天体計測学特論 I
Observational Astronomy I

Lecture 04:
Basic properties of electromagnetic wave and introduction to Fourier optics

Electro-magnetic wave and radiative flux

- Plane wave

- Intensity and Poynting vector

- Interference of light (with cos description)

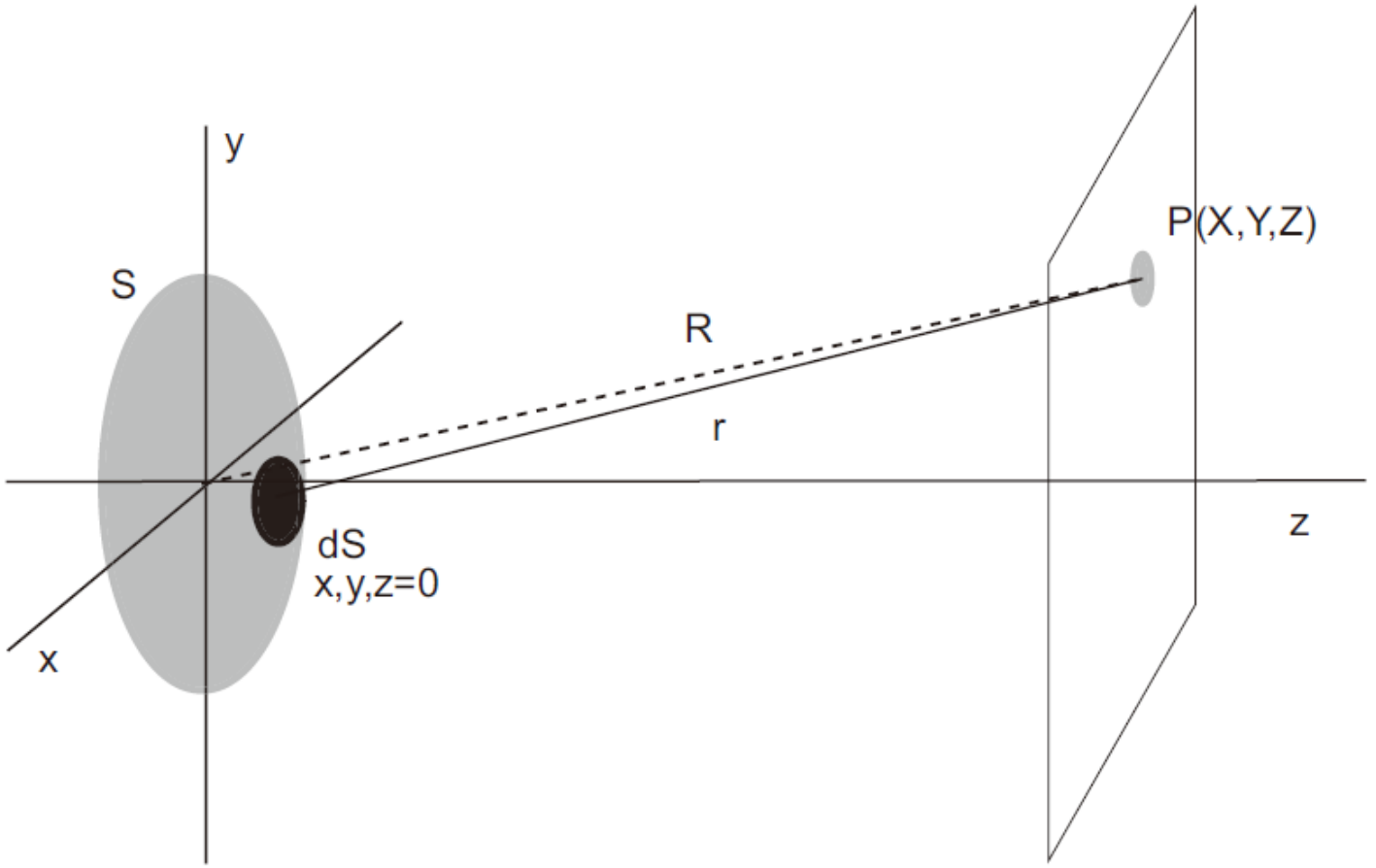
Electro-magnetic wave and radiative flux

- Plane wave described with the exponential function

- Interference of light (with exp description)

Diffraction pattern

- Configuration



Diffraction

- Spherical wave

$$E_{\mathrm{A}} \exp[i(wt - kr)]/r$$

- Electric field on the screen

$$E(X,Y) = \int\int_{\mathrm{Aperture}} \frac{E_A \exp[i(wt-kr)]}{r} \mathrm{d}x\mathrm{d}y$$

$$r = \sqrt{Z^2 + (x-X)^2 + (y-Y)^2} = Z + \frac{1}{2Z}[(x-X)^2 + (y-Y)^2] - \frac{1}{8Z^3}[(x-X)^2 + (y-Y)^2]^2 + \ldots$$

- Fresnel diffraction
  - When $r \sim R \sim Z$ and $R^3 \gg \frac{1}{8\lambda}[(x-X)^2 + (y-Y)^2]^2$

$$E(X,Y) = \frac{E_A \exp[i(wt-kR)]}{R} \int\int_{\mathrm{Aperture}} \exp[\frac{-ik((x^2+y^2-2xX-2yY))}{2R}]\mathrm{d}x\mathrm{d}y$$

Fresnel diffraction by an edge

- From “光とフーリエ変換”　谷田貝豊彦

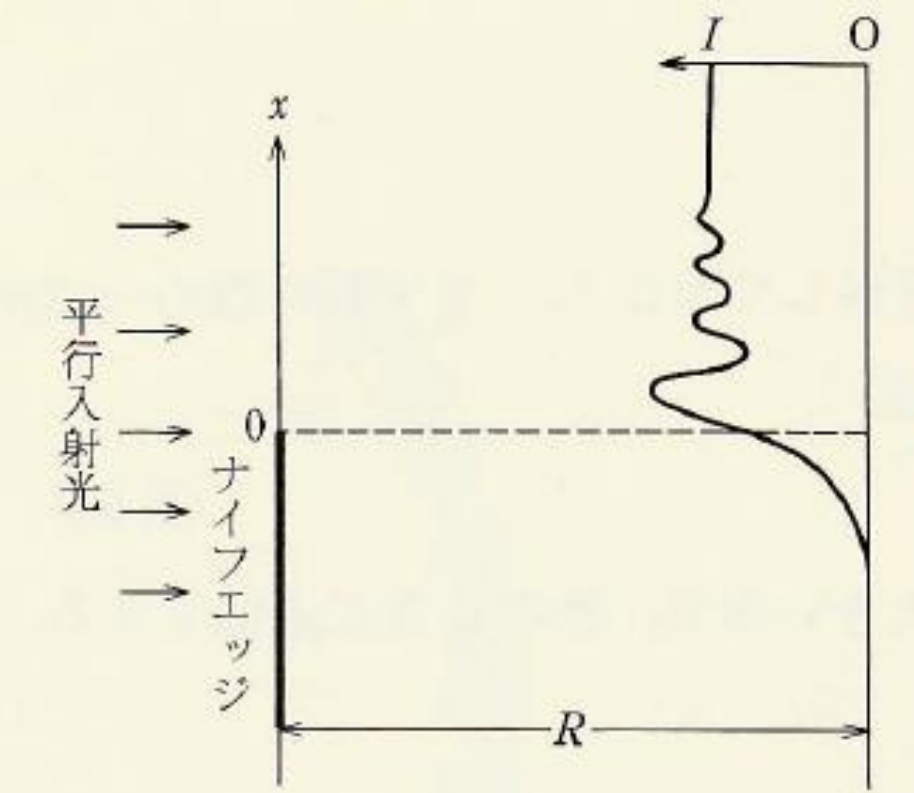


図 2.9　ナイフエッジによるフレネル回折

Fresnel diffraction by a slit

- From “光とフーリエ変換”　谷田貝豊彦

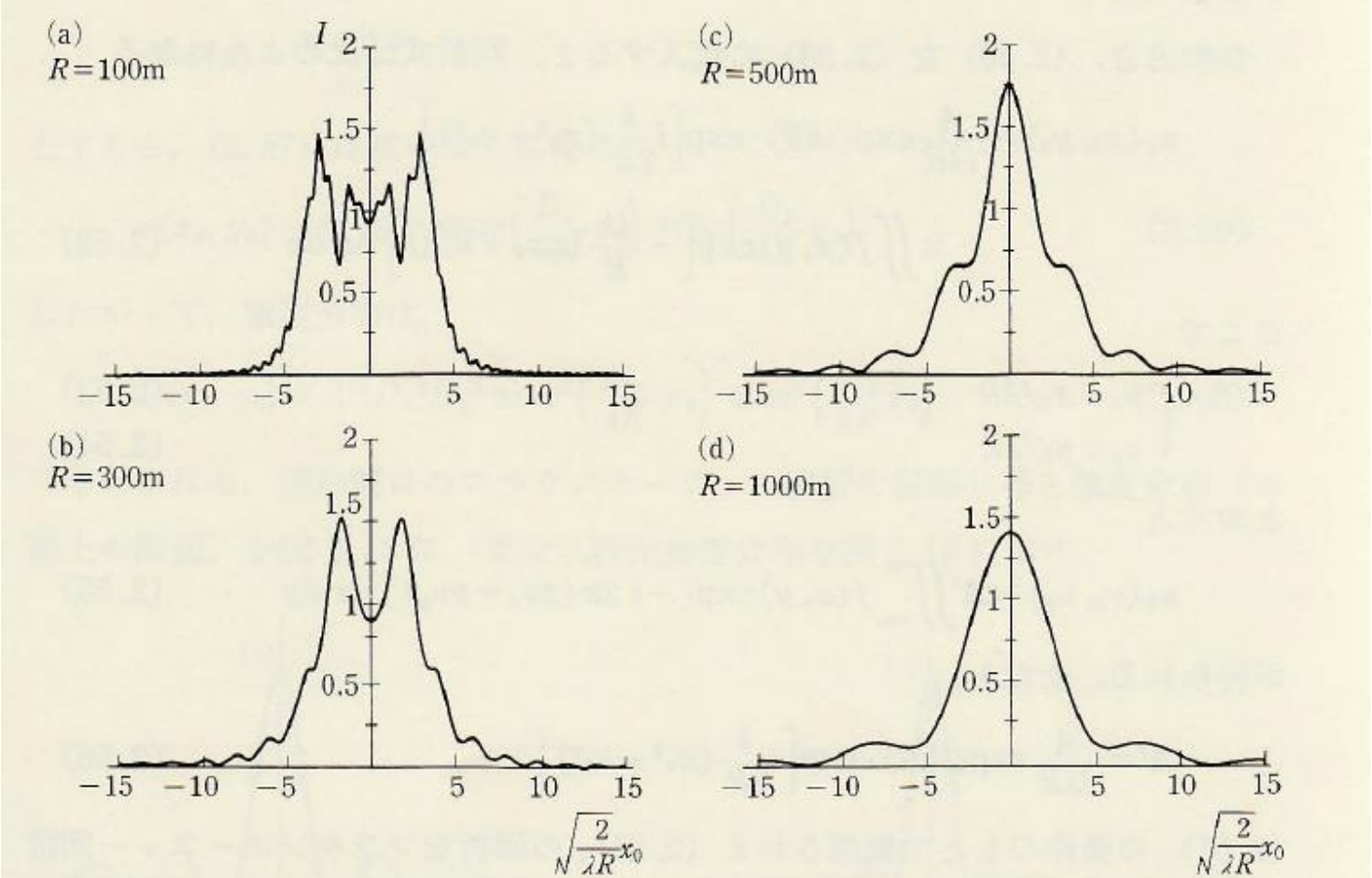
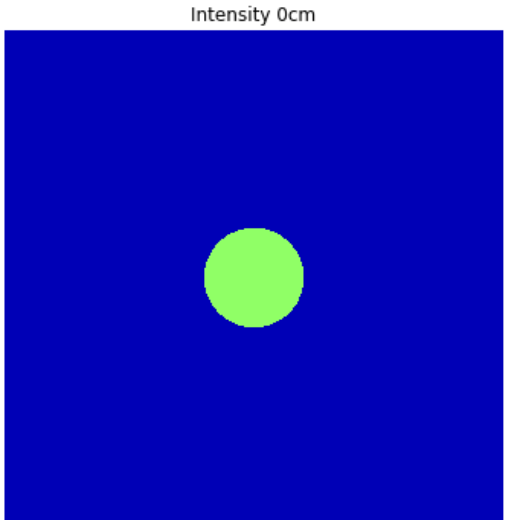
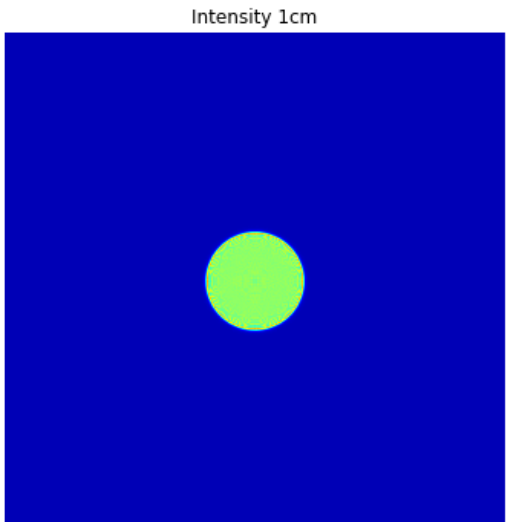


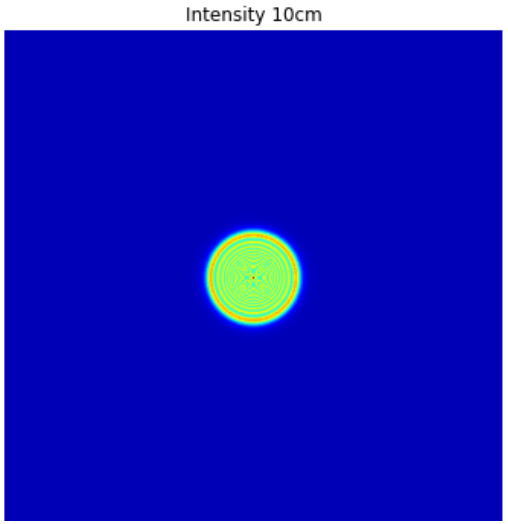
図 2.11　スリットによるフレネル回折（スリット間隔 10 mm, λ=0.63 μm）

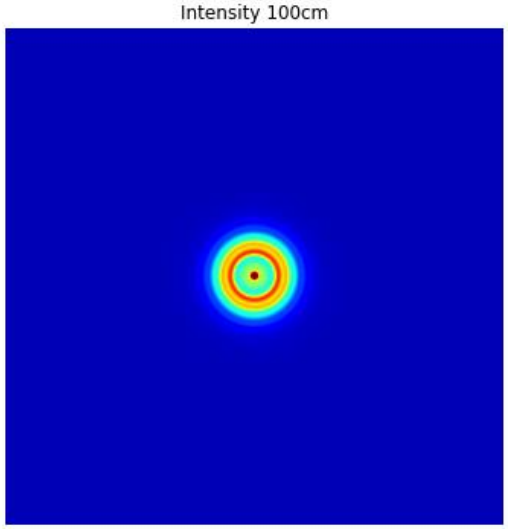
Fresnel diffraction pattern of a circular aperture

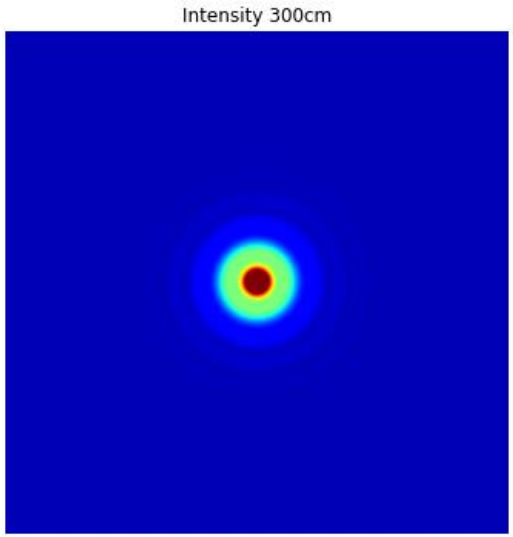
- Fresnel diffraction of illumination of a r=1mm hole (10mm x 10mm simulation) from 0m to 10m.

