The M-σ Relation in Galactic Bulges,
and
Determinations of Their Intrinsic Scatter

Gultekin et al. 2009, ApJ, 698, 198-221.

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0. Main results in this paper.

- Updated version of the M_{BH}-σ_* relation: $\log_{10}(M_{BH}/M_\odot) = \alpha + \beta \log_{10}(\sigma_*/200 \text{ km/s})$.
- The M_{BH}-$L_{bulge,V}$ relation for elliptical galaxies: $\log_{10}(M_{BH}/M_\odot) = \alpha + \beta \log_{10}(L_{bulge,V}/10^{11}L_{\odot,V})$.
- $(\alpha, \beta, \varepsilon_0)_{M\text{-}\sigma}=(8.12, 4.24, 0.44)$, $(\alpha, \beta, \varepsilon_0)_{M\text{-}L}=(8.23, 3.96, 0.31)$, where ε_0 is magnitude of the intrinsic scatter in this relation.
- The lognormal distribution(Gaussian in $\log_{10}(M_{BH})$) is favored in the distribution of this scatter(intrinsic + observational).
- The prevailing criterion($R_{infl}/d_{res}>1$) causes the systematic bias in the sense that larger α, shallower β, smaller ε_0.
- BH mass function are estimated from these relations.

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1. Introduction –'Co-evolution'–

- Many observation have shown that BH mass correlate well with the host stellar velocity dispersion or bulge luminosity.

  This correlation is called 'Magorrian relation' or 'M-σ relation'.
- This relation strongly suggests that a close link between SMBH formation, galaxy formation and AGN activity.

  This mutual evolutional paradigm is often termed 'co-evolution'.
- The related physical processes are not understood enough. Then, we expect that (i)its information must be printed in the M-σ relation and the intrinsic scatter in this relation, and (ii)we can get these information by comparing these observational results to many predictions by theories(semi-analytic models).

1. Introduction –'BH demographics'–

- Direct M_{BH} measurements require detailed observations for us. If we use this relation, however, we can roughly estimate M_{BH} easily from host σ_*.
- Then, we can construct the BH mass functions(BHMFs), which is one of the most fundamental observational quantity and contains the information. (BHMF: the number density of BHs in the co-moving volume per unit BH mass interval)
- BHMFs also constrain the theories of the co-evolutional history.

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2. Sample

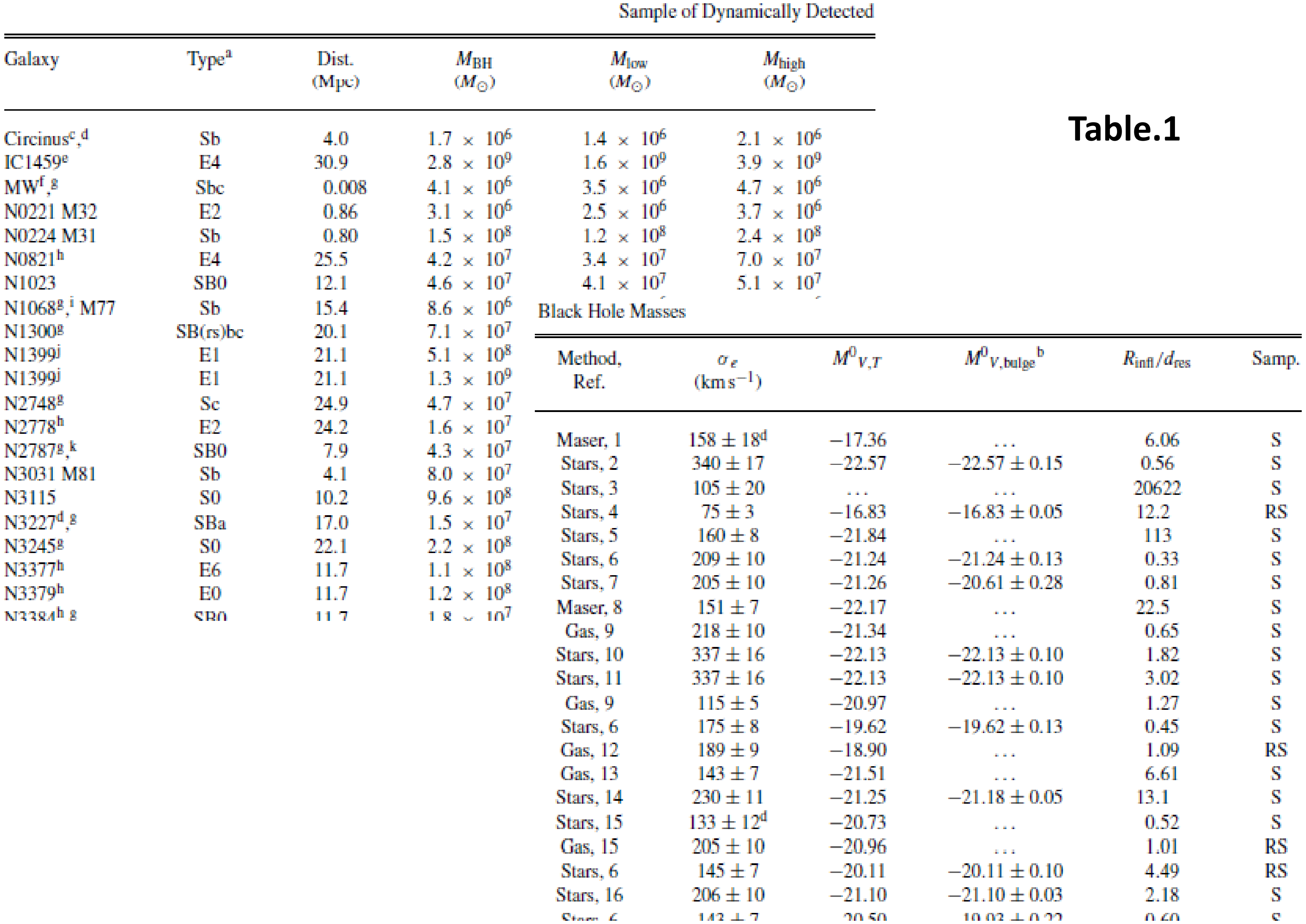
- Table.1(w/ M_{BH} measurements), Table.2(w/ upper limits only), Table.3(rejected objects).
- It was assumed that disk component less contributes to the σ_*.
- The original M_{BH} values are scaled to the preferred distances assuming $M_{BH} \propto D$ and H_0=70 km/s/Mpc.
- They did not adopt the criterion $R_{infl}/d_{res}>1$,

  where $R_{infl}=GM_{BH}/\sigma_*^2$ (R_{infl}: Sphere-of –influence of BH),

  d_{res} is spatial resolution of the kinematic observations.
  - Small R_{infl}/d_{res} lead to large M_{BH} error but not systematic error.
  - This criterion causes systematic bias to the (α, β, ε_0), not to the M_{BH}.

$R_{infl}=GM_{BH}/\sigma_*^2 \sim 11\times(M_{BH}/10^8\ M_\odot)(\sigma_*/200\ \mathrm{km/s})^{-2}$ [pc].
$\theta_{infl}=R_{infl}/D \sim 2.3\times(M_{BH}/10^8\ M_\odot)(\sigma_*/200\ \mathrm{km/s})^{-2}(D/10\ \mathrm{Mpc})^{-1}$ [’’].

