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ON THE CROSS-CORRELATION BETWEEN DARK MATTER DISTRIBUTION AND SECONDARY COSMIC MICROWAVE BACKGROUND ANISOTROPIES OF THE IONIZED INTERGALACTIC MEDIUM

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ABSTRACT

We investigate a cross-correlation between the weak gravitational lensing field of the large-scale structure κ and the secondary temperature fluctuation field Δ of the cosmic microwave background (CMB) induced by Thomson scattering of CMB photons off the ionized medium in the mildly nonlinear structure. The cross-correlation is expected to observationally unveil the biasing relation between the dark matter and ionized medium distributions in the large-scale structure. We develop a formalism for calculating the cross-correlation function and its angular power spectrum based on the small angle approximation. As a result, we find that the leading contribution to the cross-correlation comes from the secondary CMB fluctuation field induced by the quadratic Doppler effects of the bulk velocity field v of the ionized medium since the cross-correlations with the $O(v)$ Doppler effect and the Ostriker-Vishniac effect of $O(v\delta)$ are suppressed on relevant angular scales because the linear dependence on the bulk velocity field leads to cancellations between positive and negative contributions among the scatterers for the ensemble average. The magnitude of the cross-correlation can be estimated as $\langle\kappa\Delta\rangle \sim 10^{-10}$ on small angular scales ($l \gtrsim 1000$) under the currently favored cold dark matter model of the structure formation and the simplest scenario of homogeneous reionization after a given redshift of $z_{\rm ion} \gtrsim 5$. Although the magnitude turns out to be small, we find several interesting aspects of the effect. One of them is that the density-modulated quadratic Doppler effect of $O(\delta_e v^2)$ produces the cross-correlation with the weak lensing field that has *linear* dependence on the electron density fluctuation field δ_e or, equivalently, on the biasing relation between the dark matter and electron distributions. In other words, the angular power spectrum could be either positive or negative, depending on the positive biasing or antibiasing, respectively. Detection of the cross-correlation thus offers a new opportunity to observationally understand the reionization history of intergalactic medium connected to the dark matter clustering.

Subject headings: cosmic microwave background — cosmology: theory — intergalactic medium — large-scale structure of universe

1. INTRODUCTION

The absence of the hydrogen Gunn-Peterson trough in quasar (QSO) absorption spectra (Gunn & Peterson 1965) strongly indicates that the smoothly distributed hydrogen in the intergalactic medium (IGM) is already highly ionized by $z = 6$ (see, e.g., Lanzetta, Wolfe, & Turnshek 1995; Fan et al. 2000). Recently, Becker et al. (2001) reported the first detection of the Gunn-Peterson test trough in a $z = 6.28$ QSO spectrum. However, even a small fraction of neutral hydrogen in the IGM would result in an undetectable flux because of the high-absorption cross section of Lyα photons, and thus further careful investigations will be necessary. These results also imply the presence of very luminous ionizing sources such as QSOs and high-mass stars at high redshifts causing photoionizations of hydrogen by $z \gtrsim 6$. On the other hand, it is known that earlier reionization would have brought the last scattering surface to lower redshift, smoothing the intrinsic anisotropies in cosmic microwave background (CMB) radiation generated at the recombination epoch $z \sim 1000$ (Bond et al. 1991; Sugiyama, Silk, & Vittorio 1993). The recent measurements of the CMB anisotropies on subdegree angular scales (de Bernardis et al. 2000; Hanany et al. 2000; Netterfield et al. 2001) can be used to set an upper limit on reionization redshift such as $z_{\rm ion} \sim 30$ (Tegmark & Zaldarriaga 2000). Thus, under the currently favored adiabatic cold dark matter (CDM) scenario for the structure formation, reionization of the universe is expected to occur in the range of $6 \lesssim z_{\rm ion} \lesssim 30$. Revealing how reionization proceeds in the inhomogeneous universe stands out as one of the most important problems in the observational cosmology (see, e.g., see Barkana & Loeb 2001 for a recent review).

Secondary CMB anisotropies should be induced by scattering of CMB photons off electrons in the ionized medium undergoing bulk motion in the large-scale structure. It is therefore expected that the secondary anisotropies can provide a laboratory for studying structure formation in the reionized epoch. Kaiser (1984) investigated the CMB distortion induced by the bulk velocity to the pure linear order of $O(v)$ and found that the induced anisotropies are small at arcminute scales resulting from cancellations between positive and negative shifts along the line of sight since the gravitationally induced bulk velocities are irrotational. Ostriker & Vishniac (1986) then pointed out that the coupling of the Doppler effect with spatial variations in the free electron density produces microkelvin anisotropies on arcminute scales on the order of $O(\delta v)$, which is the so-called Ostriker-Vishniac (OV) effect, because the effect is a second-order contribution of perturbations that does not suffer from the

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cancellation (see also Vishniac 1987; Efstathiou 1988; Hu & White 1996; Jaffe & Kamionkowski 1998). For the same reason, spatial inhomogeneities of the ionization fraction associated with a realistic patchy reionization scenario should lead to the secondary CMB anisotropies by the Doppler effect (Aghanim et al. 1996; Gruzinov & Hu 1998; Knox, Scoccimarro, & Dodelson 1998; Benson et al. 2001; Gnedin & Jaffe 2001). In particular, Benson et al. (2001) investigated the secondary CMB fluctuations under a realistic model of the reionization of the IGM caused by starts in high-redshift galaxies based on a semianalytic model of galaxy formation and a high-resolution N-body simulation of the dark matter clustering. They showed that measurements of CMB anisotropies could actually put stringent constraints on the reionization processes. On the other hand, from theoretical aspects, it is not so evident as to answer the problem that the second-order effect of perturbations produces the dominant contribution to the secondary CMB fluctuations. Hu, Scott, & Silk (1994, hereafter HSS; see also Dodelson & Jubas 1995) derived a complete Boltzmann equation including the quadratic Doppler effects $O(v^2)$ of bulk velocity that governs redshift evolution of the CMB temperature fluctuation field. Consequently, they found that although the quadratic Doppler effect should induce spectral distortion of the CMB like the thermal Sunyaev-Zel'dovich (SZ) effect (Zel'dovich & Sunyaev 1969) as a new qualitative feature, the magnitude is always smaller than that of the OV effect for the CDM models.

A great uncertainty in models for structure formation in the reionized epoch is the extent and nature of reionization because this requires the understanding of various complicated astrophysical processes such as the nature of first ionizing sources, the fraction of ionizing photons that escape the sources to become available for photoionization of the hydrogen in the IGM, and the radiative balance between the escaped emissivity and the recombination rate in the inhomogeneous IGM. For example, even a naive question of which high-density or low-density region in the large-scale structure becomes photoionized first has still not been answered definitely. Assuming ab initio that the distribution of the ionization fraction is given as a function of gas density, Miralde-Escudé, Hanehnelt, & Rees (2000, hereafter MHR) suggested that the UV radiation may escape into and ionize the low-density region, where the recombination rates are the lowest, even though it is expected that the ionizing sources could be associated with the peaks in the density field. This conclusion has also been confirmed by recent numerical simulations, taking into account the radiative transfer approximately (Gnedin 2000).

While the relation between the dark matter and ionized medium distributions is thus a crucial clue to understanding the reionization history connected to the gravitational instability scenario of structure formation, the relation seems difficult to directly observe by methods so far proposed. Therefore, the purpose of this paper is to investigate the cross-correlation between the dark matter distribution and the secondary CMB temperature fluctuation field induced by the ionized medium in the mildly nonlinear structure. It is now recognized that the weak gravitational lensing effect due to the large-scale structure can be a unique tool to observationally reconstruct the dark matter distribution (see, e.g., Bartelmann & Schneider 2001 for a review). There are two feasible methods often considered in the literature; one is using the distortion effects on distant galaxy images that have been actually detected (see, e.g., Van Waerbeke et al. 2000b), and the other is using the weak lensing effects on the CMB map (Seljak & Zaldarriaga 1999, 2000; Zaldarriaga & Seljak 1999; Takada, Komatsu, & Futamase 2000; Zaldarriaga 2000; Van Waerbeke, Bernardeau, & Benabed 2000a; Takada & Futamase 2001; Schmalzing, Takada, & Futamase 2000; Takada 2001; Hu 2001; Hu & Okamoto 2001). In particular, it is expected that the method of using the distortion effects on the CMB map developed by Seljak & Zaldarriaga (1999; see also Zaldarriaga & Seljak 1999) allows us to reconstruct the projected dark matter distribution with high accuracy. Based on these considerations, in this paper we assume that the weak lensing field is a priori reconstructed by those methods, for example, using the future CMB data from the satellite mission *Planck Surveyor*. Then, taking a cross-correlation between the weak lensing field and the CMB map including the secondary fluctuations would be a challenging test. Furthermore, the cross-correlation method could improve the signal-to-noise ratio (S/N) of the Doppler effect compared to the original S/N in the CMB map itself if the secondary CMB field correlates with the weak lensing field, which is analogous to, for example, the cross-correlation between the CMB temperature fluctuation and polarization fields (Hu & White 1996; see also Peiris & Spergel 2000 and Van Waerbeke et al. 2000a for other examples). In this paper, based on the small angle approximation, we first develop a formalism for calculating the cross-correlation function and its angular power spectrum between the weak lensing field and the secondary CMB fluctuation field including the Doppler effects up to the second order of the bulk velocity field. We then estimate the magnitude of the cross-correlation under the currently favored CDM models and the simplest assumption of homogeneous reionization history after some epoch $z_{\rm ion} \gtrsim 5$.