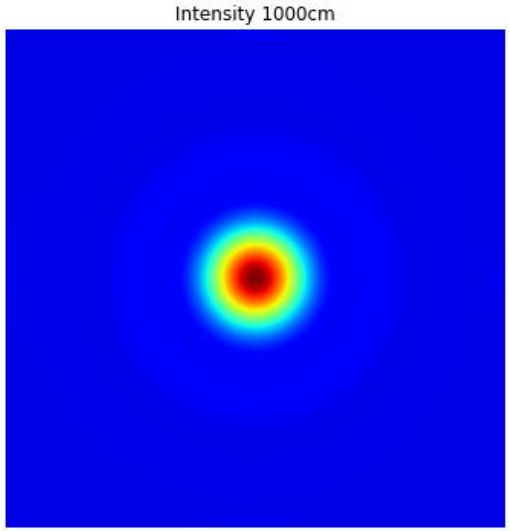






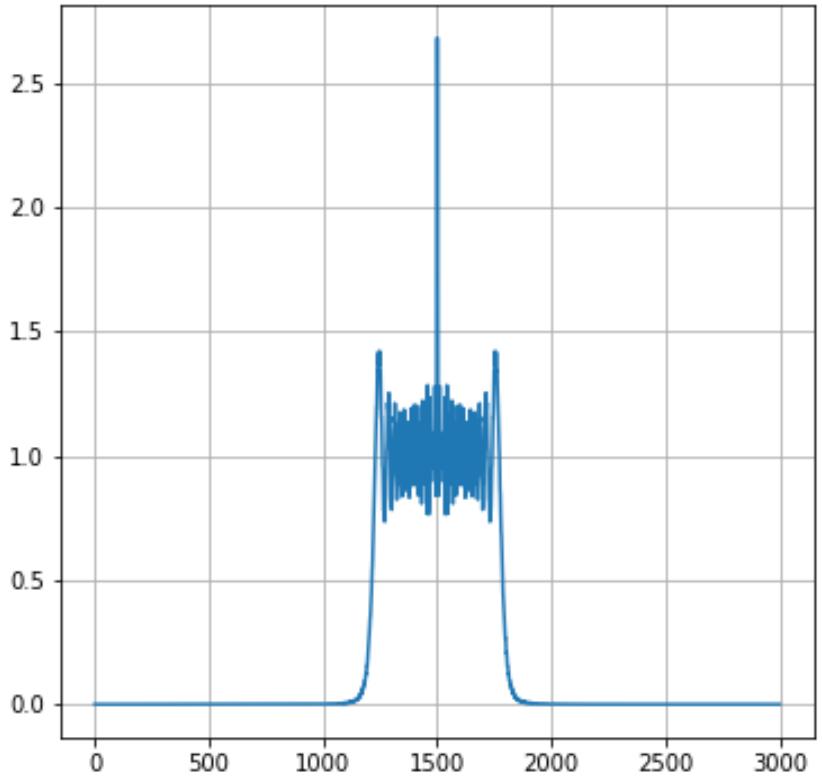


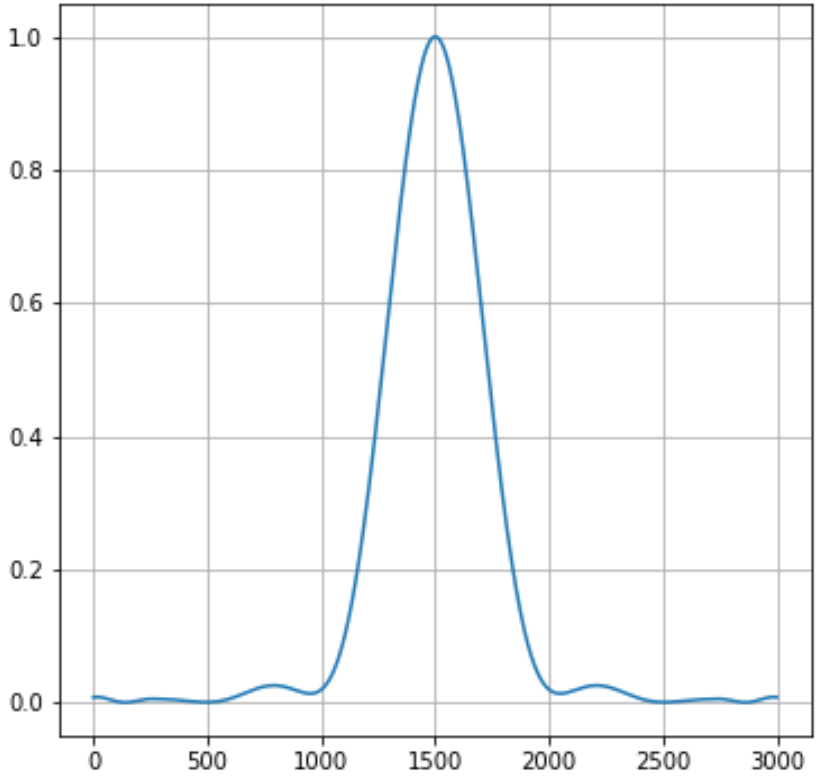




Fresnel diffraction pattern of a circular aperture

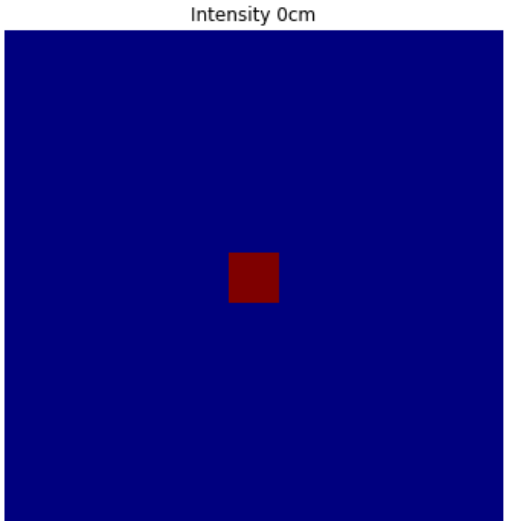
- Fresnel diffraction of illumination of a r=1mm hole (10mm x 10mm simulation with 3000 x 3000 pixel) from 0m to 10m.

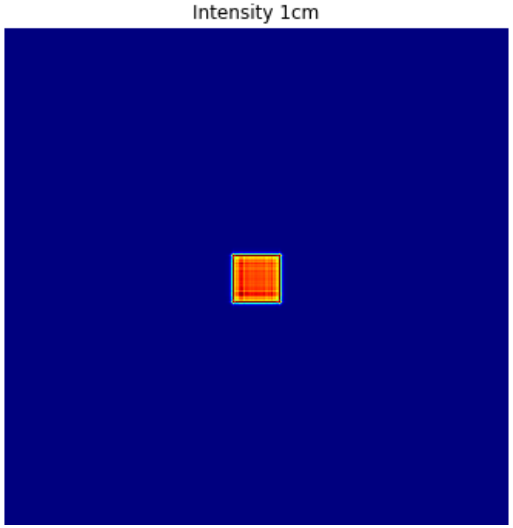


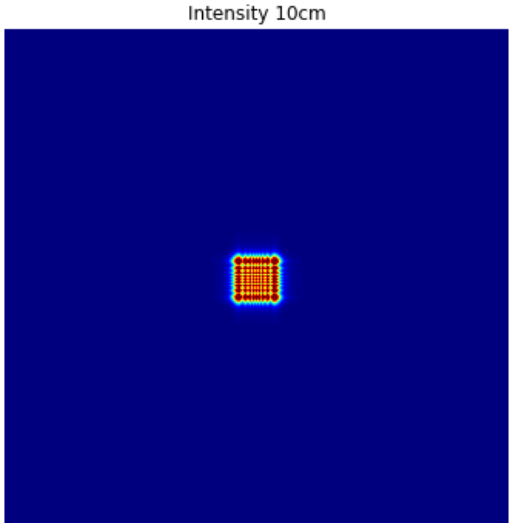


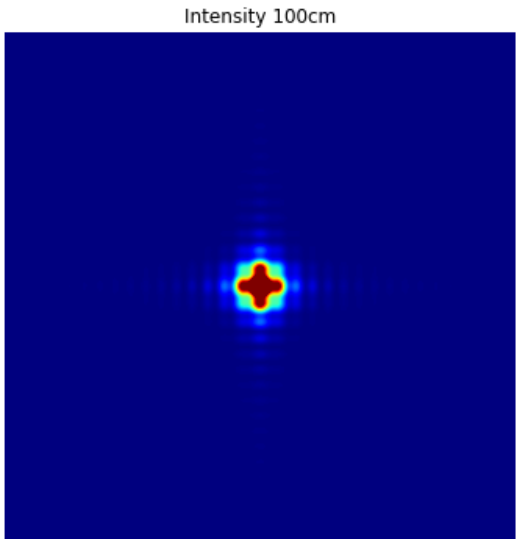
Fresnel diffraction pattern of a square aperture

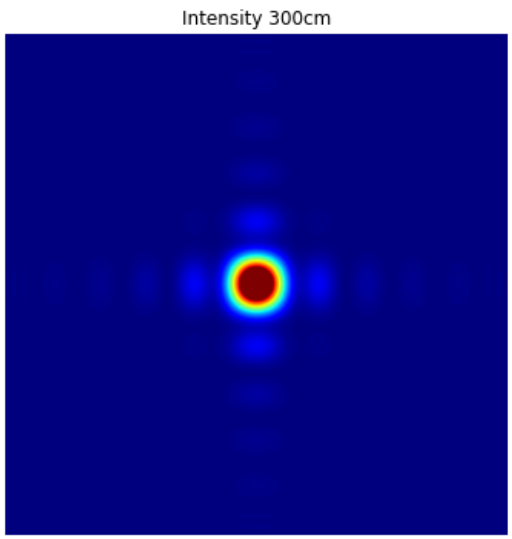
- Fresnel diffraction of illumination of a l=1mm hole (10mm x 10mm simulation) from 0m to 10m.

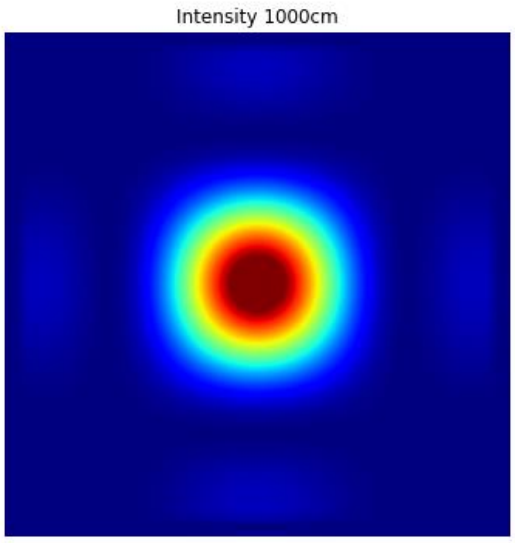












Diffraction

- Fresnel diffraction
  - When $r \sim R \sim Z$ and $R^3 \gg \frac{1}{8\lambda}[(x-X)^2+(y-Y)^2]^2$

$$E(X,Y) = \frac{E_A \exp[i(wt-kR)]}{R} \int\int_{\text{Aperture}} \exp[\frac{-ik((x^2+y^2-2xX-2yY))}{2R}]\mathrm{d}x\mathrm{d}y$$

- Fraunhoffer diffraction
  - When $R \gg \frac{1}{2\lambda}(x^2+y^2)$

$$E(X,Y) = \frac{E_A \exp[i(wt-kR)]}{R} \int\int_{\text{Aperture}} \exp[\frac{ik(xX+yY)}{R}]\mathrm{d}x\mathrm{d}y$$

Fraunhoffer diffraction pattern of slits

- From “Optics” Hecht

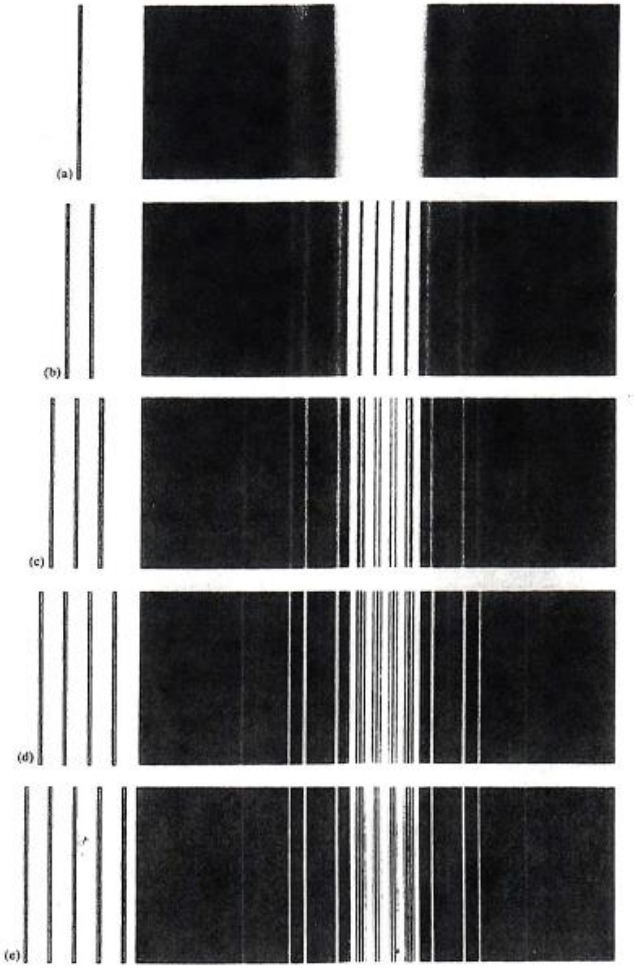


Figure 10.20 Diffraction patterns for slit systems shown at left.