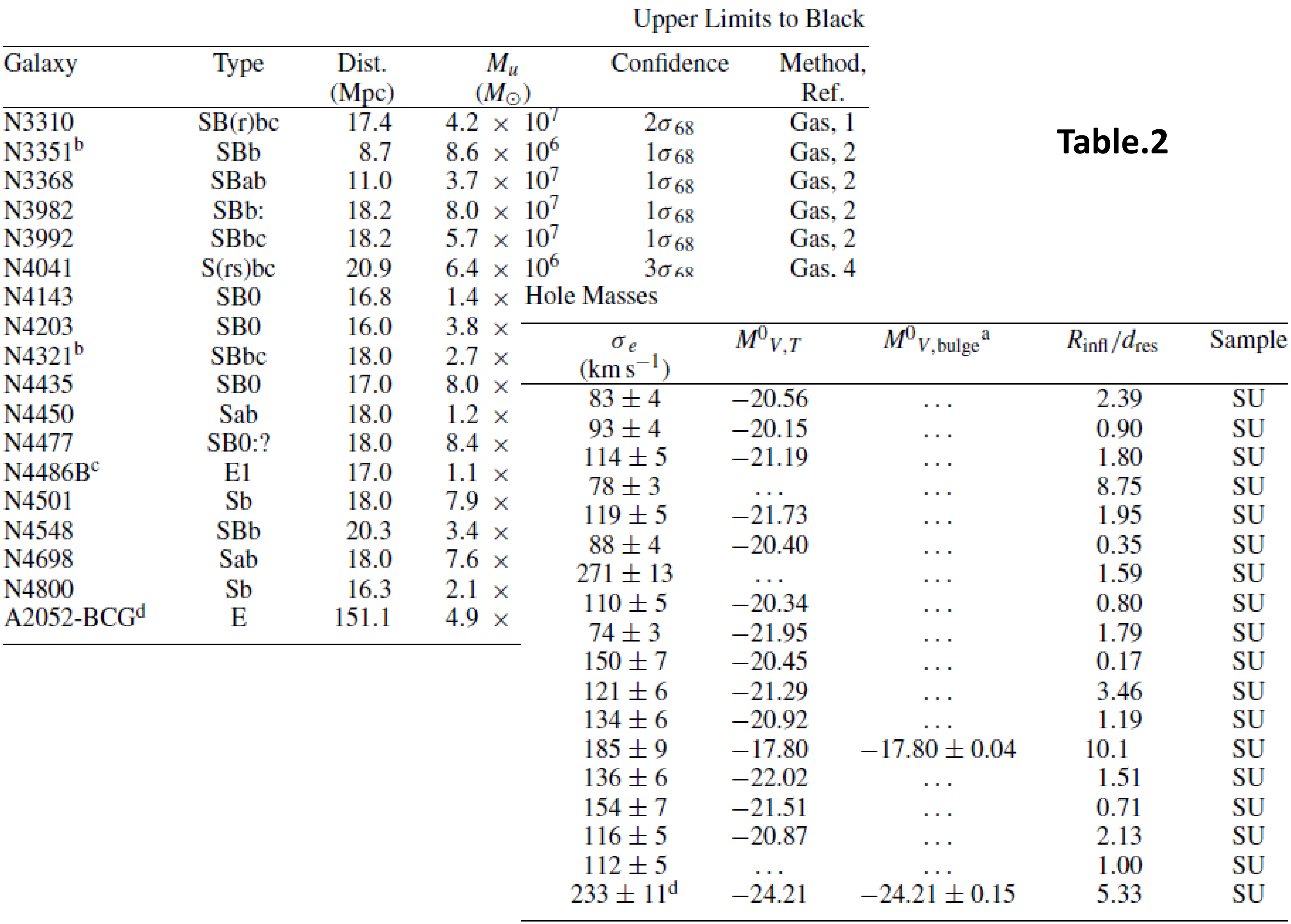
Table.1



Sample of Dynamically Detected Black Hole Masses

Galaxy	Type[a]	Dist. (Mpc)	$M_{\rm BH}$ ($M_\odot$)	$M_{\rm low}$ ($M_\odot$)	$M_{\rm high}$ ($M_\odot$)	Method, Ref.	σ_e (km s^{-1})	$M^0{}_{V,T}$	$M^0{}_{V,\rm bulge}$[b]	$R_{\rm infl}/d_{\rm res}$	Samp.
Circinus[c,d]	Sb	4.0	1.7×10^6	1.4×10^6	2.1×10^6	Maser, 1	158 ± 18[d]	−17.36	…	6.06	S
IC1459[e]	E4	30.9	2.8×10^9	1.6×10^9	3.9×10^9	Stars, 2	340 ± 17	−22.57	−22.57 ± 0.15	0.56	S
MW[f,g]	Sbc	0.008	4.1×10^6	3.5×10^6	4.7×10^6	Stars, 3	105 ± 20	…	…	20622	S
N0221 M32	E2	0.86	3.1×10^6	2.5×10^6	3.7×10^6	Stars, 4	75 ± 3	−16.83	−16.83 ± 0.05	12.2	RS
N0224 M31	Sb	0.80	1.5×10^8	1.2×10^8	2.4×10^8	Stars, 5	160 ± 8	−21.84	…	113	S
N0821[h]	E4	25.5	4.2×10^7	3.4×10^7	7.0×10^7	Stars, 6	209 ± 10	−21.24	−21.24 ± 0.13	0.33	S
N1023	SB0	12.1	4.6×10^7	4.1×10^7	5.1×10^7	Stars, 7	205 ± 10	−21.26	−20.61 ± 0.28	0.81	S
N1068[g,i] M77	Sb	15.4	8.6×10^6			Maser, 8	151 ± 7	−22.17	…	22.5	S
N1300[g]	SB(rs)bc	20.1	7.1×10^7			Gas, 9	218 ± 10	−21.34	…	0.65	S
N1399[j]	E1	21.1	5.1×10^8			Stars, 10	337 ± 16	−22.13	−22.13 ± 0.10	1.82	S
N1399[j]	E1	21.1	1.3×10^9			Stars, 11	337 ± 16	−22.13	−22.13 ± 0.10	3.02	S
N2748[g]	Sc	24.9	4.7×10^7			Gas, 9	115 ± 5	−20.97	…	1.27	S
N2778[h]	E2	24.2	1.6×10^7			Stars, 6	175 ± 8	−19.62	−19.62 ± 0.13	0.45	S
N2787[g,k]	SB0	7.9	4.3×10^7			Gas, 12	189 ± 9	−18.90	…	1.09	RS
N3031 M81	Sb	4.1	8.0×10^7			Gas, 13	143 ± 7	−21.51	…	6.61	S
N3115	S0	10.2	9.6×10^8			Stars, 14	230 ± 11	−21.25	−21.18 ± 0.05	13.1	S
N3227[d,g]	SBa	17.0	1.5×10^7			Stars, 15	133 ± 12[d]	−20.73	…	0.52	S
N3245[g]	S0	22.1	2.2×10^8			Gas, 15	205 ± 10	−20.96	…	1.01	RS
N3377[h]	E6	11.7	1.1×10^8			Stars, 6	145 ± 7	−20.11	−20.11 ± 0.10	4.49	RS
N3379[h]	E0	11.7	1.2×10^8			Stars, 16	206 ± 10	−21.10	−21.10 ± 0.03	2.18	S
N3384[h,g]	SB0	11.7	1.8×10^7			Stars, 6	143 ± 7	20.50	19.93 ± 0.22	0.60	S

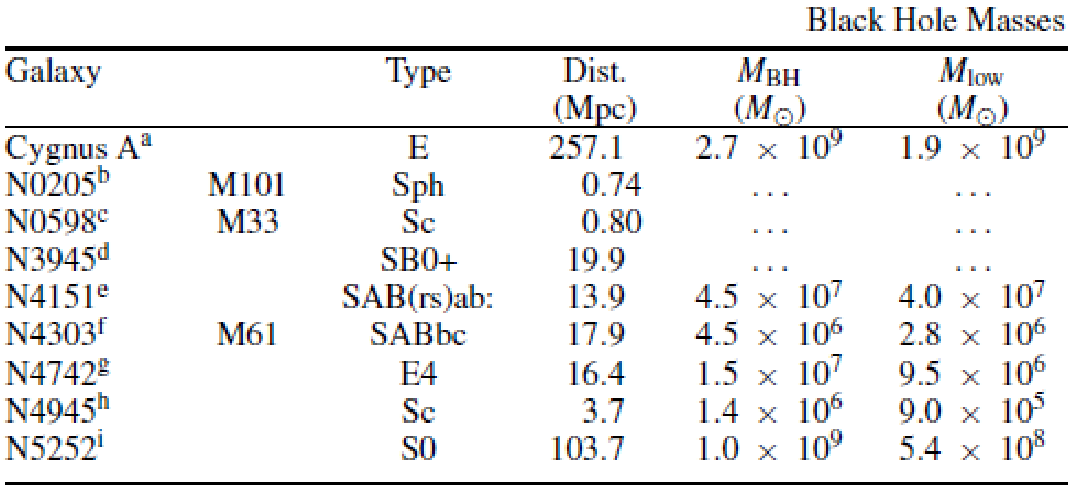
Table.2

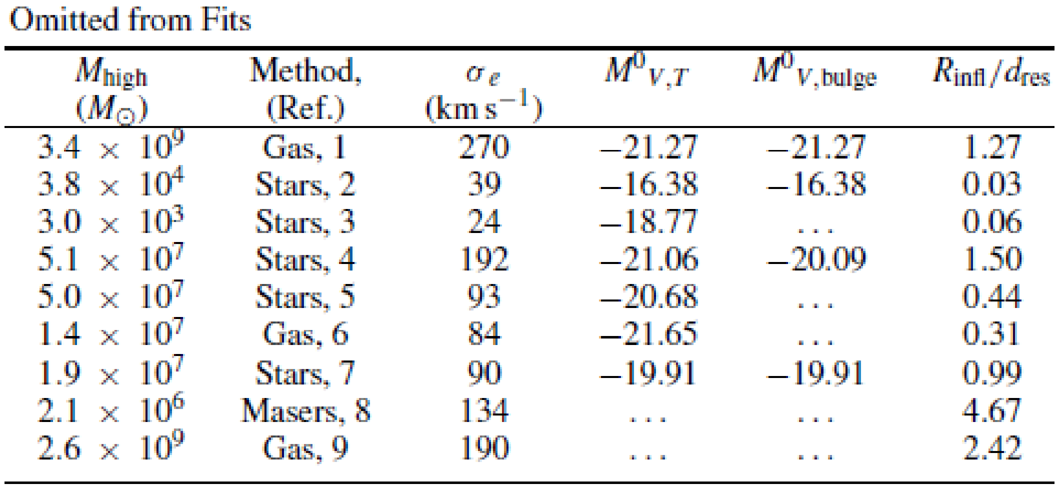


Upper Limits to Black Hole Masses

Galaxy	Type	Dist. (Mpc)	M_u $(M_\odot)$	Confidence	Method, Ref.	σ_e (km s^{-1})	$M^0{}_{V,T}$	$M^0{}_{V,\mathrm{bulge}}$[a]	$R_{\mathrm{infl}}/d_{\mathrm{res}}$	Sample
N3310	SB(r)bc	17.4	4.2×10^7	$2\sigma_{68}$	Gas, 1	83 ± 4	−20.56	...	2.39	SU
N3351[b]	SBb	8.7	8.6×10^6	$1\sigma_{68}$	Gas, 2	93 ± 4	−20.15	...	0.90	SU
N3368	SBab	11.0	3.7×10^7	$1\sigma_{68}$	Gas, 2	114 ± 5	−21.19	...	1.80	SU
N3982	SBb:	18.2	8.0×10^7	$1\sigma_{68}$	Gas, 2	78 ± 3	...	...	8.75	SU
N3992	SBbc	18.2	5.7×10^7	$1\sigma_{68}$	Gas, 2	119 ± 5	−21.73	...	1.95	SU
N4041	S(rs)bc	20.9	6.4×10^6	$3\sigma_{68}$	Gas, 4	88 ± 4	−20.40	...	0.35	SU
N4143	SB0	16.8	$1.4 \times$			271 ± 13	...	...	1.59	SU
N4203	SB0	16.0	$3.8 \times$			110 ± 5	−20.34	...	0.80	SU
N4321[b]	SBbc	18.0	$2.7 \times$			74 ± 3	−21.95	...	1.79	SU
N4435	SB0	17.0	$8.0 \times$			150 ± 7	−20.45	...	0.17	SU
N4450	Sab	18.0	$1.2 \times$			121 ± 6	−21.29	...	3.46	SU
N4477	SB0:?	18.0	$8.4 \times$			134 ± 6	−20.92	...	1.19	SU
N4486B[c]	E1	17.0	$1.1 \times$			185 ± 9	−17.80	-17.80 ± 0.04	10.1	SU
N4501	Sb	18.0	$7.9 \times$			136 ± 6	−22.02	...	1.51	SU
N4548	SBb	20.3	$3.4 \times$			154 ± 7	−21.51	...	0.71	SU
N4698	Sab	18.0	$7.6 \times$			116 ± 5	−20.87	...	2.13	SU
N4800	Sb	16.3	$2.1 \times$			112 ± 5	...	...	1.00	SU
A2052-BCG[d]	E	151.1	$4.9 \times$			233 ± 11[d]	−24.21	-24.21 ± 0.15	5.33	SU

Table.3





Black Hole Masses Omitted from Fits

Galaxy		Type	Dist. (Mpc)	$M_{\rm BH}$ ($M_\odot$)	$M_{\rm low}$ ($M_\odot$)	$M_{\rm high}$ ($M_\odot$)	Method, (Ref.)	σ_e (km s^{-1})	$M^0{}_{V,T}$	$M^0{}_{V,\rm bulge}$	$R_{\rm infl}/d_{\rm res}$
Cygnus A[a]		E	257.1	2.7×10^9	1.9×10^9	3.4×10^9	Gas, 1	270	−21.27	−21.27	1.27
N0205[b]	M101	Sph	0.74	…	…	3.8×10^4	Stars, 2	39	−16.38	−16.38	0.03
N0598[c]	M33	Sc	0.80	…	…	3.0×10^3	Stars, 3	24	−18.77	…	0.06
N3945[d]		SB0+	19.9	…	…	5.1×10^7	Stars, 4	192	−21.06	−20.09	1.50
N4151[e]		SAB(rs)ab:	13.9	4.5×10^7	4.0×10^7	5.0×10^7	Stars, 5	93	−20.68	…	0.44
N4303[f]	M61	SABbc	17.9	4.5×10^6	2.8×10^6	1.4×10^7	Gas, 6	84	−21.65	…	0.31
N4742[g]		E4	16.4	1.5×10^7	9.5×10^6	1.9×10^7	Stars, 7	90	−19.91	−19.91	0.99
N4945[h]		Sc	3.7	1.4×10^6	9.0×10^5	2.1×10^6	Masers, 8	134	…	…	4.67
N5252[i]		S0	103.7	1.0×10^9	5.4×10^8	2.6×10^9	Gas, 9	190	…	…	2.42

2. Sample

- SU: full sample including Table.1 + 2.
- S: sample including only Table.1.
- RS: ‘restricted sample’ which satisfy the following:
  - $R_{infl}/d_{res}>1$(R_{infl} is well resolved),
  - without the upper limits,
  - without the objects deemed suspicious by Ferrarese & Ford(2005), or Tremaine+(2002),
  - without the objects with multiple measurements that inconsistent with each other,
  - with subjective judgment that the quality of M_{BH} measurements is adequate.

2-1. Velocity Dispersion

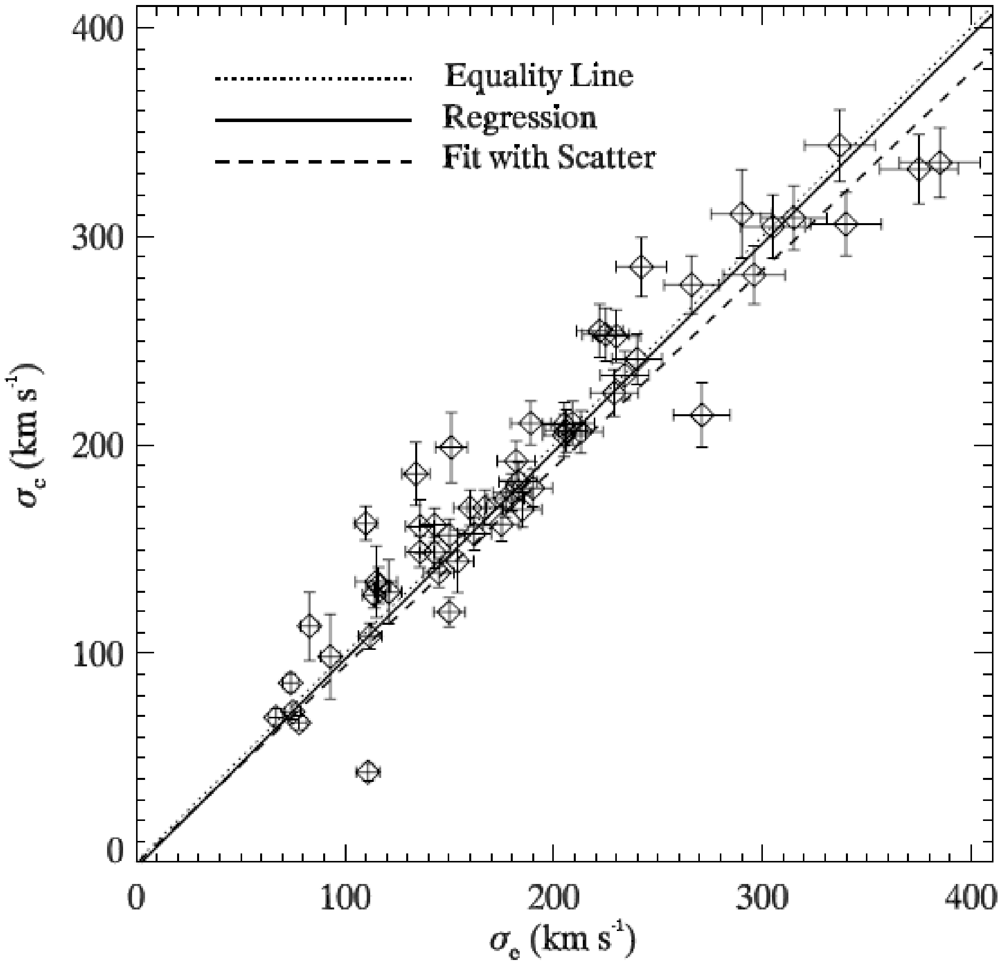
- Definition of the effective velocity dispersion:

$$\sigma_e^2 \equiv \frac{\int_0^{R_e} (\sigma^2 + V^2) I(r) dr}{\int_0^{R_e} I(r) dr},$$

  V: rotational component of the spheroid, I(r): intensity at projected radius r, Re: effective radius.

