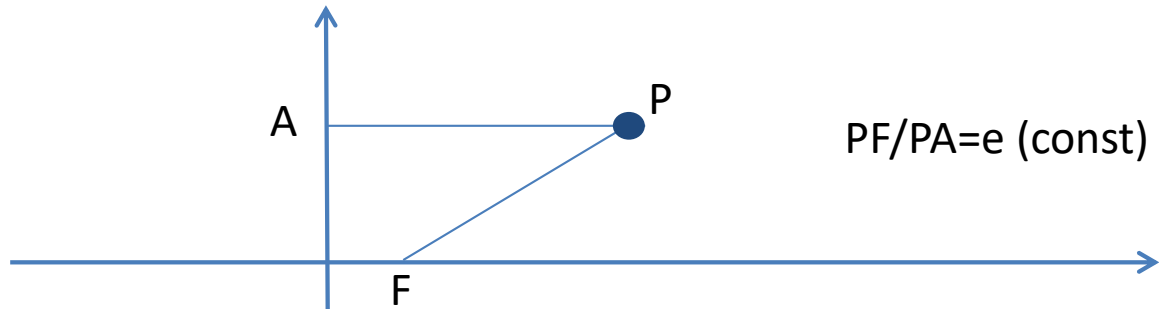
天体計測学特論 I
Observational Astronomy I

Lecture 02:
Optical systems for observations

Geometrical optics

Geometry

- Hyperbola / Ellipse



Collimating/Focusing : Optical surface

- Hyperboloidal / Ellipsoidal optical surfaces

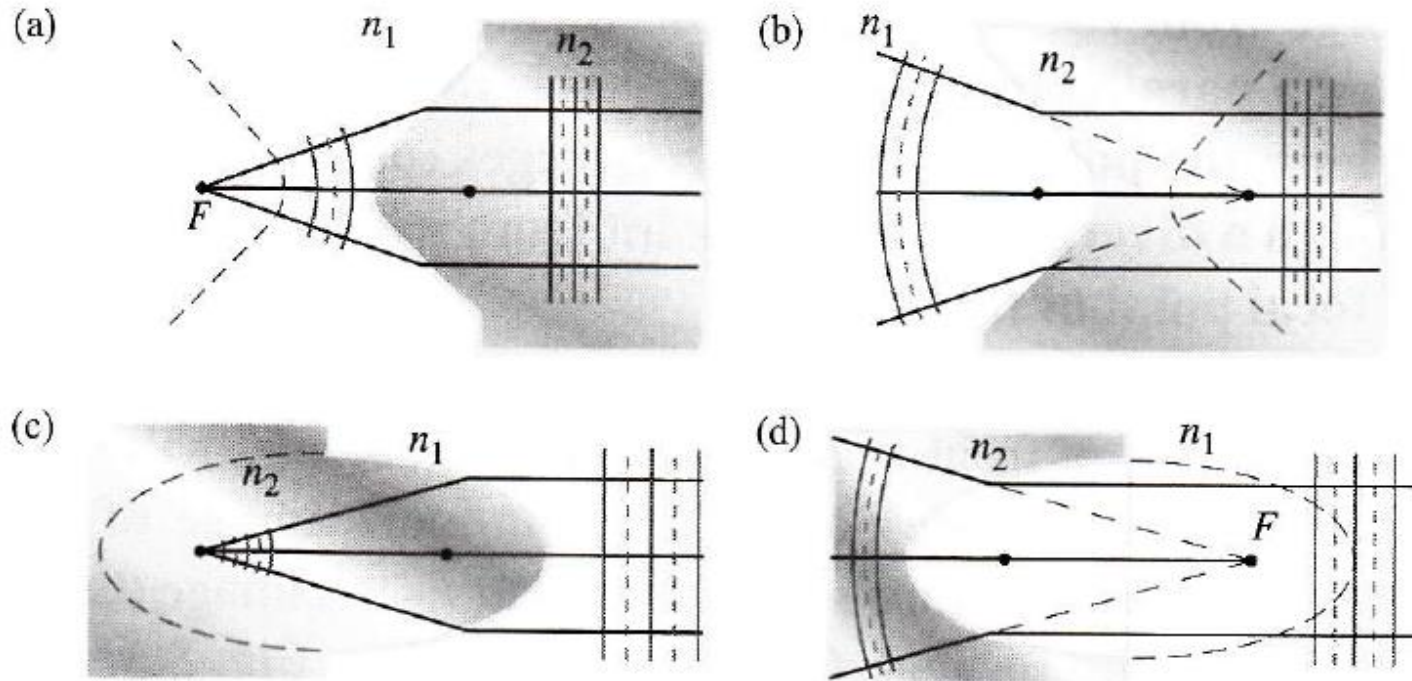
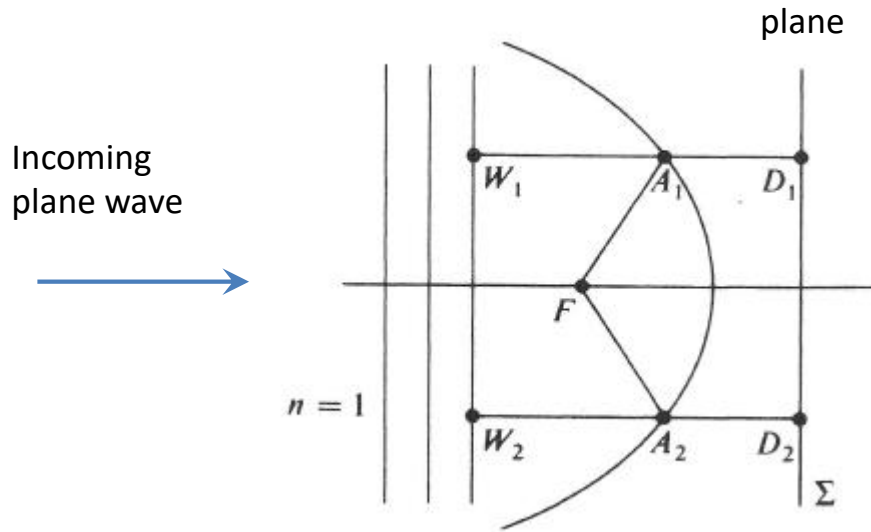


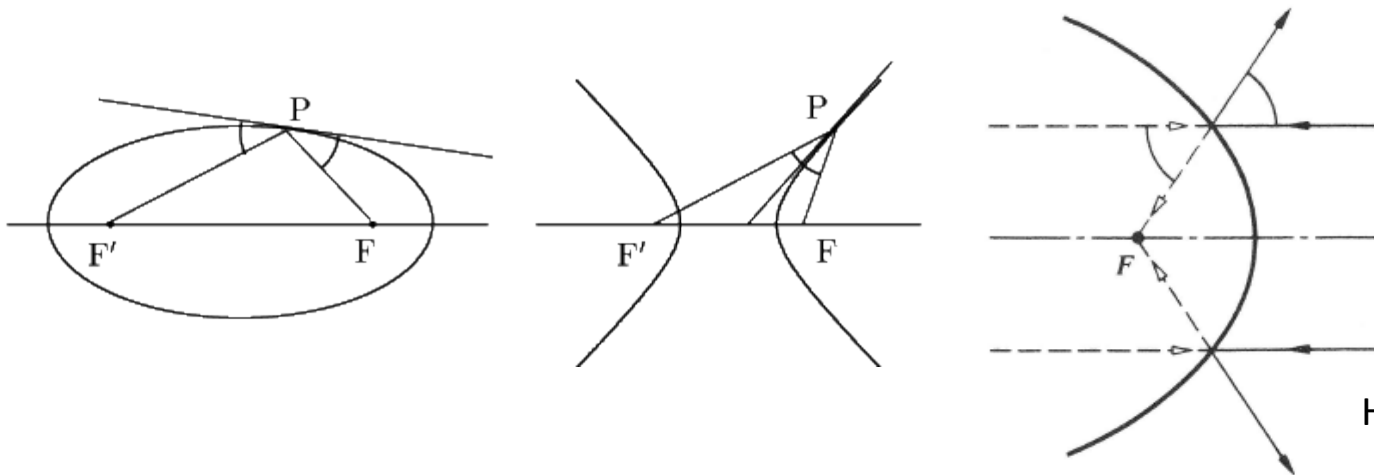
Figure 5.4 (a) and (b) Hyperboloidal and (c) and (d) ellipsoidal refracting surfaces ($n_2 > n_1$) in cross section.

Collimating / Focusing : Mirror

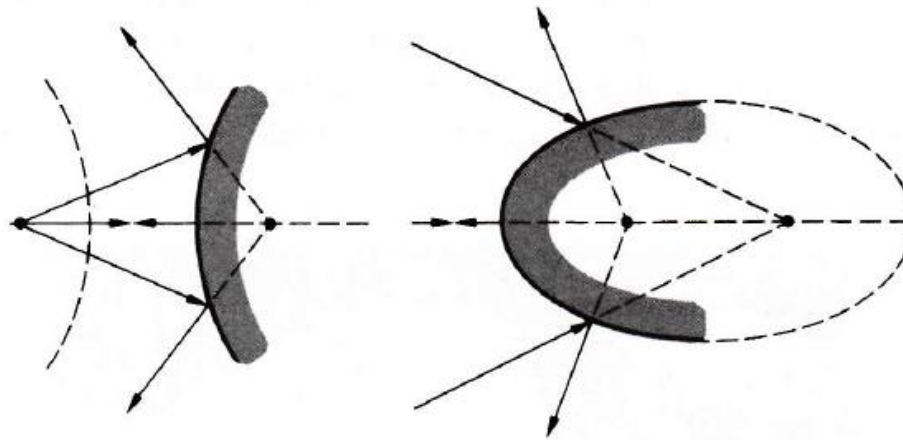
- Ellipse / Parabola / Hyperbola



$AF = e AD$, e : eccentricity
 $K = -e^2$, K : conic constant
 $y^2 - 2Rx + (K+1)x^2 = 0$