Fraunhoffer diffraction pattern of a circular aperture

- Circular aperture with $x = \rho\cos\theta, y = \rho\sin\theta, X = r\cos\phi, Y = r\sin\phi$

$$E(r) = \frac{E_A \exp[i(wt - kR)]}{R} \int_0^a \rho \int_0^{2\pi} \exp[i(k\rho r/R)\cos(\theta - \phi)] d\rho d\theta$$

$$E(r) = \frac{E_A \exp[i(wt - kR)]}{R} 2\pi a^2 (\frac{R}{kar}) J_1(\frac{kar}{R})$$

$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp(ix\cos\alpha) d\alpha \qquad J_1(x) = \frac{1}{x} \int_0^x x' J_0(x') dx'$$

- Illumination we see on the screen $< |E(r)|^2 >$

$$I(r) = \frac{2E_A^2 A^2}{R^2} [\frac{J_1(kar/R)}{kar/R}]^2$$

with pi a^2 =A

$$I(0) = \frac{E_A^2 A^2}{2R^2}$$

Fraunhoffer diffraction pattern of a circular aperture

- From “Optics” Hecht

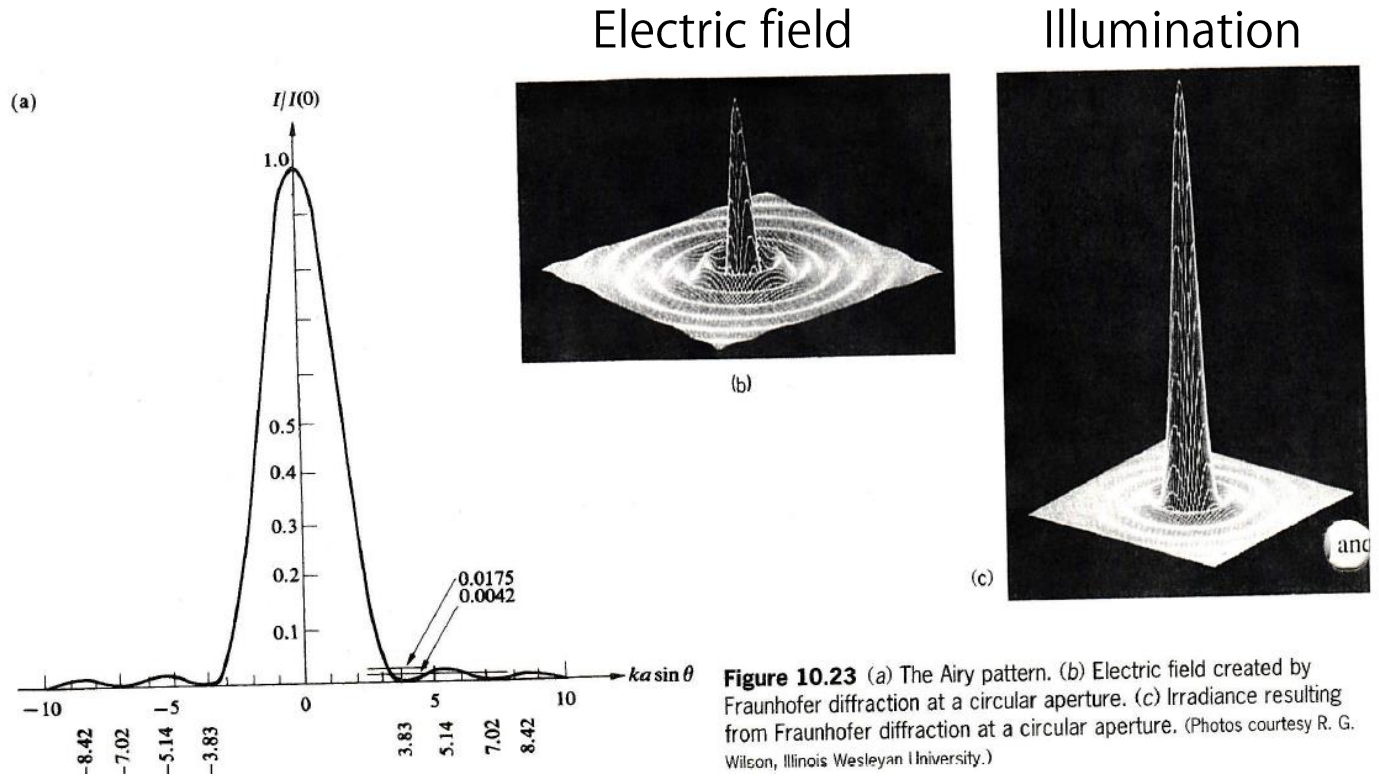
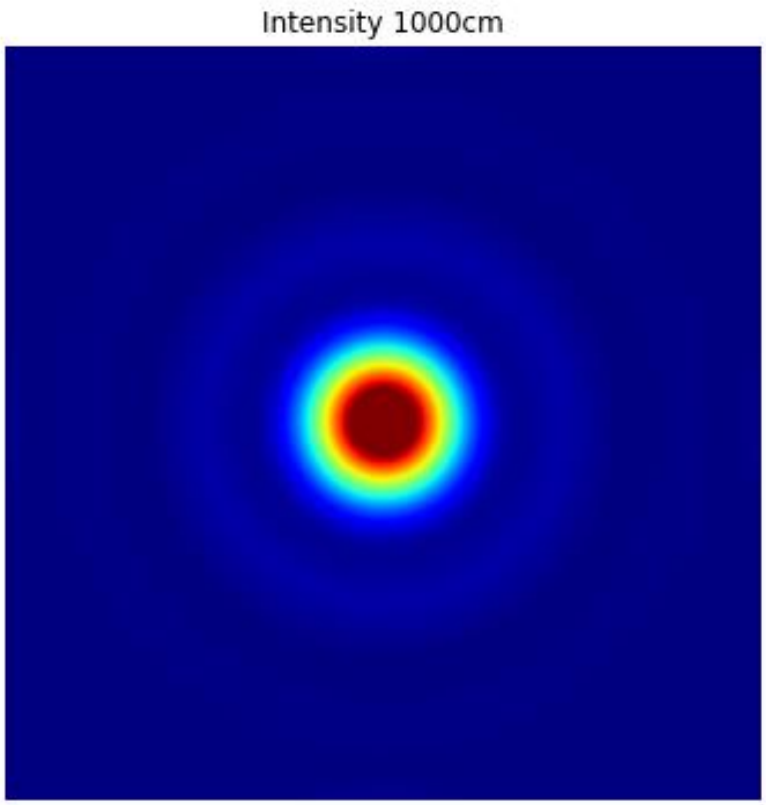
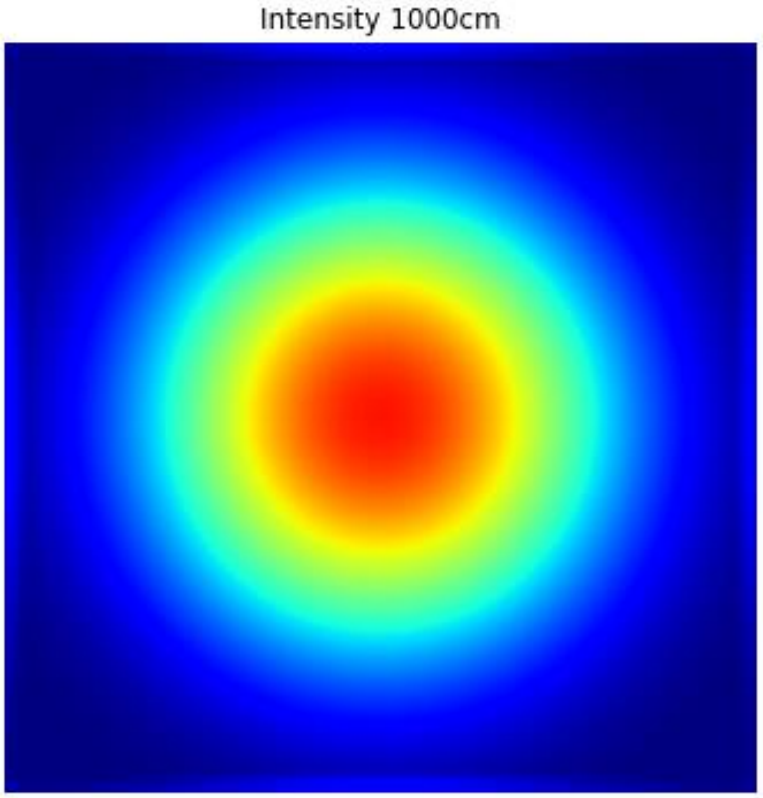


Figure 10.23 (a) The Airy pattern. (b) Electric field created by Fraunhofer diffraction at a circular aperture. (c) Irradiance resulting from Fraunhofer diffraction at a circular aperture. (Photos courtesy R. G. Wilson, Illinois Wesleyan University.)

Fraunhoffer diffraction pattern of a circular aperture

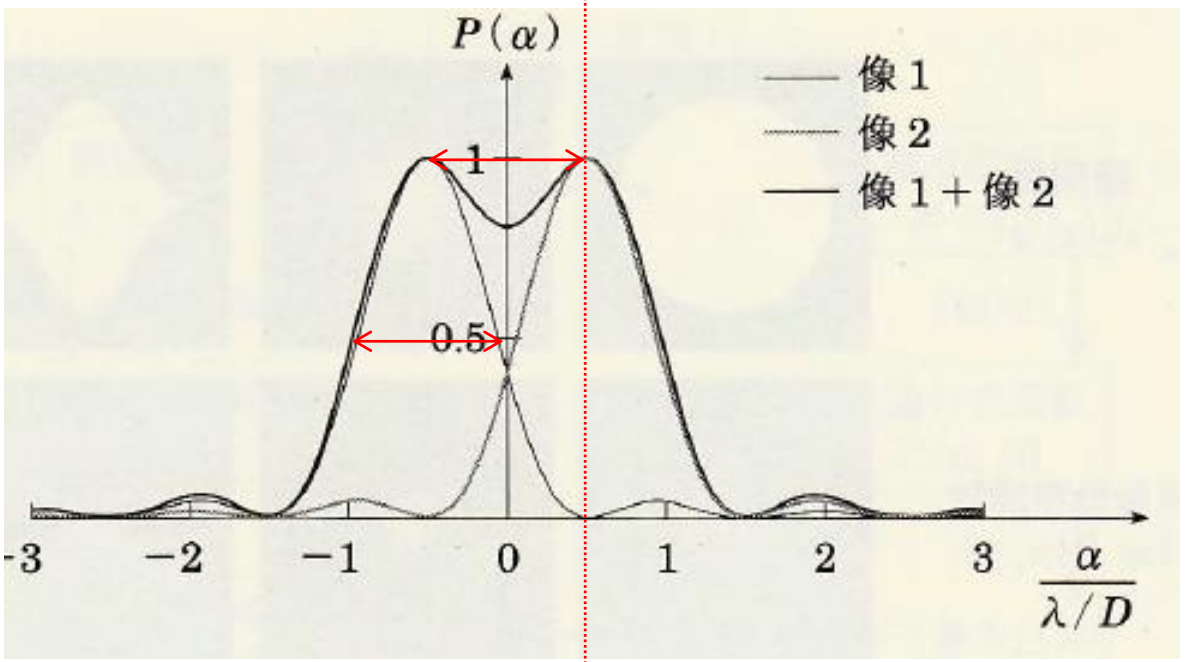
- Different aperture size : r=1mm vs. r=0.3mm