- If we could not calculate the σ_e, we alternatively used the central velocity dispersion σ_c(found in *HyperLEDA*).

2-1. Velocity Dispersion



They judged these is no systematic bias in σ_e and σ_c .

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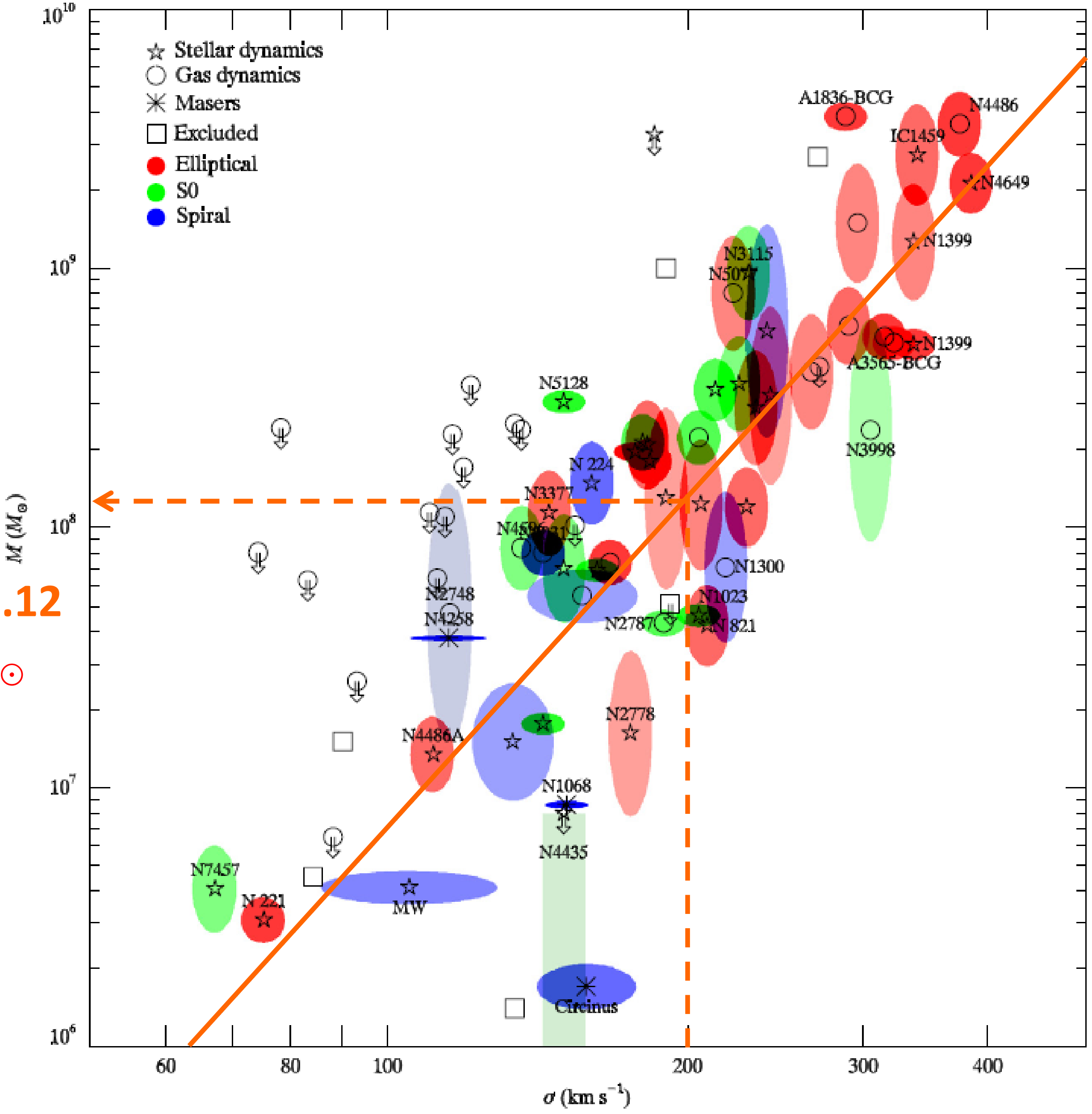
3-1. the M-σ relation.

- Best fit parameters of the form:

$$\log(M/M_\odot) = \alpha + \beta \log(\sigma_e / 200 \text{ km s}^{-1}),$$

  are $(\alpha, \beta, \varepsilon_0)$=(8.12, 4.24, 0.44).
- Applied to full sample(SU sample).
- Assumed that the lognormal intrinsic scatter and the lognormal observational error distribution(‘GG’).