The layout of this paper is as follows: In § 2 we review the relevant properties and parameters of the adiabatic CDM scenario for structure formation. In § 3 we briefly review the Boltzmann equation including the Doppler effect up to the second order of bulk velocity and discuss the physical implications of the effect, following HSS. In § 4 we develop a formalism for calculating the cross-correlation function and its angular power spectrum between the weak lensing field and the secondary CMB temperature fluctuation field induced by the bulk motion of the ionized medium in the large-scale structure. Results for the currently favored CDM models are presented in § 5. In § 6 we present the discussions and conclusions. Throughout this paper, we employ the unit of $c = 1$ for the light speed c.

2. PRELIMINARIES

2.1. *Cosmological Background Model*

The expansion rate of the universe can be described by the Friedmann equation:

$$H(t) \equiv \frac{1}{a}\frac{da}{dt} = H_0\sqrt{\Omega_{m0}a^{-3} + \Omega_{\lambda 0}}\,, \qquad (1)$$

where $a(t)$ denotes the scale factor, $H_0 = 100\ h$ km s^{-1} Mpc^{-1} is the Hubble constant, and Ω_{m0} and $\Omega_{\lambda 0}$ denote the density parameters of the present-day nonrelativistic matter and the contribution of the cosmological constant relative to the critical density $\rho_{\rm cr0} = 3H_0^2/8\pi G$, respectively. Here

we have set $a(t_0) = 1$ at present for convenience. Throughout this paper, we consider only a flat universe model as supported by the recent high-precision CMB experiments (see, e.g., de Bernardis et al. 2000; Netterfield et al. 2001), and then we have $\Omega_{\lambda 0} = 1 - \Omega_{m0}$. The conformal time that we will often use in the following discussions is defined by $d\eta = a^{-1}\, dt$. It is convenient to introduce the comoving angular diameter distance $r(\eta)$ with respect to η or, equivalently, redshift z, and for a flat universe model, it is given by

$$r(\eta) = \eta_0 - \eta = \int_0^z \frac{dz}{H(z)}\,, \tag{2}$$

where η_0 is the present conformal time and z is given by $1 + z = 1/a$.

It is currently believed that the reionization of the universe occurred at $6 \lesssim z_{\rm ion} \lesssim 30$ such that the reionized medium is optically thin to the Thomson scattering of CMB photons, which is characterized as $\tau < 1$ in terms of the optical depth τ. Assuming homogeneous ionization after some reionization epoch $z_{\rm ion}$, τ can be analytically expressed as a function of z for a flat universe model:

$$\tau \equiv \int_t^{t_0} dt'\, \bar{n}_e \sigma_{\rm T} = \frac{2}{3}\frac{\tau_{\rm H}\bar{X}_e}{\Omega_{m0}}\left[\sqrt{1 - \Omega_{m0} + \Omega_{m0}(1+z)^3} - 1\right], \tag{3}$$

with

$$\tau_{\rm H} \equiv \bar{n}_{e0}\sigma_{\rm T} H_0^{-1} = 0.0692(1 - Y)\Omega_{b0} h\,. \tag{4}$$

Here $\sigma_{\rm T}$ is the Thomson cross section, $\bar{n}_e$ is the mean number density of free electron, $\bar{X}_e$ is the mean ionization fraction, which does not depend on time here, Ω_{b0} is the present-day density parameter of baryon in units of the critical density, $\tau_{\rm H}$ is the optical depth to Thomson scattering to the Hubble distance today under the assumption of full hydrogen ionization and homogeneous distribution of baryon, and Y is the primordial helium fraction. We fix $Y = 0.24$ throughout this paper.

Again, in the case of homogeneous and full hydrogen ionization, the visibility function that gives the probability of last scattering within $d\eta$ of η is expressed as

$$g(\eta) = \dot{\tau} e^{-\tau} = \bar{n}_e \sigma_{\rm T} a e^{-\tau} = \tau_{\rm H} H_0 a^{-2} e^{-\tau}\,, \tag{5}$$

where the overdot denotes the derivative with respect to the conformal time η.

2.2. *Perturbation Theory of the Large-Scale Structure Formation*

The evolution of the large-scale structure after the decoupling epoch $z_* \sim 1000$ is characterized by the time evolution and scale dependence of the density contrast field $\delta(\boldsymbol{x}, t)$, mainly caused by the dark matter distribution. The gravitational instability then leads to bulk motions of structure with the velocity field $\boldsymbol{v}(\boldsymbol{x})$, and we expect that the ionized baryonic medium also follows the bulk motion on large scales, leading to the Doppler effects on CMB photons through the Compton scattering by electrons. Since we focus our investigations on the secondary CMB fluctuations up to the second order of v, we need to consider the fields of δ and v up to the second order that are valid in the mildly nonlinear regime such as $\delta \lesssim 1$:

$$\begin{aligned}\delta(\boldsymbol{x}) &= \delta^{(1)}(\boldsymbol{x}) + \delta^{(2)}(\boldsymbol{x}) + \cdots\,,\\ \boldsymbol{v}(\boldsymbol{x}) &= \boldsymbol{v}^{(1)}(\boldsymbol{x}) + \boldsymbol{v}^{(2)}(\boldsymbol{x}) + \cdots\,.\end{aligned} \tag{6}$$

The quantities with superscripts "(1)" and "(2)" denote the first- and second-order perturbation quantities, respectively. The time dependences of these fields can be expressed in terms of cosmological parameters based on the (Eulerian) perturbation theory (see, e.g., Peebles 1980). In the linear regime, the time dependence of $\delta^{(1)}$ is given by the growing factor as $\delta^{(1)}(\boldsymbol{x}, t) = D(t)\delta^{(1)}(\boldsymbol{x})$ with

$$D(t) \propto H(t)\int_{a(t)}^1 \frac{da}{(Ha)^3}\,, \tag{7}$$

where we have neglected the radiation energy contribution and will normalize $D(t_0) = 1$ at the present time t_0 for convenience in the following discussion.

The Fourier transformation of $\delta^{(1)}(\boldsymbol{x})$ is expressed as

$$\delta^{(1)}(\boldsymbol{x}) = \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\delta^{(1)}_{\boldsymbol{k}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,. \tag{8}$$

If the linear density fluctuations are Gaussian, the statistical properties can be completely specified by the power spectrum $P(k)$ defined by

$$\langle \delta_{\boldsymbol{k}}\delta^*_{\boldsymbol{k}'}\rangle \equiv (2\pi)^3 P(k)\delta^3(\boldsymbol{k} - \boldsymbol{k}')\,, \tag{9}$$

where the angle brackets denote an ensemble average over all realizations. Note that the time evolution of the power spectrum is given by $P(k, t) = D^2(t)P(k)$. In this paper, assuming the adiabatic CDM scenario, we employ the Harrison-Zel'dovich spectrum and the BBKS transfer function (Bardeen et al. 1986) with the shape parameter given by Sugiyama (1995) as for the shape of $P(k)$. The free parameter is then the normalization of $P(k)$ only, which is conventionally expressed in terms of the rms mass fluctuations of a sphere of $8\,h^{-1}$ Mpc radius, i.e., σ_8. From the continuity equation, we can obtain the Fourier components of the velocity field $\boldsymbol{v}^{(1)}_{\boldsymbol{k}}$ at the first order as

$$\boldsymbol{v}^{(1)}_{\boldsymbol{k}}(t) = \frac{i\boldsymbol{k}}{k^2}\dot{D}\delta^{(1)}_{\boldsymbol{k}}\,. \tag{10}$$