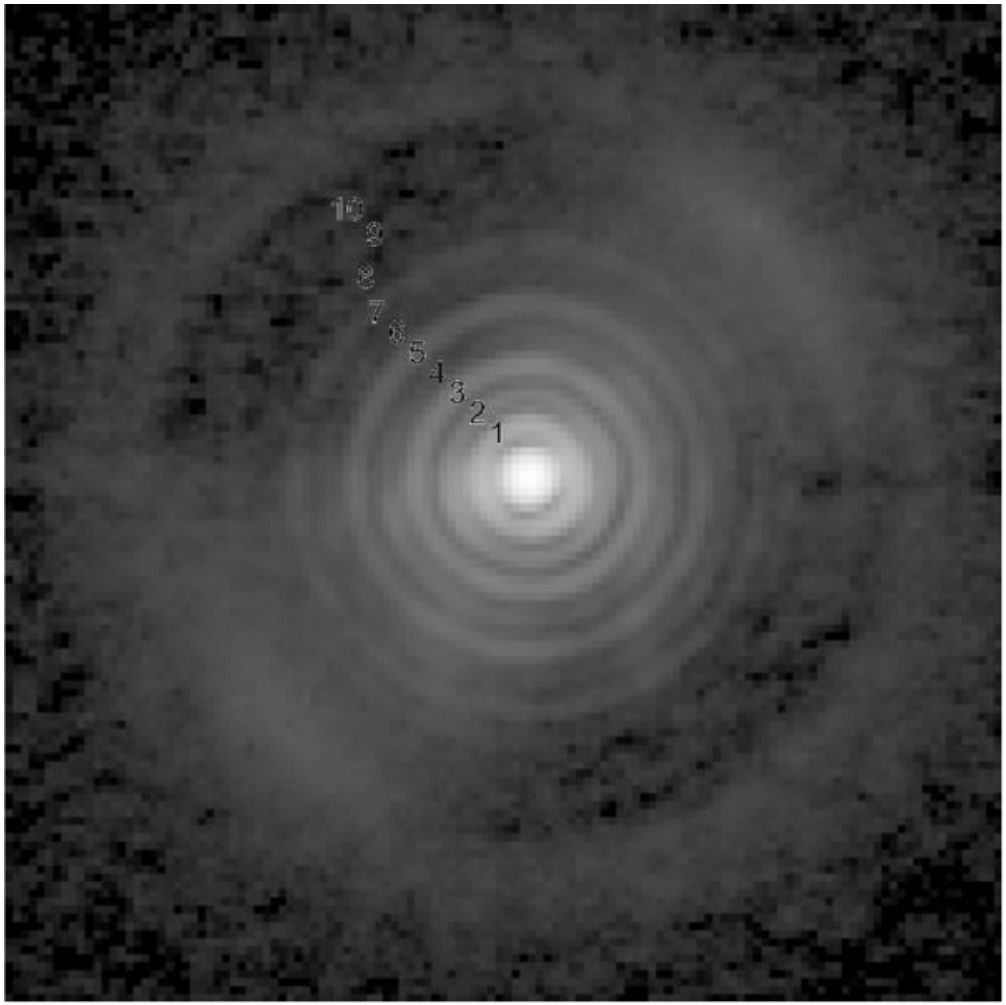
Rayleigh's criterion

- From "宇宙の観測 I"
- spatial resolution limit = 1.22 lambda / D



Diffraction pattern of the LBG telescope

- 8th magnitude star taken with an Extreme AO system
  - From lbto.org



Introduction to Fourier optics

- In case if the phase at the aperture is not constant

$$E_A(x,y) = E_0(x,y)\exp(i\phi(x,y))$$

$$E(X,Y) = \int\int E_A(x,y)\exp[ik(Xx+Yy)/R]dxdy$$

$$f_y = kY/R、\ f_x = kX/R$$

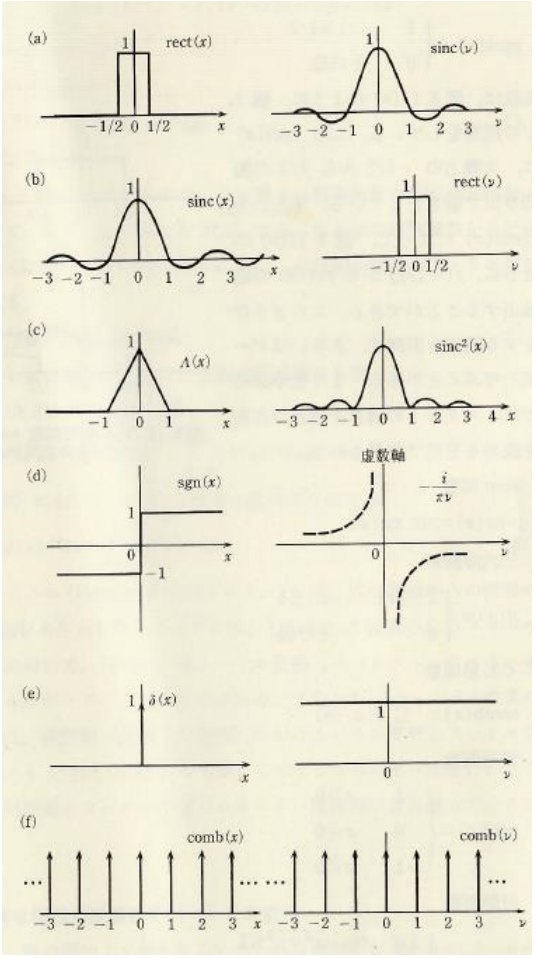
$$E(f_x,f_y) = \int\int E_A(x,y)\exp[i(f_x x + f_y y)]dxdy$$

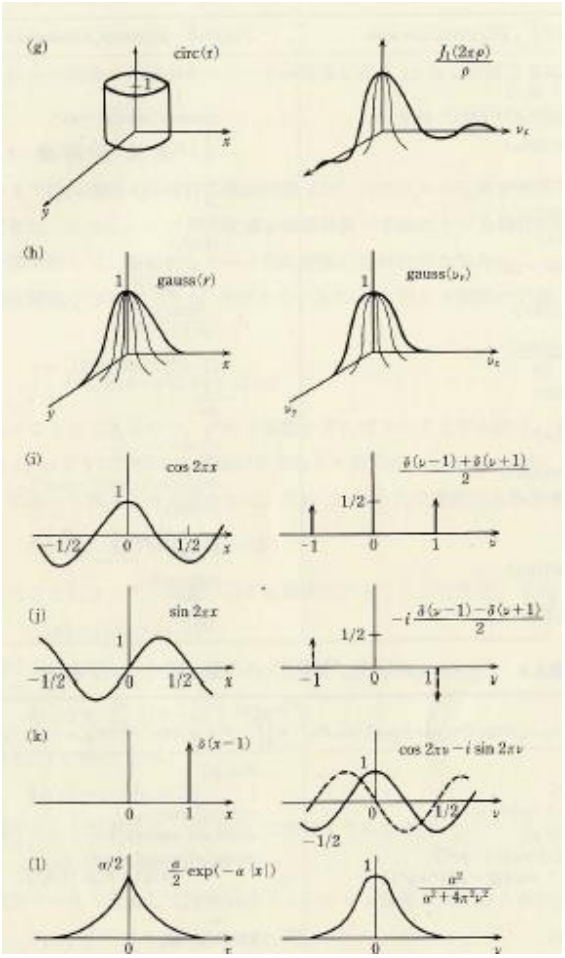
- If you are interested in make your own calculation with a fast Fourier transform programming library (FFTW etc.), check a calculation code in a jupyternotebook format.

Fourier transform

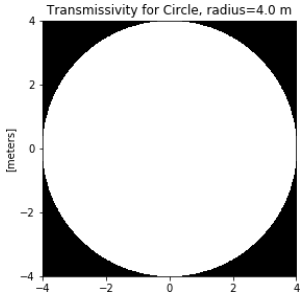
- Therefore, if the illumination has a shape (aperture), phase is constant, resulting electric field is determined by Fourier transform of each 2D function.

$$E(f_x, f_y) = \int\int E_A(x, y) \exp[i(f_x x + f_y y)] dx dy$$



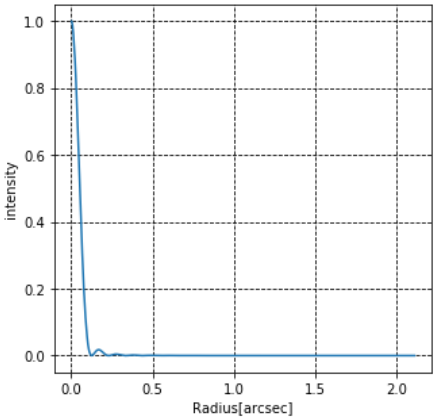


Aperture vs. PSF properties

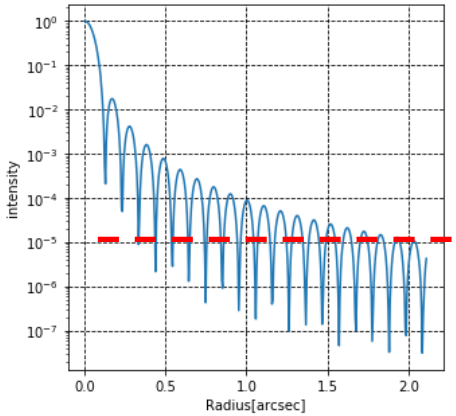


8m circular aperture

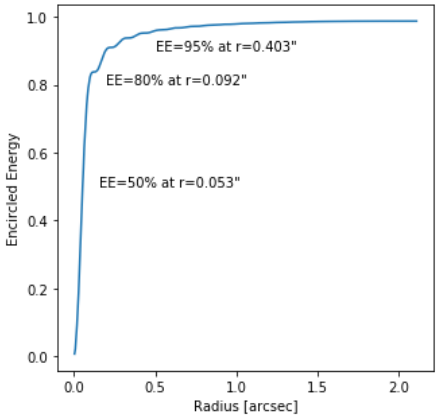
profile

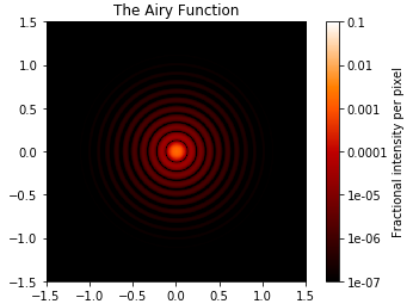


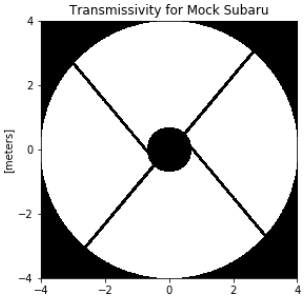
log-profile



Encircle-Energy

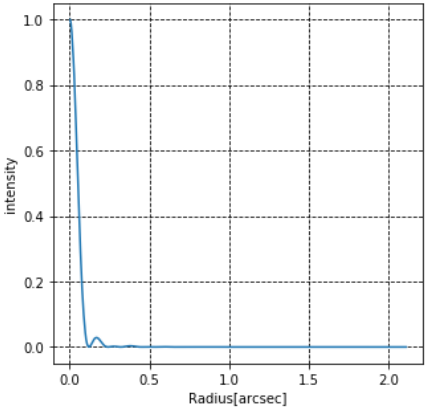




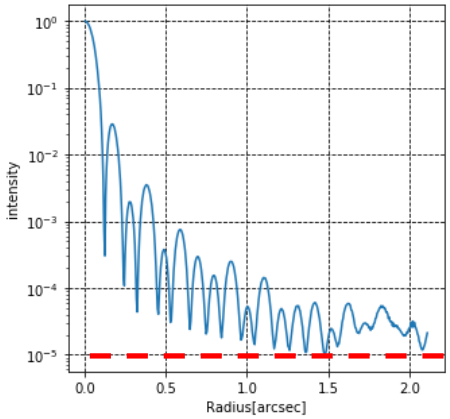


8m + secondary mirror (mock Subaru)