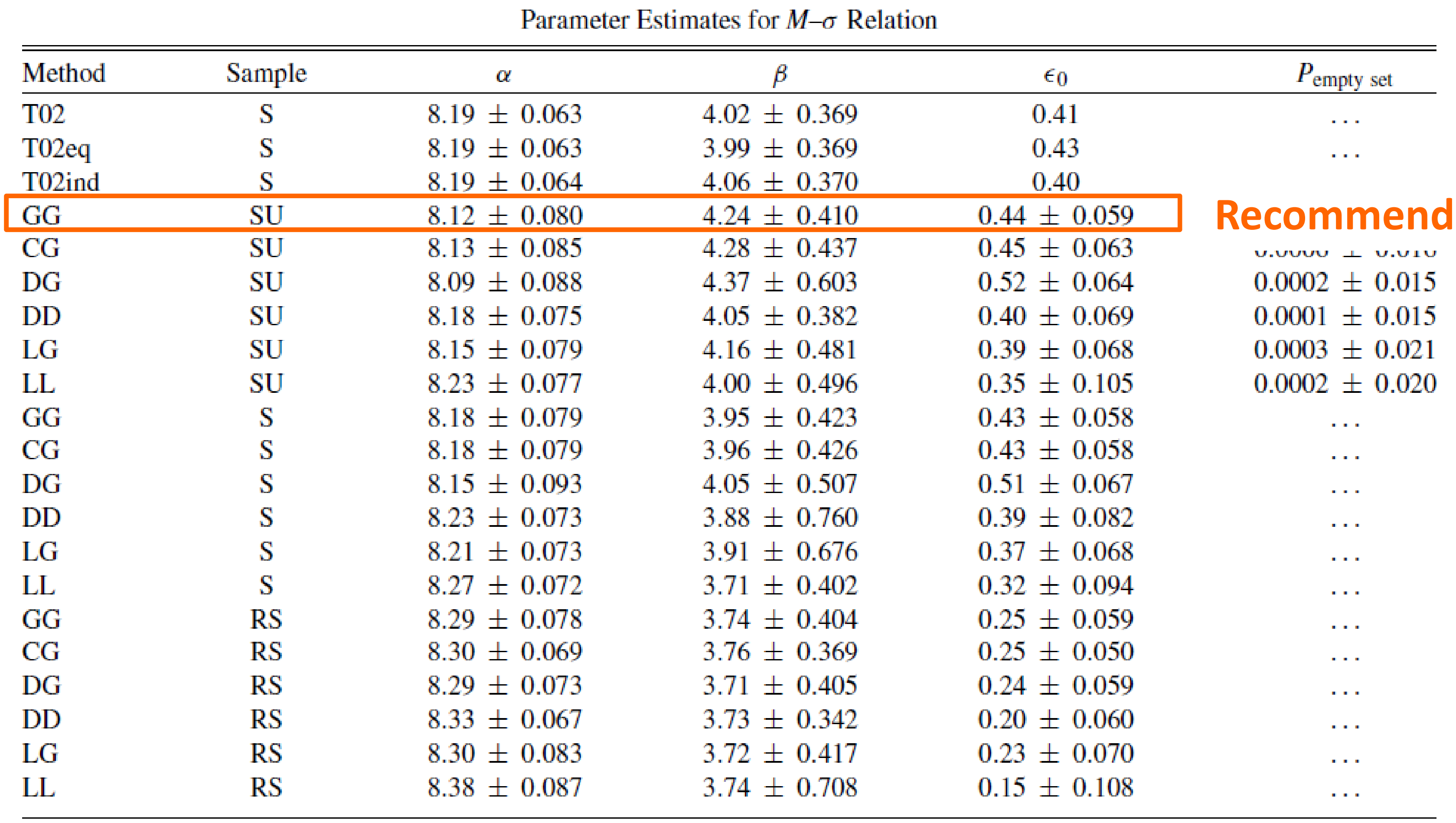


slope β=4.24

intercept α=8.12

$1.3 \times 10^8\ M_\odot$

Table.6



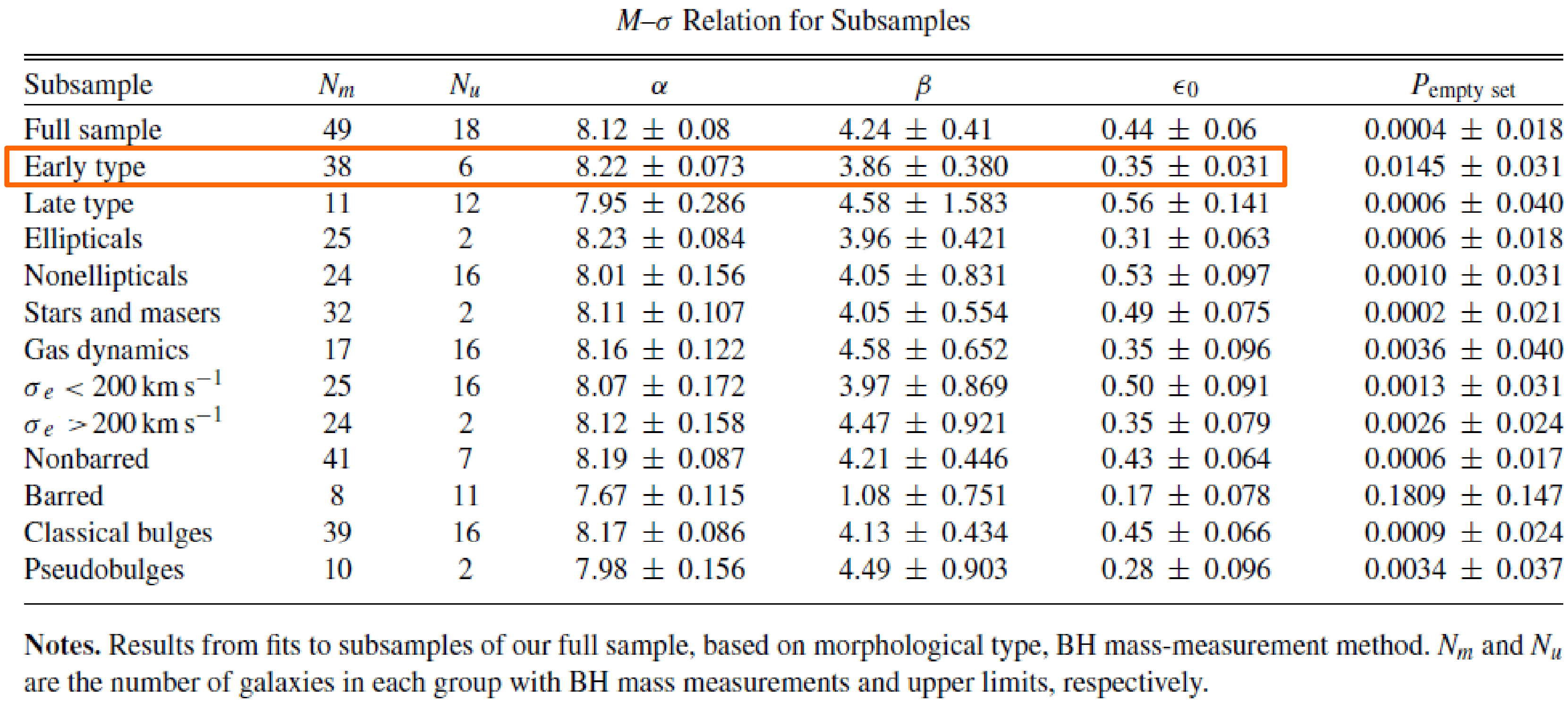
Parameter Estimates for M–σ Relation

Method	Sample	α	β	ϵ_0	$P_{\text{empty set}}$
T02	S	8.19 ± 0.063	4.02 ± 0.369	0.41	…
T02eq	S	8.19 ± 0.063	3.99 ± 0.369	0.43	…
T02ind	S	8.19 ± 0.064	4.06 ± 0.370	0.40	
GG	SU	8.12 ± 0.080	4.24 ± 0.410	0.44 ± 0.059	
CG	SU	8.13 ± 0.085	4.28 ± 0.437	0.45 ± 0.063	0.0000 ± 0.016
DG	SU	8.09 ± 0.088	4.37 ± 0.603	0.52 ± 0.064	0.0002 ± 0.015
DD	SU	8.18 ± 0.075	4.05 ± 0.382	0.40 ± 0.069	0.0001 ± 0.015
LG	SU	8.15 ± 0.079	4.16 ± 0.481	0.39 ± 0.068	0.0003 ± 0.021
LL	SU	8.23 ± 0.077	4.00 ± 0.496	0.35 ± 0.105	0.0002 ± 0.020
GG	S	8.18 ± 0.079	3.95 ± 0.423	0.43 ± 0.058	…
CG	S	8.18 ± 0.079	3.96 ± 0.426	0.43 ± 0.058	…
DG	S	8.15 ± 0.093	4.05 ± 0.507	0.51 ± 0.067	…
DD	S	8.23 ± 0.073	3.88 ± 0.760	0.39 ± 0.082	…
LG	S	8.21 ± 0.073	3.91 ± 0.676	0.37 ± 0.068	…
LL	S	8.27 ± 0.072	3.71 ± 0.402	0.32 ± 0.094	…
GG	RS	8.29 ± 0.078	3.74 ± 0.404	0.25 ± 0.059	…
CG	RS	8.30 ± 0.069	3.76 ± 0.369	0.25 ± 0.050	…
DG	RS	8.29 ± 0.073	3.71 ± 0.405	0.24 ± 0.059	…
DD	RS	8.33 ± 0.067	3.73 ± 0.342	0.20 ± 0.060	…
LG	RS	8.30 ± 0.083	3.72 ± 0.417	0.23 ± 0.070	…
LL	RS	8.38 ± 0.087	3.74 ± 0.708	0.15 ± 0.108	…

Recommended!

Intercept(α), slope(β) and intrinsic RMS scatter(ε_0) are not affected significantly by assumed distribution function.

Table.4



M–σ Relation for Subsamples

Subsample	N_m	N_u	α	β	ϵ_0	$P_{\text{empty set}}$
Full sample	49	18	8.12 ± 0.08	4.24 ± 0.41	0.44 ± 0.06	0.0004 ± 0.018
Early type	38	6	8.22 ± 0.073	3.86 ± 0.380	0.35 ± 0.031	0.0145 ± 0.031
Late type	11	12	7.95 ± 0.286	4.58 ± 1.583	0.56 ± 0.141	0.0006 ± 0.040
Ellipticals	25	2	8.23 ± 0.084	3.96 ± 0.421	0.31 ± 0.063	0.0006 ± 0.018
Nonellipticals	24	16	8.01 ± 0.156	4.05 ± 0.831	0.53 ± 0.097	0.0010 ± 0.031
Stars and masers	32	2	8.11 ± 0.107	4.05 ± 0.554	0.49 ± 0.075	0.0002 ± 0.021
Gas dynamics	17	16	8.16 ± 0.122	4.58 ± 0.652	0.35 ± 0.096	0.0036 ± 0.040
$\sigma_e < 200\,\mathrm{km\,s^{-1}}$	25	16	8.07 ± 0.172	3.97 ± 0.869	0.50 ± 0.091	0.0013 ± 0.031
$\sigma_e > 200\,\mathrm{km\,s^{-1}}$	24	2	8.12 ± 0.158	4.47 ± 0.921	0.35 ± 0.079	0.0026 ± 0.024
Nonbarred	41	7	8.19 ± 0.087	4.21 ± 0.446	0.43 ± 0.064	0.0006 ± 0.017
Barred	8	11	7.67 ± 0.115	1.08 ± 0.751	0.17 ± 0.078	0.1809 ± 0.147
Classical bulges	39	16	8.17 ± 0.086	4.13 ± 0.434	0.45 ± 0.066	0.0009 ± 0.024
Pseudobulges	10	2	7.98 ± 0.156	4.49 ± 0.903	0.28 ± 0.096	0.0034 ± 0.037

Notes. Results from fits to subsamples of our full sample, based on morphological type, BH mass-measurement method. N_m and N_u are the number of galaxies in each group with BH mass measurements and upper limits, respectively.

Intercept(α), slope(β) and intrinsic RMS scatter(ε_0)
are affected significantly by the choice of objects.
Particularly, late-type galaxies have a significant impact.
(Suggestions from this will be mentioned later)

3-2. The intrinsic scatter in the M-σ.

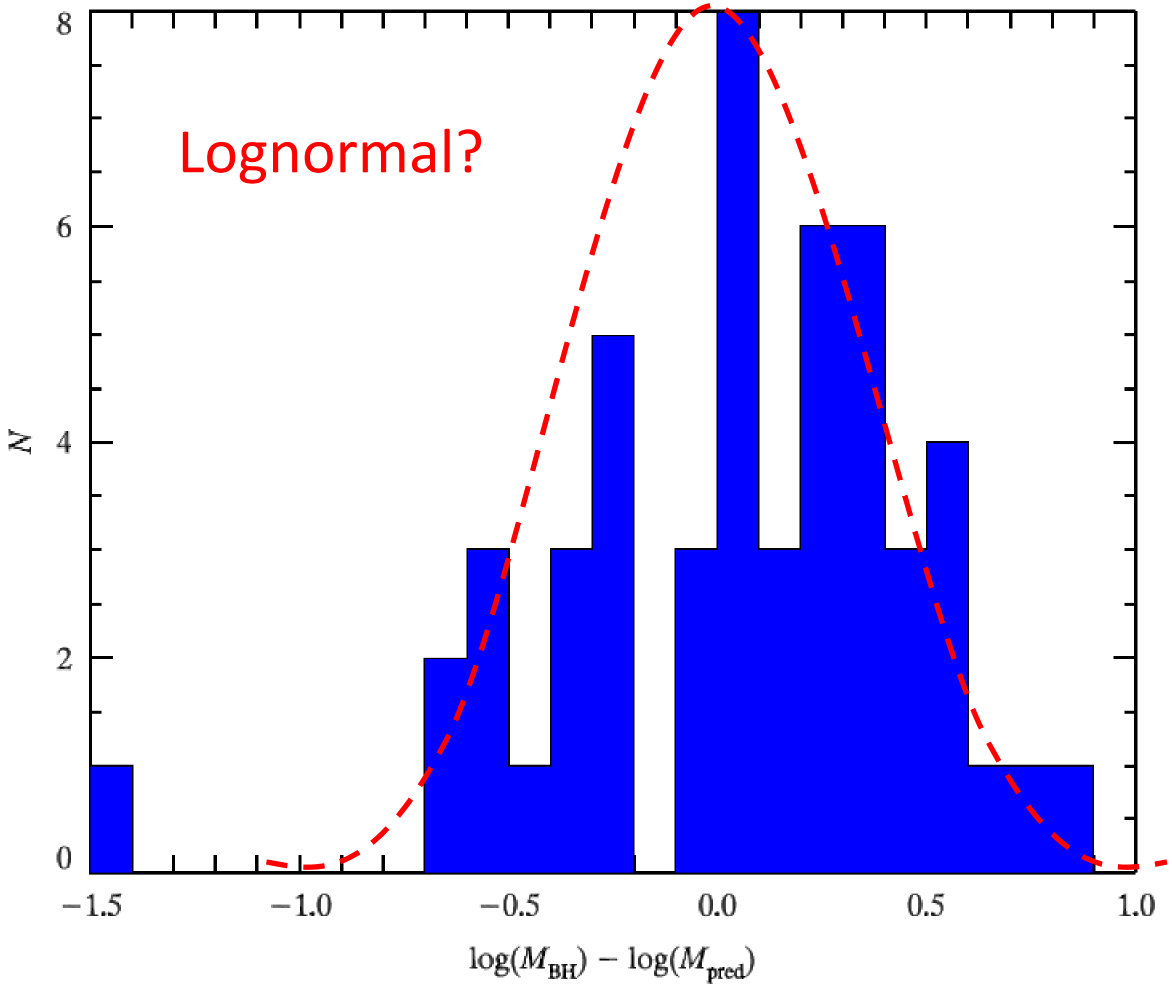


Figure 3. Histogram of residuals from the best-fit –σ relation in sample S.