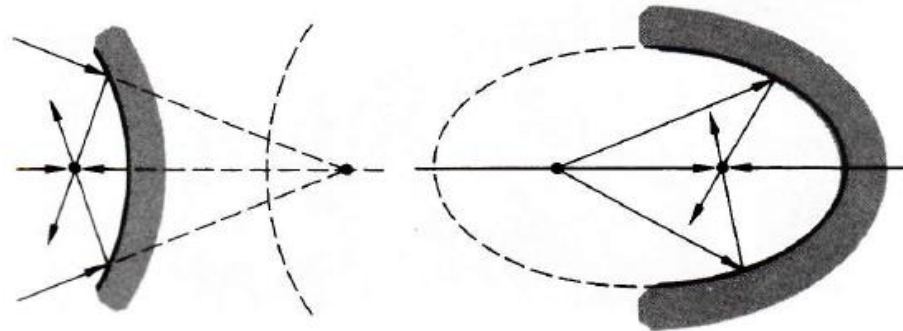


Focus with Hyperbolic / Elliptical Mirrors



(a) Convex hyperbolic

(b) Convex elliptical

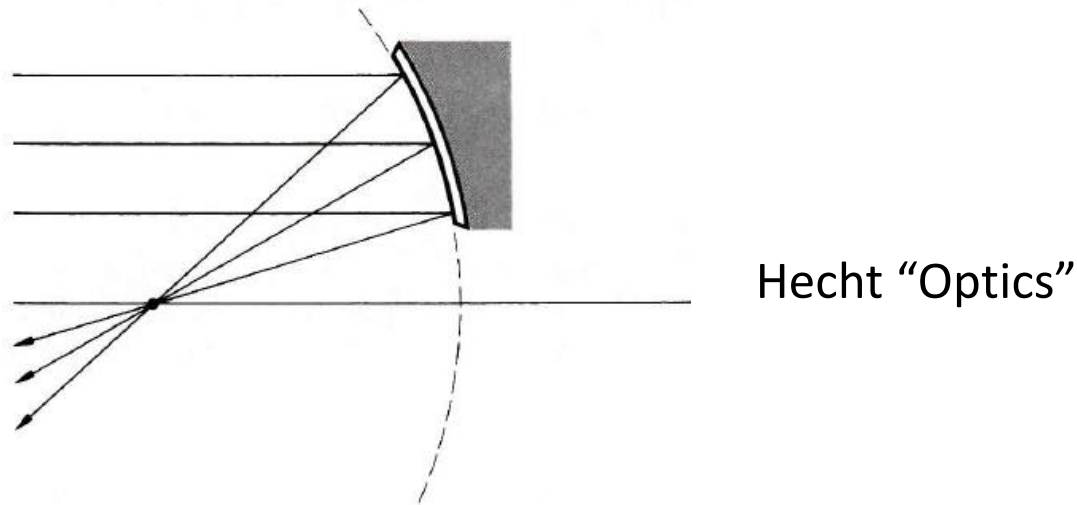


(c) Concave hyperbolic

(d) Concave elliptical

Collimating / focusing with parabolic mirrors

- Parabolic and off-axis parabolic mirror



From Edmund optics

Spherical mirrors

- (Spherical) aberration of a spherical mirror

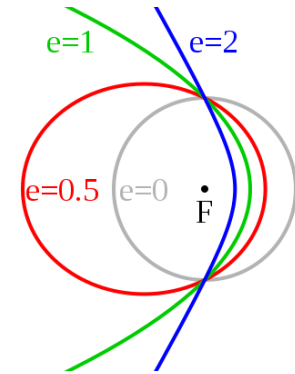
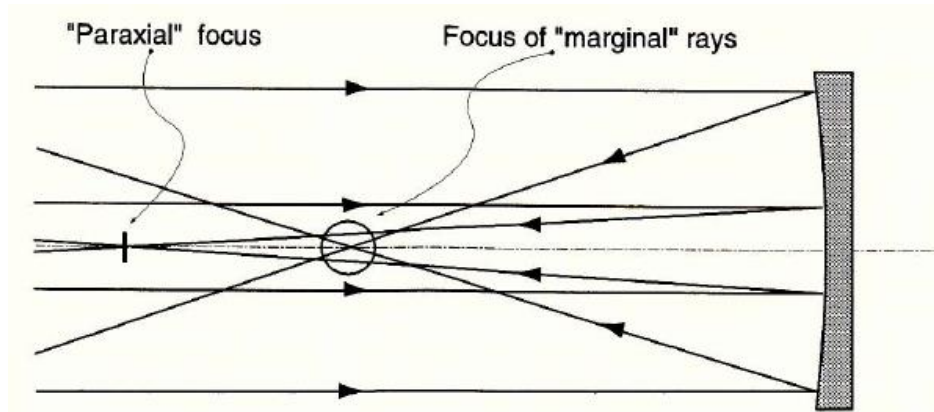
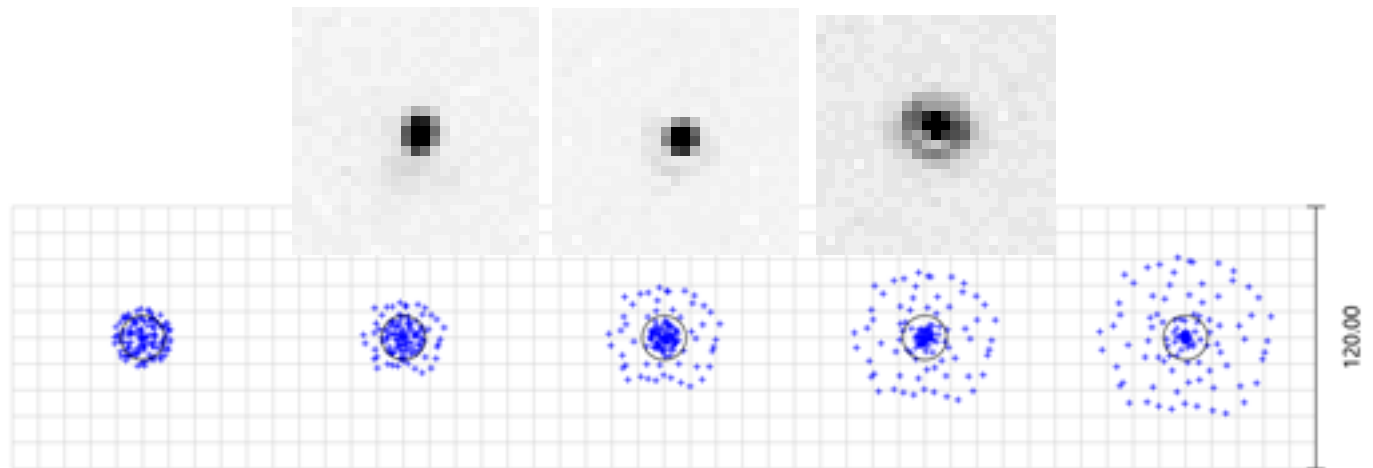


Fig. 1.2. Spherical aberration of a spherical concave telescope mirror. "Paraxial" rays are nominally at a negligible height from the axis

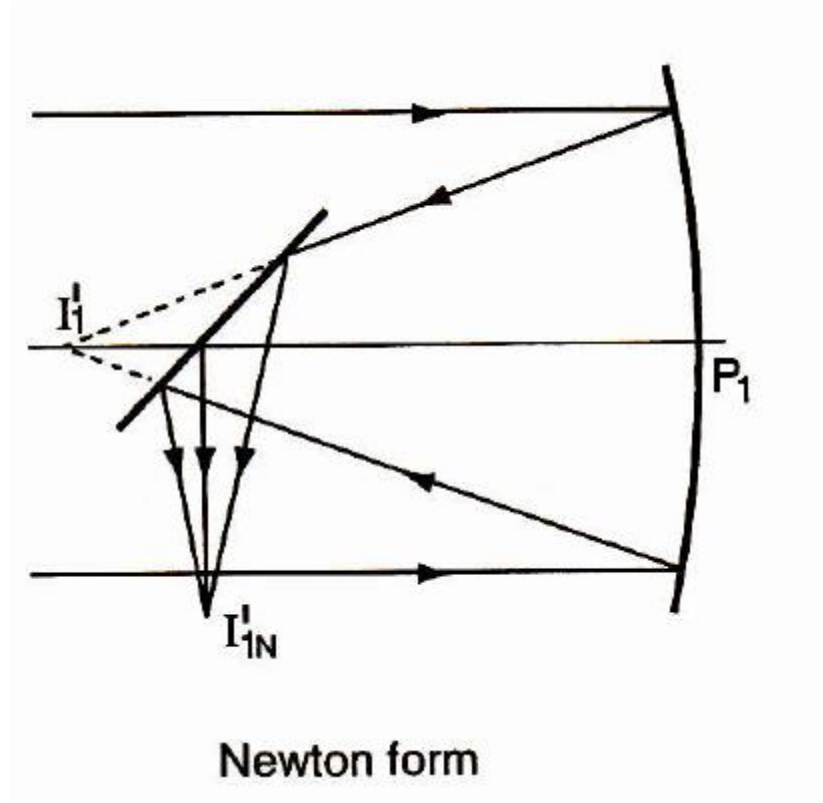
Wilson "Reflecting Telescope Optics I"

0.0000, 0.0000 mm



Reflective telescope (0) :

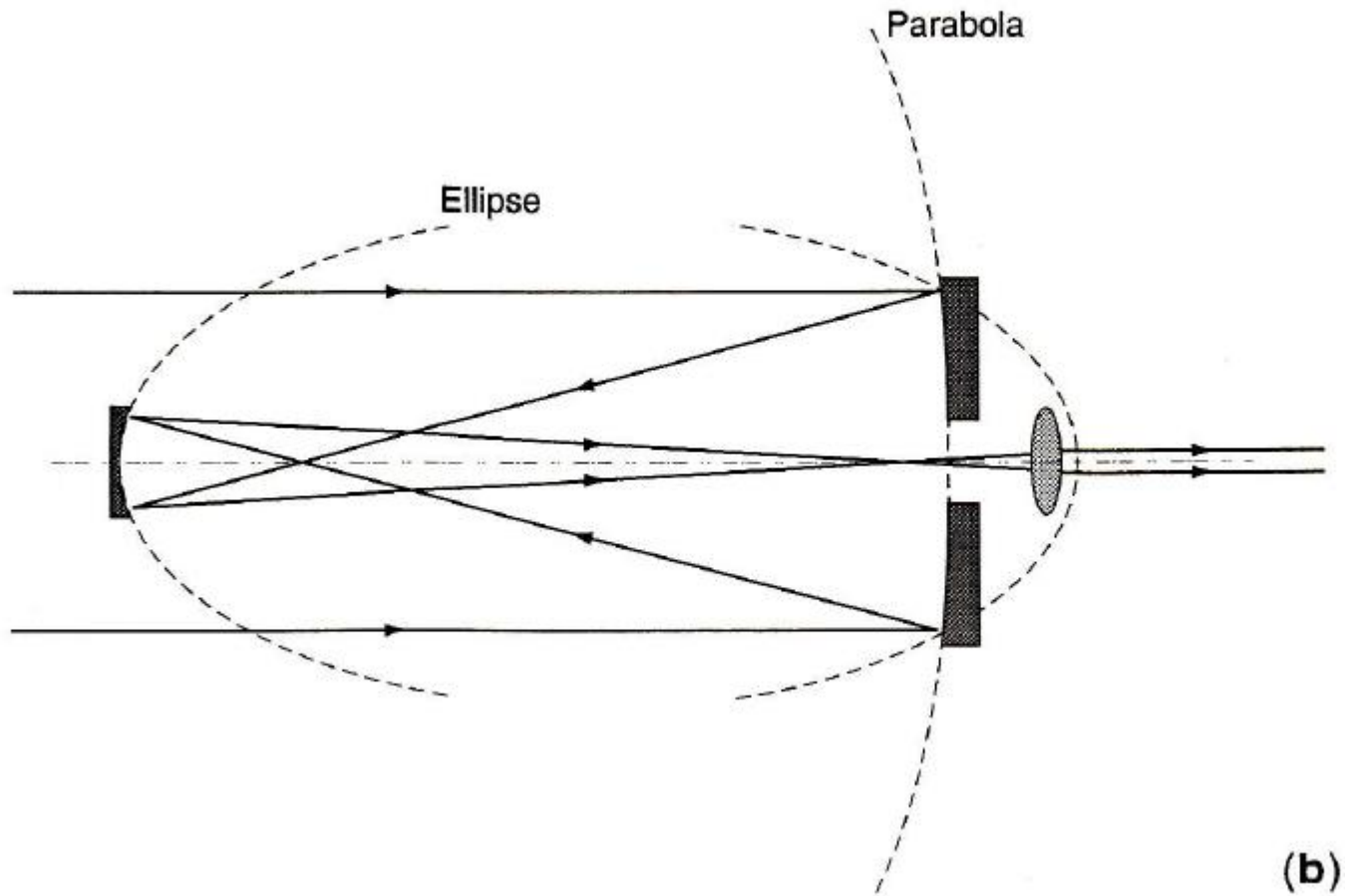
- Newtonian telescope (M1:~Parabolic)



Wilson "Reflecting Telescope Optics I"