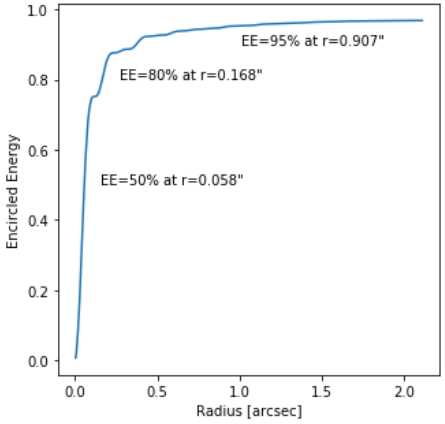
profile

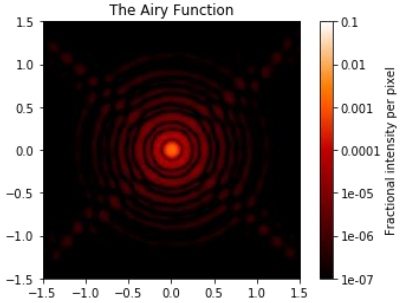


log-profile



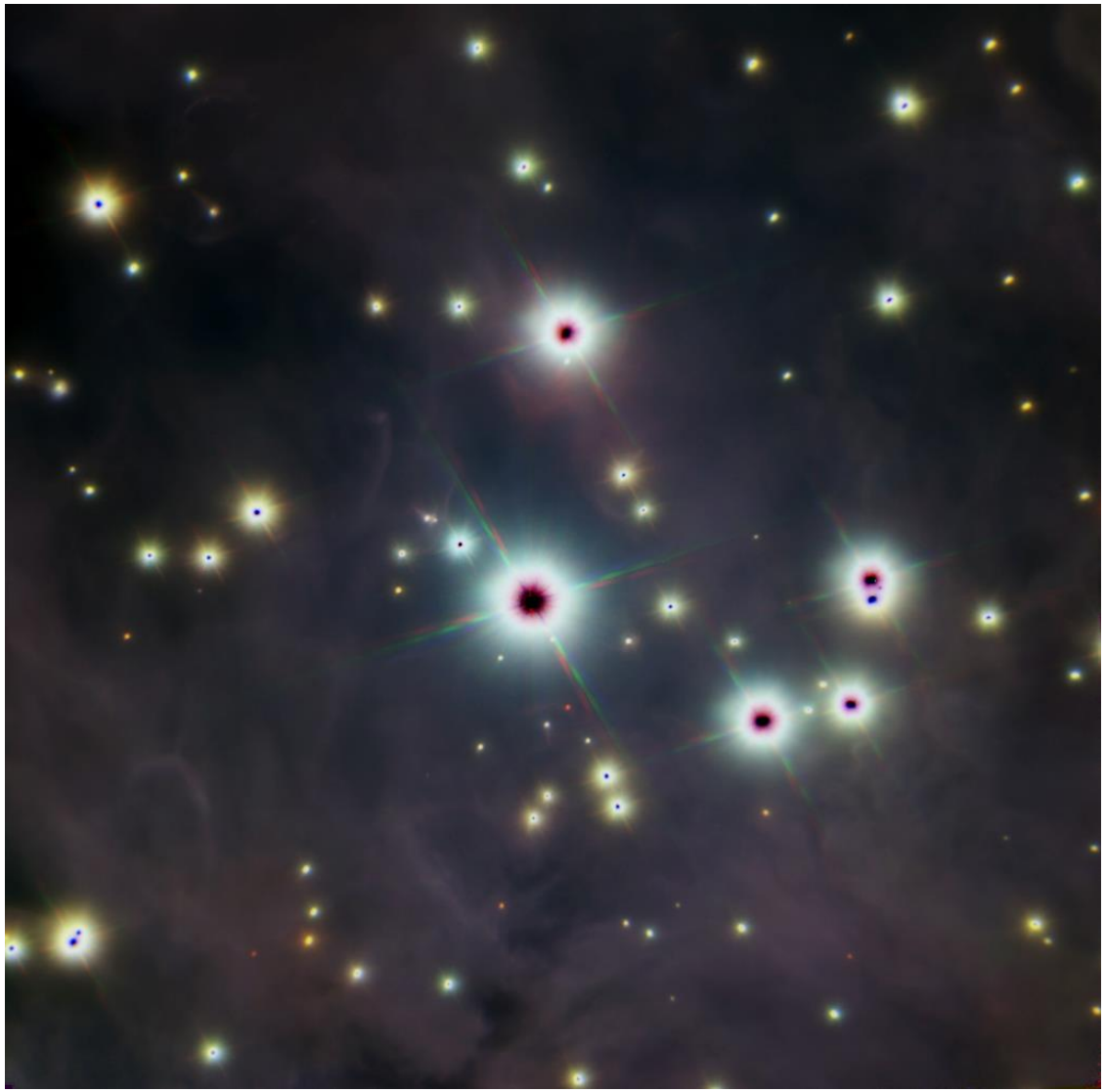
Encircle-Energy



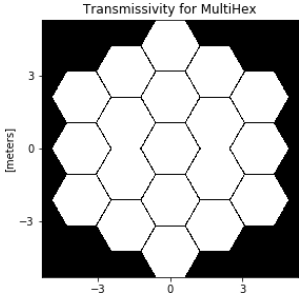


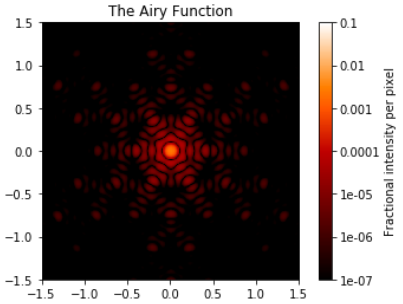
Subaru PSF in NIR

- Orion nebula trapezium with Subaru + IRCS + AO188



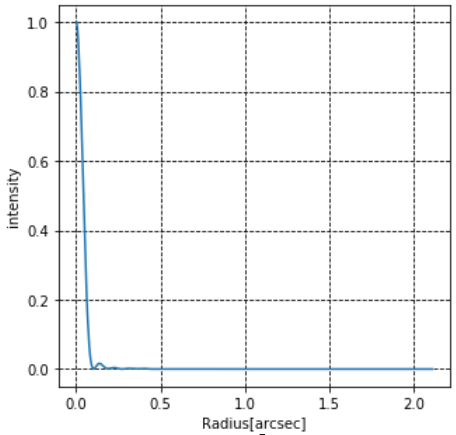
Aperture vs. PSF properties



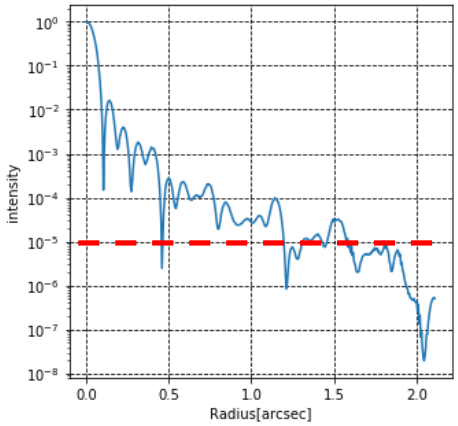


8m hex aperture

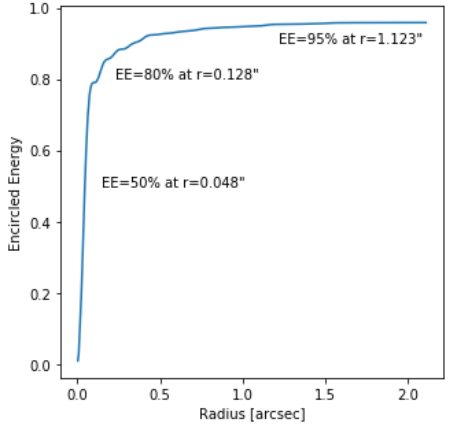
profile

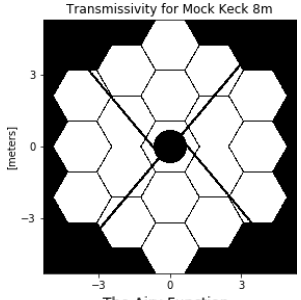


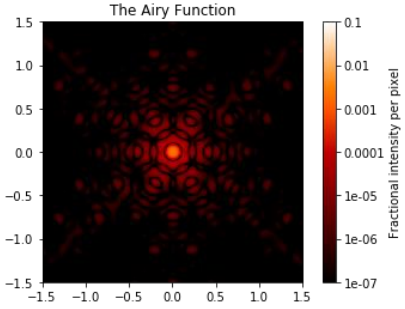
log-profile



Encircle-Energy

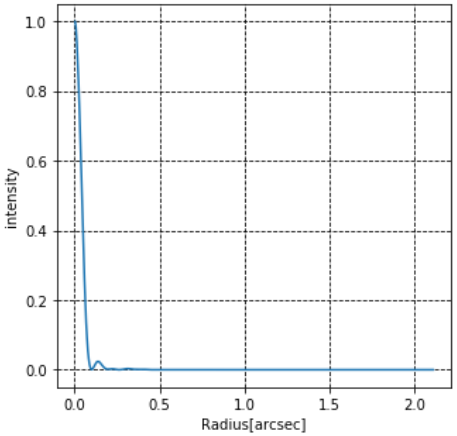




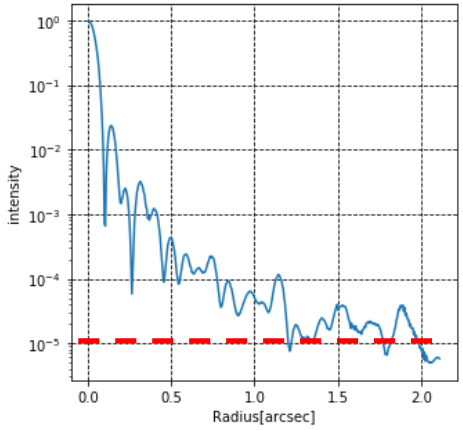


8m hex + secondary mirror (mock Keck)

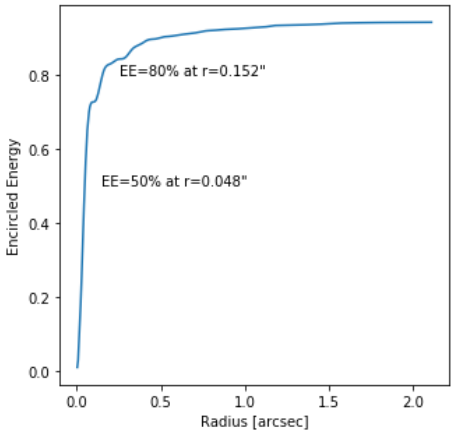
profile



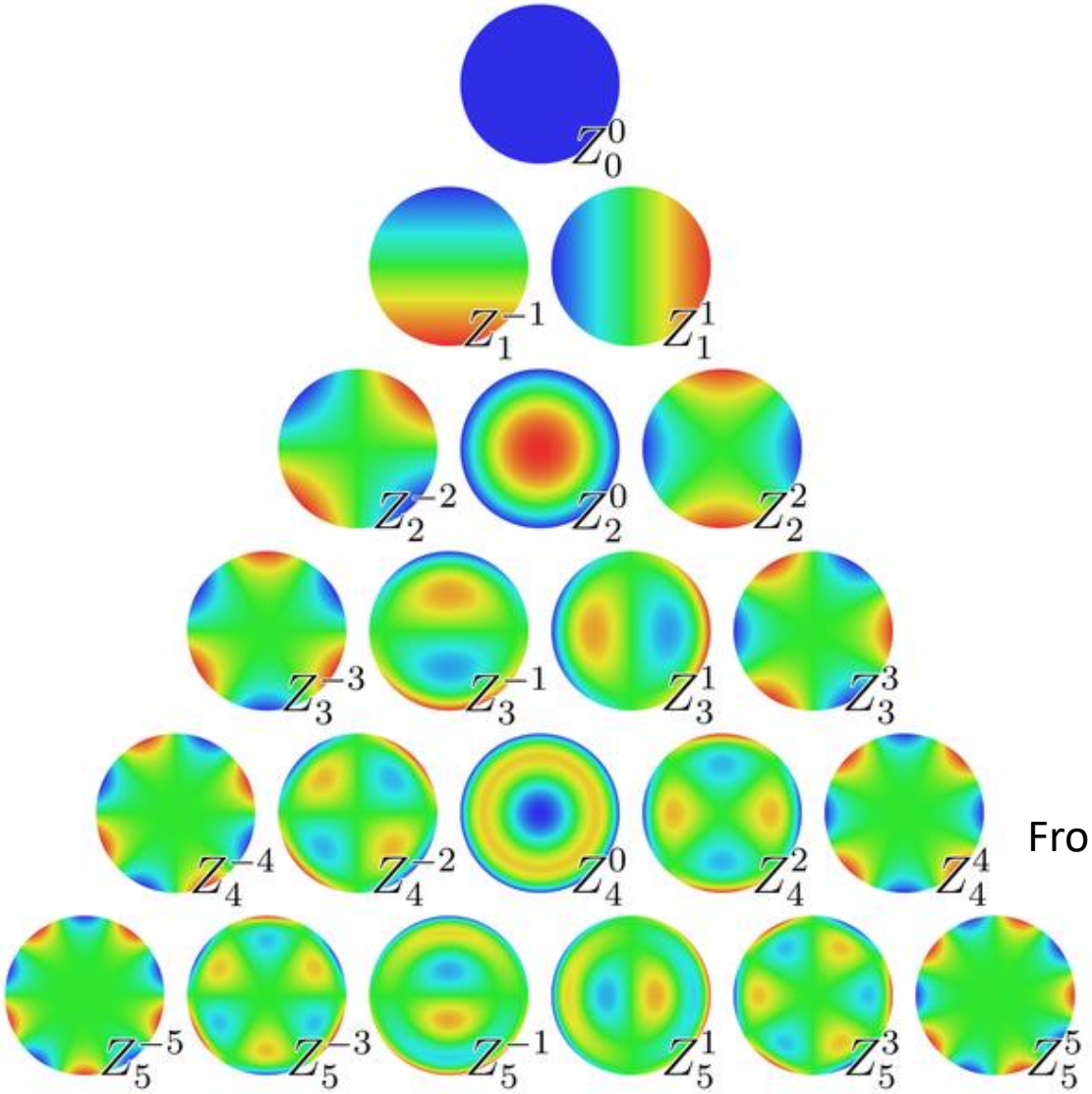
log-profile



Encircle-Energy



Zernike polynomials : orthogonal in circular area with unit radius

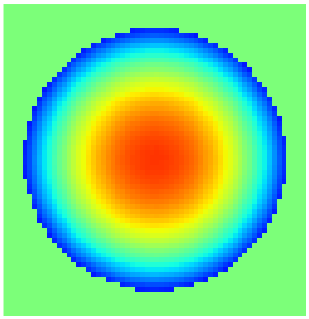


From en.wikipedia.org

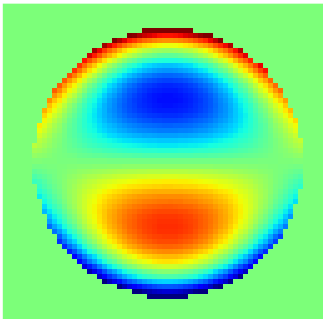
Third order Seidel aberration and Zernike polynomial: phase vs. PSF shape

Phase (1mm aperture)

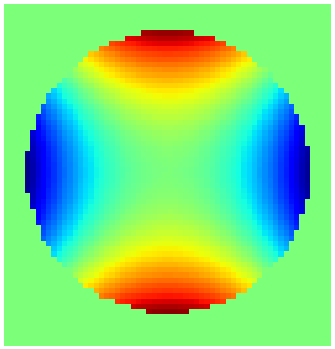
De-focus



Coma



Astigmatism



Trefoil

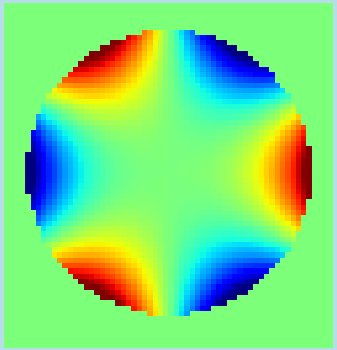
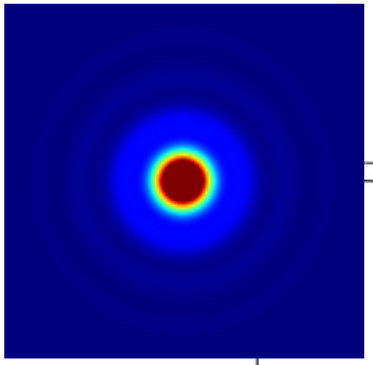
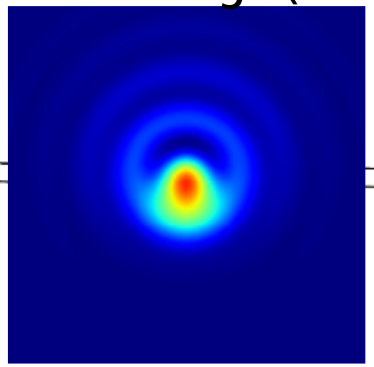
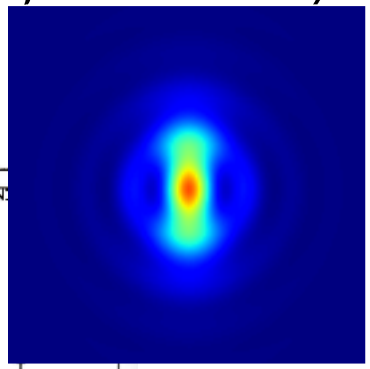
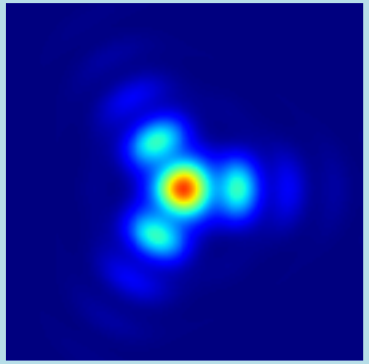


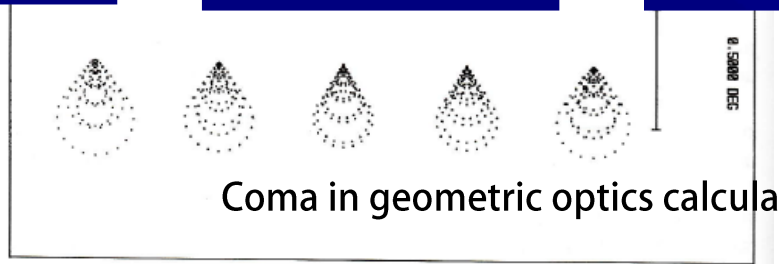
Image (far field, 3m distance)