intrinsic scatter

\+

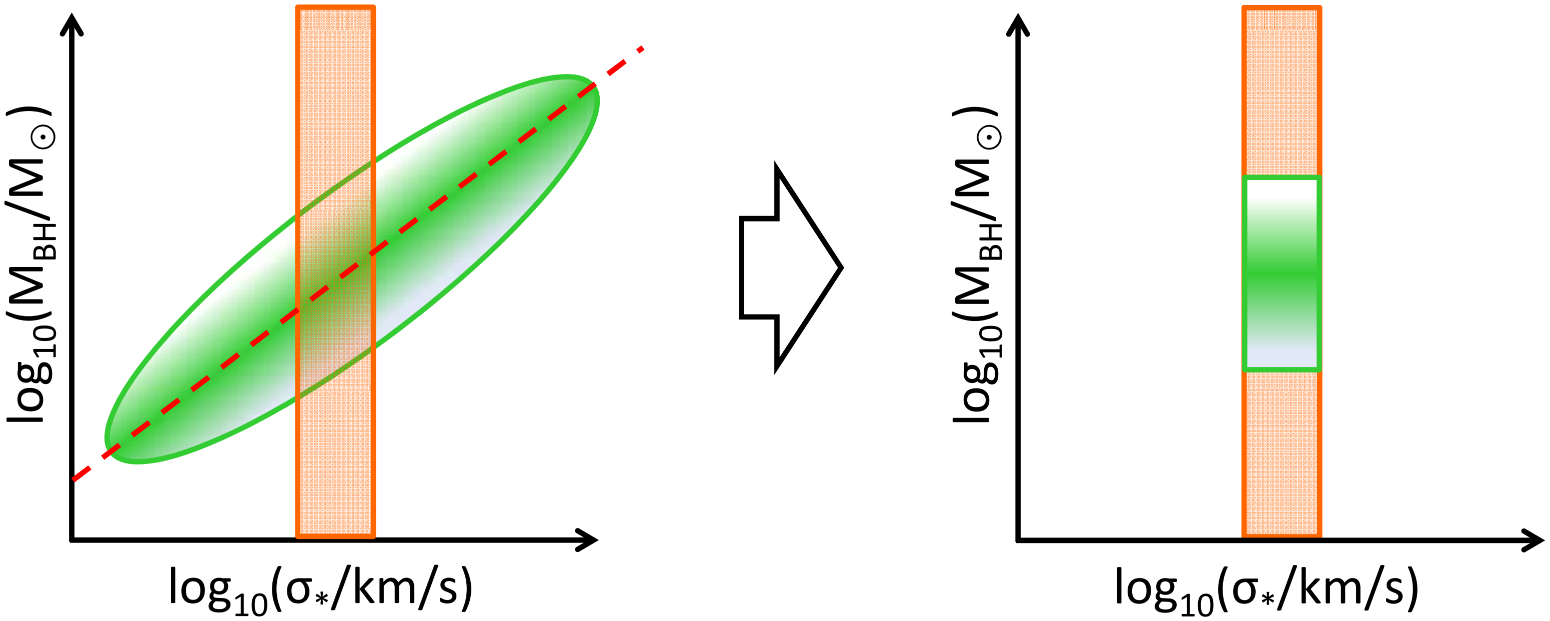
observational error

=

total distribution

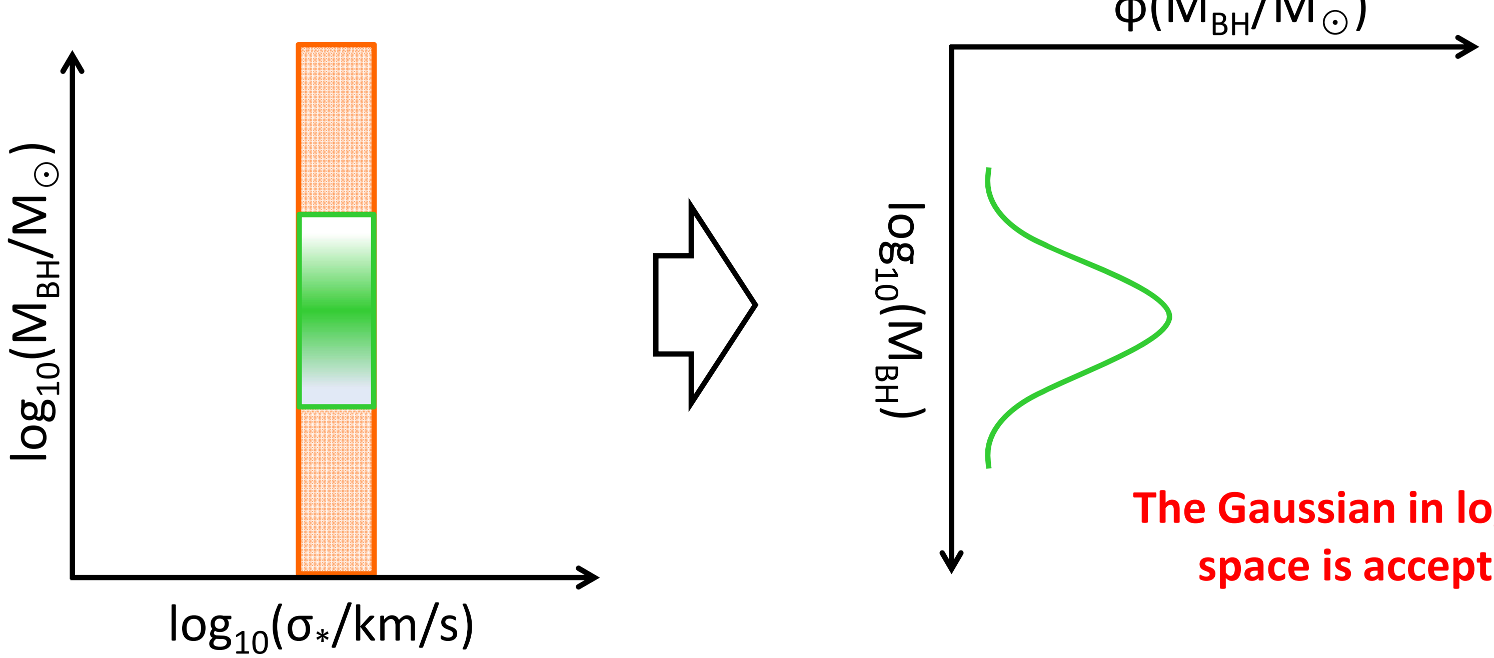
3-2. *The intrinsic scatter in the M-σ.*

They limited the σ_* range to $165 < (\sigma_*/\mathrm{km/s}) < 235$ in order to neglect the slope, and performed the Anderson-Darling test for normality with unknown center and intrinsic scatter.



19 galaxies are in this σ_* range.

- They found that lognormal(Gaussian in $\log_{10}(M_{BH})$) is acceptable, but normal(Gaussian in M_{BH}) is not acceptable.
- They compared various distribution functions(Gaussian, Lorentzian, Double-sided exponential, Double Gaussian, Gaussian with different standard deviations above and below the mean) in $\log_{10}(M_{BH})$ space and found that the Gaussian is favored.



The Gaussian in $\log_{10}(M_{BH})$ space is acceptable!

3-3. Log-quadratic fits to the M-σ.

- Best fit parameters of the form

$$\log\left(\frac{M_{\rm BH}}{M_\odot}\right) = \alpha + \beta \log\left(\frac{\sigma_e}{200\ {\rm km\ s^{-1}}}\right) + \gamma \left[\log\left(\frac{\sigma_e}{200\ {\rm km\ s^{-1}}}\right)\right]^2 .$$

  are (α, β, γ, ε_0)=(8.08, 4.47, 1.72, 0.44).
- Applied to full sample(SU sample).
- There is no description about the distribution function. Probably, the ‘GG’ is assumed.
- The odd ratio showed that the log-linear fit is favored.

$$\mathcal{R}_{ab} = \frac{\int \mathcal{L}_a(a_1, a_2, \ldots, a_m) P_a(a_1, a_2, \ldots, a_m) da_1 da_2 \ldots da_m}{\int \mathcal{L}_b(b_1, b_2, \ldots, b_n) P_b(b_1, b_2, \ldots, b_n) db_1 db_2 \ldots db_n},$$

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- To understand the intrinsic scatter correctly, we need to evaluate all systematic(observational) errors correctly.
- The prevailing idea that the poor resolution in determining the M_{BH} leads to systematic bias has not been justified.
- The poor resolution certainly increases the error but it does not cause systematic biases.
- So, they did not adopt the criterion: $R_{infl}/d_{res}>1$. Furthermore, they found the bias caused by this criterion.

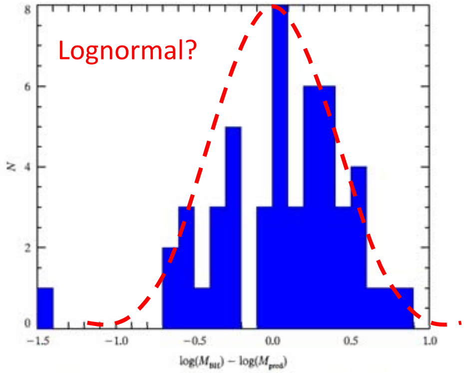


Figure 3. Histogram of residuals from the best-fit –σ relation in sample S.

intrinsic scatter
+
observational error
=
total distribution

4-1. Discussion on biases 1.

Details of their Monte Carlo simulation:

(1)They give the lognormal distribution with standard deviation (SD) 0.2 dex to the log(σ_*/200 km/s) .

(2)They convert σ_* to M_{BH} from the M-σ(α=4, β=8, ϵ_0=0.3), with the lognormal intrinsic scatter.

(3)They give the observational lognormal error with 0.2 dex SD to the converted M_{BH}.

(4)They also give the 5% minimum error to the σ_*.

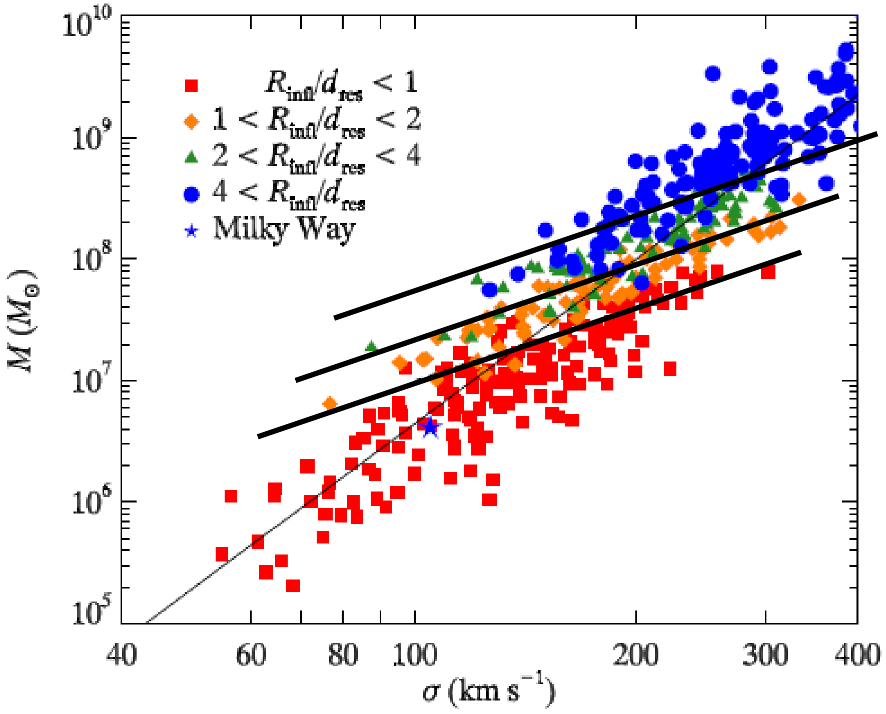
(5)They used the d_{res}=0''.1.

(6)They distributed the galaxies uniformly in volume out to 30 Mpc.

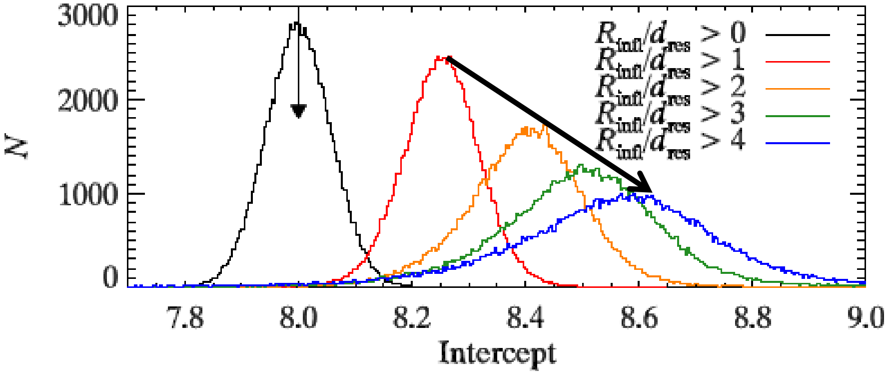
(7)They repeated for 10^5 realizations for each sample discussed below.

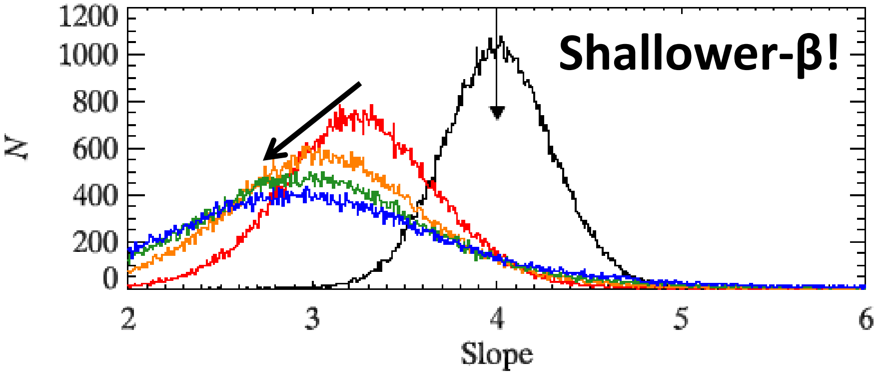
Their simulation showed that this criterion cause systematic bias:

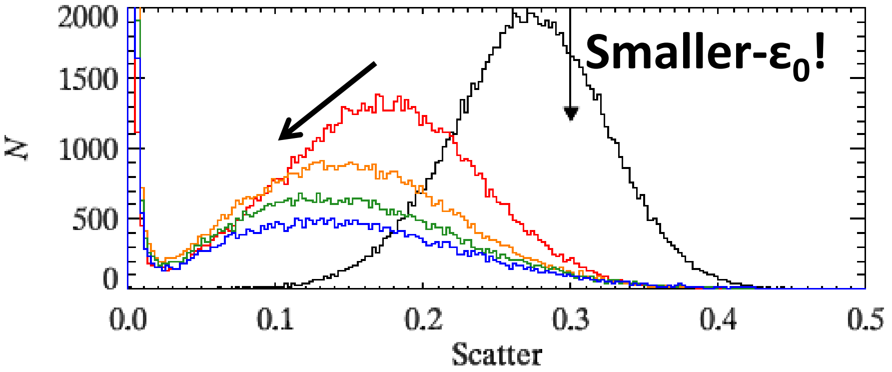
Bias!



Higher-α!

