Braneworld Black holes

Pau Figueras

DAMTP
University of Cambridge

JGRG 21, Tohoku University, Sendai

27th of September, 2011

Based on:

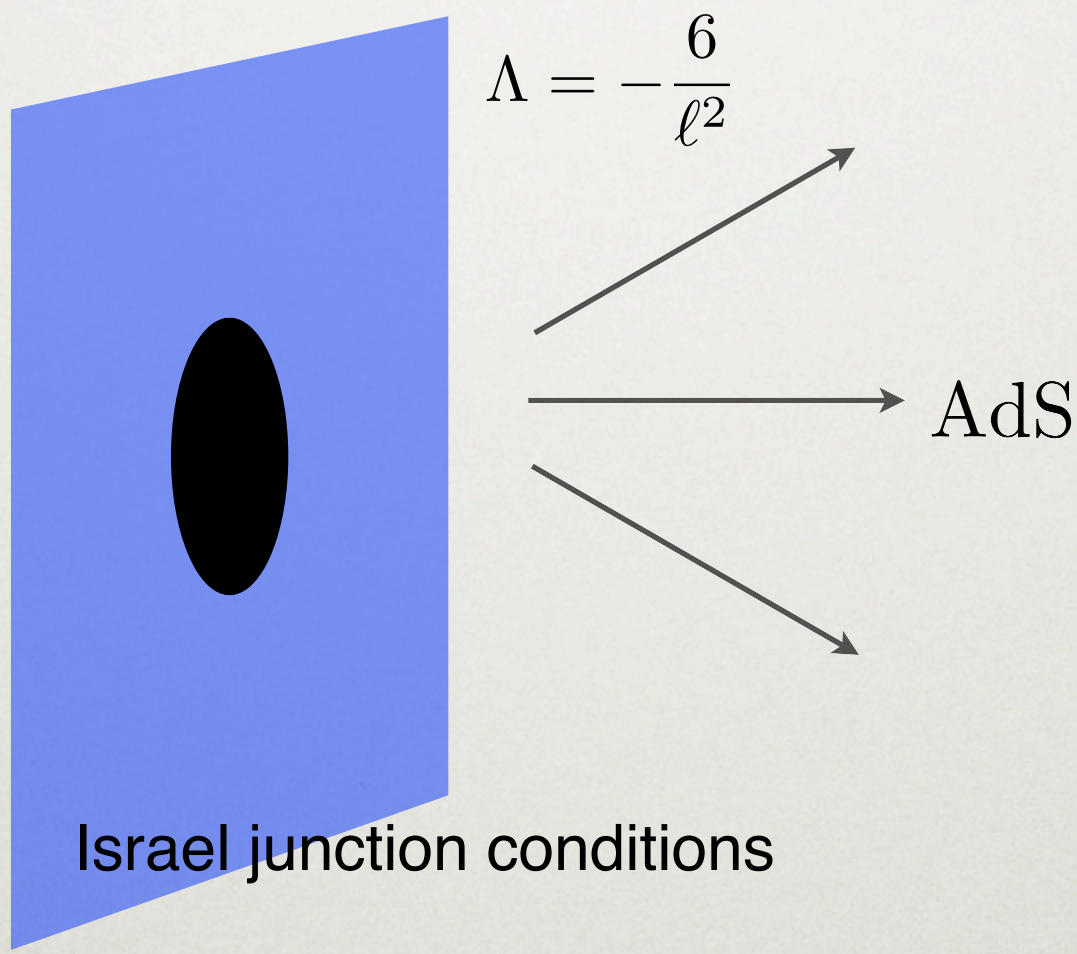
- PF, Lucietti and Wiseman: [1104.4489]
- PF & Wiseman: [1105.2558]
- PF & Wiseman: [11??.????]

Outline of the Talk

- Review of RSII braneworlds
- The method
- Gravitational dual to $\mathcal{N} = 4$ SYM on Schwarzschild
- Braneworld black holes in RSII
- Summary

The Randall-Sundrum (RSII) model

Consider the 4+1 dimensional asymptotically AdS spacetime. Cut off the geometry near the boundary of AdS and glue a copy of it onto this surface.



The RSII model offers a remarkable alternative to compactification: on scales much larger than ℓ, 4d gravity is recovered on the brane. [Randall and Sundrum; Garriga and Tanaka; Giddings, Katz and Randall]

The Randall-Sundrum (RSII) model

- The RSII model offers an alternative to compactification: in the linear regime and on scales much larger than ℓ, 4d gravity is recovered.
- The gravitational potential on the brane goes like [Garriga and Tanaka; Giddings, Katz and Randall]

$$\bar{h}_{tt} \sim \frac{1}{r} + \frac{2\,\ell^2}{3\,r^3}$$

and therefore there is no mass gap.

The Randall-Sundrum (RSII) model

• The RSII model offers an alternative to compactification: in the linear regime and on scales much larger than ℓ, 4d gravity is recovered.
• The gravitational potential on the brane goes like [Garriga and Tanaka; Giddings, Katz and Randall]

$$\bar{h}_{tt} \sim \frac{1}{r} + \frac{2\,\ell^2}{3\,r^3}$$

and therefore there is no mass gap.

• What about in the strong field regime?
• For scales much smaller than ℓ, 5d gravity is recovered. In particular, a small ($R_4 << \ell$) black hole on the brane will look like 5d (AF) Schwarzschild.
• **Do we recover 4d gravity on the brane for large black holes?**

The Randall-Sundrum (RSII) model

• The RSII model offers an alternative to compactification: in the linear regime and on scales much larger than ℓ, 4d gravity is recovered.
• The gravitational potential on the brane goes like [Garriga and Tanaka; Giddings, Katz and Randall]

$$\bar{h}_{tt} \sim \frac{1}{r} + \frac{2\,\ell^2}{3\,r^3}$$

and therefore there is no mass gap.