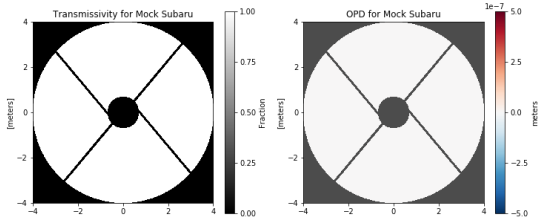


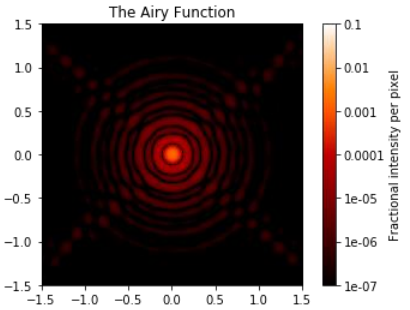


Coma in geometric optics calculation

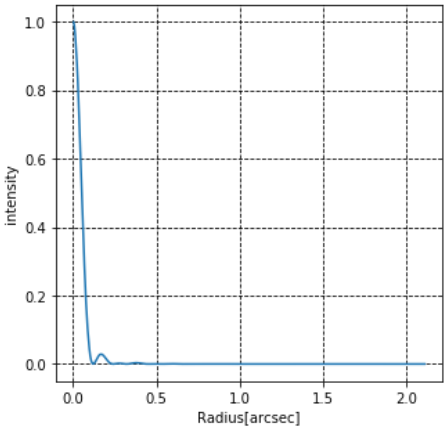
Adding phase error (coma)

8m mock Subaru

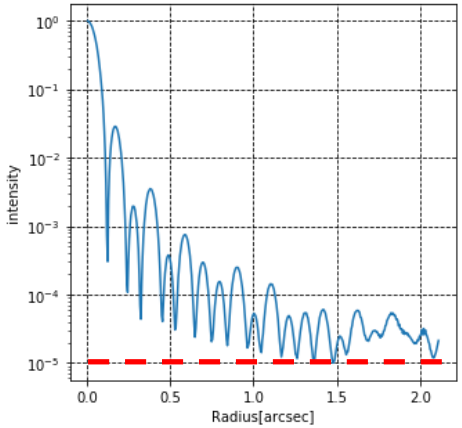




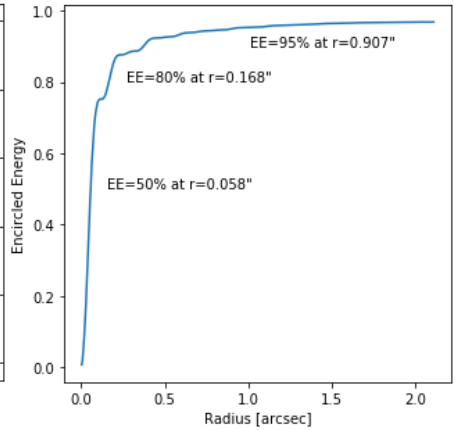
profile



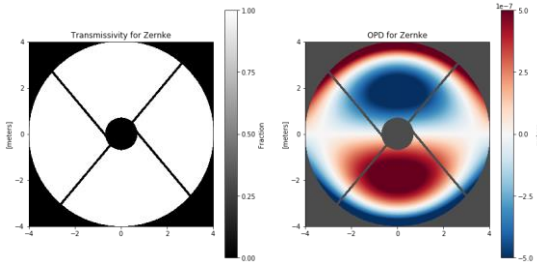
log-profile

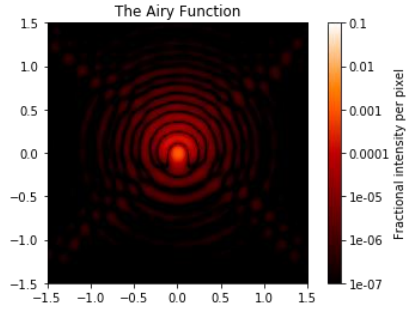


Encircle-Energy

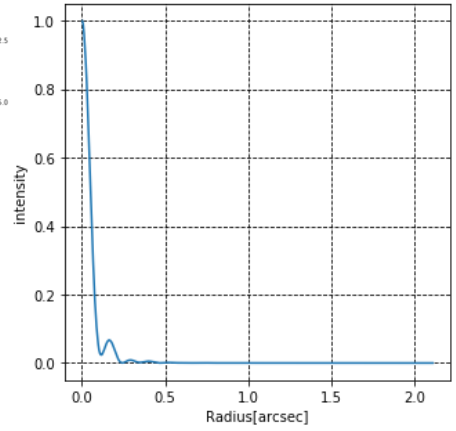


8m mock Subaru + Coma

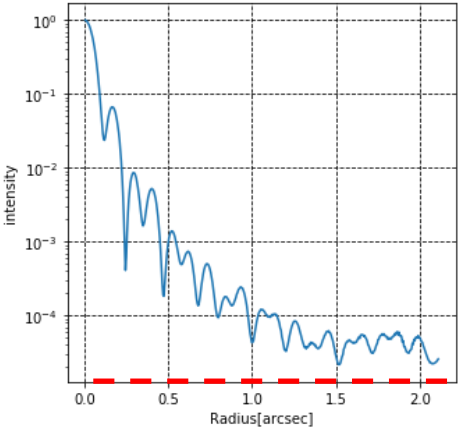




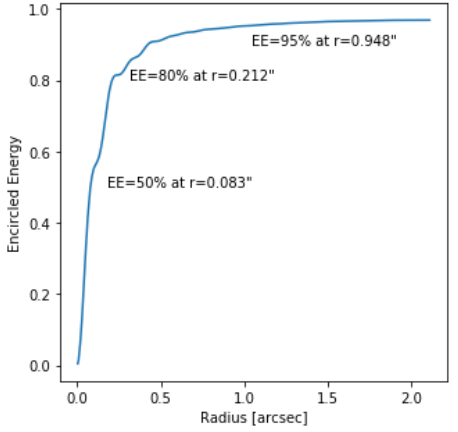
profile



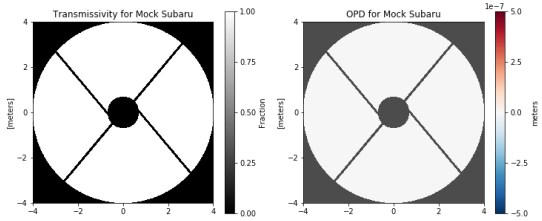
log-profile

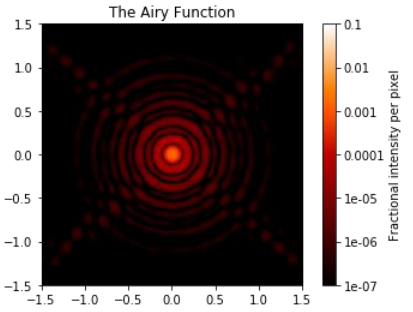


Encircle-Energy



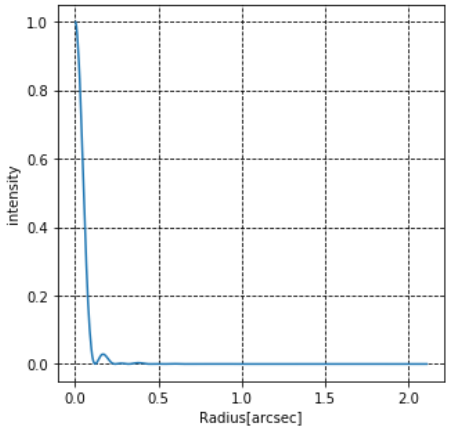
Adding phase error (spherical)



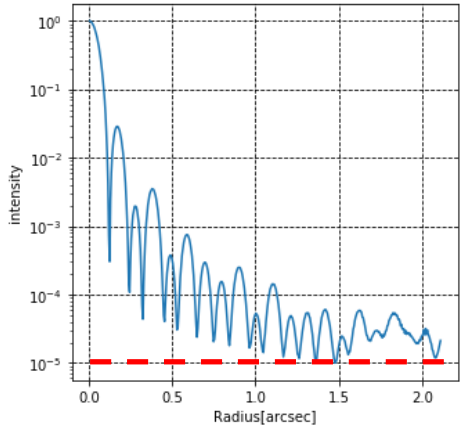


8m mock Subaru

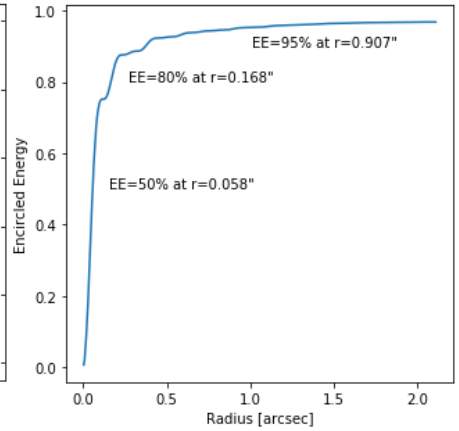
profile

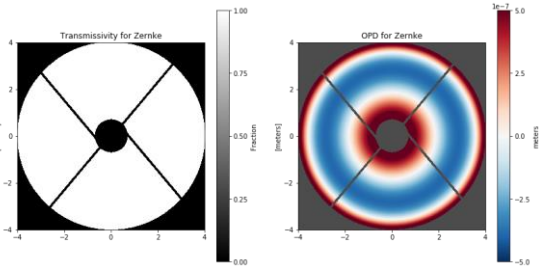


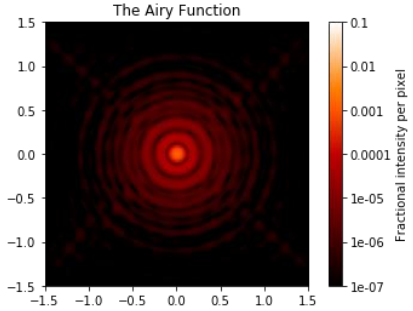
log-profile



Encircle-Energy

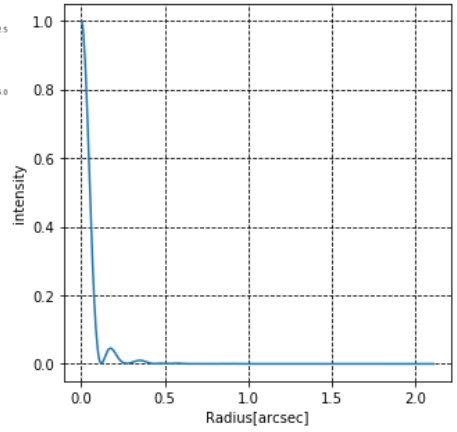




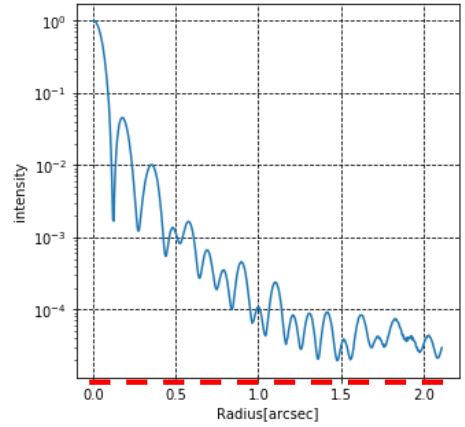


8m mock Subaru + Spherical

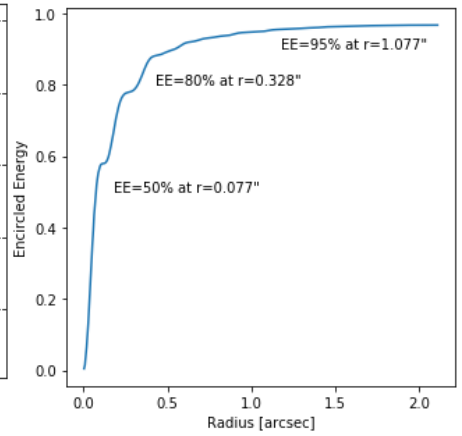
profile



log-profile

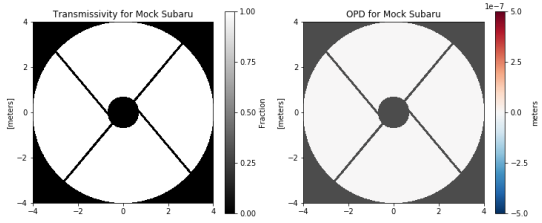


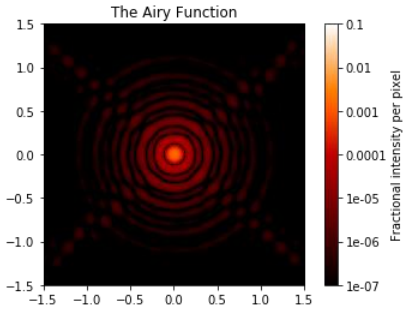
Encircle-Energy



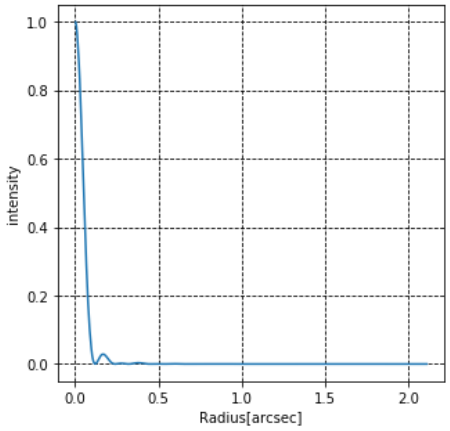
Adding phase error (phase14)