Likewise, based on the perturbation theory, we can obtain the second-order Fourier components of density fluctuation and bulk velocity fields in terms of $\delta^{(1)}_{\boldsymbol{k}}$ and D (see, e.g., Peebles 1980):

$$\begin{aligned}\delta^{(2)}_{\boldsymbol{k}}(t) &= D^2\int \frac{d^3\boldsymbol{k}'}{(2\pi)^3}F(\boldsymbol{k}', \boldsymbol{k} - \boldsymbol{k}')\delta^{(1)}_{\boldsymbol{k}'}\delta^{(1)}_{\boldsymbol{k}-\boldsymbol{k}'}\,,\\ \boldsymbol{v}^{(2)}_{\boldsymbol{k}}(t) &= \frac{i\boldsymbol{k}}{k^2}D\dot{D}\int \frac{d^3\boldsymbol{k}'}{(2\pi)^3}G(\boldsymbol{k}', \boldsymbol{k} - \boldsymbol{k}')\delta^{(1)}_{\boldsymbol{k}'}\delta^{(1)}_{\boldsymbol{k}-\boldsymbol{k}'}\,,\end{aligned} \tag{11}$$

with

$$\begin{aligned}F(\boldsymbol{k}_1, \boldsymbol{k}_2) &\equiv \frac{5}{7} + \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1 k_2} + \frac{2}{7}\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1 k_2}\right)^2\,,\\ G(\boldsymbol{k}_1, \boldsymbol{k}_2) &\equiv \frac{3}{7} + \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1 k_2} + \frac{4}{7}\left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1 k_2}\right)^2\,,\end{aligned} \tag{12}$$

where we have neglected extremely weak dependences of the functions F and G on cosmological parameters Ω_{m0} and $\Omega_{\lambda 0}$

for simplicity and considered the scalar-type perturbations only. It is worth noting that even if perturbations start from Gaussian in the linear regime, the mildly nonlinear evolution leads to nonvanishing third-order moments such as $\langle\delta^3\rangle$ and $\langle\delta v^2\rangle$ because those quantities are essentially the fourth-order moments of linear perturbation quantities like $\langle(\delta^{(1)})^4\rangle$.

3. BOLTZMANN EQUATION FOR SECOND-ORDER COMPTON SCATTERING

Here, following HSS, we briefly review the derivation of the secondary CMB temperature fluctuation field induced by the Doppler effect of the ionized medium up to the second order $O(v^2)$ of the bulk velocity. Let us first write down the Boltzmann equation in terms of the CMB photon energy p and direction γ^i in the presence of Compton scattering between the CMB photons and free electrons that could lead to energy changes,

$$\frac{\partial f}{\partial t}+\frac{\partial f}{\partial x^i}\frac{dx^i}{dt}+\frac{\partial f}{\partial p}\frac{dp}{dt}+\frac{\partial f}{\partial \gamma^i}\,d\gamma^i\,dt=C(x,\,p)\;, \quad (13)$$

where f is the photon occupation number and $C(x, p)$ is the collision term due to the Compton scattering. HSS derived the collision term that leads to the Doppler effect up to the second order $O(v^2)$ as

$$C_v(x,\,p)=n_e\sigma_{\rm T}\left(-\boldsymbol{\gamma}\cdot\boldsymbol{v}p\frac{\partial f}{\partial p}+\left\{\left[(\boldsymbol{\gamma}\cdot\boldsymbol{v})^2+v^2\right]p\frac{\partial f}{\partial p}+\left[\frac{3}{20}v^2+\frac{11}{20}(\boldsymbol{\gamma}\cdot\boldsymbol{v})^2\right]p^2\frac{\partial^2 f}{\partial p^2}\right\}\right), \quad (14)$$

where we have ignored the term of thermal electron scattering, leading to the thermal SZ effect. The first term in the large parentheses on the right-hand side represents the Doppler effect of $O(v)$ that produces the OV effect from the coupling with the electron density fluctuation field δ_e at the order $O(v\delta_e)$. The other terms give the quadratic Doppler effect of $O(v^2)$. It is interesting that the effect leads to energy exchanges between CMB photons and electrons like the thermal SZ effect and therefore causes the Compton y-distortion. As shown by HSS, we can get more physical insight into this effect by considering the limit of many scattering blobs or, equivalently, the multiple scattering limit. If we can average equation (14) over the direction of bulk velocity field $\boldsymbol{v}$, the $O(v)$ Doppler effect primarily cancels in this case, and then we can obtain

$$C_v(x,\,p)=n_e\sigma_{\rm T}\frac{\langle v^2\rangle}{3}\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^4\frac{\partial f}{\partial p}\right). \quad (15)$$

If introducing the effective temperature $T_{\rm eff}$ defined by $\langle v^2\rangle=3T_{\rm eff}/m_e$ (m_e is the electron mass), this collision term reduces to the same form as that in the Kompaneets equation with the thermal temperature T_e of the hot electron, which gives the thermal SZ effect. This implies that many ionized blobs distributed in the large-scale structure can be considered like hot electrons inside one cluster, and thus those blobs induce the energy exchange to the CMB photons characterized by the effective temperature $T_{\rm eff}$. Under the CDM scenario for the structure formation, the amplitude of Compton y-distortion induced by the quadratic Doppler effect is below that due to the thermal SZ effect mainly caused by intracluster hot plasma. This is because we can typically estimate $T_{\rm eff}\sim 10^{-1}$ eV for the gravitationally induced bulk flow and $T_e\sim$ keV for intracluster hot plasma, and because of the fact that $T_e\gg T_{\rm eff}$ leads to this conclusion, even though clusters are rare objects in the universe.

In this paper we do not employ the many scattering blobs limit for the purpose of generality, as done by HSS. Namely, we shall no longer be concerned with the frequency dependence of the temperature fluctuation field. Integration of equation (13) over photon energy p leads to the Boltzmann equation up to the order $O(v^2)$ to govern the time evolution of the temperature fluctuation field $\Delta(\equiv\delta T/T_{\rm CMB})$ along the line of sight in the direction of γ:

$$\dot{\Delta}+\gamma^i\partial_i\Delta=n_e\sigma_{\rm T}a\left[\Delta_0-\Delta+\boldsymbol{\gamma}\cdot\boldsymbol{v}-v^2+7(\boldsymbol{\gamma}\cdot\boldsymbol{v})^2\right], \quad (16)$$

where we have neglected the gravitational effect and the correction due to the quadrupole temperature fluctuations. By solving the equation above formally, we can obtain the secondary fluctuation field induced by the Doppler effect of ionized medium during propagations of the CMB photons from the decoupling $z_*\sim 1000$ to present:

$$\Delta(\eta_0,\,\gamma)=\int_{\eta_*}^{\eta_0}d\eta g(\eta)X_e(\eta)[1+\delta_e(\boldsymbol{x},\,\eta)]\times\left[\boldsymbol{\gamma}\cdot\boldsymbol{v}-v^2+7(\boldsymbol{\gamma}\cdot\boldsymbol{v})^2\right], \quad (17)$$

where η_* is the conformal time at the decoupling epoch $z_*\approx 1000$, $g(\eta)$ is the visibility function given by equation (5), and we have used the fact that the *free* electron density field can be expressed as

$$n_e(\boldsymbol{x},\,\eta)=\bar{\boldsymbol{n}}_e(\eta)[1+\delta_e(\boldsymbol{x},\,\eta)]=\bar{\boldsymbol{n}}_{\rm H0}a^{-3}(\eta)X_e(\eta)(1+\delta_e)\;. \quad (18)$$

$X_e(\eta)$ is the ionization fraction as a function of η, δ_e is the density fluctuation field of the electron along the line of sight, and $\bar{\boldsymbol{n}}_{\rm H0}$ is the mean hydrogen number density at present given by $\bar{\boldsymbol{n}}_{\rm H0}=1.12\times 10^{-5}$ cm $\Omega_{b0}\,h^2(1-Y)-3$. Note that as explicitly introduced by Gnedin & Jaffe (2001), n_e can also be expressed as $n_e=X_e(1+\delta_X)n_{\rm H}=X_e\bar{\boldsymbol{n}}_{\rm H}[1+\delta_X(\boldsymbol{x},\,\eta)][1+\delta_b(\boldsymbol{x},\,\eta)]$, where δ_X and δ_b are the fluctuation fields of the ionization fraction and the baryon density, respectively, as a function of the spatial position and time. It is thus clear that δ_e is different from δ_b in a general reionization history. Note that we will deal with δ_e throughout this paper.

4. FORMALISM: CROSS-CORRELATION BETWEEN DOPPLER EFFECTS ON CMB AND THE WEAK LENSING FIELD

In this section we develop a formalism for calculating the cross-correlation function and its angular power spectrum between the weak lensing field and the secondary CMB anisotropies field induced by the Doppler effect of the ionized medium.