• What about in the strong field regime?
• For scales much smaller than ℓ, 5d gravity is recovered. In particular, a small ($R_4 << \ell$) black hole on the brane will look like 5d (AF) Schwarzschild.
• **Do we recover 4d gravity on the brane for large black holes?**

• Interpretation in AdS/CFT: The black hole solutions localised on the brane in the RSII model which are found solving the classical bulk equations in AdS_{D+1} with brane boundary conditions correspond to quantum-corrected black holes in D-dimensions. [Tanaka ; Emparan, Fabbri and Kaloper]
• Conjecture: ***No large, static and non-extremal black hole on the brane should exist*** [Tanaka ; Emparan, Fabbri and Kaloper]. Counter argument by [Fitzpatrick, Randall and Wiseman]

The Randall-Sundrum (RSII) model

Analytical progress very difficult [Shiromizu, Maeda, Sasaki; Charmousis, Gregory;....]

Summary of numerical previous work (Relativistic stars were constructed [Wiseman]):

• Kudoh, Tanaka and Nakamura ('03): only small ($R_4/\ell \leq 0.3$) black holes were found.
• Kudoh ('06): up to intermediate size black holes ($R_5/\ell \leq 2.$) were found in D=6.
• Yoshino ('08): no static black hole at all was found. One possible interpretation: no static black hole (no matter the size) on the brane exists.
• Kaus and Reall ('09): the near horizon geometry of *extremal* braneworld black holes of arbitrary size was found. (no Hawking radiation expected in this case anyway)

The Method

We want to solve:

$$R_{\mu\nu} = 0$$

for a *static* black hole spacetimes ($\mathcal{M}$, g) in D dimensions.

- Superficially we have $D(D+1)/2$ equations for the same number of metric components but because of the Bianchi identity there are only $D(D-1)/2$ non-trivial equations.

- Gauge fixing is necessary in order to have a (strongly) elliptic system of equations.

- Methods for solving PDEs:
  - Elliptic: boundary value problem.
  - Hyperbolic/parabolic: initial value problem.

THE METHOD

Introduced by [Headrick, Kitchen and Wiseman] for the static case and [Adam, Kitchen and Wiseman] for the stationary case (see also [PF, Lucietti and Wiseman]).

The Method

Introduced by [Headrick, Kitchen and Wiseman] for the static case and [Adam, Kitchen and Wiseman] for the stationary case (see also [PF, Lucietti and Wiseman]).

Instead of considering the Einstein equations, we consider a characteristic version of it (the Harmonic Einstein equation) which is manifestly elliptic:

$$R^H_{\mu\nu} = 0 \qquad R^H_{\mu\nu} = R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} \qquad \xi^\mu = g^{\alpha\beta}(\Gamma^\mu_{\alpha\beta} - \bar{\Gamma}^\mu_{\alpha\beta})$$

where $\bar{\Gamma}$ is the Levi-Civita connection associated to a reference metric $\bar{g}$ on the manifold.

Note:

- $R^H_{\mu\nu} = 0$ is strongly elliptic: $R^H_{\mu\nu} \sim -\frac{1}{2} g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu}$
- Analogous to harmonic gauge: $\xi^\mu = 0 \quad \Rightarrow \quad \Delta_g x^\mu = H^\mu = -g^{\alpha\beta}\bar{\Gamma}^\mu_{\alpha\beta}$
- There are no constraints to worry about.

The Method

Comments/Remarks:

• Since the term proportional to Λ in the Einstein equations has no derivatives we can simply added to the Einstein Harmonic equation without affecting its elliptic character:

$$R^H_{\mu\nu} \equiv R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 0$$

• Using the Bianchi identity, ξ^μ obeys $\nabla^2\xi_\mu + R_\mu{}^\nu\xi_\nu = 0$

The Method

Comments/Remarks:

• Since the term proportional to Λ in the Einstein equations has no derivatives we can simply added to the Einstein Harmonic equation without affecting its elliptic character:

$$R^H_{\mu\nu} \equiv R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)} - \Lambda g_{\mu\nu} = 0$$

• Using the Bianchi identity, ξ^μ obeys $\nabla^2\xi_\mu + R_\mu{}^\nu\xi_\nu = 0$

Ultimately we want to solve the original Einstein equations

• **Dynamical case**: choosing $\xi^\mu = 0$ and $\partial_t \xi^\mu = 0$ on a Cauchy surface Σ ensures that the solutions to $R^H{}_{\mu\nu} = 0$ are Einstein!

• **Elliptic case**: solve $R^H{}_{\mu\nu} = 0$ subject to BCs compatible with $\xi^\mu = 0$.

• A solution $R_{\mu\nu} = 0$ in the gauge $\xi^\mu = 0$ certainly implies $R^H{}_{\mu\nu} = 0$ but the converse is not true: there can be solutions $R^H{}_{\mu\nu} = 0$ with non-trivial $\xi^\mu = 0$ called Ricci solitons.

• What boundary conditions should we impose on ξ^μ in order to find Einstein metrics?

The Method

• In favourable circumstances one can in fact prove that only Einstein solutions exist on a given manifold:

- Bourguignon ('79) and Perelman ('02): no solitons exist on compact manifolds.
- For various asymptotics (AF, KK, AdS) one can prove that no Ricci solitons can exist. [PF, Lucietti and Wiseman]

• For the brane boundary conditions in the RSII model we ***cannot*** prove that no solitons exist.

• Since $R^H{}_{\mu\nu} = 0$ is elliptic and if the boundary conditions are compatible with the ellipticity of the problem, then every solution should be locally unique.

• Therefore, an Einstein solution can always be distinguished from a Ricci soliton.

Solving the equations

• **Method 1**: local relaxation (diffusion) ⇒ Ricci-DeTurck flow

$$\frac{\partial}{\partial\lambda}\, g_{\mu\nu} = -2\, R^H_{\mu\nu}$$

➡ evolve the metric until one reaches a fixed point.

Comments:

• Very easy to implement!
• It is diffeomorphic to Ricci flow,

$$\frac{\partial}{\partial\lambda}\, g_{\mu\nu} = -2\, R_{\mu\nu}$$

since $\delta g_{\mu\nu} = \nabla_{(\mu}\xi_{\nu)}\,\delta\lambda$ is a diffeomorphism.
⇒ the trajectory in the space of geometries is independent of the choice of reference metric!

Solving the equations

Consider perturbations around a fixed point $Ric[g_0] = 0$: $g \rightarrow g_0 + \delta g$.