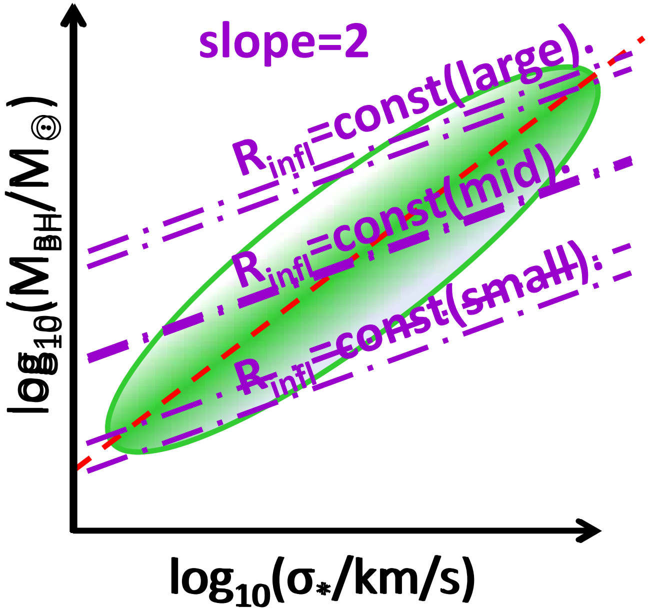


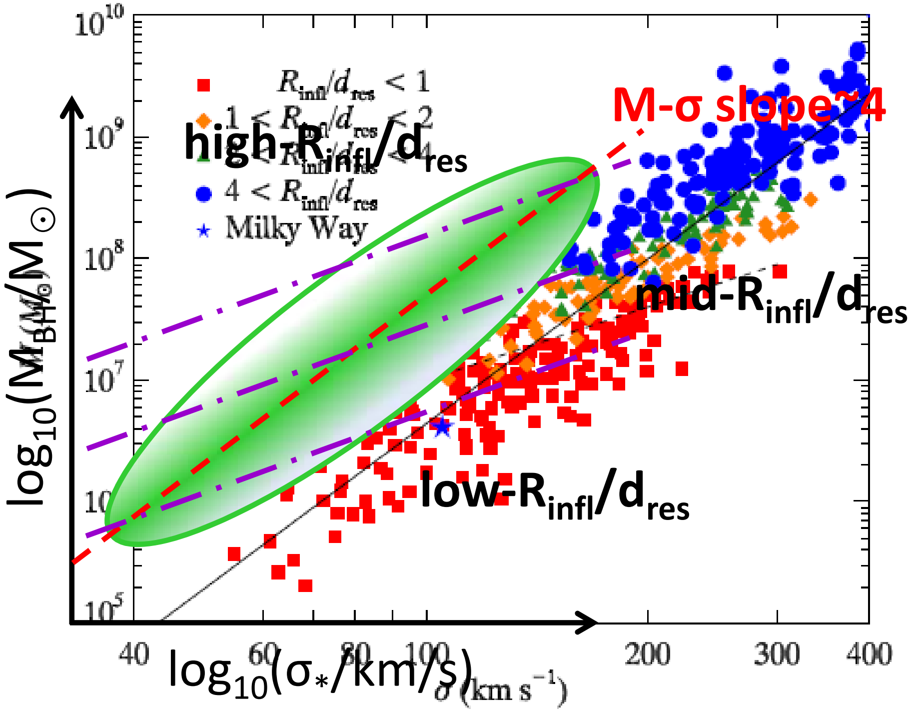

Shallower-β!


Smaller-ε_0!

They found that this bias originates in the definition of R_{infl} and the scatter in the M-σ:

$R_{infl}=GM_{BH}/\sigma_*^2$, → R_{infl} is constant along the lines that have slope 2 in the log(σ*)-log(M_{BH}) plane.





The criterion $R_{infl}/d_{res}>1$ leads to higher intercept(α), shallower slope(β), smaller intrinsic scatter(ε_0)!

4-2. Discussion on biases 2.

They tried alternative criterion:

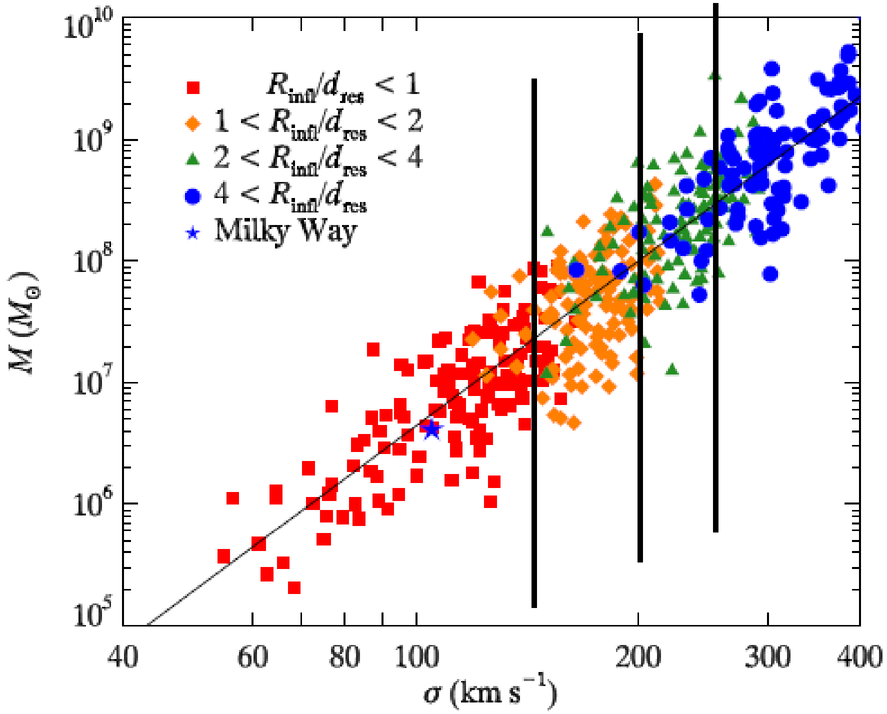
$R_{infl}=GM_{BH}/\sigma_*^2$, where M_{BH} and σ_* are observed value.

$R_{exp}=GM_{BH}/\sigma_*^2$, where M_{BH} is expected value from the M-σ, and σ_* is observed value.

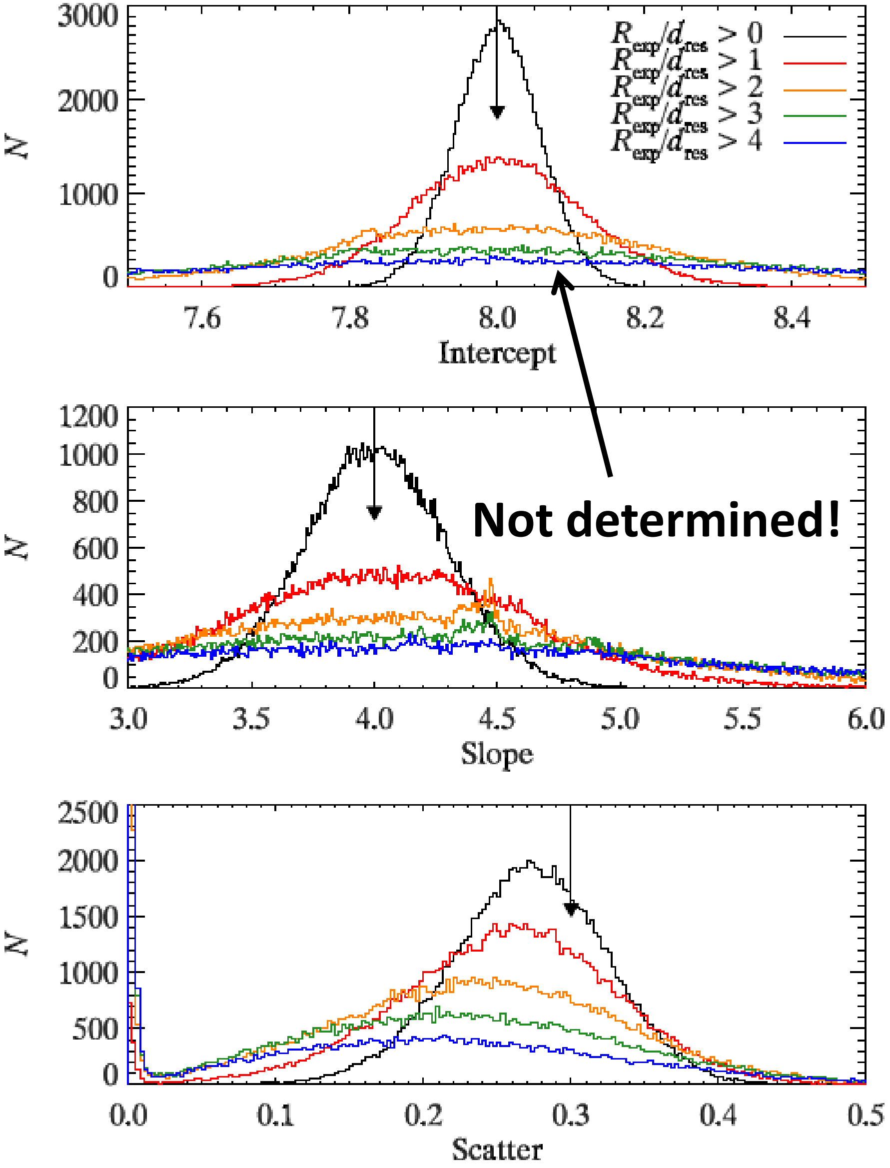
Fitting procedures are the following:

(1) Fit the parameters to the sample without the criterion.
→ $(\alpha, \beta, \varepsilon_0)$ are obtained.
(2) Calculate the R_{exp} from the expected M_{BH} and observed σ_*.
(3) Select the sample by the criterion $R_{exp}/d_{res}>1$.
(4) Fit the parameters again to the sample with $R_{exp}/d_{res}>1$.
(5) Iterate these processes until parameters converge.
→ Finally, they can get converged $(\alpha, \beta, \varepsilon_0)$ values(?).

They found that this can resolve the bias but causes another problem.

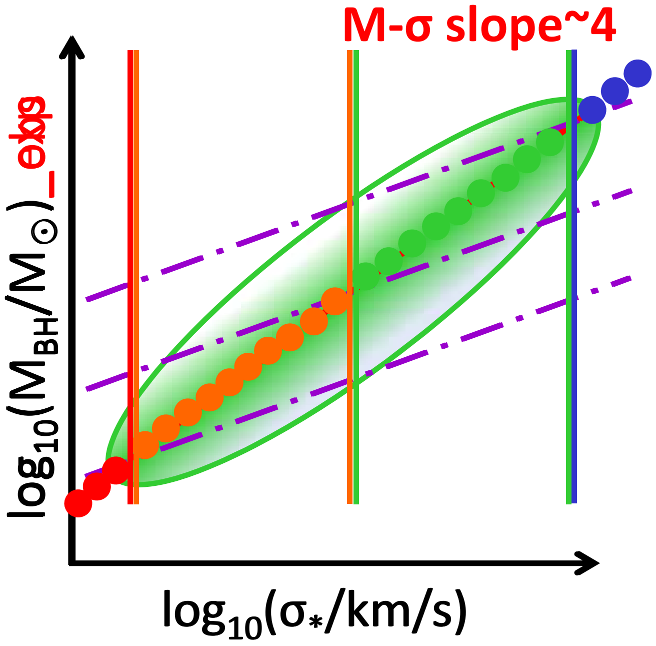


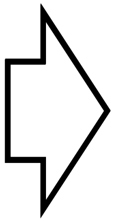
Bias!