4.1. *Weak Lensing Field*

The weak gravitational lensing due to the large-scale structure is now recognized as a unique tool for directly measuring the dark matter distribution. Seljak & Zaldar-

riaga (1999; see also Zaldarriaga & Seljak 1999) developed a useful method for reconstructing the projected field of dark matter fluctuations between the last scattering surface and the present from the lensing distortion on the product of gradients of the temperature field. This method could be successfully performed by the satellite mission *Planck*. It has also been shown that a new power of temperature fluctuations generated by the weak lensing effect on arcminute scales below the Silk damping scale can be directly used to reconstruct the projected dark matter field, provided that we have data with arcminute-scale spatial resolution and sensitivity of $\sim 10\ \mu$K on an arcminute scale (Seljak & Zaldarriaga 2000; Zaldarriaga 2000; Hu 2001) and that we can remove other secondary signals such as the SZ effect by taking advantage of the specific spectral properties or by combining the CMB measurement with the X-ray or SZ effect itself. Furthermore, if measurements of the CMB polarization become possible, combinations of the lensing effects on the temperature fluctuation and the *E*- and *B*-type polarization fields can improve the reconstruction (Zaldarriaga & Seljak 1998; Hu & Okamoto 2001). This is because the lensing effect induces the *B*-type polarization pattern in the observed sky from the nonlinear coupling to the primary *E*-mode, even if the primary CMB contains the *E*-mode only, as suggested by standard inflationary scenarios, and there is negligible cosmological contamination of the polarization field in the arcminute regime, unlike the temperature fluctuations (Hu 2000a).

The projected dark matter field is called the convergence field κ and can be expressed in the form of a weighted projection of the three-dimensional density fluctuation field along the line of sight:

$$\kappa \equiv \int_{\eta_*}^{\eta_0} d\eta W(\eta_*, \eta)\delta(\boldsymbol{x}, \eta) , \tag{19}$$

where $W(\eta_*, \eta)$ is the lensing weight function defined by

$$W(\eta_*, \eta) \equiv \frac{3}{2}\Omega_{m0}H_0^2 \frac{r(\eta - \eta_*)}{r(\eta_*)} r(\eta)a^{-1} . \tag{20}$$

On the other hand, the lensing distortion effect on distant galaxy images can be similarly used to measure the relatively low redshift dark matter distribution such as $z \lesssim 2$, depending on the typical redshift of source galaxies (Bartelmann & Schneider 2001). In this case, the lower limit in equation (19) becomes the conformal time corresponding to the redshift of source galaxies z_s. However, we should bear in mind that the lensing effect on galaxies is relatively irrelevant for the purpose of this paper because this method usually restricts the convergence field to low redshifts (see, e.g., Van Waerbeke et al. 2000b), while the reionization signal in the CMB map is imprinted at moderate and high redshifts. The weak lensing effects on CMB thus has a great advantage of probing the dark matter distribution up to high redshift, which cannot be attained by any other means (see also Takada et al. 2000; Takada & Futamase 2001; Takada 2001). In this paper we assume that the convergence field $\kappa(\boldsymbol{\gamma})$ with respect to direction $\boldsymbol{\gamma}$ is a priori reconstructed from measurements of the lensing effects on the CMB.

4.2. *Doppler Effect of $O(v)$ and $O(v\delta_e)$*

From equation (17), the secondary temperature fluctuation field induced by the $O(v)$ Doppler effect can be written as

$$\Delta(\gamma) = \int_{\eta_{\rm ion}}^{\eta_0} d\eta g(\eta) X_e (1 + \delta_e) \boldsymbol{\gamma} \cdot \boldsymbol{v} . \tag{21}$$

The pure linear-order Doppler effect $\Delta \sim O(v)$ is suppressed on small scales because the gravitational instability generates irrotational flow, leading to cancellations between positive and negative Doppler shifts along the line of sight (Kaiser 1984). For this reason, the leading contribution to the secondary temperature fluctuations due to the Doppler effect comes from second-order perturbations that do not suffer from the cancellation like the OV effect of $O(v\delta_e)$ (Ostriker & Vishniac 1986; Vishniac 1987; Efstathiou 1988; Hu & White 1996; Jaffe & Kamionkowski 1998) and the patchy ionization effect of $O(v\delta_X)$ (Aghanim et al. 1996; Knox, Scoccimarro, & Dodelson 1998; Gruzinov & Hu 1998; Benson et al. 2001), where δ_X is the spatial inhomogeneities of the ionization fraction X_e as discussed following equation (18).

From equations (19) and (21), a cross-correlation between the convergence field and the secondary temperature fluctuation field due to the Doppler effects of $O(v)$ and $O(v\delta_e)$ depends on the moments $\langle \delta v \rangle$ and $\langle \delta v \delta_e \rangle$ of perturbations, respectively, where the latter moment seems to produce some contributions in the quasi-nonlinear regime, as discussed in § 2.2. However, it is clear that since those moments have linear dependence on the bulk velocity field v, the cross-correlation is suppressed on relevant small angular scales because of the cancellations between positive and negative contributions from the bulk velocity field for the ensemble average. Note that the two-point correlation function of the OV effect or, equivalently, its angular power spectrum arises from the quadratic contribution of the velocity field as $O(v^2\delta^2)$, whereas it does not suffer from the cancellation (Ostriker & Vishniac 1986; Vishniac 1987; Efstathiou 1988).

4.3. *Quadratic Doppler Effect of $O(v^2)$*

We next consider secondary temperature fluctuations induced by the quadratic Doppler effect of the bulk velocity. In the case of assuming the homogeneous ionization history, the secondary fluctuations are divided into two contributions: the Doppler effect of $O(v^2)$ and the density-modulated effect of $O(v^2\delta_e)$. As will be shown later, the effect of $O(v^2\delta_e)$ generally produces dominant contribution on small angular scales $l \gtrsim 1000$, where the nonlinear structures with $\delta \gtrsim 1$ are important. From equation (17), the secondary fluctuation field of $O(v^2)$ is given by

$$\Delta^{v^2}(\gamma) = \int_{\eta_{\rm ion}}^{\eta_0} d\eta g(\eta) X_e \left[-v^2 + 7(\boldsymbol{\gamma} \cdot \boldsymbol{v})^2 \right] . \tag{22}$$

The bulk flow in adiabatic CDM models arises mainly from the linear regime. In the ΛCDM model, half the contribution to $\langle v^2 \rangle$ comes from scales $k \lesssim 0.07\ h\ {\rm Mpc}^{-1}$, and the density fluctuations go nonlinear at $k \gtrsim 0.15\ h\ {\rm Mpc}^{-1}$. This is the reason that the secondary CMB anisotropies by the Doppler effects of $O(v^2)$ and $O(v^2\delta_e)$ are an order of magnitude smaller than the anisotropies by the OV effect of $O(v\delta_e)$ on relevant angular scales. However, because of the fact that there is no cross-correlation between the convergence field and the OV effect, the quadratic Doppler effects can provide the primary contribution to the cross-correlation between

the convergence field and the secondary CMB temperature fluctuations induced by the ionized medium.

Since we are interested in the cross-correlation on small angular scales such as $l \gtrsim 100$, it is useful to employ the small angle approximation originally developed by Bond & Efstathiou (1987). In this case, the unit vector $\boldsymbol{\gamma}$ in equation (22), which denotes the direction of line of sight, can be considered to have components as $\boldsymbol{\gamma} \approx [\theta_x, \theta_y, (1-\theta^2)^{1/2}]$ for $|\theta_x|, |\theta_y| \ll 1$ in the small region around the north pole on the celestial sphere. It is then convenient to introduce the two-dimensional vector $\boldsymbol{\theta}$, which has $\boldsymbol{\theta} = (\theta_x, \theta_y, 0)$. Within the context of the small angle approximation, the cross-correlation function between the convergence field κ and the temperature fluctuation field Δ^{v^2} separated by the angle of θ is defined by

$$C^{\kappa v^2}(\theta) \equiv \langle \kappa(\gamma_a)\Delta^{v^2}(\gamma_b)\rangle_{\boldsymbol{\gamma}_a\cdot\boldsymbol{\gamma}_b=\cos\theta} \approx \langle \kappa(\boldsymbol{\theta}_a)\Delta^{v^2}(\boldsymbol{\theta}_b)\rangle_{|\boldsymbol{\theta}_a-\boldsymbol{\theta}_b|\approx\theta} , \quad (23)$$

where $\boldsymbol{\theta}_a$ and $\boldsymbol{\theta}_b$ are the two-dimensional vectors corresponding to the unit vectors γ_a and γ_b for κ and Δ^{v^2}, respectively. Furthermore, if using the two-dimensional Fourier transformation as, for example, $\kappa(\boldsymbol{\theta}) = \int d^2\boldsymbol{l}/(2\pi)^2 \kappa_l e^{i\boldsymbol{l}\cdot\boldsymbol{\theta}}$, the cross-correlation function can be expressed in terms of its angular power spectrum by the Hankel transform of zeroth order as

$$C^{\kappa v^2}(\theta) = \int \frac{ldl}{2\pi} C_l^{\kappa v^2} J_0(l\theta) , \quad (24)$$

where $J_0(x)$ is the zeroth-order Bessel function. The angular power spectrum $C_l^{\kappa v^2}$ can also be expressed in terms of the two-dimensional Fourier components of κ and Δ^{v^2} as

$$\langle \kappa_{\boldsymbol{l}} \Delta^{v^2}_{\boldsymbol{l}'} \rangle \equiv C_l^{\kappa v^2}(2\pi)^2 \delta^2(\boldsymbol{l}+\boldsymbol{l}') . \quad (25)$$