Their evolution under the Ricci-DeTurck flow is given by

$$\delta \dot{g}_{\mu\nu} = -\Delta_L \delta g_{\mu\nu}$$

Therefore, a fixed point is stable (or attractive) iff Δ_L is positive.

But for many black hole spacetimes Δ_L has negative modes [Gross, Perry and Yaffe], and hence for generic initial data Ricci flow will ***NOT*** converge to the desired fixed point.

For a black hole spacetime with n negative modes, one has to tune an n parameter set of initial data

$\Rightarrow$ Ricci flow is not very useful if $n > 1$!

Solving the equations

Method 2: Newton's method. Iteratively replace

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu} \quad \text{with} \quad h_{\mu\nu} = -\Delta_H^{-1} R^H_{\mu\nu}$$

where Δ_H is the linearisation of R^H.

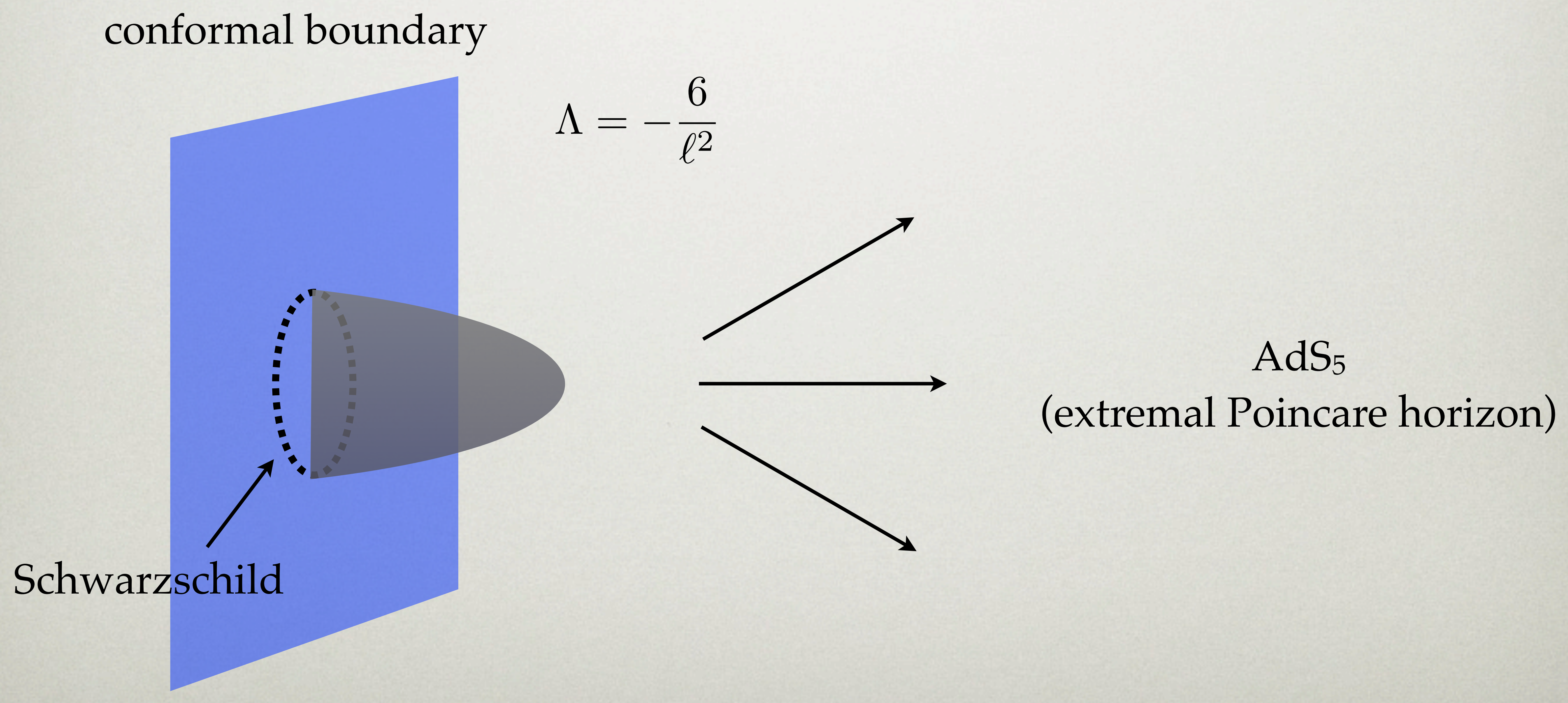
Comments:

• *Advantages*:
- Fast convergence.
- No problems with -ve modes (only zero modes cause trouble).

• *Disadvantages*:
- Harder to implement than Ricci Flow.
- Non-geometric in nature and the trajectory in the space of geometries depends on the choice of reference metric.
- The basin of attraction depends on the reference metric and in practice it can be rather small.

Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Goal: use AdS/CFT to construct the gravitational dual of $\mathcal{N}$=4 SYM on Schwarzschild such that far from the black hole the theory is in a vacuum state.



Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Why this AdS/CFT solution is relevant to the braneworld black hole problem?

1. The arguments of non-existence of Tanaka and Emparan et al. apply to this case.
2. This solution turns out to be much cleaner and easy to find.
3. *One can prove analytically that no solitons can exist in this case!*
4. The AdS/CFT solution corresponds to the infinite radius limit of a braneworld black hole.

➡ it is more difficult to argue that it doesn't exist!

Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

We can choose coordinates in order to make the isometries manifest (∂_τ and axis of symmetry) to simplify the problem. This introduces fictitious boundaries at the fixed points and extra boundary conditions follow from requiring smoothness of the original metric.

$\Rightarrow$ compatible with non-existence of solitons.

Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

We can choose coordinates in order to make the isometries manifest (∂_τ and axis of symmetry) to simplify the problem. This introduces fictitious boundaries at the fixed points and extra boundary conditions follow from requiring smoothness of the original metric.
$\Rightarrow$ compatible with non-existence of solitons.