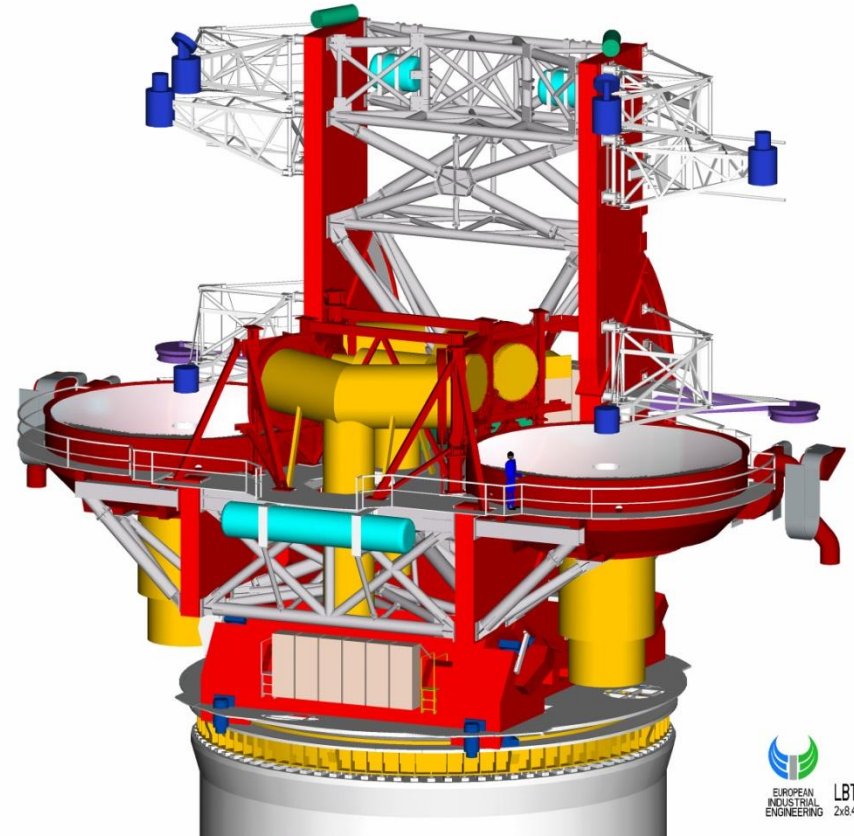
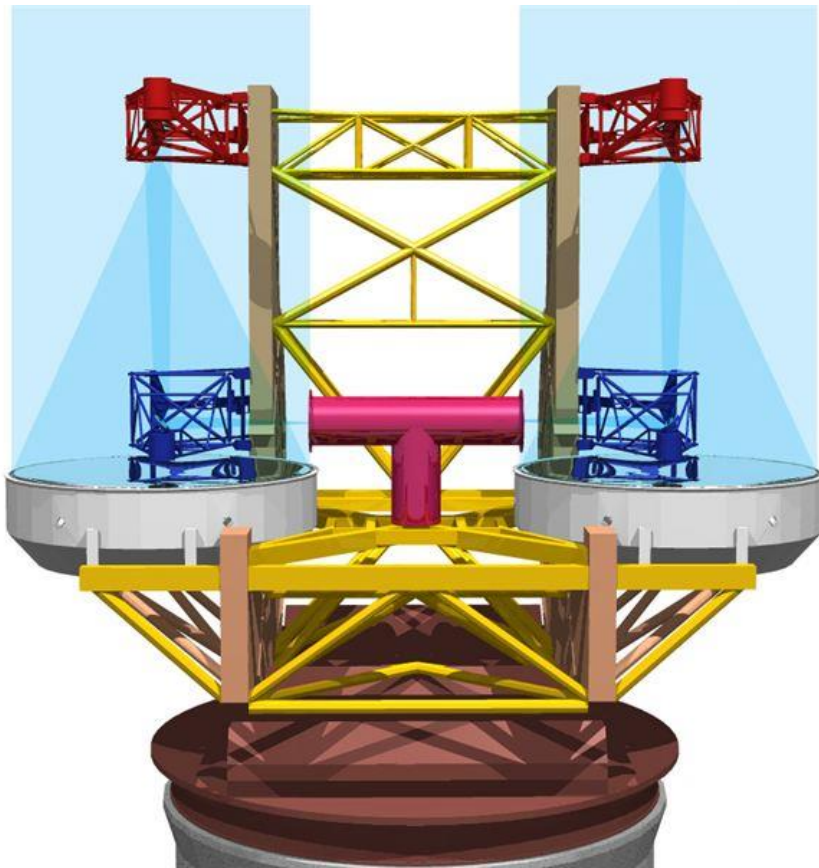
Reflective telescope (1) : Gregorian

- The concept



Reflective telescope (1) : Gregorian

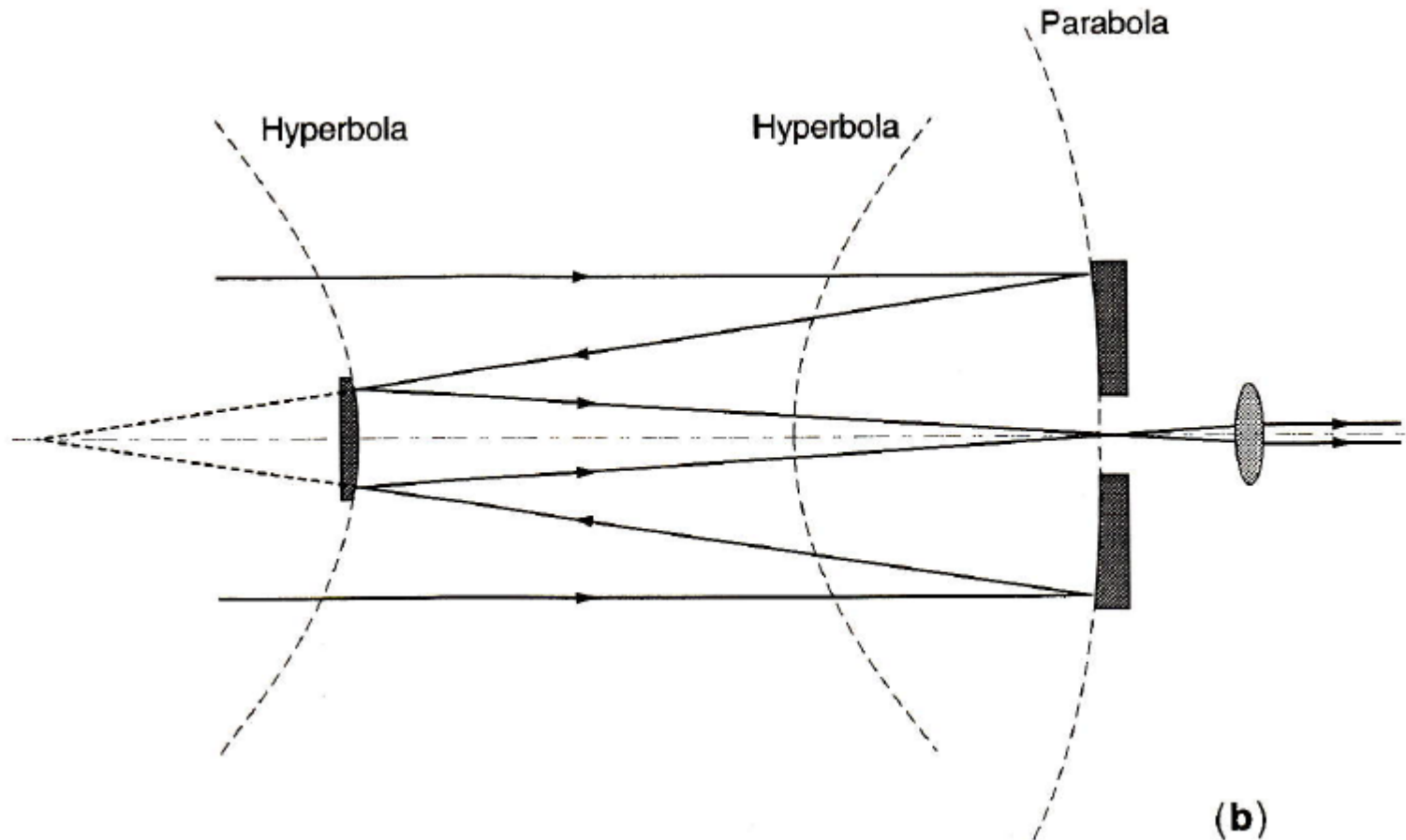
- Large Binocular Telescope



From lbt.org

Reflective telescope (2) : Cassegrain

- The concept



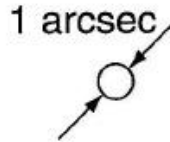
Spot diagram for a Cassegrain telescope

Prescription Data

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 Date : Fri Mar 16 1995

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 Gaussian Factor : 0.00000
 Eff. Focal Len. : 38501.7
 Total Track : 8832.9
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 Working F/# : 11.0802
 Obj. Space N.A. : 1.75e-007
 Stop Radius : 1750
 Parax. Ima. Hgt. : 107.997
 PRAX: Hgt. : 0
 Entr. Pup. Dia. : 3500
 Entr. Pup. Pos. : 0
 Exit Pup. Dia. : 349.344
 Exit Pup. Pos. : -10443.2
 Maximum Field : 0.25
 Primary Wave : 0.500000
 Lens Units : millimeters
 Angular Mag. : 3.68676



Fields

Field Type: Angle in degrees
 # X-Value Y-Value Weight
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 2 0.00000 0.150000 1.00000
 3 0.00000 0.080000 1.00000

Wavelengths

Units: Microns
 # Value Weight
 1 0.500000 1.000000

SURFACE DATA SUMMARY:

Surf	Type	Radius	Thickness	Class	Diameter	Conic
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1	STANDARD	-15400	-5933.3	MIRROR	3506.868	-1
2	EVENASPHER	-4416.7	8833.905	MIRROR	857.5355	-1
3	STANDARD	-1954.713	0		336.4275	0

SURFACE DATA DETAIL:

Surface 0B : STANDARD
 Surface 0T : STANDARD
 Surface 2 : EVENASPHER
 Coeff on x 2 : 0
 Coeff on x 4 : 1.81341e-011
 Coeff on x 6 : -5.80367e-020
 Coeff on x 8 : 0
 Coeff on x 10 : 0
 Coeff on x 12 : 0
 Coeff on x 14 : 0
 Coeff on x 16 : 0
 Surface 2A : STANDARD

INDEX OF REFRACTION DATA:

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1	MIRROR	1.00000000
2	MIRROR	1.00000000
3		1.00000000

(a)

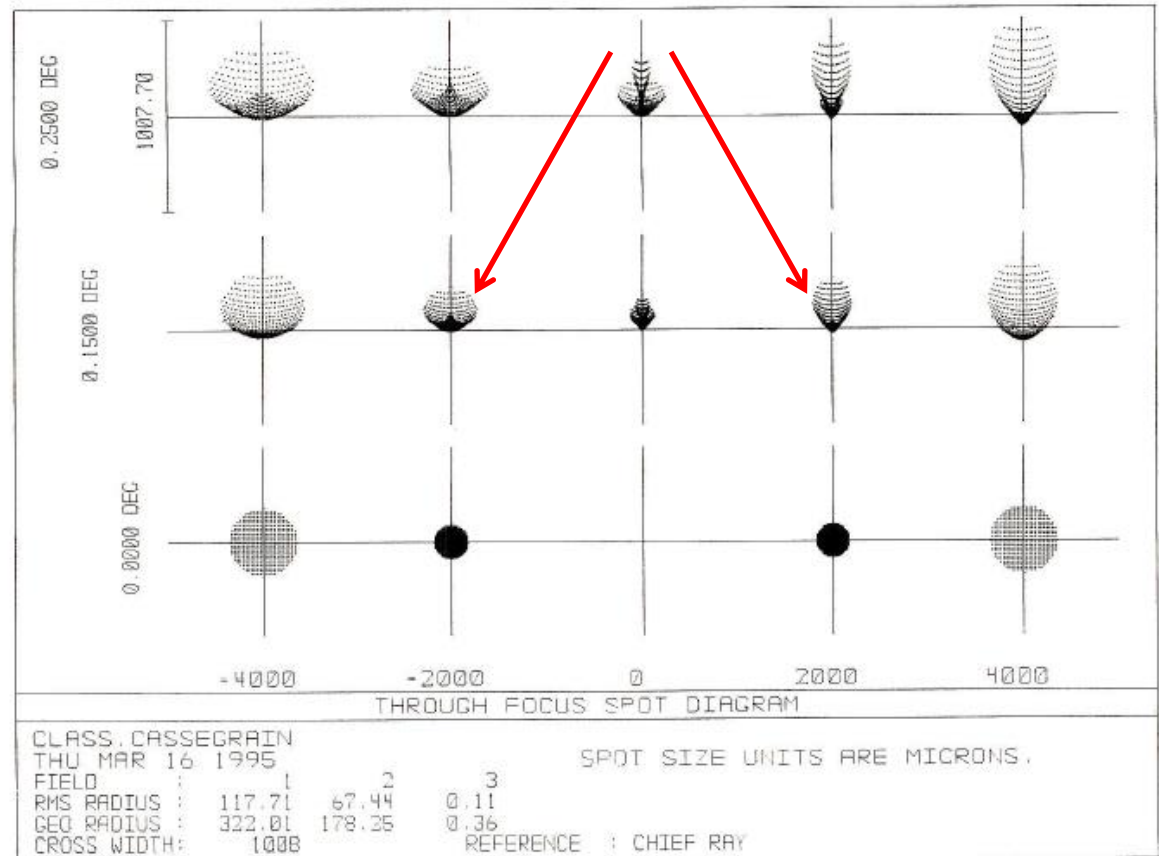
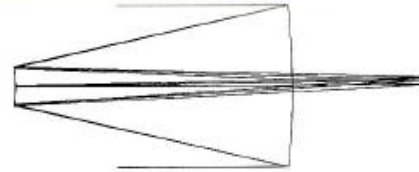


Fig. 3.4. (a) Spot-diagrams for a classical Cassegrain telescope with the geometry of the ESO 3.5 m NTT ($f/11$; $m_2 = -5$) for an optimum field curvature $r_c = -1955$ mm (concave to the incident light)

Wavefront and image aberration

- Spherical wavefront forms an ideal image.
- Distorted wavefront (W') results in the aberration in the image plane.