From equations (19) and (22), the cross-correlation function (eq. [23]) can be further calculated as

$$C^{\kappa v^2}(\theta) = \int_{\eta_*}^{\eta_0} d\eta \int_{\eta_{\rm ion}}^{\eta_0} d\eta'\, W(\eta,\eta_*) g(\eta') \langle \delta(\eta,\boldsymbol{\theta}_a) X_e q_e(\eta',\boldsymbol{\theta}_b)\rangle \approx \langle X_e\rangle \int d\eta \int d\eta'\, W(\eta,\eta_*) g(\eta') \langle \delta(\eta,\boldsymbol{\theta}_a) q_e(\eta',\boldsymbol{\theta}_b)\rangle , \quad (26)$$

where $q_e(\boldsymbol{\theta}_b) \equiv -v^2 + 7(\boldsymbol{\gamma}_b\cdot\boldsymbol{v})^2$, with $\gamma_b \approx (\theta_{bx}, \theta_{by}, 1)$ and the functions g and W given by equations (5) and (20), respectively. In the second line we have replaced the ionization fraction of X_e, which generally depends on conformal time η, with the average fraction $\langle X_e\rangle$ between $\eta_{\rm ion} \le \eta \le \eta_0$, assuming that $X_e(\eta)$ is a slowly varying function with respect to the conformal time after a given reionized epoch $z_{\rm ion}$. From equation (6), the ensemble average $\langle \delta(\boldsymbol{\theta}_a, \eta) q_e\rangle$ can be rewritten in terms of third-order moments of perturbations as

$$\langle \delta(\boldsymbol{\theta}_a) q_e(\boldsymbol{\theta}_b)\rangle \approx 2\left\langle \delta^{(1)}(\boldsymbol{\theta}_a)(-v_i^{(1)}v_i^{(2)} + 7v_z^{(1)}v_z^{(2)})(\boldsymbol{\theta}_b)\right\rangle + \left\langle \delta^{(2)}(\boldsymbol{\theta}_a)(-v_i^{(1)}v_i^{(1)} + 7v_z^{(1)}v_z^{(1)})(\boldsymbol{\theta}_b)\right\rangle + O[(\delta^{(1)})^6] . \quad (27)$$

Here we have used $\langle \delta^{(1)} v^{(1)} v^{(1)}\rangle = 0$, and from the fact that the propagation direction of CMB photons is almost parallel to the $\hat{z}$-direction in our setting, we have used $\boldsymbol{\gamma}_b\cdot\boldsymbol{v} \approx v_z$, where v_z denotes the z-component of bulk velocity field. Therefore, using equations (10) and (11), we can obtain

$$\langle \delta(\boldsymbol{\theta}_a) q_e(\boldsymbol{\theta}_b)\rangle = D(\eta)\left(D\dot{D}^2\right)(\eta') \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \int \frac{d^3\boldsymbol{k}'}{(2\pi)^3} \times \frac{7k_z'(k_z+k_z') - k'^2 - \boldsymbol{k}\cdot\boldsymbol{k}'}{k'^2|\boldsymbol{k}+\boldsymbol{k}'|^2} \times [4G(-\boldsymbol{k}, \boldsymbol{k}+\boldsymbol{k}') + F(-\boldsymbol{k}', \boldsymbol{k}+\boldsymbol{k}')] \times P(k)P(|\boldsymbol{k}+\boldsymbol{k}'|) e^{i\boldsymbol{k}_\perp\cdot(r\boldsymbol{\theta}_a - r'\boldsymbol{\theta}_b) - ik_z(\eta-\eta')} , \quad (28)$$

where $\boldsymbol{k}_\perp$ are the (x, y)-components of $\boldsymbol{k}$ that are perpendicular to the line of sight and r and r' denote the comoving angular distances from the present time to η and η', respectively. One can readily see that the integration of the term $e^{-ik_z(\eta-\eta')}$ over k_z on the right-hand side of equation (28) leads to the Dirac delta function $\delta(\eta-\eta')$ as long as we are concerned with wavelength modes much smaller than the comoving Hubble distance characterized by η. This leads to the consequence that the dominant contribution to the cross-correlation function arises from structures of dark matter and ionized medium that are distributed at the same redshift and are separated by θ on the observed sky. This is the so-called Fourier space analogue of Limber's equation (Kaiser 1992; Jaffe & Kamionkowski 1998). It is known that this flat-sky approximation causes errors no greater than $O(0.1\%)$ at $l \gtrsim 1000$ (Hu 2000b; Jaffe & Kamionkowski 1998). Under this approximation, the cross-correlation function $C^{\kappa v^2}(\theta)$ can be calculated as

$$C^{\kappa v^2}(\theta) \approx \langle X_e\rangle \int_{\eta_{\rm ion}}^{\eta_0} d\eta g(\eta) W(\eta,\eta_*) D^2 \dot{D}^2 \times \int \frac{k_\perp dk_\perp}{2\pi} S(k_\perp) J_0(k_\perp r\theta) , \quad (29)$$

with

$$S(k) \equiv 2\int_0^\infty dk' \int_0^{2\pi} d\phi \int_0^1 d\mu \times \frac{k'^2(7\mu^2-1) - k'k\cos\phi\sqrt{1-\mu^2}}{k^2+k'^2+2kk'\cos\phi\sqrt{1-\mu^2}} \times [4G(k, k', \phi, \mu) + F(k, k', \phi, \mu)]P(k') \times P\left(\sqrt{k^2+k'^2+2kk'\cos\phi\sqrt{1-\mu^2}}\right) , \quad (30)$$

where we have assumed that $\boldsymbol{k}_\perp$ is along the x-coordinate from the statistical isotropy and then $\boldsymbol{k}_\perp\cdot\boldsymbol{k}' = k_\perp k'\cos\phi\cos\theta$.

Hence, from equations (24) and (29), we can finally derive the angular power spectrum of the cross-correlation between the convergence field and the secondary temperature fluctuation field by the quadratic Doppler effect of $O(v^2)$:

$$C_l^{\kappa v^2} = \langle X_e\rangle \int_{\eta_*}^{\eta_0} d\eta g(\eta) W(\eta, \eta_*) r^{-2}(\eta) \dot{D}^2 D^2 S\left(k = \frac{l}{r}\right) . \quad (31)$$

This is the first main result of this paper. It should be noted that the integrand quantity in $C_l^{\kappa v^2}$ depends on the cosmological parameters and the dark matter power spectrum.

Namely, it is not affected by the unknown biasing relation between the dark matter and ionized medium distributions. In this sense, we can accurately predict the magnitude of the integrand quantity once we fix the dark matter power spectrum such as the CDM model and the cosmological parameters, which will be well constrained in the near future by other observations such as measurements of the CMB anisotropies, the weak lensing survey, and so on. We therefore expect that measurements of $C_l^{\kappa v^2}$ can set precise constraints on the average ionization fraction $\langle X_e \rangle$ between the reionization epoch and the present.

4.4. *Density-modulated Quadratic Doppler Effect of* $O(v^2\delta_e)$

Next we consider the secondary temperature fluctuation field induced by the density-modulated quadratic Doppler effect of $\Delta \sim O(\delta_e v^2)$. From equation (17) we have

$$\Delta^{\delta v^2}(\boldsymbol{\theta}) = \int d\eta g(\eta) X_e \delta_e \left[-v^2 + 7(\boldsymbol{\gamma} \cdot \boldsymbol{v})^2 \right] . \quad (32)$$

In the following discussion, to relate δ_e to the dark matter fluctuation field δ, we introduce the linear biasing parameter b expressed as $\delta_e(\boldsymbol{x}, \eta) \equiv b\delta(\boldsymbol{x}, \eta)$ for simplicity. Note that b generally depends on time and spatial position.