General metric ansatz:

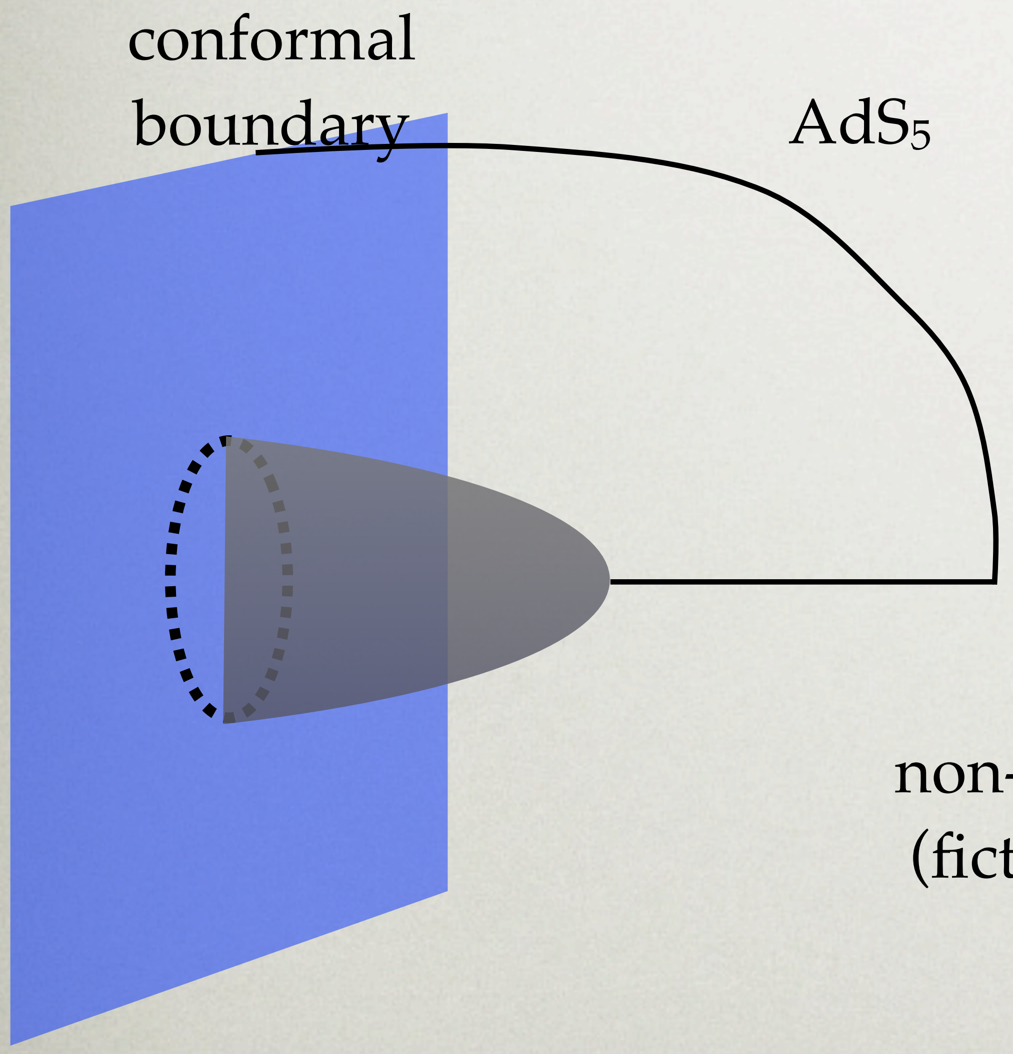
$$ds^2 = \frac{\ell^2}{(1-x^2)^2}\left(4r^2 f^2 e^T\, d\tau^2 + x^2 g\, e^S\, d\Omega^2_{(2)} + \frac{4}{f^2}\, e^{T+r^2 f\, A} dr^2 + \frac{4}{g}\, e^{S+x^2 B} dx^2 + \frac{2\, r\, x}{f}\, F\, dr\, dx\right)$$