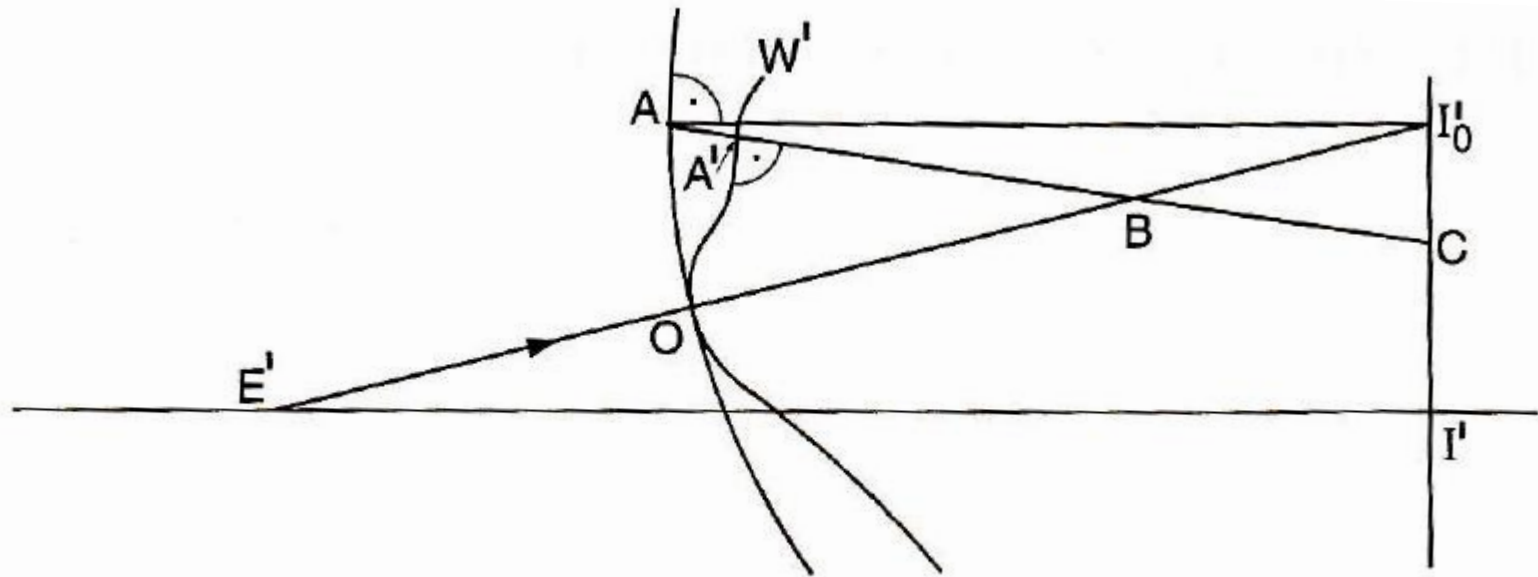


Fig. 3.1. Wavefront, longitudinal and lateral aberration

Parabola (ideal) to the third order approximation

- Expanding the equation for a sphere

$$z = r - r \left(1 - \frac{y^2}{r^2} \right)^{1/2}$$

Wilson "Reflecting Telescope Optics I"

$$z_{Sph} = \frac{c}{2}y^2 + \frac{c^3}{8}y^4 + \frac{c^5}{16}y^6 + \dots,$$

$$c = 1/r.$$

Parabola

- Elliptical and hyperbolic surfaces are described with

$$z_{Ell} = \frac{c}{2}y^2 + \frac{c^2}{8a}y^4 + \frac{c^3}{16a^2}y^6 + \dots$$

$$z_{Hyp} = \frac{c}{2}y^2 - \frac{c^2}{8a}y^4 + \frac{c^3}{16a^2}y^6 - \dots$$

$$c = \frac{a}{b^2} = \frac{1}{a(1 - \varepsilon^2)},$$

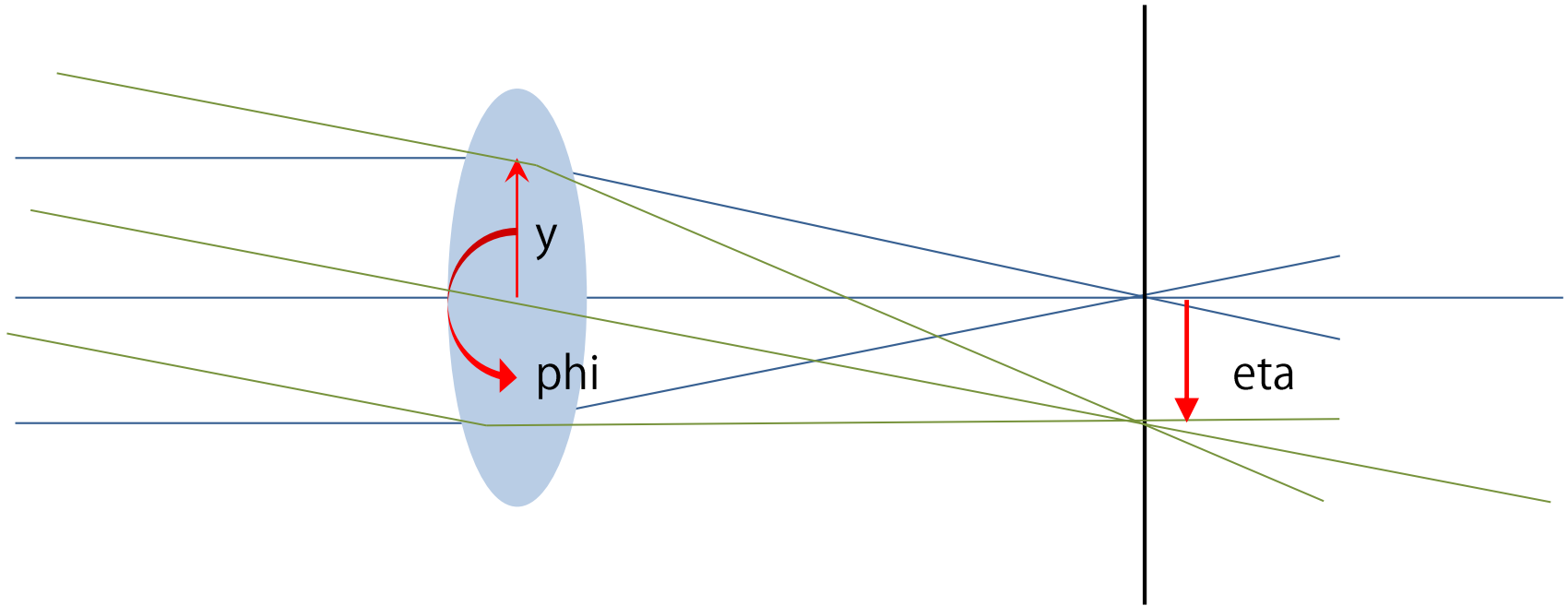
- All of them are the same in the first term (first order), different from the second term (third order).
- All of them can be described by the Schwarzschild (conic) constant b_s

$$b_s = -\varepsilon^2$$

$$z = \frac{c}{2}y^2 + \frac{c^3}{8}(1 + b_s)y^4 + \frac{c^5}{16}(1 + b_s)^2y^6 + \dots$$

Describing the wavefront distortion

- Coordinate for an on-axis optics
 - y : the aperture radius
 - η : the field radius
 - ϕ : azimuth angle



The Seidel description of the wavefront error

- Third order aberrations :

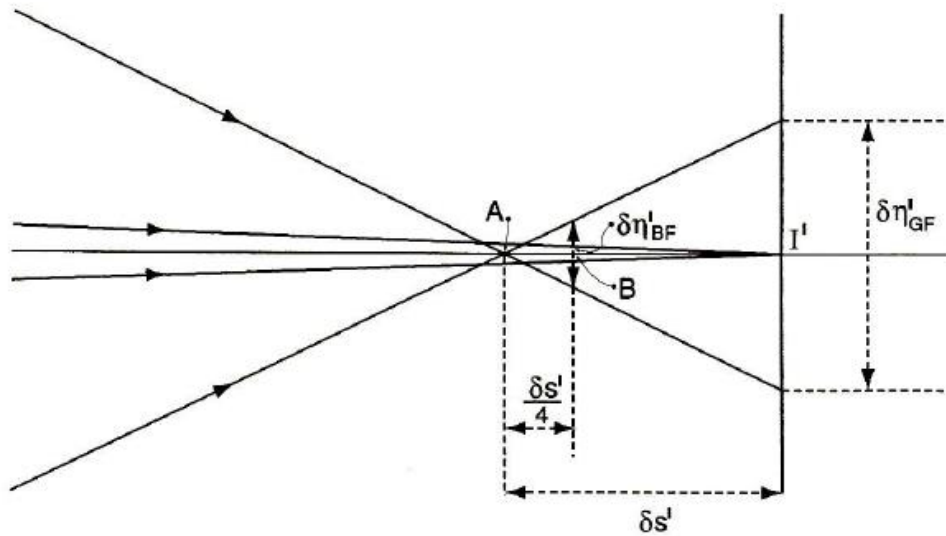
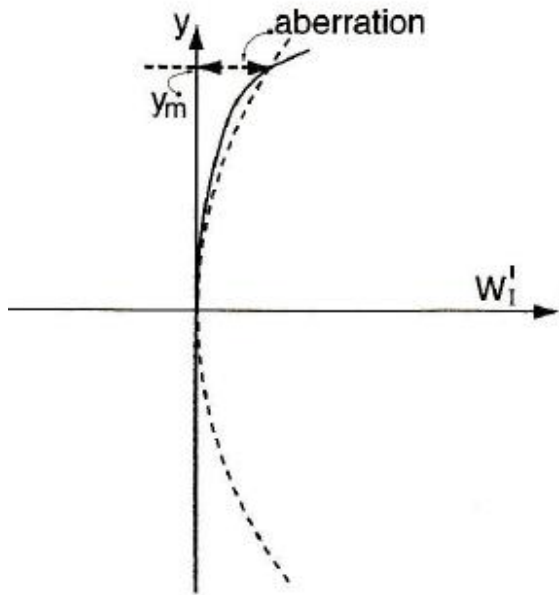
(spherical) + (coma) + (astigmatism) + (field curvature) + (field distortion)

$$\begin{aligned} W'_3(y_1, \eta') = & \frac{1}{8} \left(\frac{y_1}{y_{m1}} \right)^4 \sum S_I + \frac{1}{2} \left(\frac{y_1}{y_{m1}} \right)^3 \left(\frac{\eta'}{\eta'_m} \right) \sum S_{II} \cos \phi \\ & + \frac{1}{4} \left(\frac{y_1}{y_{m1}} \right)^2 \left(\frac{\eta'}{\eta'_m} \right)^2 \left[\underbrace{(3 \sum S_{III} + \sum S_{IV}) \cos^2 \phi}_{\text{astigmatism}} \right. \\ & \quad \left. + \underbrace{(\sum S_{III} + \sum S_{IV}) \sin^2 \phi}_{\text{field curvature}} \right] \\ & + \frac{1}{2} \left(\frac{y_1}{y_{m1}} \right) \left(\frac{\eta'}{\eta'_m} \right)^3 \sum S_V \cos \phi \end{aligned}$$