It is relatively straightforward to calculate the cross-correlation function between the convergence field and the temperature fluctuations field $\Delta^{\delta v^2}$:

$$\begin{aligned} C^{\kappa,\delta v^2}(\theta) &\equiv \left\langle \kappa(\boldsymbol{\theta}_a) \Delta^{\delta v^2}(\boldsymbol{\theta}_b) \right\rangle = \int d\eta \int d\eta' W(\eta, \eta_*) g(\eta') \\ &\quad \times \left\langle X_e b \delta(\eta, \boldsymbol{\theta}_a) \left\{ \delta \left[-v^2 + 7(\boldsymbol{\gamma}_b \cdot \boldsymbol{v})^2 \right] \right\} (\eta', \boldsymbol{\theta}_b) \right\rangle \\ &\approx \langle X_e b \rangle \int d\eta \int d\eta' W(\eta, \eta_*) g(\eta') \\ &\quad \times \left\langle \delta(\eta, \boldsymbol{\theta}_a) \left\{ \delta \left[-v^2 + 7(\boldsymbol{\gamma}_b \cdot \boldsymbol{v})^2 \right] \right\} (\eta', \boldsymbol{\theta}_b) \right\rangle \\ &= \langle X_e b \rangle \int d\eta \int d\eta' W(\eta, \eta_*) g(\eta') \langle \delta\delta \rangle \\ &\quad \times \left\langle -v^2 + 7(\boldsymbol{\gamma}_b \cdot \boldsymbol{v})^2 \right\rangle \\ &= \langle X_e b \rangle \int d\eta \int d\eta' W(\eta, \eta_*) g(\eta') \\ &\quad \times \langle \delta(\eta, \boldsymbol{\theta}_a) \delta(\eta', \boldsymbol{\theta}_b) \rangle \tfrac{4}{3} \langle v^2 \rangle , \end{aligned} \quad (33)$$

where $\langle X_e b \rangle$ is the average quantity of $X_e b$ between the reionization epoch and present. After the similar procedure as used in the derivation of equation (31), we can obtain the angular power spectrum

$$\begin{aligned} C_l^{\kappa,\delta v^2} &\approx 4 \langle X_e b \rangle \int_{\eta_{\rm ion}}^{\eta_0} d\eta W(\eta, \eta_*) g(\eta) r^{-2}(\eta) \\ &\quad \times D^2 P \left(k = \frac{l}{r} \right) v_{\rm rms}^2(\eta) , \end{aligned} \quad (34)$$

where $v_{\rm rms}$ is the rms of the present-day bulk velocity field and can be expressed in terms of the linear density power spectrum as

$$v_{\rm rms}^2(\eta) = \frac{\dot{D}^2(\eta)}{3} \int \frac{dk'}{2\pi^2} P(k') . \quad (35)$$

Here the factor $\frac{1}{3}$ is due to the three-dimensional freedom of the bulk velocity field such that $v_{\rm rms}$ represents the rms of one-dimensional motion. The most important result of equation (34) is that the angular power spectrum $C_l^{\kappa,\delta v^2}$ is proportional to not the quadratic but the linear term of $\langle X_e b \rangle$ for all value of l and therefore could be either positive or negative, depending on the sign of $\langle X_e b \rangle$ or, more specifically, on the sign of $\langle b \rangle$ since the integrand quantity in equation (34) is always positive. The quantity $\langle X_e b \rangle$ should be a crucial clue to understanding how the ionized medium distribution is related to the dark matter distribution in the large-scale structure: which still remains largely uncertain from both theoretical and observational aspects. The measurements of $C_l^{\kappa,\delta v^2}$ can thus be a new direct probe of revealing the projected biasing relation. It is worth noting that the angular power spectrum of the OV effect depends on the quadratic term $O(X_e^2 b^2)$ and is always positive irrespective of the sign of $\langle X_e b \rangle$.

The simple assumption that the ionized medium traces the dark matter distribution allows us to place the upper limit on $C_l^{\kappa,\delta v^2}$ using the nonlinear dark matter power spectrum, which has been well studied with N-body simulations for the full range of adiabatic CDM models. Equation (34) can be then rewritten as

$$\begin{aligned} C_l^{\kappa,\delta v^2} &= 4 \langle X_e b \rangle \int_{\eta_{\rm ion}}^{\eta_0} d\eta W(\eta, \eta_*) g(\eta) r^{-2} \\ &\quad \times P_{\rm NL} \left(k = \frac{l}{r}, \eta \right) v_{\rm rms}^2(\eta) , \end{aligned} \quad (36)$$

where $P_{\rm NL}(k, \eta)$ is the power spectrum of nonlinear density fluctuations for which we will employ the fitting formula developed by Peacock & Dodds (1996). Note that we here consider a possible case in which electrons in a reionized medium with nonlinear density fluctuations are distributed along the line of sight moving according to a large-scale coherent bulk velocity field, as suggested by the CDM power spectrum. This is, in fact, an essential mechanism that the Ostriker-Vishniac effect can avoid cancellations between blue- and redshifts along the line of sight from the coupling of the density and velocity fields. Based on this consideration, in Figure 3 we use the nonlinear power spectrum only for the calculation of $C_l^{\kappa,\delta v^2}$ and the second-order perturbation theory for that of $C_l^{\kappa v^2}$ because the latter arises from the ensemble average of the density fluctuation field and the bulk velocity filed on the mildly nonlinear regime.

5. RESULTS

We present numerical results for the angular power spectra given by equations (31) and (34) for the ΛCDM model. We adopt a set of cosmological parameters of $\Omega_{m0} = 0.3$, $\Omega_{\lambda 0} = 0.7$, $\Omega_{b0} = 0.05$, $h = 0.7$, and $\sigma_8 = 1$, which is consistent with the recent CMB experiments (Netterfield et al. 2001) and the cluster abundance (Eke, Cole, & Frenk 1996; Kitayama & Suto 1997). We simply consider the homogeneous hydrogen ionization model after a given reionization epoch in the range of $5 < z_{\rm ion} < 30$. In this case we will consider $X_e = $ const. with respect to time during $0 \le z \le z_{\rm ion}$, and equations (3) and (5) can be used for calculations of the optical depth and visibility function, respectively. Although a more realistic model is a patchy or inhomogeneous reionization (see, e.g., Benson et al. 2001), our model allows us to compute the magnitude of effects in the simplest way.

We first estimate the order of magnitude of secondary temperature fluctuations by the quadratic Doppler effect.

Using the ΛCDM model above, the present-day rms of the bulk velocity field can be estimated from equation (35) as $v_{\rm rms}(t_0) \approx 1.19 \times 10^{-3}$ in the unit of $c = 1$. Equation (17) tells us that the magnitude of temperature fluctuations induced by the quadratic Doppler effect can be roughly estimated as $\Delta^{v^2} \sim \tau v_{\rm rms}^2 \sim 10^{-9}$ to 10^{-8} for $z_{\rm ion} = 10$ since we have $\tau \sim 10^{-3}$–10^{-2} for the model considered in this paper. Similarly, the magnitude of temperature fluctuations induced by the OV effect can be estimated as $\Delta^{\rm OV} \sim \tau v_{\rm rms}\delta \sim 10^{-6}$, which is consistent with the numerical results done by Hu & White (1996; see also Efstathiou 1988 and Jaffe & Kamionkowski 1998). We thus conclude that $\Delta^{v^2} \ll \Delta^{\rm OV}$ in the observed CMB map itself and that the magnitude of Δ^{v^2} is much below the sensitivities of the *Microwave Anistropy Probe* and the *Planck Surveyor*.