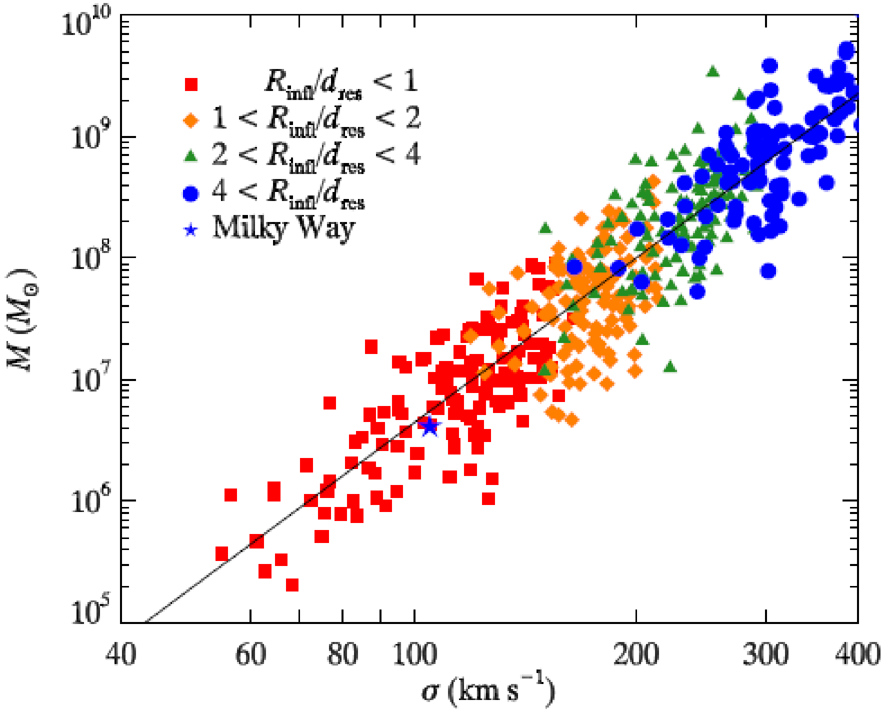


Not determined!

They found that the high-R_{exp}/d_{res} is equivalent to high-σ_*.







Observed M_{BH} have the scatter.

Expected M_{BH} have **no scatter!**

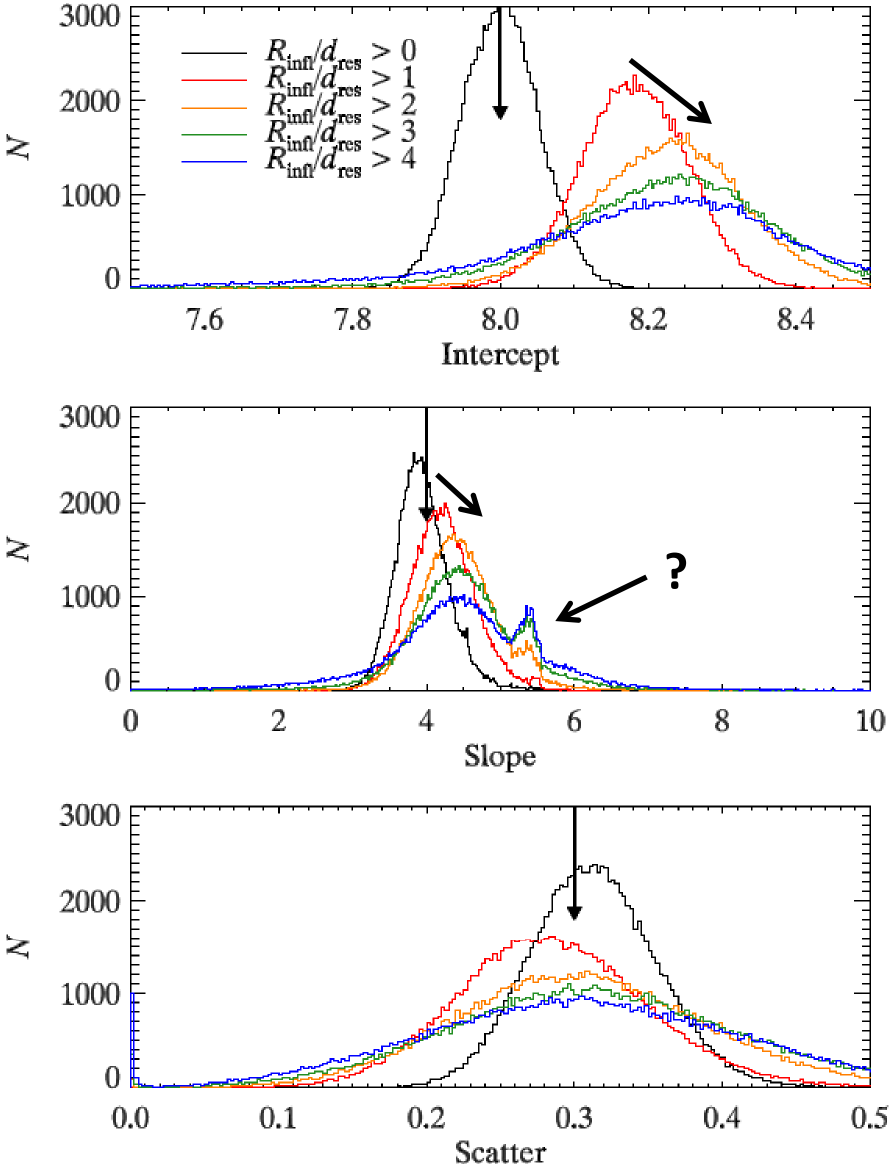
So, the previous bias will disappear.

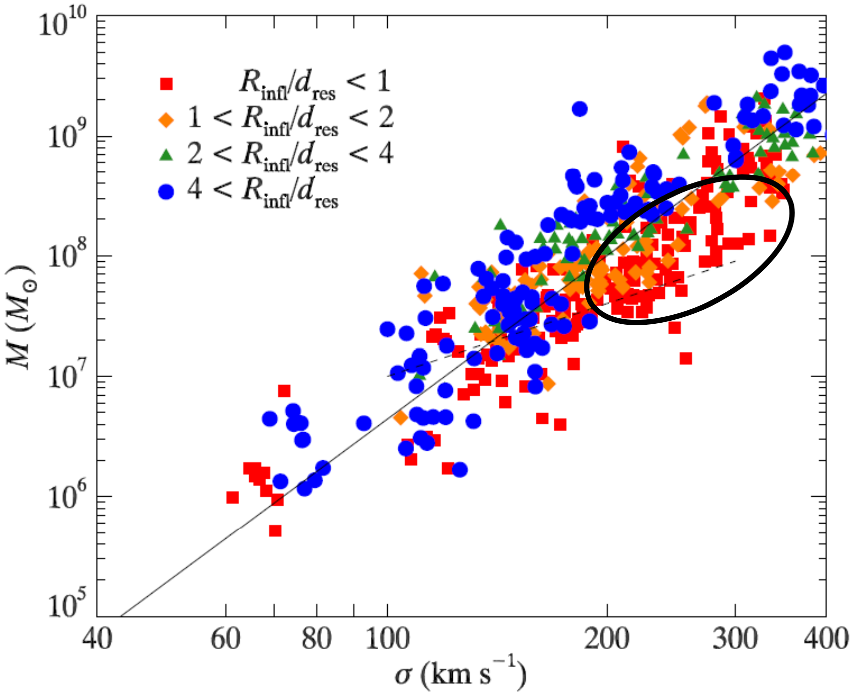
High-R_{exp}/d_{res} (high-σ_*) leads to large uncertainties in intercept(α), slope(β), intrinsic scatter(ε_0).

4-3. Discussion on biases 3.

They used the same criterion:
$R_{exp}=GM_{BH}/\sigma_*^2$, where M_{BH} is expected value from
the M-σ, and σ_* is observed value.
and the same fitting procedures.
But, they used ***observed*** σ_*, error in σ_*, D, error in M_{BH}
and instrumental resolution.

They found that this causes the other problem.





4-4. Their conclusion about the biases.

They concluded that

(1)any previous papers has not verified that the poor resolution causes the systematic biases to M_{BH},

(2)any criterion about the resolution causes biases,

(3)it seems the best choice that we don’t use these criterion!

4-5. Other possible biases.

The list of the other possible biases:

(1)Error in the stellar dynamical models,
(2)The unmodeled contribution from the dark matter halo,
(3)The uncertainty in the inclination and nongravitational forces,
(4)Possibility of the non-equilibrium state,
(5)The uncertainty in how to handle the gas kinematics of AGN,
(6)The uncertainty in the effect of projection.

comparison of the ε_0.

- They compared (i)the fitting methodology, (ii)data version of the same object, (iii)sample objects to that of Tremaine et al.(2002).
- They concluded that the difference of ε_0 is originated primarily in (iii), particularly if the spirals are included, secondary in (ii).
- The ε_0 are almost consistent with that of T02 if only early-type galaxies are considered.
- This suggests that

  (i) the spirals have the unaccounted observational errors,

  (ii) the spirals actually have the large intrinsic scatter.

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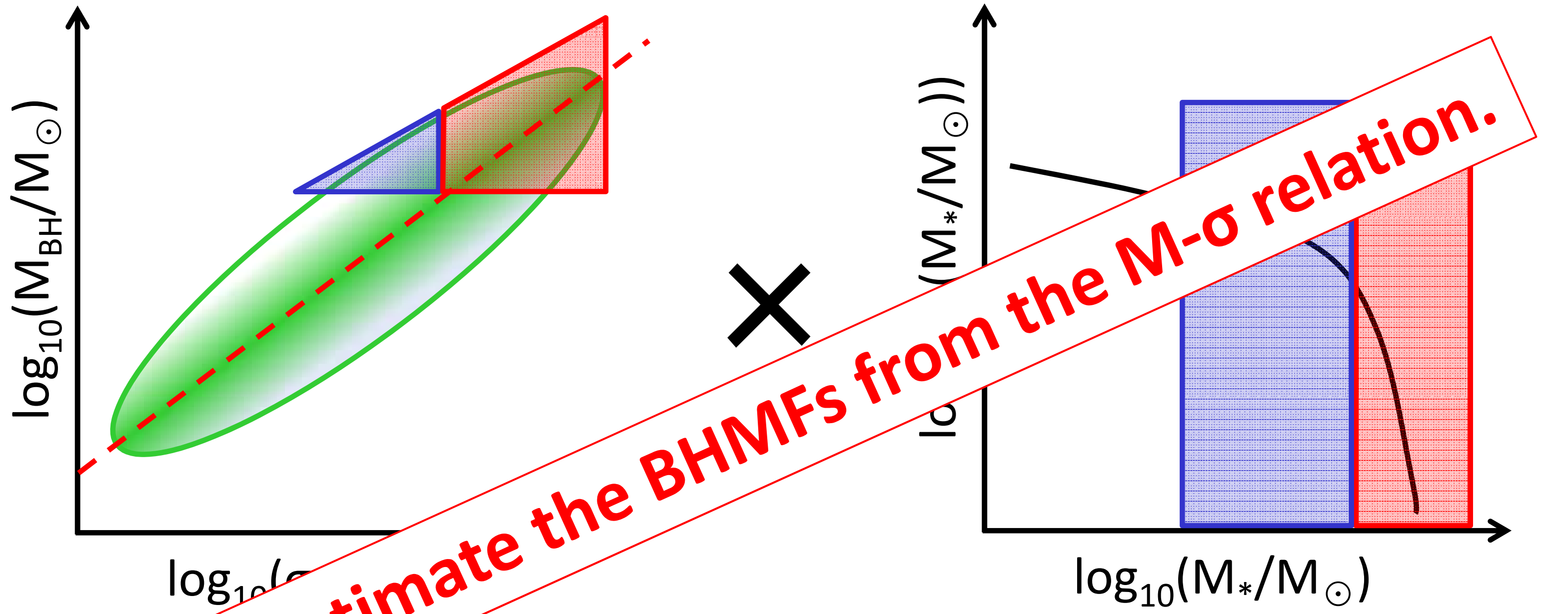
5. Black hole mass function(BHMF).