Spherical aberration (SI) : 球面収差

$$\frac{1}{8} \left(\frac{y_1}{y_{m1}} \right)^4 \sum S_I$$

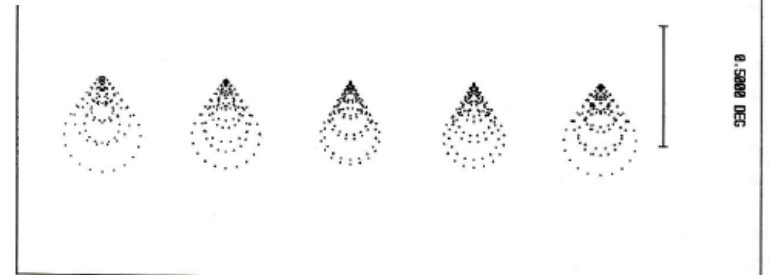
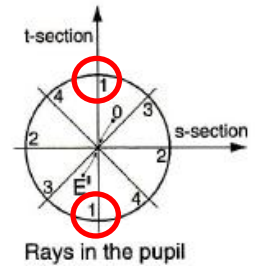
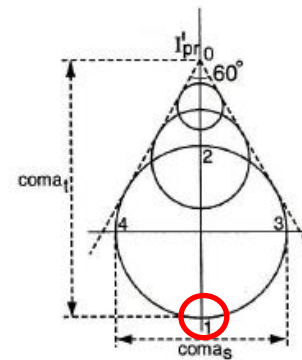
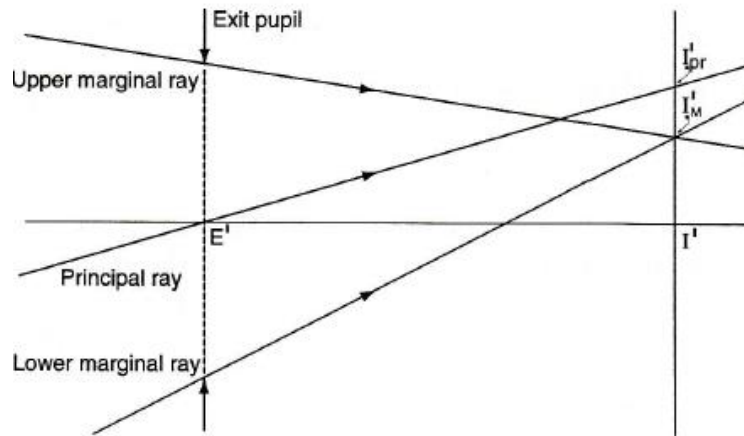
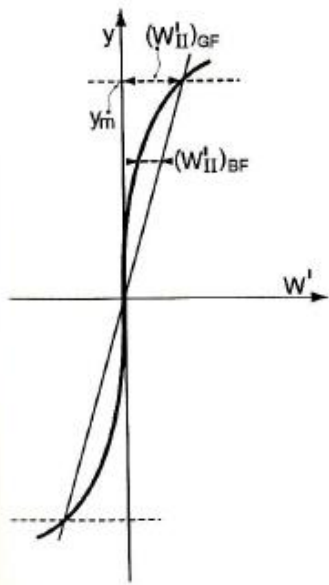
- The size of the aberration does not depend on the image height (η)



Coma (SII) : コマ収差

$$\frac{1}{2} \left(\frac{y_1}{y_{m1}} \right)^3 \left(\frac{\eta'}{\eta'_m} \right) \sum S_{II} \cos \phi$$

- Dependent on Φ : asymmetric
- The size of the aberration depends on the image height (η), no-coma aberration at the center of FoV.



Astigmatism (SIII) : 非点収差

$$(W'_{III} + W'_{IV})_{GF} = \frac{1}{4} \left(\frac{y}{y_m} \right)^2 \left(\frac{\eta'}{\eta'_m} \right)^2 \left[(3S_{III} + S_{IV}) \cos^2 \phi + (S_{III} + S_{IV}) \sin^2 \phi \right]$$

$$(W'_{III} + W'_{IV})_{GF} = \frac{1}{4} \left(\frac{y}{y_m} \right)^2 S_{III} \cos 2\phi + \frac{1}{4} \left(\frac{y}{y_m} \right)^2 (2S_{III} + S_{IV})$$

η term ignored

- Depends on 2Φ : horizontal vs. vertical
- Depends on image radius (η).

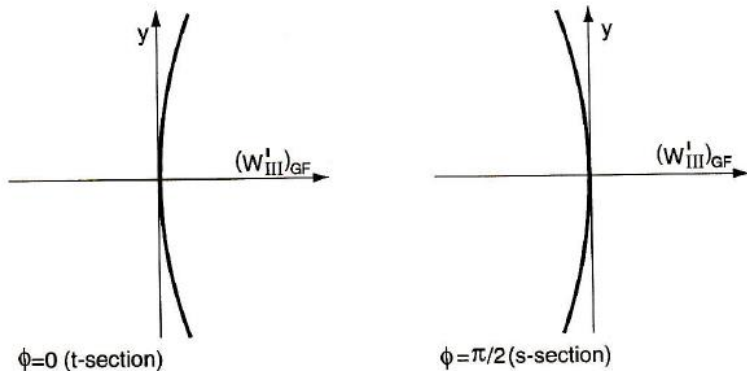


Fig. 3.19. Third order astigmatism: wavefront aberration reversal in the t - and s -sections due to the $\cos 2\phi$ term

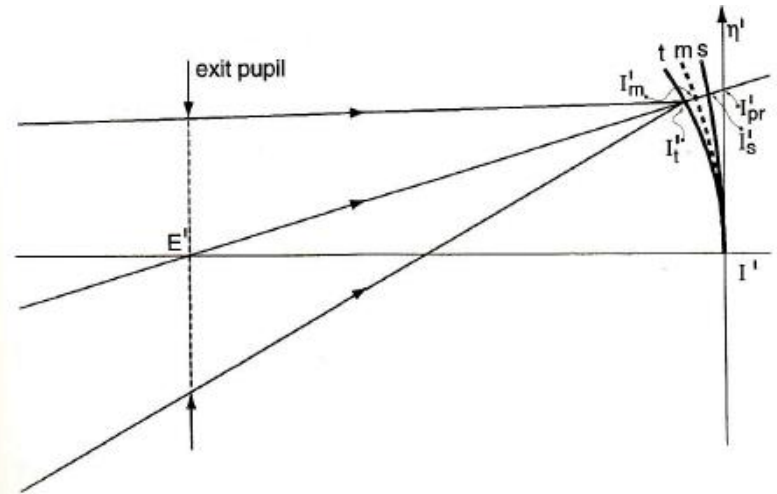
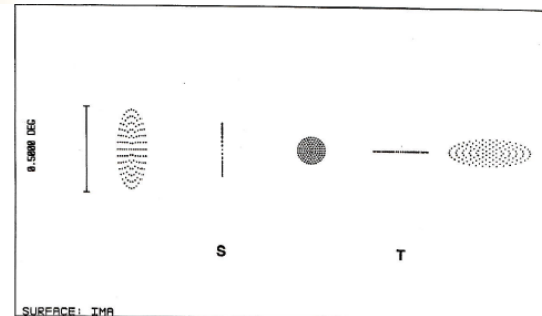


Fig. 3.20. Third order astigmatism: astigmatic surfaces and lines



Field curvature (S_{IV}) : 像面湾曲

$$(W'_{III} + W'_{IV})_{GF} = \frac{1}{4} \left(\frac{y}{y_m} \right)^2 \left(\frac{\eta'}{\eta'_m} \right)^2 \left[(3S_{III} + S_{IV}) \cos^2 \phi + (S_{III} + S_{IV}) \sin^2 \phi \right]$$

η term ignored

$$(W'_{III} + W'_{IV})_{GF} = \frac{1}{4} \left(\frac{y}{y_m} \right)^2 S_{III} \cos 2\phi + \frac{1}{4} \left(\frac{y}{y_m} \right)^2 (2S_{III} + S_{IV})$$

- Off-focus (y^2) depends on the image radius (η).

Distortion (Sv) : 像面歪曲

- Tilted wavefront ($y^* \cos(\phi)$) = image shift
- Depends on image radius (η^3)

$$+\frac{1}{2} \left(\frac{y_1}{y_{m1}} \right) \left(\frac{\eta'}{\eta'_m} \right)^3 \sum S_V \cos \phi$$

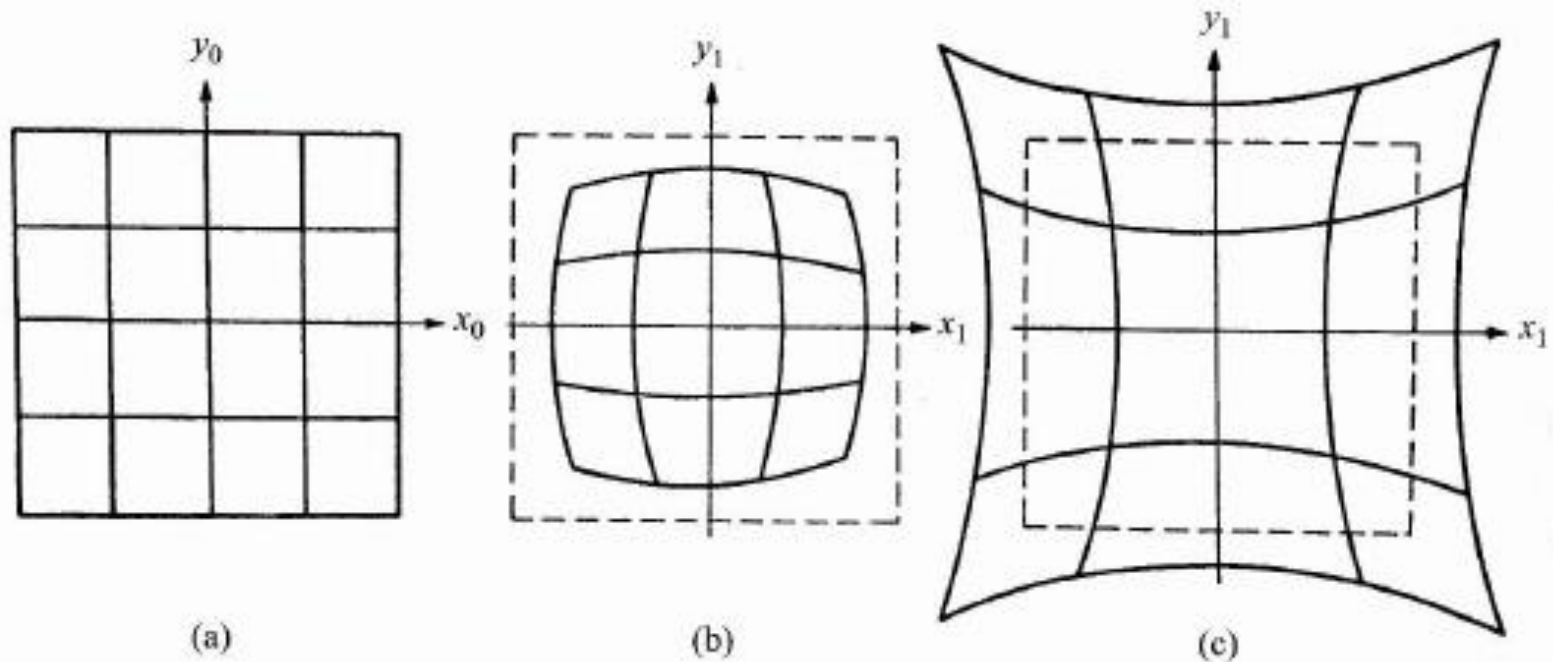


Fig. 5.8 (a) Object. (b) Image in the presence of barrel distortion ($E > 0$). (c) Image in the presence of pincushion distortion ($E < 0$).