Figure 1 shows the angular power spectra of the cross-correlations between the weak lensing field and the secondary temperature fluctuations induced by the quadratic Doppler effect of $O(v^2)$ (*dashed lines*) and the density-modulated effect of $O(\delta_e v^2)$ (*solid line*), which are computed using equations (31) and (34), respectively. We here assumed $b = 1$ for the unknown biasing parameter between the dark matter and electron distributions in the large-scale structure. The magnitudes of the Doppler effects of both $O(v^2)$ and $O(\delta_e v^2)$ are comparable on $l \lesssim 10^2$ in the case of no biasing $b = 1$. This is because the leading contribution to the cross-correlation between the weak lensing field of $O(\delta)$ and the Doppler effect of $O(v^2)$ comes from the fourth-order moments of linear perturbations like $O(\delta^{(1)2}v^{(1)2})$ resulting from the vanishing of the third-order moments $\langle \delta^{(1)} v^{(1)2} \rangle = 0$, while the cross-correlation from the density-modulated Doppler effect of $O(\delta_e v^2)$ also arises from the moments of $\langle \delta^{(1)} v^{(1)2} \rangle$. It is clear that the contribution from the density-modulated effect of $O(v^2\delta_e)$ dominates the effect of $O(v^2)$ on angular scales of $l \gtrsim 100$. The total magnitudes of the angular power spectra peak around $l \approx 1500$ with a power of $C_l^{\kappa \rm DP} l^2/2\pi \sim 10^{-10}$ for the ΛCDM model and for the five cases of reionization epoch $z_{\rm ion} = 5$, 7, 10, 15, and 20. The peak location in l-space results from the shape of CDM power spectrum. On the other hand, Cooray (2000) investigated the angular power spectrum of the cross-correlation between the weak lensing field and the thermal SZ temperature fluctuation field based on the halo approach of dark matter clustering (Komatsu & Kitayama 1999) and the simple model of intracluster gas distribution. The magnitude of the angular power spectrum from the SZ effect amounts to $C_l^{\kappa \rm SZ} l^2 \sim 10^{-8}$ at $l \approx 2000$ for models similar to those considered in this paper. The temperature fluctuation induced by the quadratic Doppler effect is over an order of magnitude smaller than that by the SZ effect because of $\Delta^{v^2} \sim 10^{-9}$ to 10^{-8} and $\Delta^{\rm SZ} \sim 10^{-6}$. From this fact, it seems unlikely that the magnitude of the cross-correlation between κ and the quadratic Doppler effect is smaller only by $\sim 10^{-2}$ than that of the SZ effect. This is because the cross-correlation with the Doppler effect comes from wide-range redshift structures such as $0 < z < z_{\rm ion}$, while the SZ effect has dominant contributions from low-redshift massive dark halos such as clusters of galaxies with $M \gtrsim 10^{14}\ M_\odot$ at $z \lesssim 0.5$ (Cooray 2000). These results therefore imply that if the weak lensing field is a priori reconstructed by measurements of lensing effects on the CMB, the cross-correlation could improve the S/N of the quadratic Doppler effect, even though the angular power spectrum itself of the Doppler effect has a small power of $C_l^{\rm DP} l^2 \lesssim 10^{-17}$ only. This is an analogous statement to the cross-correlation between the primary CMB temperature fluctuation and polarization fields (see, e.g., Hu & White 1997).

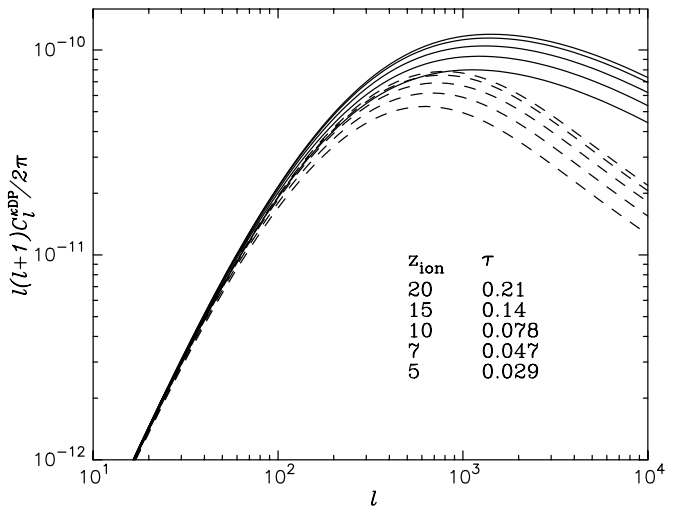

FIG. 1.—Angular power spectra of the cross-correlation between the weak lensing field κ and the secondary CMB fluctuations field Δ induced by the quadratic Doppler effects of ionized medium. These spectra are calculated under the simple assumptions that the gas traces the dark matter and the full hydrogen ionization after a given reionization epoch; $X_e = 1$ and $b = 1$ for the ionization fraction and the biasing parameter. The solid and dashed lines show contributions from the pure quadratic Doppler effect of $O(v^2)$ and the density-modulated effect of $O(v^2\delta_e)$ given by eqs. (31) and (34), respectively. We here considered five cases of the reionization epoch as $z_{\rm ion} = 5$, 7, 10, 15, and 20.

To explicitly illustrate how the cross-correlation considered could be useful to reveal the biasing relation between the dark matter and electron distributions, we here employ a toy model of the antibiasing relation, motivated by the reionization scenario of the low-density region prior to the high-density region as suggested by MHR. The homogeneous full hydrogen ionization with $X_e = 1$ seems to lead to $\delta_e = \delta_b$ from $n_e = n_{\rm H}$, which seems inconsistent with the antibiasing relation, and thus we here assume the partly homogeneous ionization fraction of $X_e = 0.5$ after $z_{\rm ion}$. Although the antibiasing process depends on how reionization proceeds in the large-scale structure in detail, we consider the simplest model of the linear and constant antibiasing relation given by $\delta_e = b\delta$ with $b = -0.8$. It should then be noted that the condition of $\delta_e \geq -1$ leads to $\delta < 1/|b|$. Accordingly, we have taken into account the range of δ for the calculation of the ensemble average of $\langle \delta_e \delta \rangle$ used in equation (34). Figure 2 shows the result of this toy model, and one can readily see that we should observe the negative total angular power spectra (*solid lines*) of the cross-correlation between the weak lensing field and the secondary temperature fluctuation field induced by the Doppler effect on $l \gtrsim 2000$. Note that this critical scale of l depends on the value of the antibiasing parameter b because the cancellation between the spectra from the quadratic Doppler effect of $O(v^2)$ and the density-modulated effect of $O(v^2\delta_e)$ also depends on b. As expected, the cross-correlation function could thus be a direct probe of the antibiasing relation, which cannot be attained by measurements of the angular power spectrum of the $O(v)$ Doppler effect such as the OV effect because of the quadratic dependence on the biasing relation.

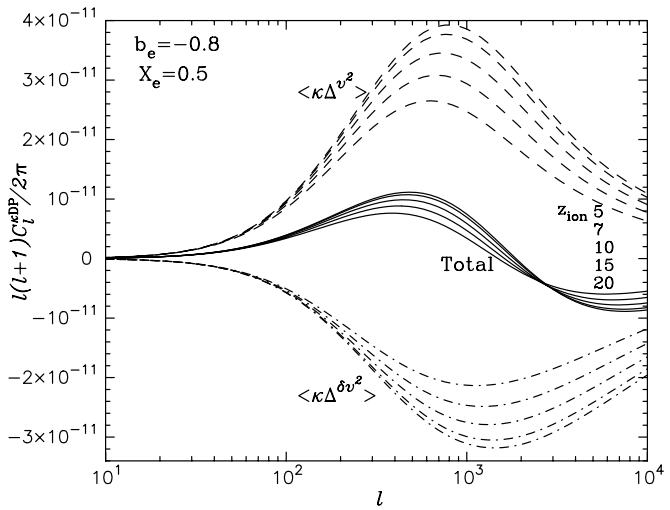

FIG. 2.—Toy model of cross-correlation angular power spectrum for the simplest model of the antibiasing relation between the dark matter and electron distributions given by $b = -0.8$. We here considered the reionization epochs of $z_{\rm ion} = 5$, 7, 10, 15, and 20 and assumed that $X_e = 0.5$ for the ionization fraction after the respective reionization epoch. The dashed and dot-dashed lines show the angular power spectra of cross-correlation between the weak lensing field and the temperature fluctuation field induced by the Doppler effects of $O(v^2)$ and $O(v^2\delta_e)$, respectively, as shown in Fig. 1. The solid lines show the total angular power spectra that we would actually observe, and the spectra become negative on $l \gtrsim 2000$ for this model.

Figure 3 shows the maximal estimate of the angular power spectrum of the cross-correlation using the nonlinear dark matter power spectrum under the simplest assumption that the gas traces dark matter distribution in the large-scale structure. We here again considered the simplest model of $X_e = 1$ and $b = 1$ as in Figure 1. The nonlinear enhancement could be important on small angular scales of $l \gtrsim 10^3$. However, it is still not clear if the electron distribution traces dark matter distribution in the nonlinear regime because physical processes of the gas pressure and radiative dynamics should also play an important role in such nonlinear regions (see, e.g., Gnedin 2000). For this reason, the curves in Figure 3 provide us with upper limits on a realistic angular power spectrum of the cross-correlation.

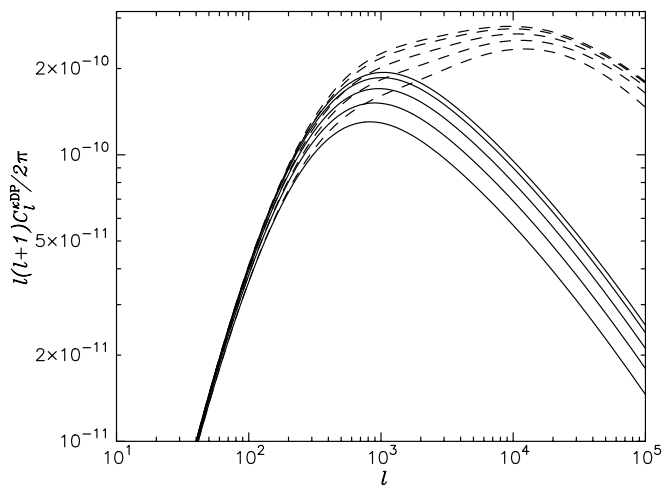

FIG. 3.—Maximal nonlinear enhancement of the angular power spectra of the cross-correlations for the same models ($X_e = b = 1$) as shown in Fig. 1. The solid and dashed lines demonstrate the total angular power spectra taking into account the linear and nonlinear dark matter power spectrum, respectively. These power spectra include both contributions from the cross-correlations of the weak lensing field with the pure quadratic Doppler effect of $O(v^2)$ and the density-modulated effect of $O(v^2\delta_e)$ as shown in Fig. 1. We used eq. (36) for the calculation of the nonlinear density-modulated spectrum. The nonlinear effect could be important at $l \gtrsim 1000$.