M-σ relation ***Mass, Luminosity function of galaxies***



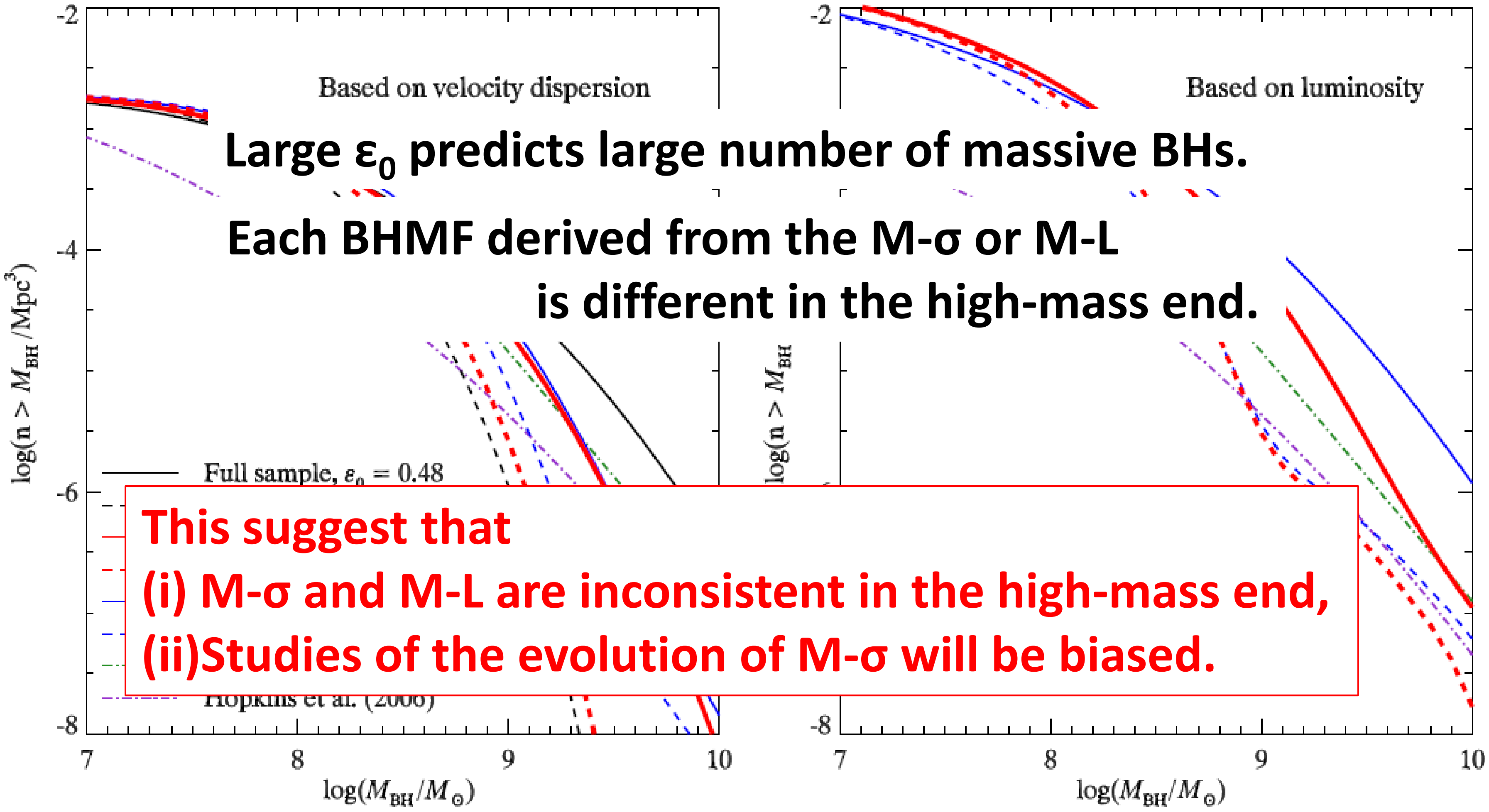
We can estimate the BHMFs from the M-σ relation.

Massive B… …ted by
(i)ma… …ts, or
(ii)med…um hosts with overmassive BHs.

(i)Massive hosts are very rare!
(ii)Medium hosts are common!

We expect that massive BHs will come largely from (ii).
Note that this expectation depends sensitively on ε_0!

5. Cumulative BHMFs from the M-σ, M-L.



Large ε_0 predicts large number of massive BHs.

Each BHMF derived from the M-σ or M-L is different in the high-mass end.

This suggest that
(i) M-σ and M-L are inconsistent in the high-mass end,
(ii)Studies of the evolution of M-σ will be biased.

Main results in this paper.

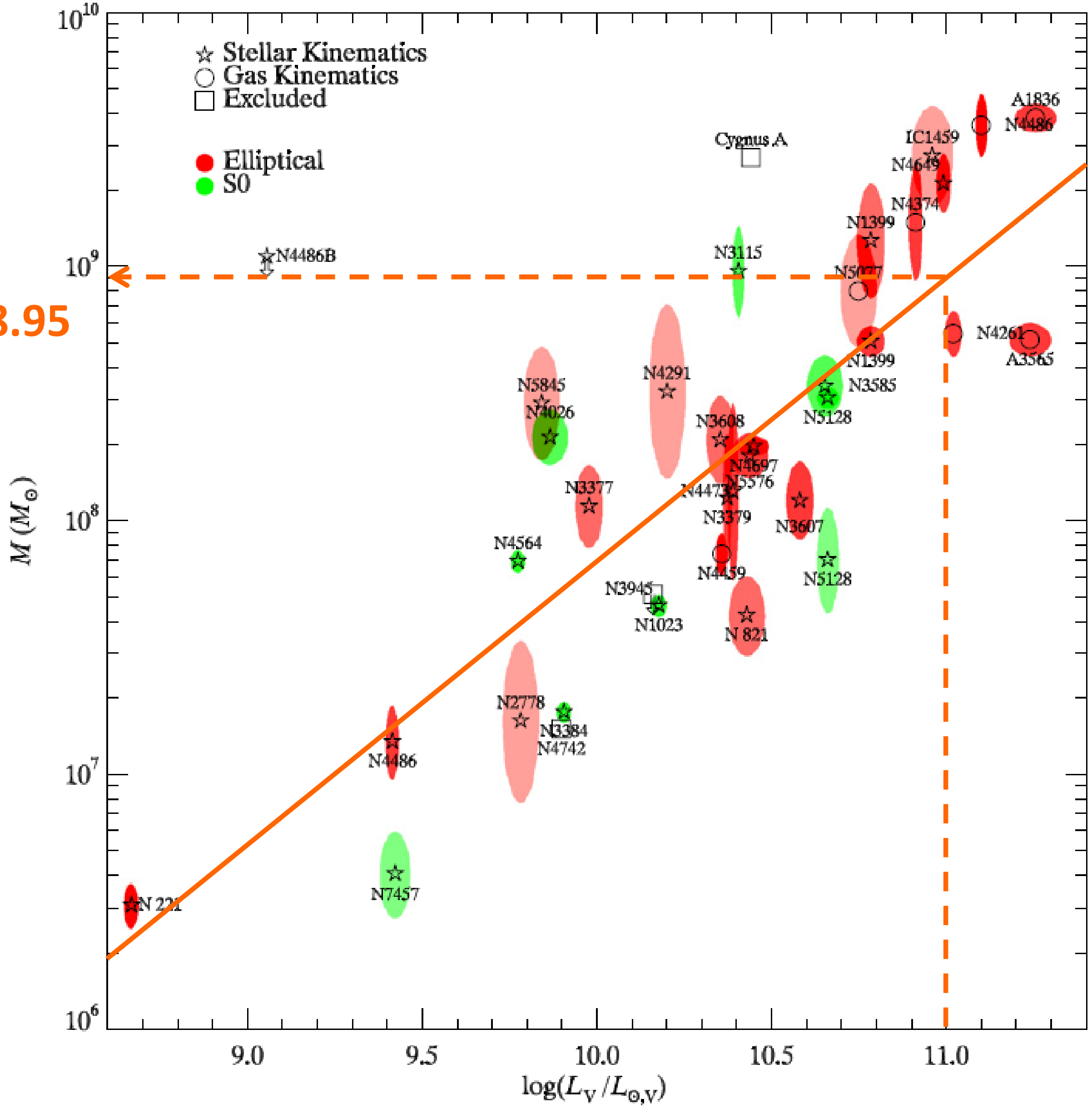
- Updated version of the M_{BH}-σ_* relation:
  $\log_{10}(M_{BH}/M_\odot) = \alpha + \beta \log_{10}(\sigma_*/200\ \mathrm{km/s})$.
- The M_{BH}-$L_{bulge,V}$ relation for elliptical galaxies:
  $\log_{10}(M_{BH}/M_\odot) = \alpha + \beta \log_{10}(L_{bulge,V}/10^{11}L_{\odot,V})$.
- $(\alpha, \beta, \varepsilon_0)_{M\text{-}\sigma}=(8.12, 4.24, 0.44)$, $(\alpha, \beta, \varepsilon_0)_{M\text{-}L}=(8.23, 3.96, 0.31)$,
  where ε_0 is magnitude of the intrinsic scatter in this relation.
- The lognormal distribution(Gaussian in $\log_{10}(M_{BH})$) is favored in the distribution of this scatter(intrinsic + observational).
- The prevailing criterion($R_{infl}/d_{res}>1$) causes the systematic bias in the sense that larger α, shallower β, smaller ε_0.
- The cumulative BHMFs derived from the M-σ, M-L relations are different in high-mass end.

2-2. Luminosities

- Extinction-corrected, bulge, V-band luminosity L_V: $\mathrm{Log}(L_V/L_{\odot,V})=0.4(4.83- M^0_{V,\ bulge})$ calculated from the extinction-corrected magnitudes $M^0_{V,\ bulge}$.
- The choice of V-band is a compromise between B and V bands.
- They did not include the spirals but included the S0s because the high confidence of bulge-disk decomposition.

3-4. the M-L relation.

- Best fit parameters of the same form. The parameters are $(\alpha, \beta, \varepsilon_0)=(8.95, 1.11, 0.38)$.
- Applied to full sample(SU sample).
- Assumed that the lognormal intrinsic scatter and the lognormal observational error distribution(‘GG’).
- Other detailed parameters are not presented.



slope β=1.11

intercept α=8.95

$10^{11}\ L_{\odot,V}$

3-5. The intrinsic scatter in the M-L.

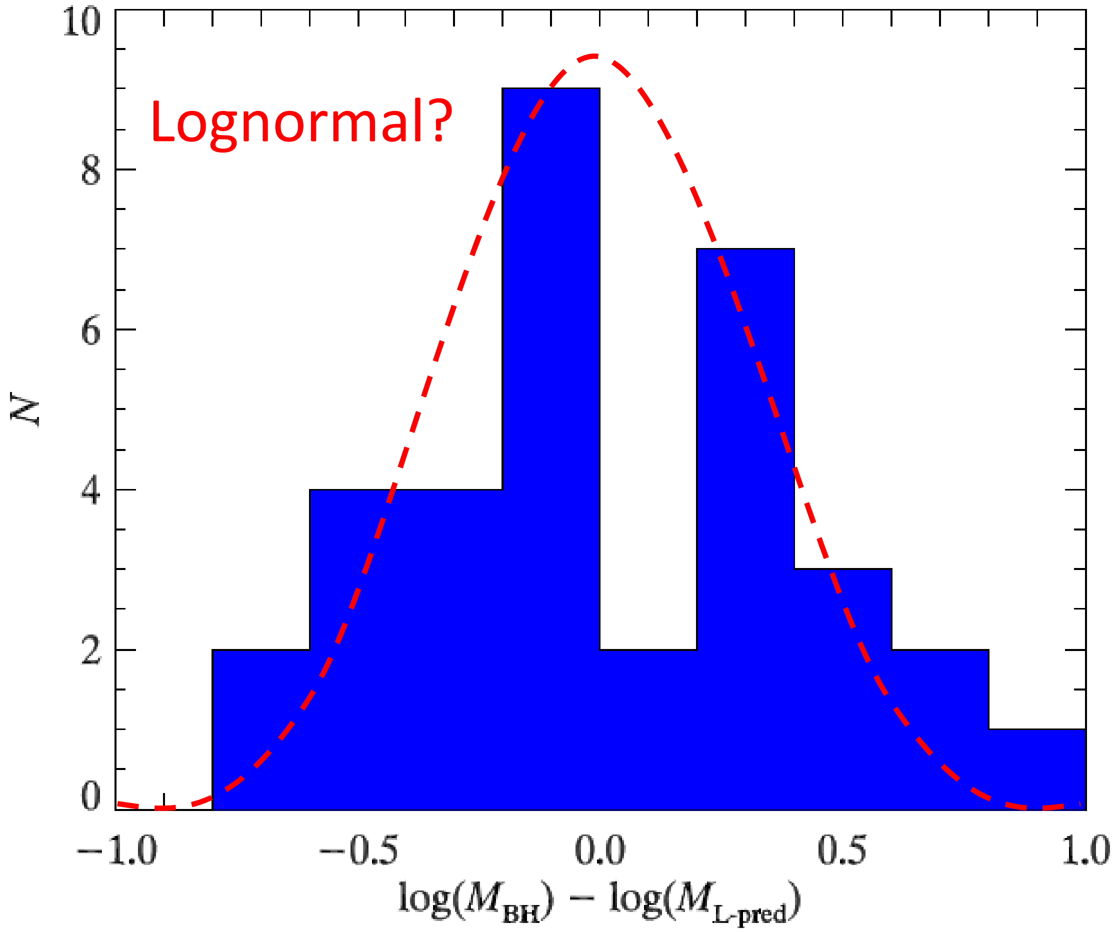


Figure 5. Histogram of residuals from best-fit *M*–*L* relation.

intrinsic scatter

+

observational error

=

total distribution