Seidel coefficient of 1-mirror system

- b_s : Schwarzschild (conic) constant
- y_1 : mirror radius
- f_1 : mirror focal length
- u_{pr1} : field radius
- s_{pr1} : distance between mirror and aperture stop (=0)

$$(S_I)_1 = - \left(\frac{y_1}{f_1'} \right)^4 \frac{f_1'}{4} (1 + b_{s1})$$

$$(S_{II})_1 = - \left(\frac{y_1}{f_1'} \right)^3 \frac{1}{4} \left[2f_1' - s_{pr1}(1 + b_{s1}) \right] u_{pr1}$$

$$(S_{III})_1 = - \left(\frac{y_1}{f_1'} \right)^2 \frac{1}{4f_1'} \left[4f_1'(f_1' - s_{pr1}) + s_{pr1}^2(1 + b_{s1}) \right] u_{pr1}^2$$

$$(S_{IV})_1 = + \frac{H^2}{f_1'}$$

- Coefficient for a multi-mirror system can be evaluated by summing a coefficient of each mirror. (ν : id of a surface)

$$\underline{\sum S_q}_{\text{total}} = \sum_{\nu} (S_q)_{\nu} ,$$

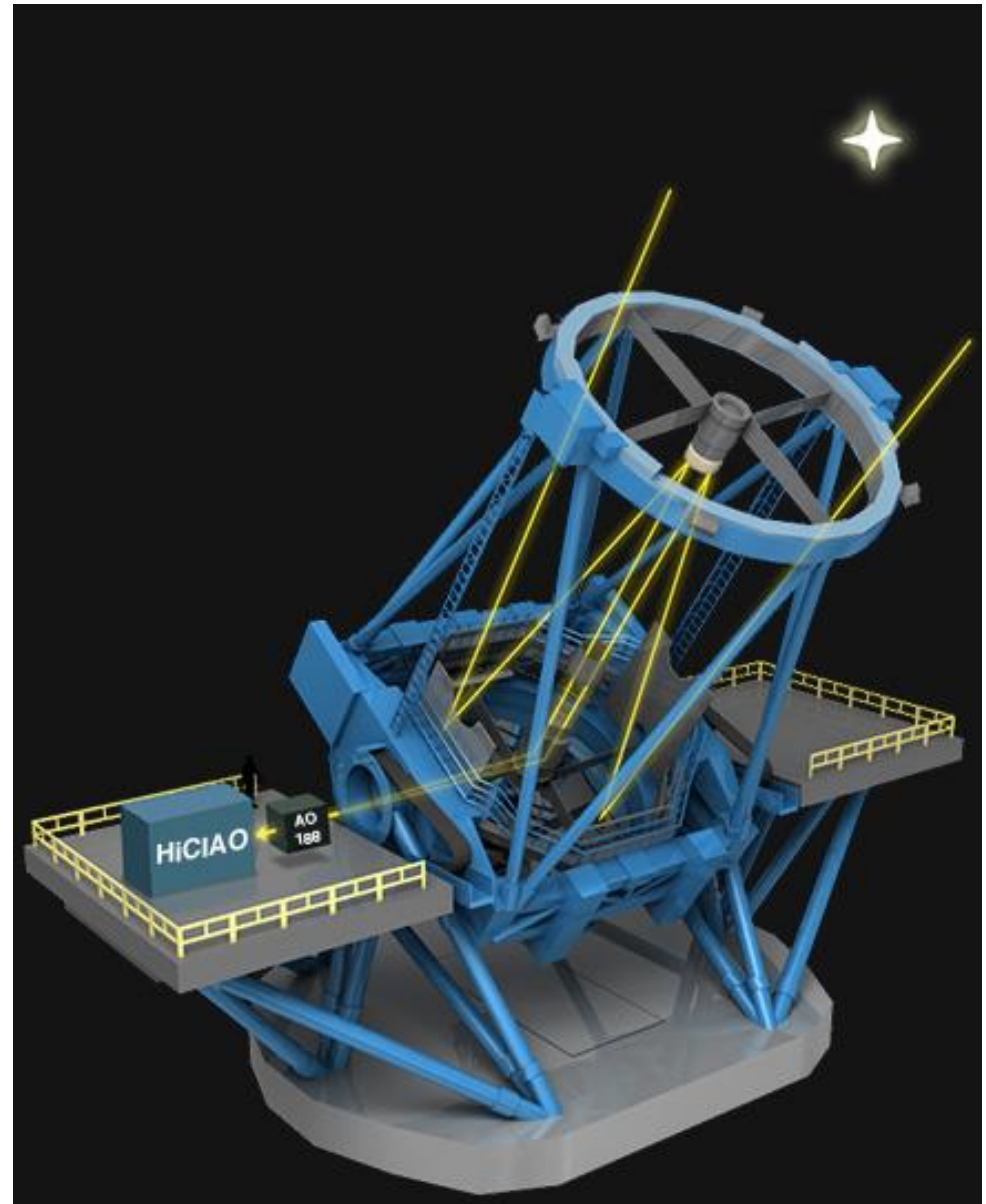
Seidel coefficient for some basic telescopes

Table 3.3. Seidel coefficients for some basic telescope systems. The asterisk denotes the aspheric contribution

	Case	Surface ν	$(S_I)_\nu$	$(S_{II})_\nu$	$(S_{III})_\nu$	$(S_{IV})_\nu = -(P_c)_\nu$	Effective field curvature $2(S_{III})_\nu + (S_{IV})_\nu$
1.	Spherical mirror (EP at primary)	1	+0.25	-0.5	+1.0	-1.0	+1.0
2.	Parabolic mirror (EP at primary)	1	+0.25	-0.5	+1.0	-1.0	+1.0
		1*	-0.25	0	0	0	0
		Sum	0	-0.5	+1.0	-1.0	+1.0
3.	Classical Cassegrain telescope (EP at primary) $m_2 = -4$	1	+16.0	-8.0	+4.0	-4.0	+4.0
		1*	-16.0	0	0	0	0
		2	-4.21875	+3.86719	-3.54493	+13.33333	+6.24347
		2*	+4.21875	+3.63281	+3.12825	0	+6.25650
		Sum	0	-0.50000	+3.58332	+9.33333	+16.49997
4.	Ritchey-Chrétien (RC) Cassegrain telescope (EP at primary) $m_2 = -4$	1	+16.0	-8.0	+4.0	-4.0	+4.0
		1*	-16.58054	0	0	0	0
		2	-4.21875	+3.86719	-3.54493	+13.33333	+6.24347
		2*	+4.79940	+4.13281	+3.55881	0	+7.11762
		Sum	0	0	+4.01388	+9.33333	+17.36109
5.	Dall-Kirkham (DK) Cassegrain telescope (EP at primary) $m_2 = -4$	1	+16.0	-8.0	+4.0	-4.0	+4.0
		1*	-11.78125	0	0	0	0
		2	-4.21875	+3.86719	-3.54493	+13.33333	+6.24347
		2*	0	0	0	0	0
		Sum	0	-4.13281	+0.45507	+9.33333	+10.24347
6.	Spherical primary (SP) Cassegrain telescope (EP at primary) $m_2 = -4$	1	+16.0	-8.0	+4.0	-4.0	+4.0
		1*	0	0	0	0	0
		2	-4.21875	+3.86719	-3.54493	+13.33333	+6.24347
		2*	-11.78125	-10.14495	-8.73592	0	-17.47184
		Sum	0	-14.27776	-8.28085	+9.33333	-7.22837
7.	Classical Gregory telescope (EP at primary) $m_2 = +4$	1	+16.0	-8.0	+4.0	-4.0	+4.0
		1*	-16.0	0	0	0	0
		2	+2.53125	+4.05469	+6.49502	-22.22222	-9.23218
		2*	-2.53125	+3.44531	-4.68945	0	-9.37890
		Sum	0	-0.50000	+5.80557	-26.22222	-14.61108
8.	Aplanatic Gregory telescope (EP at primary) $m_2 = +4$	1	+16.0	-8.0	+4.0	-4.0	+4.0
		1*	-15.63285	0	0	0	0
		2	+2.53125	+4.05469	+6.49502	-22.22222	-9.23218
		2*	-2.89860	+3.94531	-5.37000	0	-10.74000
		Sum	0	0	+5.12502	-26.22222	-15.97218
9.	3-mirror system of Korsch-Design I (EP at primary) $m_2 = -10$ $m_3 = -0.15$	1	+0.843750	-1.125000	+1.500000	-1.500000	+1.500000
		1*	-1.065604	0	0	0	0
		2	-0.229711	+0.654328	-1.863844	+5.400000	+1.672312
		2*	+0.437211	+0.874422	+1.748844	0	+3.497688
		3	+0.061250	-0.126052	+0.259411	-3.900000	-3.381178
		3*	-0.046896	-0.277699	-1.644412	0	-3.288824
		Sum	0	0	0	0	0

Reflective telescope (3) : Ritchey-Chretien

- Hyperbolic + Hyperbolic
- Subaru Telescope



From mtk.nao.ac.jp

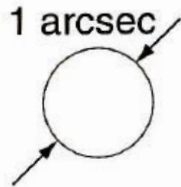
Spot diagram for a Ritchey-Chretien telescope

Prescription Data

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 Date : Fri Mar 03 1995

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 Eff. Focal Len. : 38501.7
 Total Track : 8833.89
 Image Space F/# : 11.0005
 Working F/# : 11.0005
 Obj. Space N.A. : 2.75e-007
 Stop Radius : 1750
 Parax. Ima. Hgt. : 167.937
 Parax. Mag. : 0
 Entr. Pup. Dia. : 3500
 Entr. Pup. Pos. : 0
 Exit Pupil Dia. : 349.344
 Exit Pupil Pos. : -10443.2
 Maximum Field : 0.25
 Primary Wave : 0.500000
 Lens Units : Millimeters
 Angular Mag. : 3.68676



Fields

Field Type: Angle in degrees
 # X-Value Y-Value Weight
 1 0.000000 0.250000 1.000000
 2 0.000000 0.150000 1.000000
 3 0.000000 0.000000 1.000000

Wavelengths

Units: Microns
 # Value Weight
 1 0.500000 1.000000

SURFACE DATA SUMMARY:

Surf	Type	Radius	Thickness	Glass	Diameter	Conic
OBJ	STANDARD	Infinity	Infinity		0	
STO	EVENASPH	-15400	-593.3	MIRROR	3500.865	-1
2	EVENASPH	-4416.7	8833.895	MIRROR	857.9522	-1
IMA	STANDARD	-1880.66	0		335.9057	0

SURFACE DATA DETAIL:

Surface Obj : STANDARD
 Surface STO : EVENASPH
 Coeff on r 2 : 0
 Coeff on r 4 : 8.25183e-016
 Coeff on r 6 : 0
 Coeff on r 8 : 0
 Coeff on r 10 : 0
 Coeff on r 12 : 0
 Coeff on r 14 : 0
 Coeff on r 16 : 0
 Surface 2 : EVENASPH
 Coeff on r 2 : 0
 Coeff on r 4 : 2.10756e-012
 Coeff on r 6 : -7.76e-020
 Coeff on r 8 : 0
 Coeff on r 10 : 0
 Coeff on r 12 : 0
 Coeff on r 14 : 0
 Coeff on r 16 : 0
 Surface IMA : STANDARD

INDEX OF REFRACTION DATA:

Surf Glass n
 0 0.560000
 1 MIRROR 1.00000000
 2 MIRROR 1.00000000
 3 MIRROR 1.00000000

(b)