8m mock Subaru

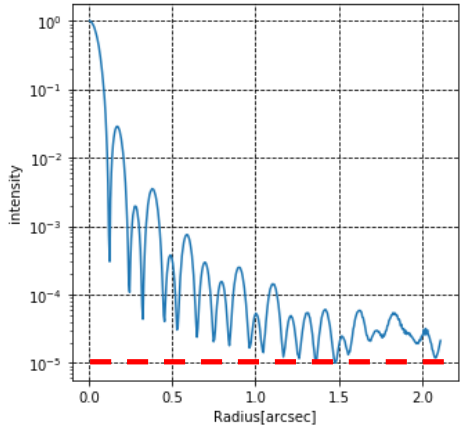




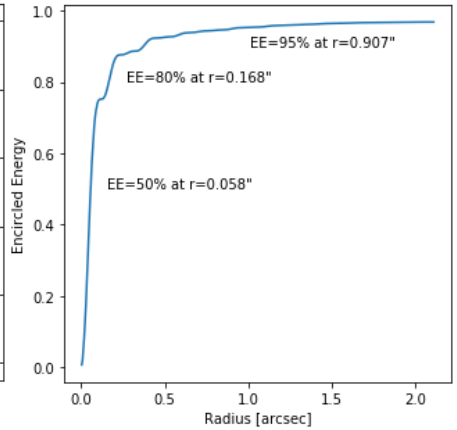
profile



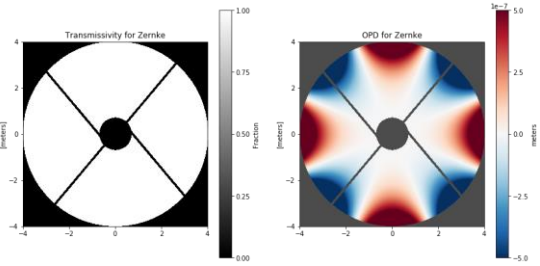
log-profile

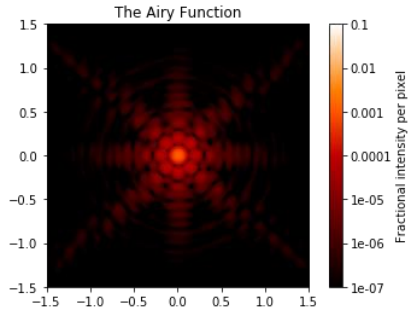


Encircle-Energy

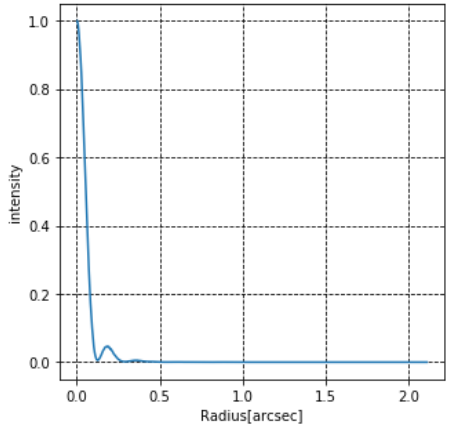


8m mock Subaru + Phase14

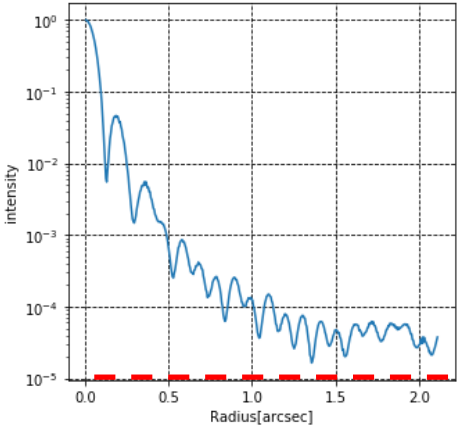




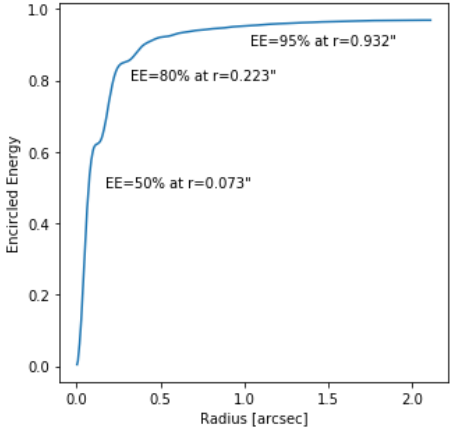
profile



log-profile

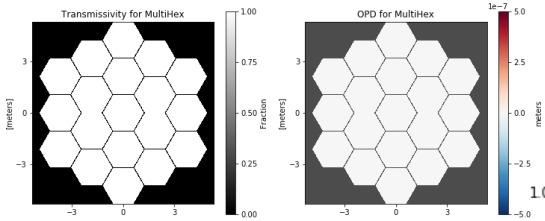


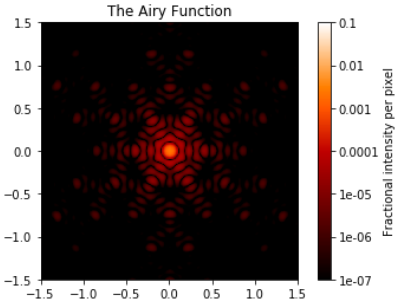
Encircle-Energy



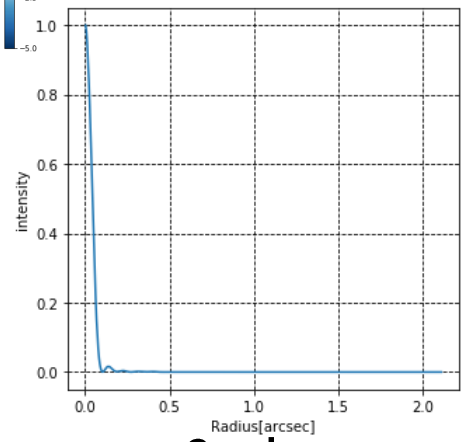
Aperture vs. PSF properties

8m hex aperture

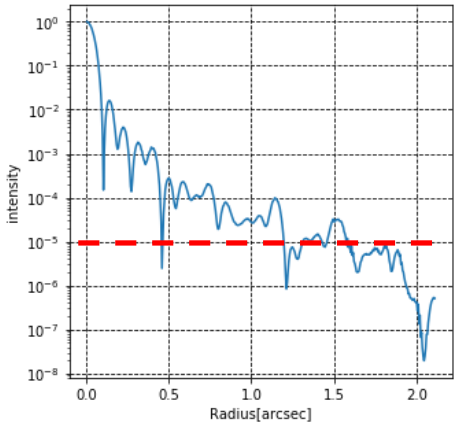




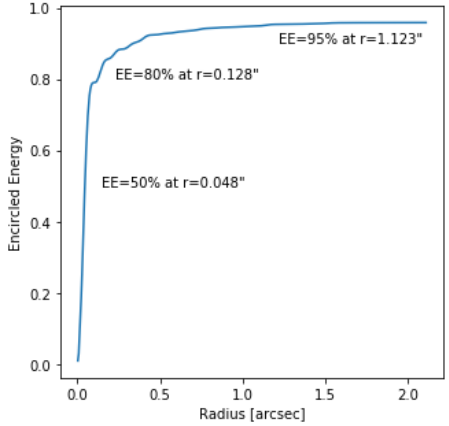
profile



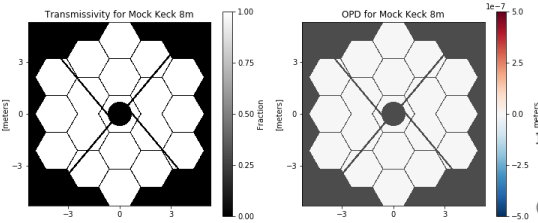
log-profile

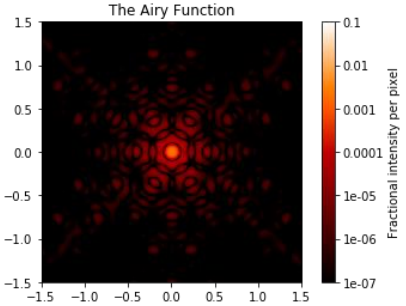


Encircle-Energy

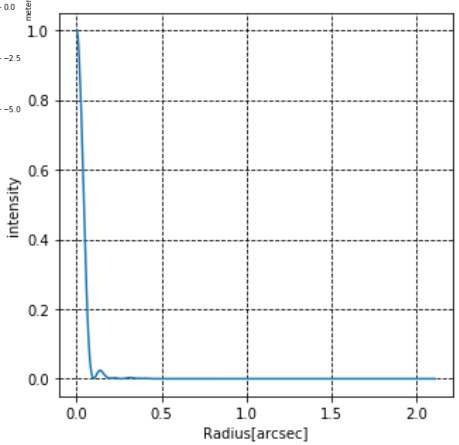


8m hex + secondary mirror (mock Keck)