6. DISCUSSION AND CONCLUSIONS

In this paper we have investigated the cross-correlation between the weak lensing field and the secondary CMB temperature fluctuations induced by the Doppler effect of the ionized medium in the reionized epoch of the universe. The main result of this paper is to develop a formalism for calculating the cross-correlation function and its angular power spectrum based on the small angle approximation. The leading order to the cross-correlation comes from the secondary CMB anisotropies due to the quadratic Doppler effects of the bulk motions since the cross-correlation with the $O(v)$ Doppler effect such as the OV effect depends linearly on the bulk velocity field and thus suffers from cancellations between positive and negative contributions from the bulk velocity field for the ensemble average. There are two main contributions to the cross-correlation function: one is contribution from the pure quadratic Doppler effect of $O(v^2)$, and the other is that from the density-modulated effect of $O(v^2\delta_e)$, where δ_e is the electron density fluctuation field. For the CDM models, the latter contribution dominates the former on angular scales of $l \gtrsim 100$ if the ionized medium traces the dark matter distribution. We found that the cross-correlation of $O(X_e v^2 \delta)$ from the Doppler effect of $O(v^2)$ is sensitive to the average ionization fraction $\langle X_e \rangle$ between the reionization epoch and the present in the sense that the bulk velocity and the weak lensing field depend only on the dark matter fluctuations, through the gravitational instability scenario, and on the cosmological parameters, which will be well constrained by the future precise data such as the primary CMB anisotropies and the weak lensing survey. On the other hand, we showed that the cross-correlation of $O(X_e v^2 \delta_e \delta)$ between the weak lensing field and the density-modulated quadratic Doppler effect of $O(v^2\delta_e)$ depends linearly on the biasing relation between the dark matter and free electron distributions, and therefore its angular power spectrum could be both positive and negative, depending on the biasing and antibiasing, respectively. If the low-density region of the IGM is ionized prior to the high-density region in the reionization history, as suggested by MHR (see also Gnedin 2000), we would measure the negative angular spectrum of the cross-correlation. Our results thus offer a new opportunity to understand the unknown problem of how reionization proceeds in the inhomogeneous intergalactic medium connected to the dark matter clustering in principle, although the magnitude of the signal is very small.

Let us briefly comment on other sources of cross-correlation between the weak lensing field and the observed CMB map. One significant source is the thermal SZ effect, and it has indeed been shown that the cross-correlation could have the magnitude of $\langle \Delta^{\rm SZ} \kappa \rangle \sim 10^{-8}$ for the similar CDM model (Cooray 2000), as investigated in this paper. The SZ effect thus produces a much larger contribution than the cross-correlation with the Doppler effect estimated as $\langle \Delta^{\rm DP} \kappa \rangle \sim 10^{-10}$. However, since the thermal SZ signal arises mainly from hot plasma in clusters of galaxies at low redshift ($z \lesssim 0.5$), it is expected that we can remove this sig-

nal from the cross-correlation maps combined with the X-ray data or the SZ maps. The other possible sources are the CMB foregrounds such as synchrotron radiation from extragalactic dust that may correlate with the dark matter distribution in the large-scale structure. Isolating the signal shown in this paper observationally from the foregrounds will be a great challenge using the different frequency dependence (Tegmark et al. 2000), although this problem is beyond the scope of this paper.

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REFERENCES

Aghanim, N., Désert, F. X., Puget, J. L., & Gispert, R. 1996, A&A, 311, 1
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Barkana, R., & Loeb, A. 2001, Phys. Rep., 349, 125
Bartelmann, M., & Schneider, P. 2001, Phys. Rep., 340, 291
Becker, R. H., et al. 2001, AJ, 122, 2850
Benson, A. J., Nusser, A., Sugiyama, N., & Lacey, C. G. 2001, MNRAS, 320, 153
Bond, J. R., & Efstathiou, G. P. 1987, MNRAS, 226, 655
Bond, J. R., Efstathiou, G. P., Lubin, P. M., & Meinhold, P. R. 1991, Phys. Rev. Lett., 66, 2179
Cooray, A. 2000, Phys. Rev. D, 62, 103506
de Bernardis, P., et al. 2000, Nature, 404, 955
Dodelson, S., & Jubas, J. M. 1995, ApJ, 439, 503
Efstathiou, G. P. 1988, in Large-Scale Motions in the Universe, ed. V. Rubin & S. J. Coyne (Princeton: Princeton Univ. Press), 299
Eke, V., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263
Fan, X., et al. 2000, AJ, 120, 1167
Gnedin, N. Y. 2000, ApJ, 535, 530
Gnedin, N. Y., & Jaffe, A. H. 2001, ApJ, 551, 3
Gruzinov, A., & Hu, W. 1998, ApJ, 508, 435
Gunn, J. E., & Peterson, B. A. 1965, ApJ, 142, 1633
Hanany, S., et al. 2000, ApJ, 545, L5
Hu, W. 2000a, ApJ, 529, 12
———. 2000b, Phys. Rev. D, 62, 043007
———. 2001, ApJ, 557, L79
Hu, W., & Okamoto, T. 2001, ApJ, submitted (astro-ph/0111606)
Hu, W., Scott, D., & Silk, J. 1994, Phys. Rev. D, 49, 648 (HSS)
Hu, W., & White, M. 1996, A&A, 315, 33
———. 1997, Phys. Rev. D, 56, 596
Jaffe, A., & Kamionkowski, M. 1998, Phys. Rev. D, 58, 043001
Kaiser, N. 1984, ApJ, 282, 374
Kaiser, N. 1992, ApJ, 388, 272
Kitayama, T., & Suto, Y. 1997, ApJ, 490, 557
Knox, L., Scoccimarro, R., & Dodelson, S. 1998, Phys. Rev. Lett., 81, 2004
Komatsu, E., & Kitayama, T. 1999, ApJ, 526, L1
Lanzetta, K. M., Wolfe, A. M., & Turnshek, D. A. 1995, ApJ, 440, 435
Miralde-Escude, J., Haehnelt, M., & Rees, M. 2000, ApJ, 530, 1 (MHR)
Netterfield, C. B., et al. 2001, ApJ, submitted (astro-ph/0104460)
Ostriker, J. P., & Vishniac, E. T. 1986, ApJ, 306, L51
Peacock, J. A., & Dodds, S. J. 1996, MNRAS, 280, L19
Peebles, P. J. E. 1980, The Large Scale Structure of the Universe (Princeton: Princeton Univ. Press)
Peiris, H. V., & Spergel, D. N. 2000, ApJ, 540, 605
Schmalzing, J., Takada, M., & Futamase, T. 2000, ApJ, 544, L83
Seljak, U., & Zaldarriaga, M. 1999, Phys. Rev. Lett., 82, 2636
———. 2000, ApJ, 538, 57
Sugiyama, N. 1995, ApJS, 100, 281
Sugiyama, N., Silk, J., & Vittorio, N. 1993, ApJ, 419, L1
Takada, M. 2001, ApJ, 558, 29
Takada, M., & Futamase, T. 2001, ApJ, 546, 620
Takada, M., Komatsu, E., & Futamase, T. 2000, ApJ, 533, L83
Tegmark, M., Eisenstein, D. J., Hu, W., & de Oliveira-Costa, A. O. 2000, ApJ, 530, 133
Tegmark, M., & Zaldarriaga, M. 2000, Phys. Rev. Lett., 85, 2240
Vishniac, T. 1987, ApJ, 322, 597
Van Waerbeke, L., Bernardeau, F., & Benabed, K. 2000a, ApJ, 540, 14
Van Waerbeke, L., et al. 2000b, A&A, 358, 30
Zaldarriaga, M. 2000, Phys. Rev. D, 62, 063510
Zaldarriaga, M., & Seljak, U. 1998, Phys. Rev. D, 58, 023003
———. 1999, Phys. Rev. D, 59, 123507
Zel'dovich, Ya. B., & Sunyaev, R. A. 1969, Ap&SS, 4, 301