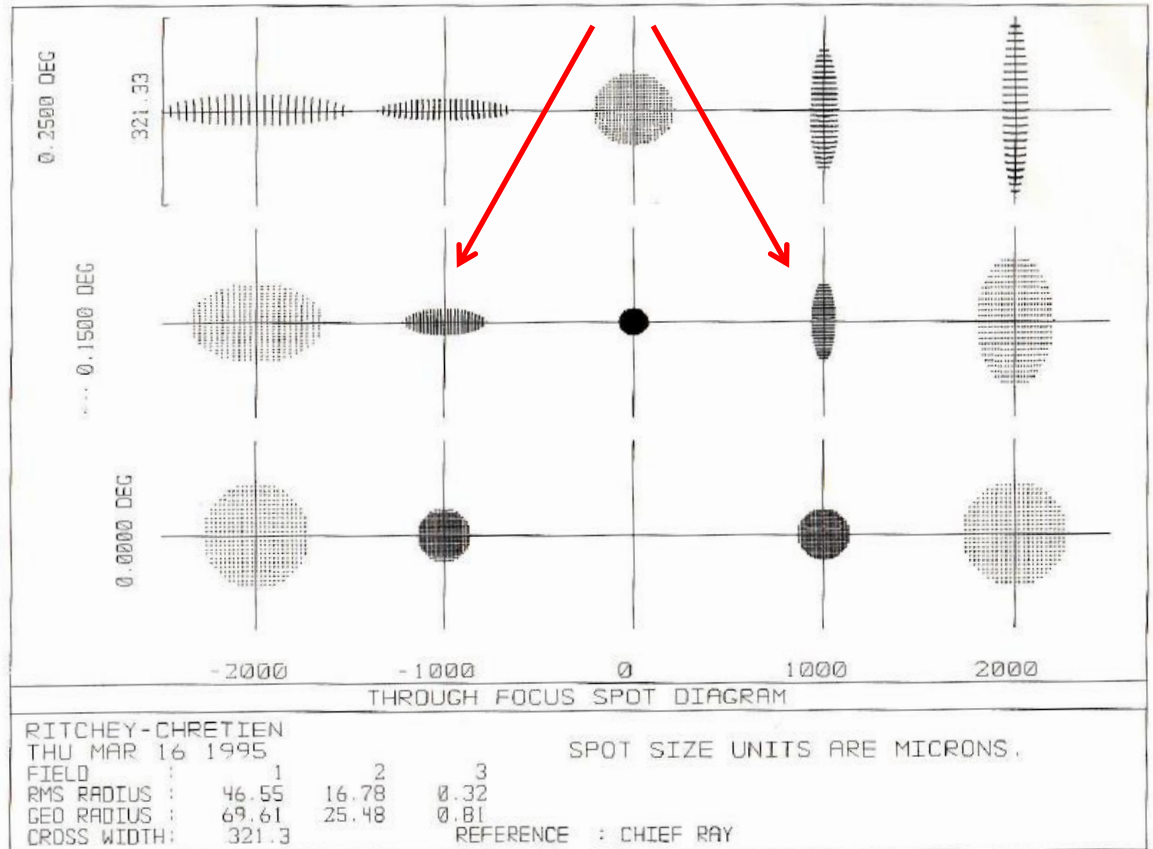
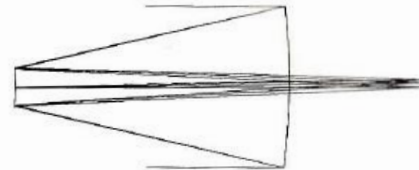
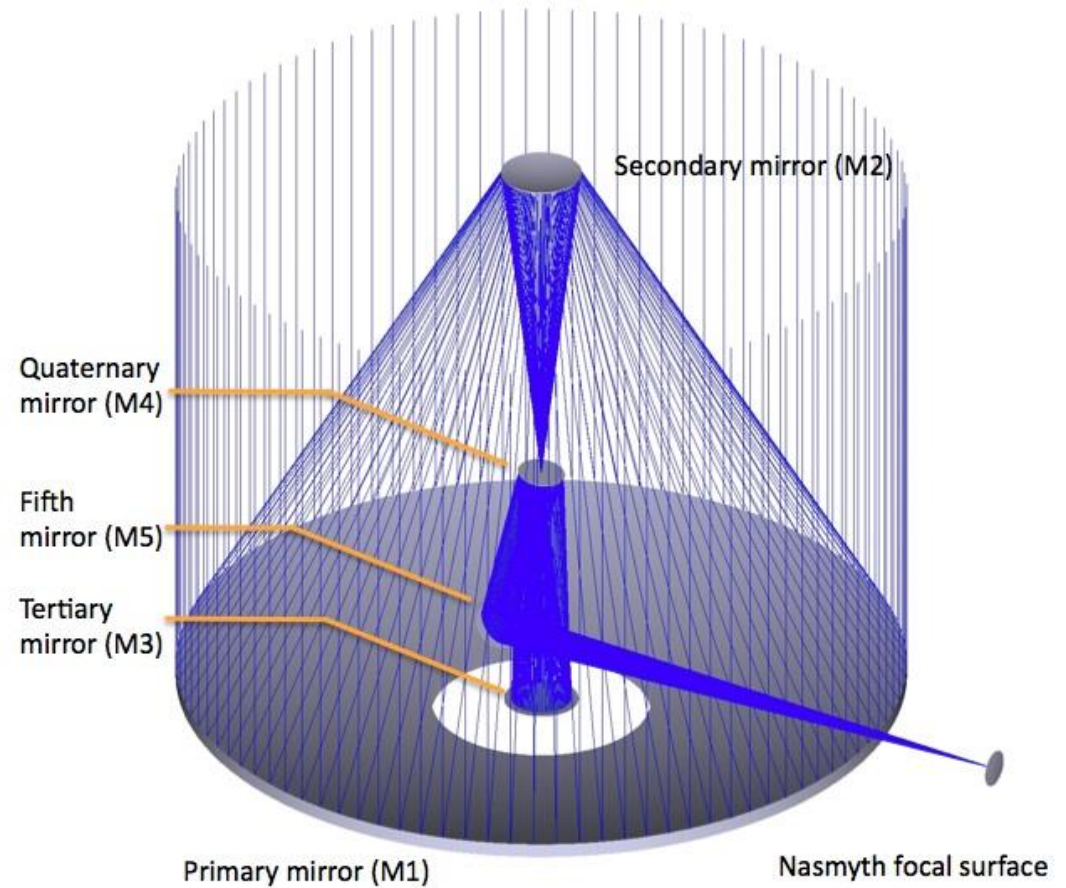
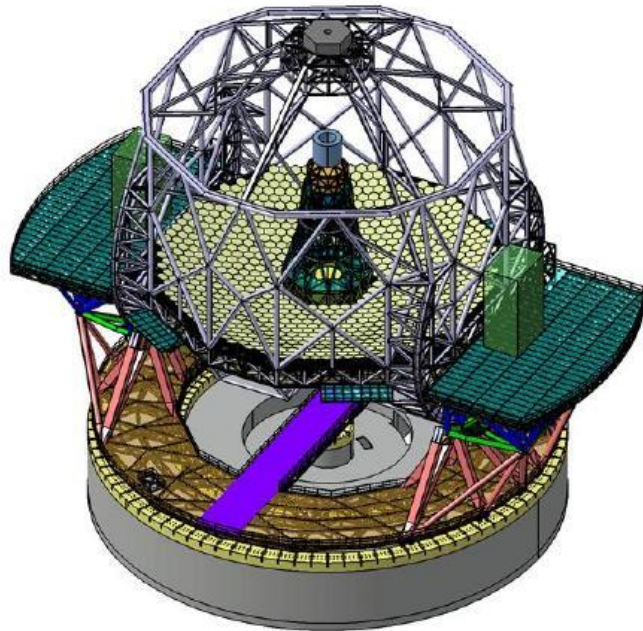


Fig. 3.4. (b) Spot-diagrams for an RC aplanatic telescope with the geometry of the ESO 3.5 m NTT ($f/11$; $m_2 = -5$) for an optimum field curvature $r_c = -1881$ mm

Reflective telescope (4) : Three-mirror anastigmat

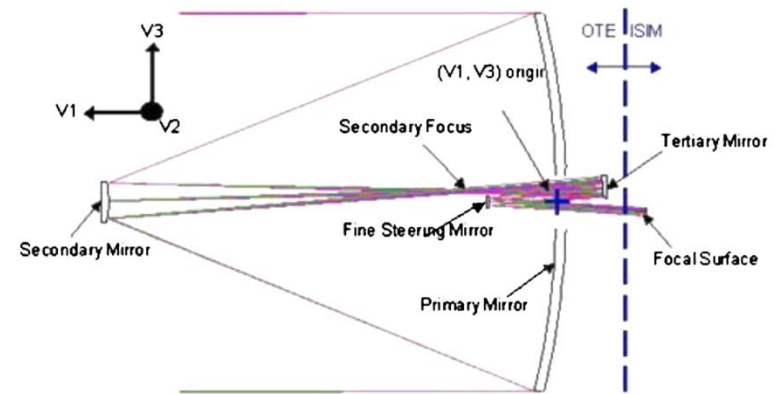
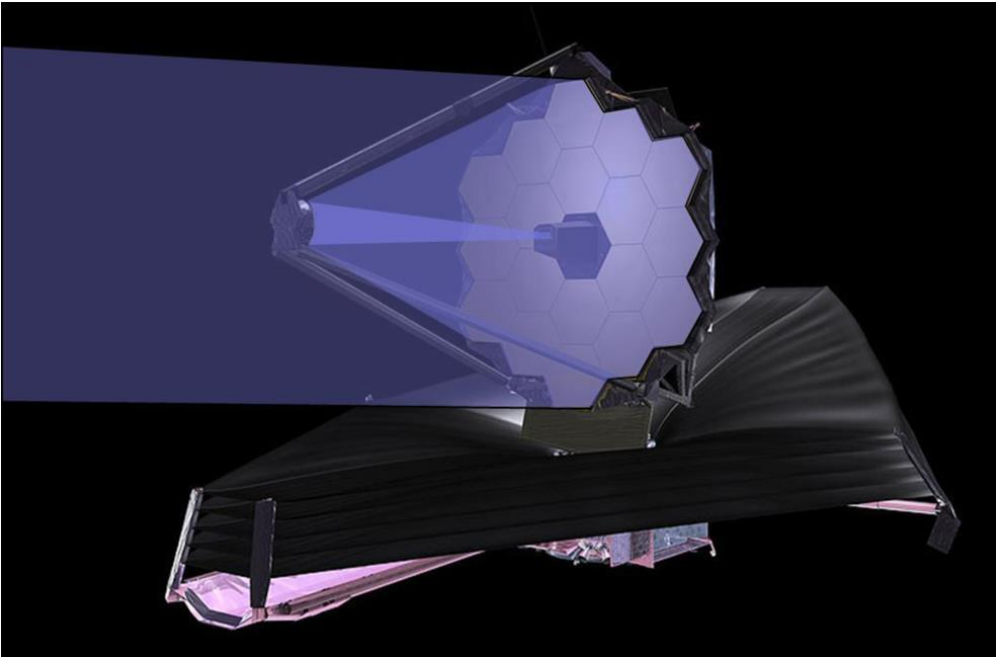
- European-Extremely Large Telescope



From eso.org

Reflective telescope (4) : Three-mirror anastigmat

- James-Webb Space telescope



Component	RoC (mm)	Surface	Conic	V1 (mm)	V2 (mm)	V3 (mm)	Phys. Size (mm)
Primary	15879.7	concave	-0.9967	0	0	0	6605.2
Secondary	1778.9	convex	-1.6598	7169.0	0	0	738
Tertiary	3016.2	concave	-0.6595	-796.3	0	-0.19	728 x 517
Fine Steering Mirror		flat		1047.8	0	-2.36	172.5

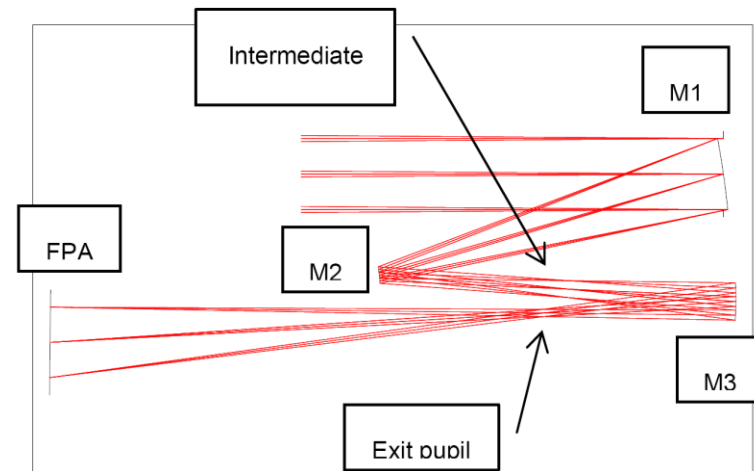
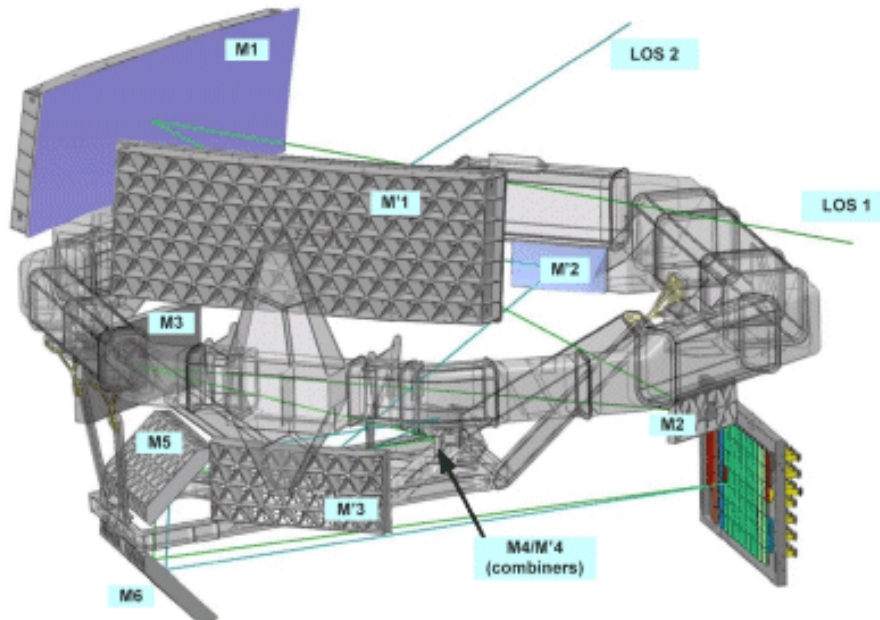
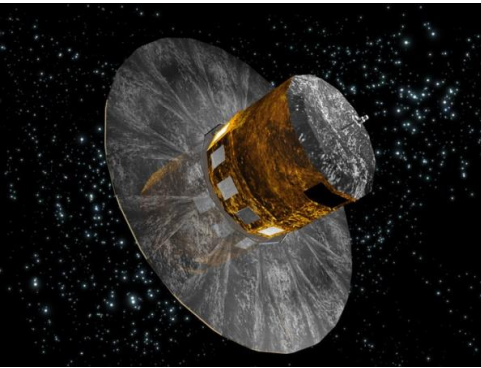
PM-SM EFL ≈ 59400 mm