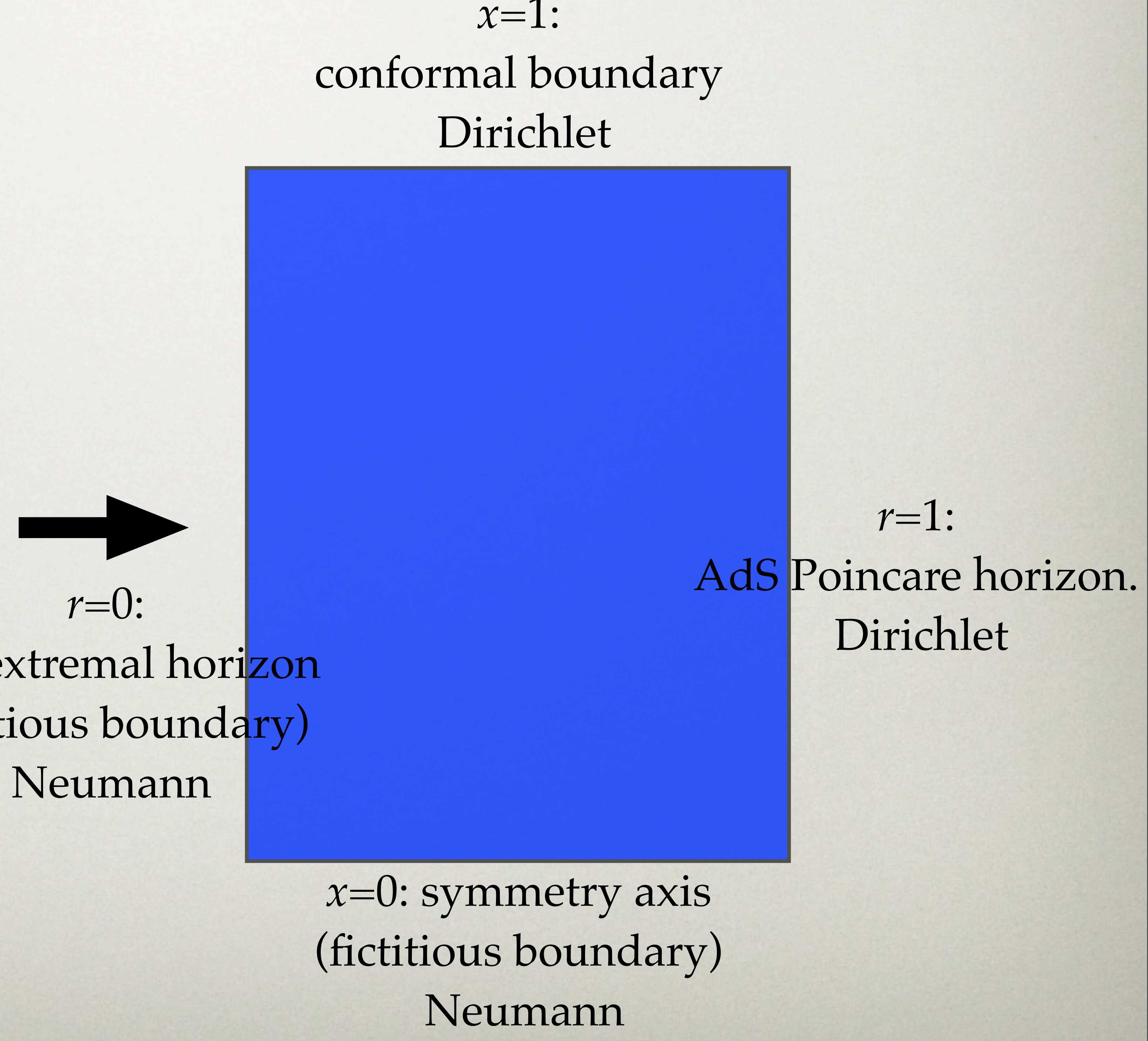
$$f = 1 - r^2\,, \qquad g = 2 - x^2$$

- *T, S, A, B, F* are functions of *r* and *x* and these are the functions we are solving for.
- Without loss of generality we can choose $0 \leq r, x \leq 1$.
- Reference metric: $T = S = A = B = F = 0$.

Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Boundary conditions:





Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Important aspects of this solution:

1. With the previous BCs we can analytically show that no Ricci soliton can exist.
2. There are no negative modes: the boundary black hole is non-dynamical.

➡ We can find the solution using Ricci Flow!

Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Embedding of the horizon geometry into hyperbolic space:

$$ds_H^2 = \frac{1}{z^2}(dz^2 + dy^2 + y^2 d\Omega_{(2)}^2)$$

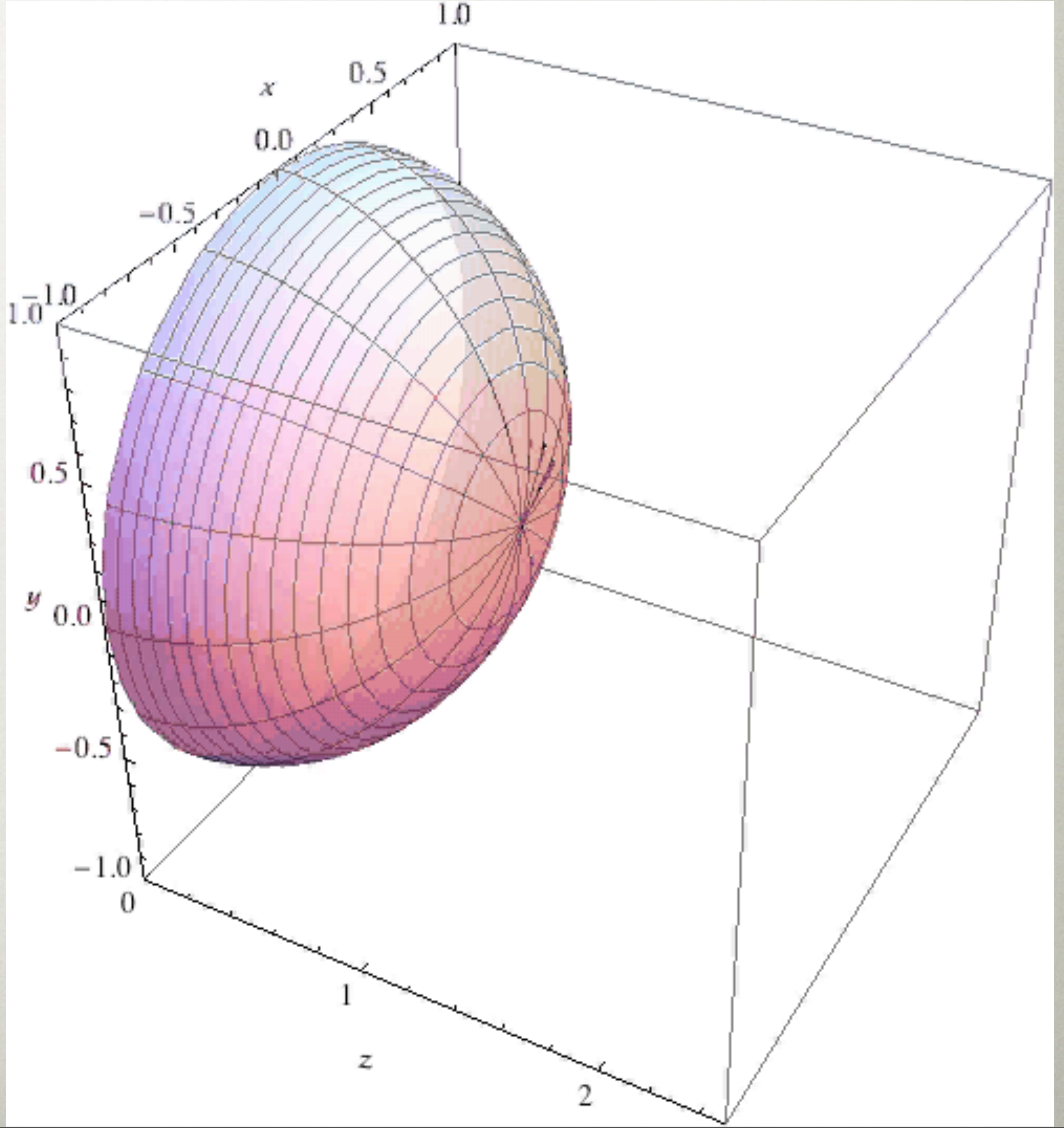
$$y = y(z)$$

Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Embedding of the horizon geometry into hyperbolic space:

$$ds^2_H = \frac{1}{z^2}(dz^2 + dy^2 + y^2 d\Omega^2_{(2)})$$

$$y = y(z)$$



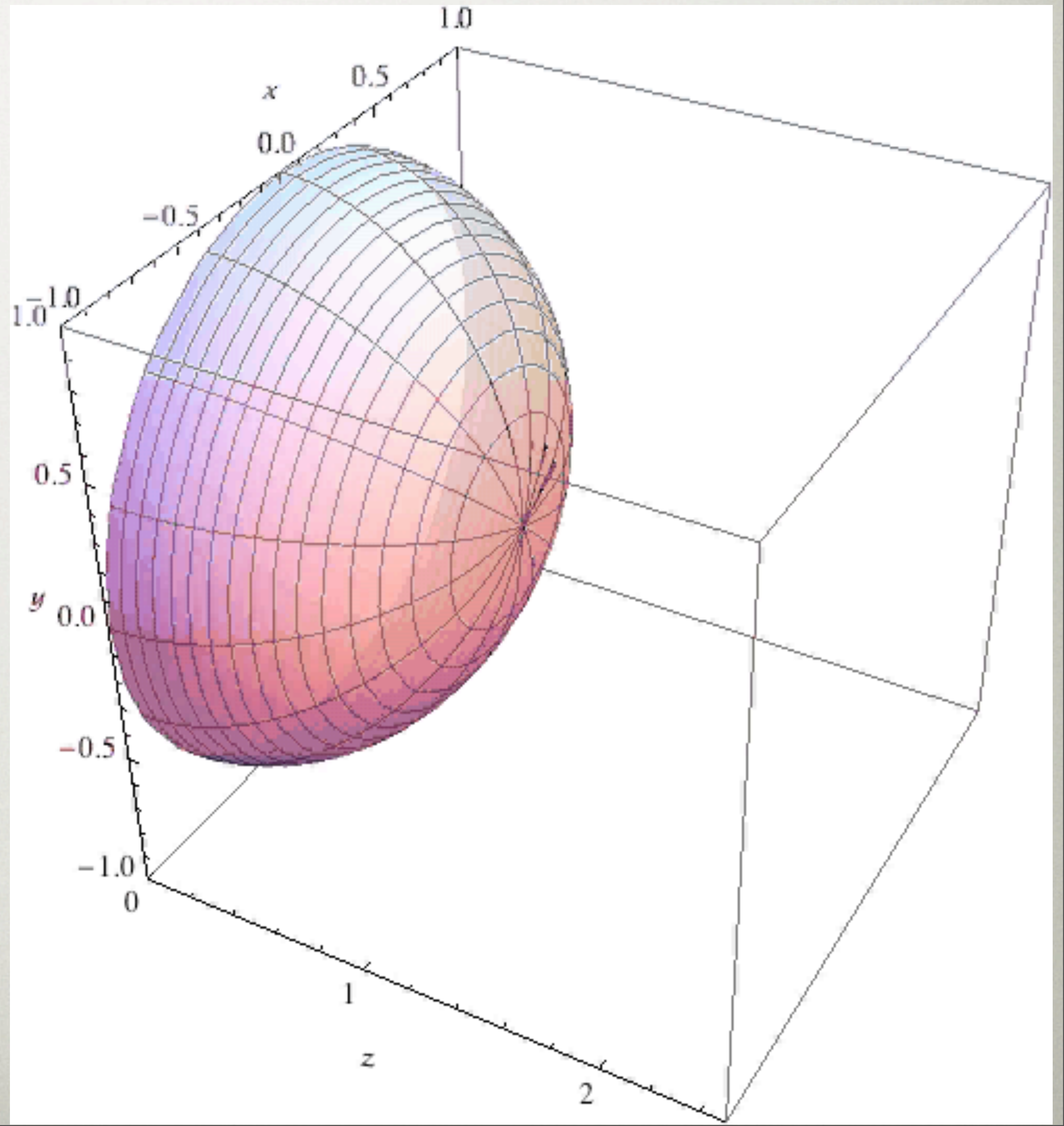
Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Embedding of the horizon geometry into hyperbolic space:

$$ds_H^2 = \frac{1}{z^2}(dz^2 + dy^2 + y^2 d\Omega_{(2)}^2)$$

$$y = y(z)$$

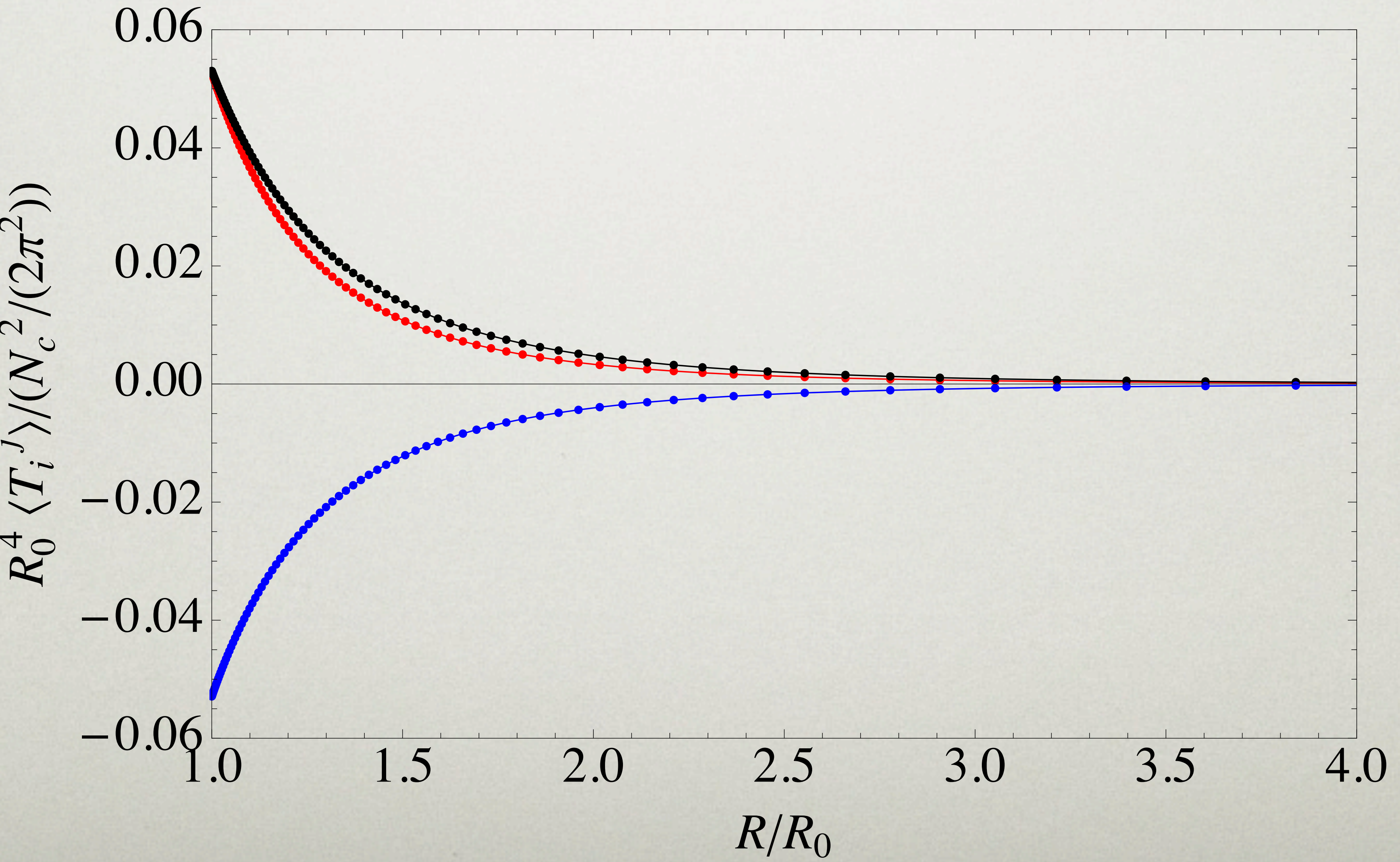
Note: the geometry only looks string-like in a small region near the boundary, too small for a GL type mode to fit on the horizon ⇒ the solution is presumably stable.



Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

$O(N_c^2)$ of the quantum stress tensor:

$$\frac{1}{N_c^2}\langle T_i{}^j\rangle = \frac{1}{2\pi^2}\,\frac{1}{R^4}\,\mathrm{diag}\left\{\frac{3\,R_0}{4\,R}\left(1-\frac{R_0}{R}\right)+t_4(R)\,,\ \frac{3\,R_0^2}{4\,R^2}-\big(t_4(R)+2\,s_4(R)\big)\,,\right.$$
$$\left.-\frac{3\,R_0}{8\,R}+s_4(R)\,,\ -\frac{3\,R_0}{8\,R}+s_4(R)\right\},$$



Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Main features and interpretation:

- Traceless: no conformal anomaly.
- Our solution corresponds to the gravitational dual of $\mathcal{N}$=4 SYM on the background of Schwarzschild in the Unruh vacuum (not Hartle-Hawking and possibly not Boulware either).
- The dual classical geometry only captures the $O(N_c^2)$ of the full quantum stress tensor, and this piece is ***static and regular*** everywhere.
- To see the usual divergences on the past horizon in the Unruh vacuum one should include bulk quantum/string corrections.

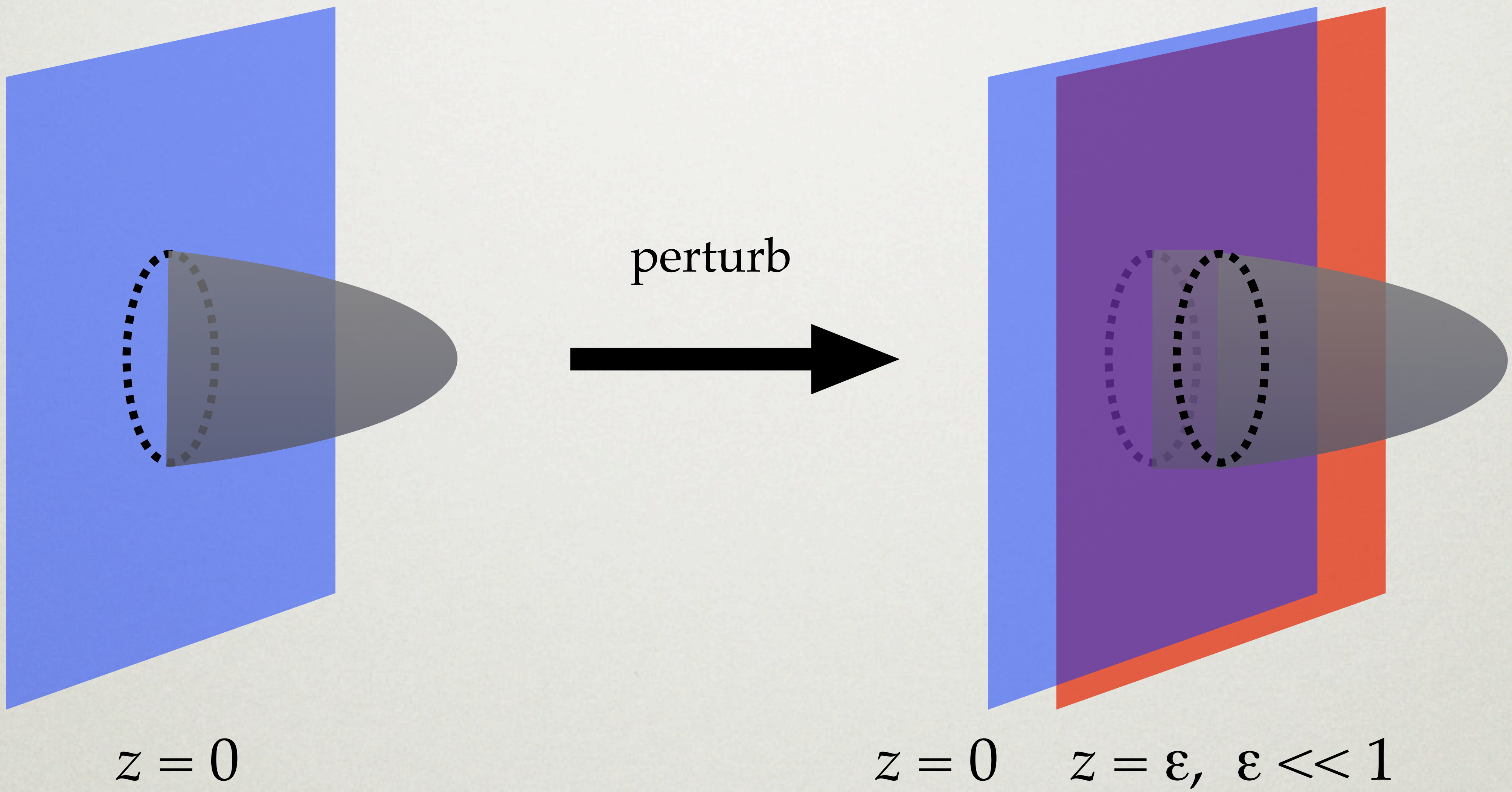
Gravity dual of $\mathcal{N}$=4 SYM on Schwarzschild