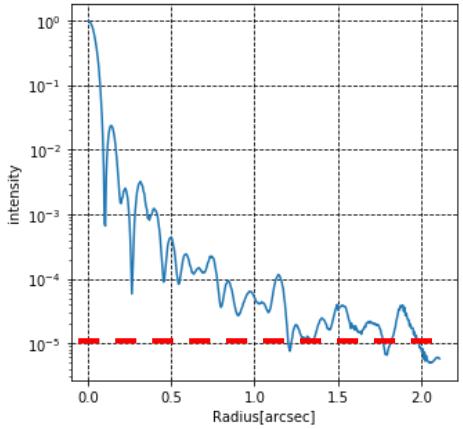




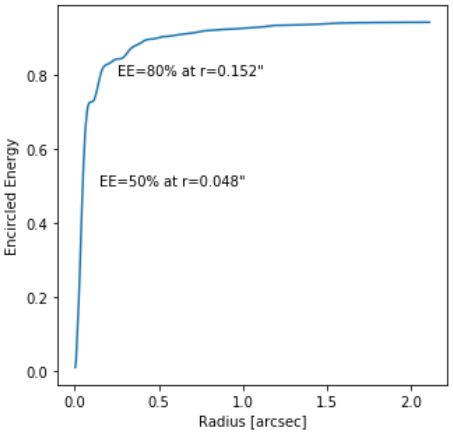
profile



log-profile

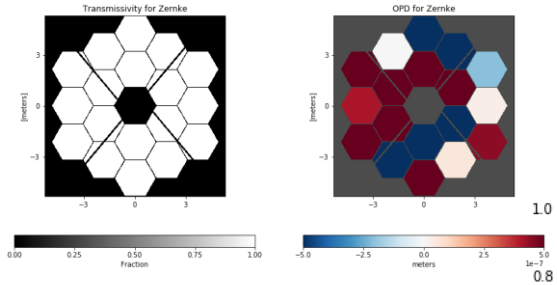


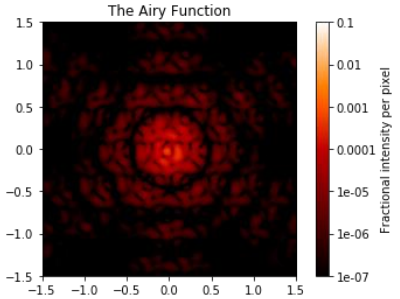
Encircle-Energy



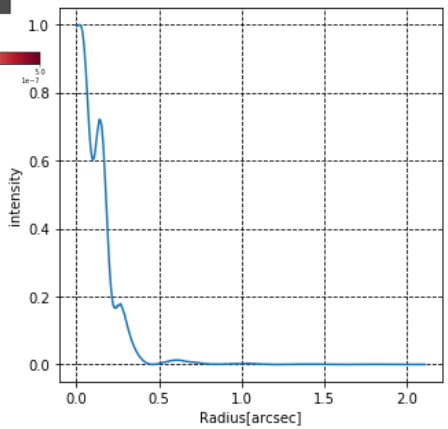
Error in phase vs. PSF properties

8m mock Keck with “phasing” error

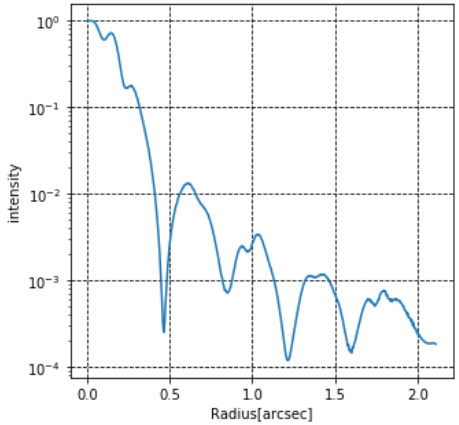




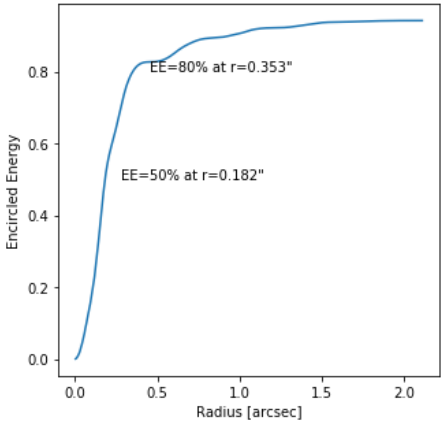
profile



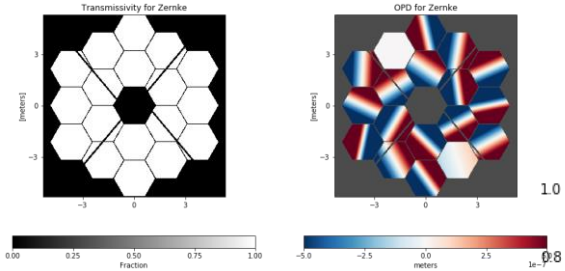
log-profile

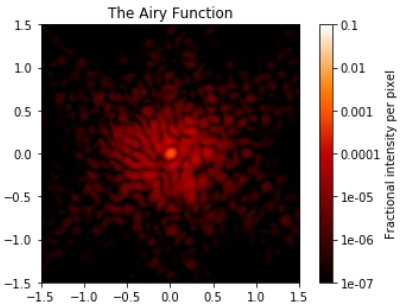


Encircle-Energy

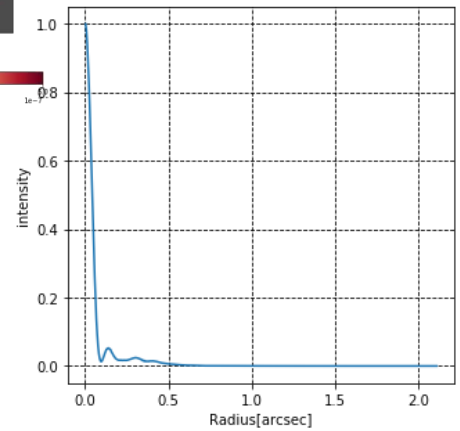


8m mock Keck with “tip-tilt” error

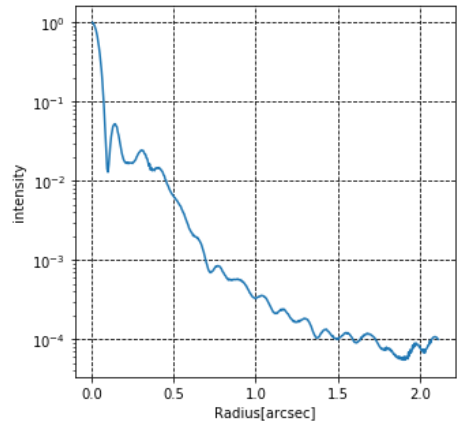




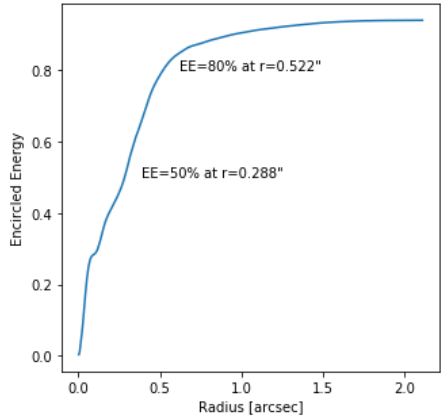
profile



log-profile

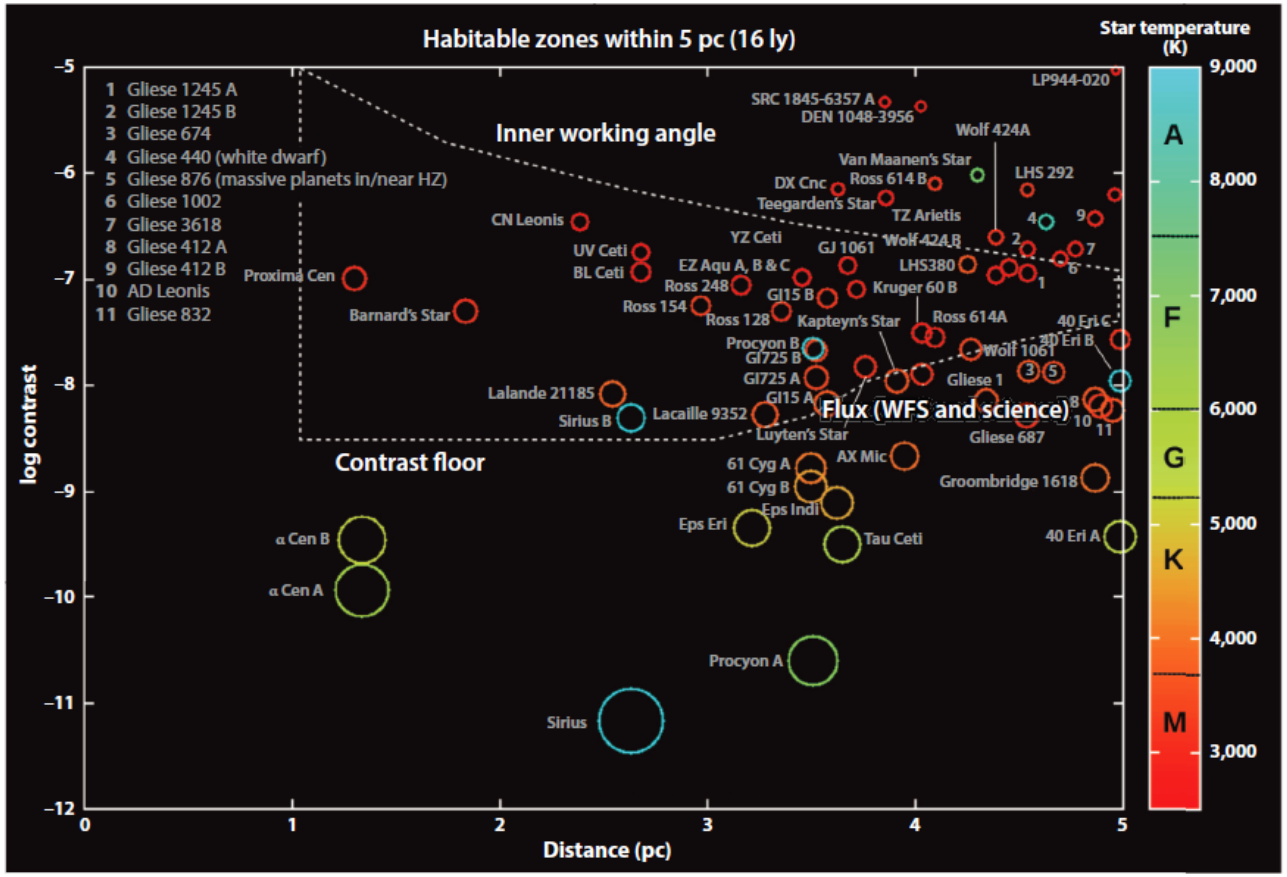


Encircle-Energy



Importance of the outskirts of PSFs

- Contrast required to detect Earth-sized planet



Guyon 2018

Figure 3

Reflected light contrast ratio (*vertical axis*) between an HZ Earth-sized planet and its host star, shown for all stars within 5 pc (distance to star on *horizontal axis*). Each circle corresponds to a star; its color and diameter encode the star effective temperature and the angular size of the HZ, respectively. The planets accessible to direct imaging with ExAO systems are within boundaries imposed by instrument performance: contrast floor, contrast performance in flux-limited regime, and coronagraph inner working angle. The location of the boundaries in this figure is arbitrary and only intended for illustrative purpose. Abbreviations: HZ, habitable zone; WFS, wavefront sensor.

Required contrast for various science targets

- From Guyon 2018

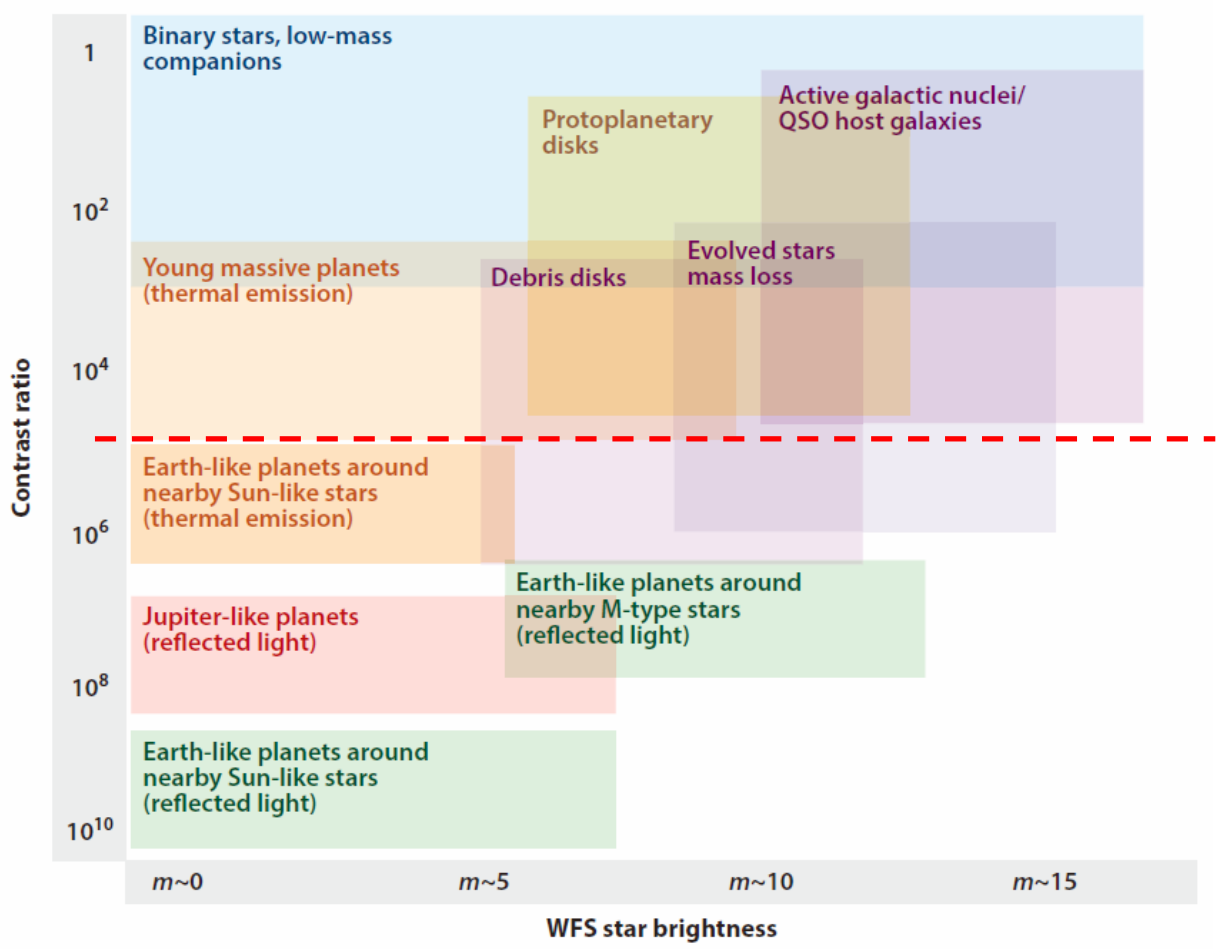
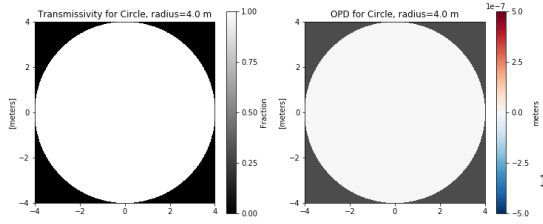


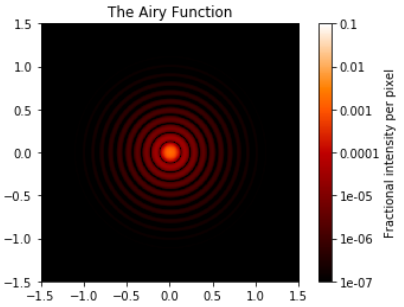
Figure 2

Selected ExAO science cases are shown here as a function of guide star brightness (*horizontal axis*, brightness decreases from left to right) and contrast ratio (*vertical axis*). Exoplanet imaging requires the highest wavefront control performance, up to $\approx 10^{-10}$ contrast for reflected light imaging of Earth-like planets around Sun-like stars. Location of boxes is approximate and highly target dependent. Abbreviations: QSO, quasi-stellar object; WFS, wavefront sensor.

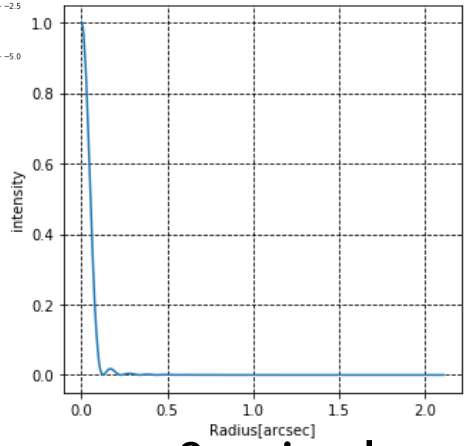
Reduce the outskirts

8m circular aperture

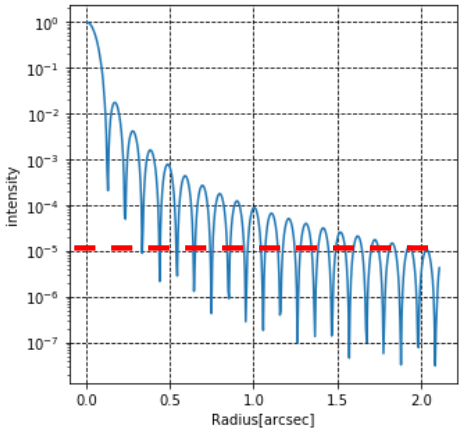




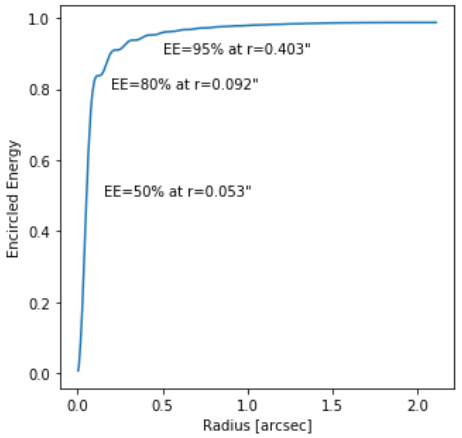
profile



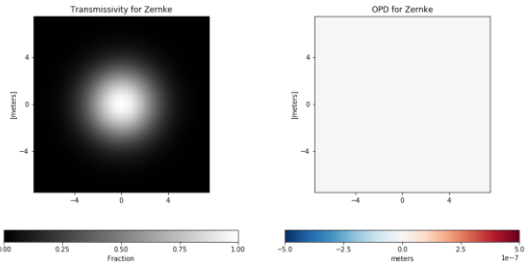
log-profile

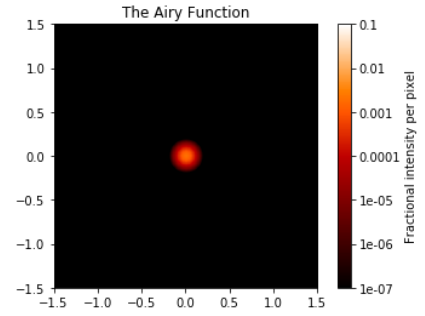


Encircle-Energy

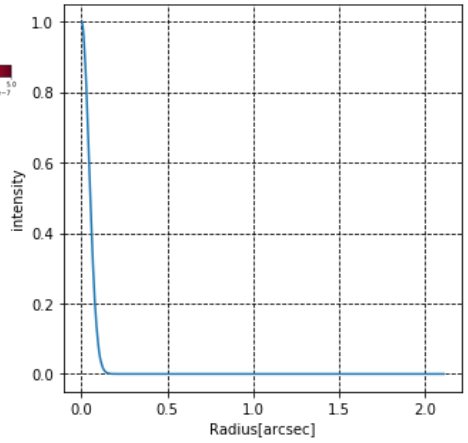


8m circular aperture + apodization

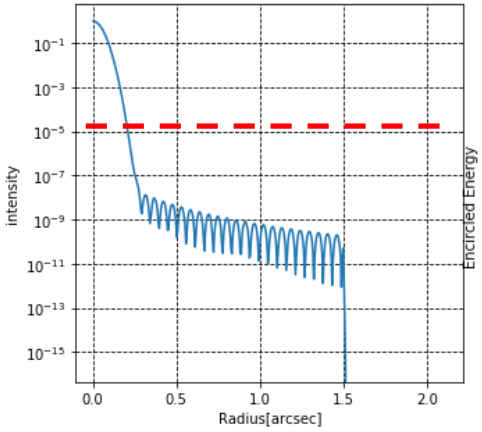




profile



log-profile



Encircle-Energy