OTE EFL ≈ 131400 mm

From SPIE digital library

Reflective telescope (5) : Three-mirror anastigmat

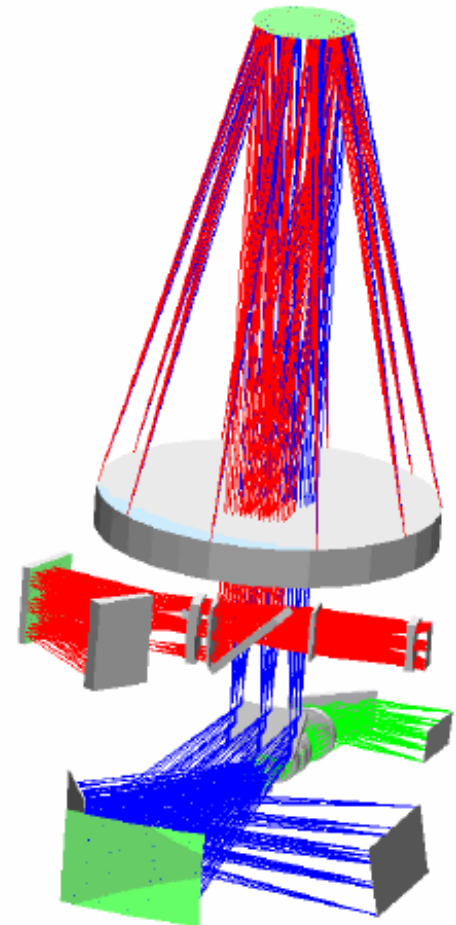
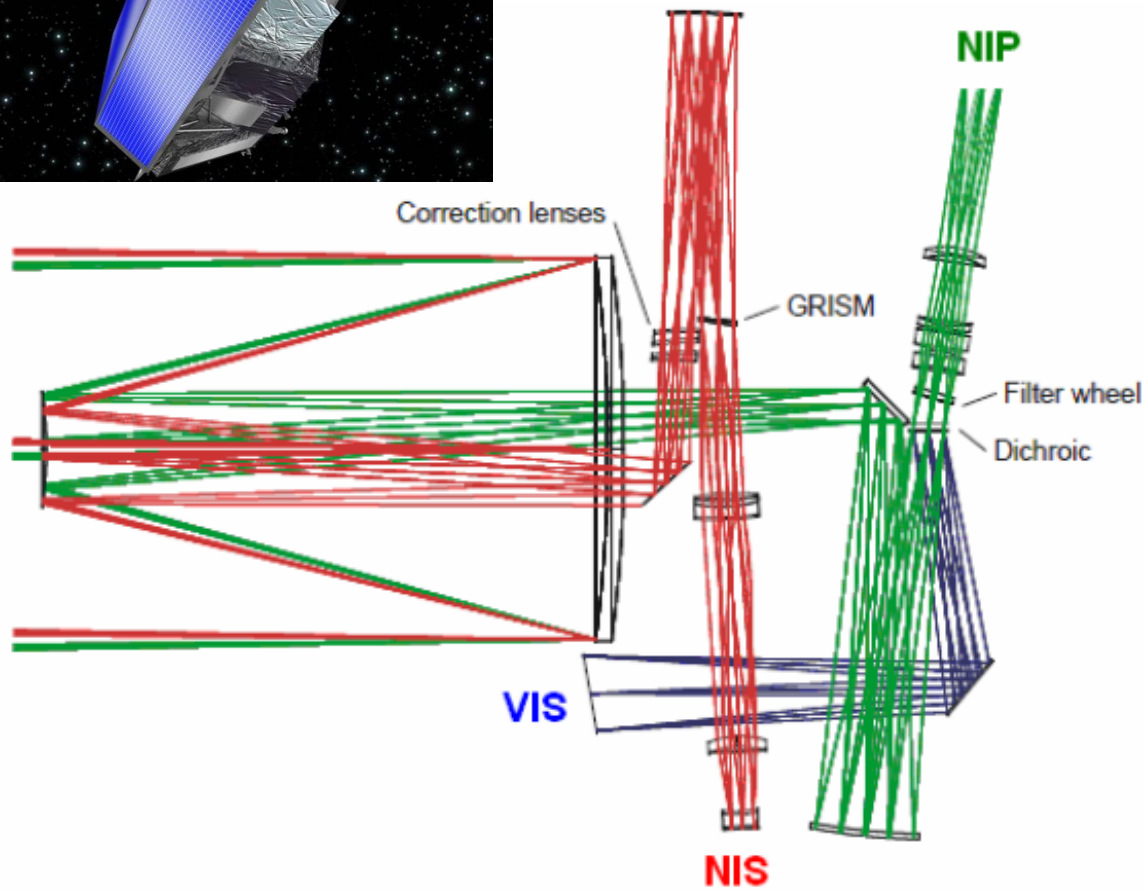
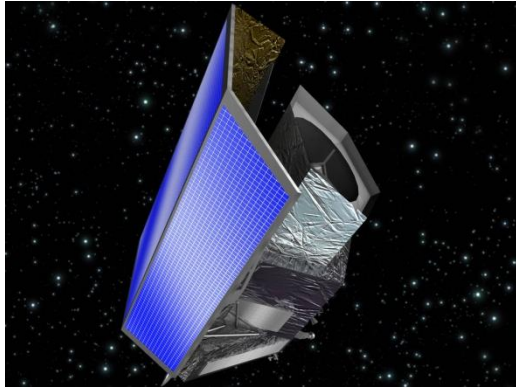
- GAIA : combine two FoVs with 106.5deg separation. 1.45x0.5m aperture with 35m focal length. 60mas x 180 mas sampling with 0.7deg x 0.7deg FoV.



From sci.esa.int

Reflective telescope (4) : Three mirror Korsch

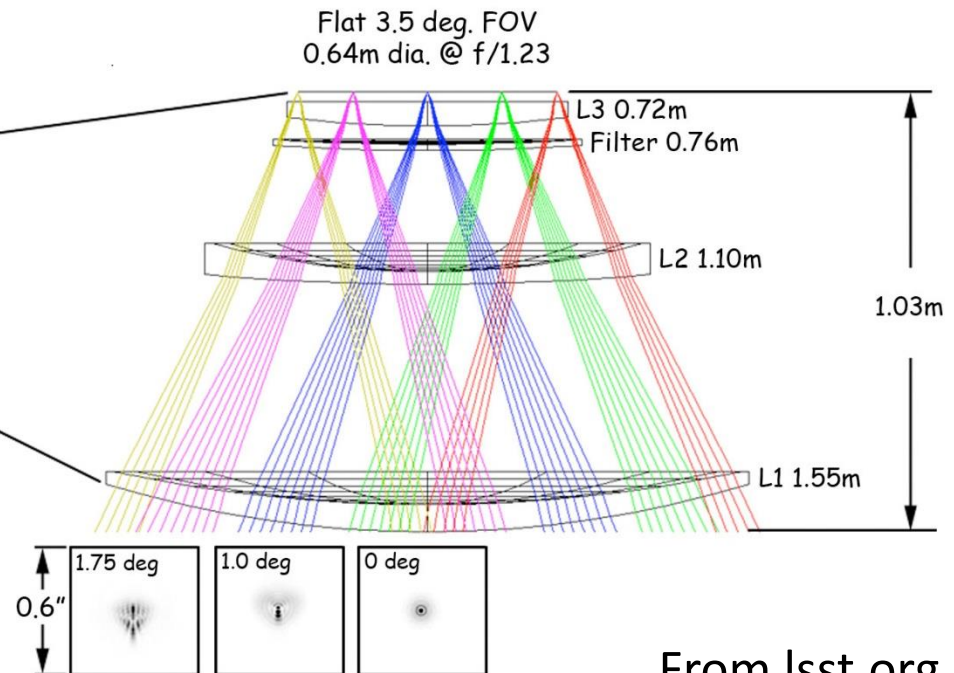
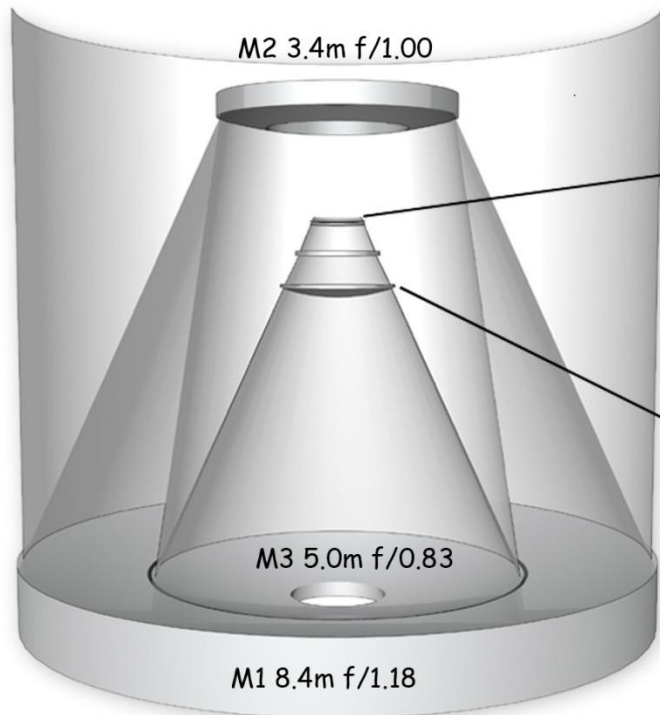
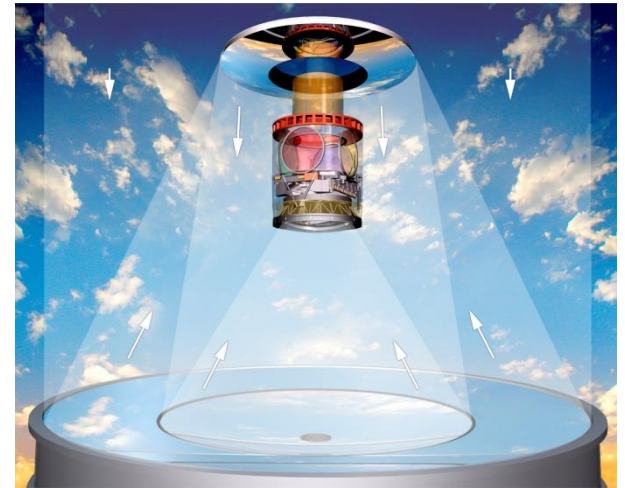
- Euclid mission



From euclid.cnes.fr

Reflective telescope (4) : Three mirrors

- Large Synoptic Survey telescope



From lsst.org