Bosenova collapse of axion cloud around a rotating black hole

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Contents

Introduction
Code
Simulation
Summary
Contents

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- Code
- Simulation
- Summary
Axions

Candidates of massive scalar fields

QCD axion

- QCD axion was introduced to solve the Strong CP problem.
- It is one of the candidates of dark matter.

String axion

- String theory predicts the existence of 10-100 axion-like massive scalar fields.
- There are various expected phenomena of string axions.

Axion field makes a bound state and causes the superradiant instability

Detweiler, PRD22 (1980), 2323.
Zouros and Eardley, Ann. Phys. 118 (1979), 139.
Bound state

Zouros and Eardley, Ann. Phys. 118 (1979), 139.

Superradiance condition:
\[ \omega < \Omega_H m \]

Potential

\[ \Phi = e^{-i\omega t} R(r) S(\theta) e^{im\phi} \]

\[ R = \frac{u}{\sqrt{r^2 + a^2}} \quad \Rightarrow \quad \frac{d^2 u}{dr^*_2} + \left[ \omega^2 - V(\omega) \right] u = 0 \]
BH-axion system

Superradiant instability
- Emission of gravitational waves
- Pair annihilation of axions

Effects of nonlinear self-interaction
- Bosenova
- Mode mixing

Arvanitaki and Dubovsky, PRD83 (2011), 044026.
Nonlinear effect

QCD axion

break

$U(1)_{PQ}$ symmetry $\rightarrow$ $Z(N)$ discrete symmetry

$V(\Phi)$ becomes periodic. $\Delta \Phi = 2\pi v_a / N = 2\pi f_a$

$$V = f_a^2 \mu^2 [1 - \cos(\Phi / f_a)]$$

$$\nabla^2 \Phi - \mu^2 f_a \sin \left( \frac{\Phi}{f_a} \right) = 0$$

$$\nabla^2 \varphi - \mu^2 \sin \varphi = 0 \quad \varphi \equiv \frac{\Phi}{f_a}$$

The similar statement holds also for string axions.
BH-axion system

Superradiant instability
- Emission of gravitational waves
- Pair annihilation of axions

Effects of nonlinear self-interaction
- Bosenova
- Mode mixing

Arvanitaki and Dubovsky, PRD83 (2011), 044026.
Bosenova in condensed matter physics

http://spot.colorado.edu/~cwieman/Bosenova.html

BEC state of Rb85 (interaction can be controlled)
Switch from repulsive interaction to attractive interaction

Wieman et al., Nature 412 (2001), 295
What we would like to do

We would like to study the phenomena caused by axion cloud generated by the superraciant instability around a rotating black hole.

In particular, we study numerically whether “Bosenova” happens when the nonlinear interaction becomes important.

We adopt the background spacetime as the Kerr spacetime, and solve the axion field as a test field.
Contents

- Introduction
- Code
- Simulation
- Summary
Stable simulation cannot be realized in Boyer-Lindquist coordinates.
First difficulty

Stable simulation cannot be realized in Boyer-Lindquist coordinates.

We use ZAMO coordinates.

\[ \Omega = \frac{d\phi}{dt} = \frac{u^\phi}{u^t} = -\frac{g_{t\phi}}{g_{\phi\phi}} \]

\[ = \frac{2Mar}{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta} \]

\[ \tilde{t} = t, \]

\[ \tilde{\phi} = \phi - \Omega(r, \theta)t, \]

\[ \tilde{r} = r, \]

\[ \tilde{\theta} = \theta, \]
Second difficulty

- ZAMO coordinates becomes more and more distorted in the time evolution
Second difficulty

ZAMO coordinates becomes more and more distorted in the time evolution

We “pull back” the coordinates

\[ nT_P \leq t \leq (n + 1)T_P: \]

\[
\begin{align*}
t^{(n)} &= t, \\
\phi^{(n)} &= \phi - \Omega(r, \theta)(t - nT_P), \\
r^{(n)} &= r, \\
\theta^{(n)} &= \theta.
\end{align*}
\]
Our 3D code

- Space direction: 6th-order finite discretization
- Time direction: 4th-order Runge-Kutta
- Grid size: \[ \Delta r_* = 0.5 \quad (M = 1) \]
  \[ \Delta \theta = \Delta \phi = \pi / 30 \]
- Courant number: \[ C = \frac{\Delta t}{\Delta r_*} = \frac{1}{20} \]
- Pure ingoing BC at the inner boundary, Fixed BC at the outer boundary
- Pullback: 7th-order Lagrange interpolation
Code check

- Comparison with semianalytic solution in the Klein-Gordon case
- Convergence
- Conserved quantities
Numerical simulation

- Sine-Gordon equation
  \[ \nabla^2 \varphi - \mu^2 \sin \varphi = 0 \]

- Setup \( a/M = 0.99, \quad M\mu = 0.4 \)

- We choose the state of axion cloud that has grown by superradiant instability of \( m = 1 \) mode.
Expected evolution

amplitude

(1)

(2)

???

time
Expected evolution

\[ \varphi^{(1)} \mid_{\text{init}} = \varphi^{(KG)} \]
\[ (\text{amplitude}) = 0.6 \]

\[ \varphi^{(2)} \mid_{\text{init}} = 1.1 \times \varphi^{(1)} \mid_{t=1000M} \]
Simulation (1)

Axion field on equatorial plane ($\phi = 0$)

$-200 \leq r_*/M \leq 200$
Simulation (1)

Energy density with respect to the tortoise coordinate

\[
t/M = 0 \quad \quad \quad \quad \quad t/M = 1000
\]

\[-200 \leq r_*/M \leq 300\]
Expected evolution

\[
\varphi^{(1)}|_{\text{init}} = \varphi^{(KG)}
\]

\[
(\text{amplitude}) = 0.6
\]

\[
\varphi^{(2)}|_{\text{init}} = 1.1 \times \varphi^{(1)}|_{t=1000M}
\]
Simulation (2)

Axion field on equatorial plane ($\phi = 0$)

$-200 \leq r_*/M \leq 300$
Simulation (2)

Energy density with respect to the tortoise coordinate

\[ \frac{dE}{dr_*} \]

\( t/M = 0 \)

\[ -200 \leq r_*/M \leq 300 \]

\( t/M = 1000 \)

\[ -200 \leq r_*/M \leq 300 \]
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- Introduction
- Code
- Simulation
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Summary

We developed a reliable code and numerically studied the behaviour of axion field around a rotating black hole.

When the nonlinear self-interaction becomes relevant, the “bosenova collapse” can be seen, but not very violent.

The final state of superradiant instability would be a quasi-stationary state.

Issues for future

Calculation of the gravitational waves emitted in bosenova.

The case where axions couple to magnetic fields.