Physical picture:

• The black hole acts as a heat source exciting the plasma around it.
• The strong interactions of the plasma are attractive and want to collapse back into the black hole.
• At $O(N_c^2)$ there is equilibrium between the radiation pressure and the attractive self-interactions of the plasma.
• The flux of radiation at infinity is an $O(1)$ effect, which is not captured by the bulk gravity approximation.

Braneworld black holes

From the AdS/CFT we can construct perturbatively very large braneworld black holes:



Israel junction conditions

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + \tilde{g}_{\mu\nu}(z,x)\, dx^\mu dx^\nu\right)$$

$$\tilde{g}_{\mu\nu}(z,x) = g^{\rm Schw}_{\mu\nu} + z^4\, t_{\mu\nu}(x)\, dx^\mu dx^\nu + O(z^6)$$

$$g_{\mu\nu} = g^{\rm Schw}_{\mu\nu} + \epsilon^2\, \delta g_{\mu\nu}$$

Braneworld black holes

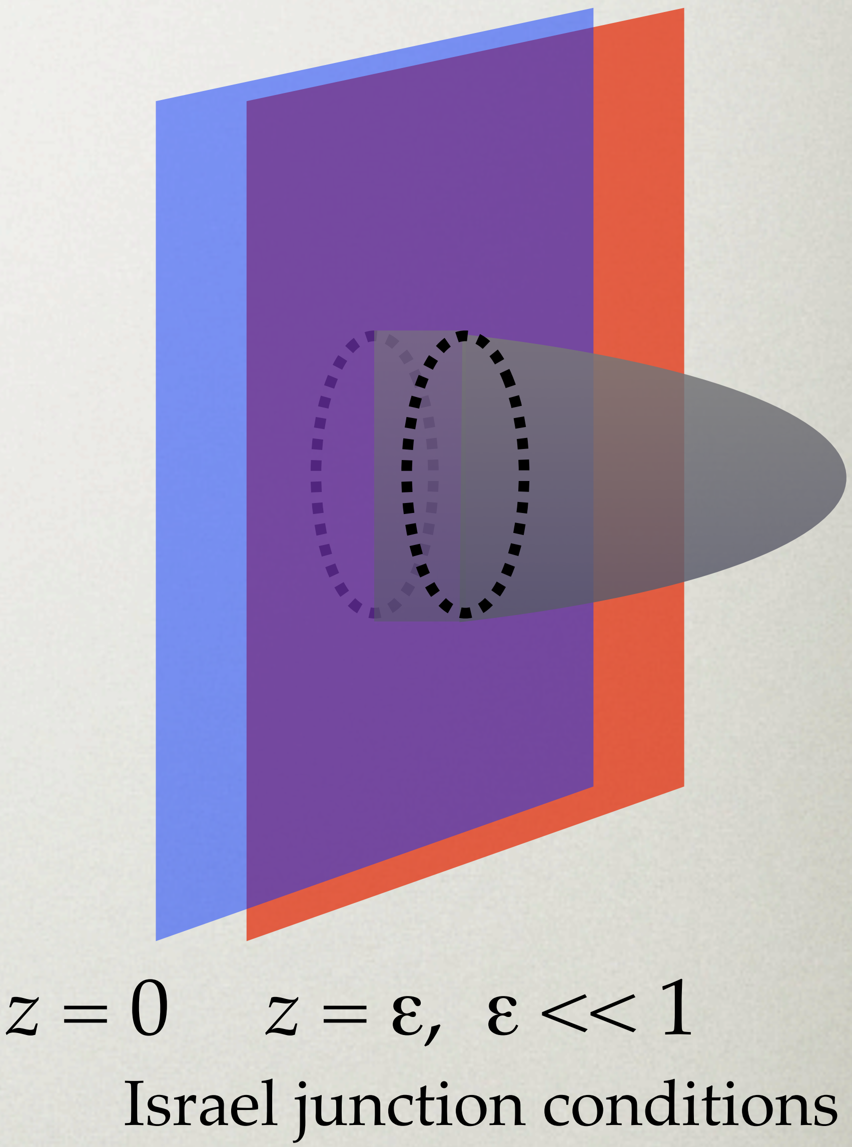
Induced metric on the brane:

$$\gamma_{\mu\nu} = \frac{\ell^2}{\epsilon^2}(g_{\mu\nu}^{\rm Schw} + \epsilon^2\,\delta g_{\mu\nu})$$

$\Rightarrow$ the metric is approximately Schw. with a radius much larger than the AdS radius ℓ

The perturbation satisfies:

$$\delta G_{\mu\nu} = 16\pi G_4 \langle T_{\mu\nu}^{\rm CFT}[g^{\rm Schw}]\rangle$$



Braneworld black holes

Metric ansatz: "close" to the AdS/CFT solution but introduce a cut off near the boundary.

$$ds^2 = \frac{\ell^2}{\Delta^2}\left(4r^2 f^2 e^T\, d\tau^2 + x^2 g\, e^S\, d\Omega^2_{(2)} + \frac{4}{f^2}\, e^{T+r^2 A} dr^2 + \frac{4}{g}\, e^{S+x^2 B} dx^2 + \frac{2\, r\, x}{f}\, F\, dr\, dx\right)$$

$$\Delta = (1-x^2) + \epsilon(1-r^2)\,, \qquad f = 1-r^2\,, \qquad g = 2-x^2$$

In the limit $\varepsilon \to 0$ we should recover the AdS/CFT solution.

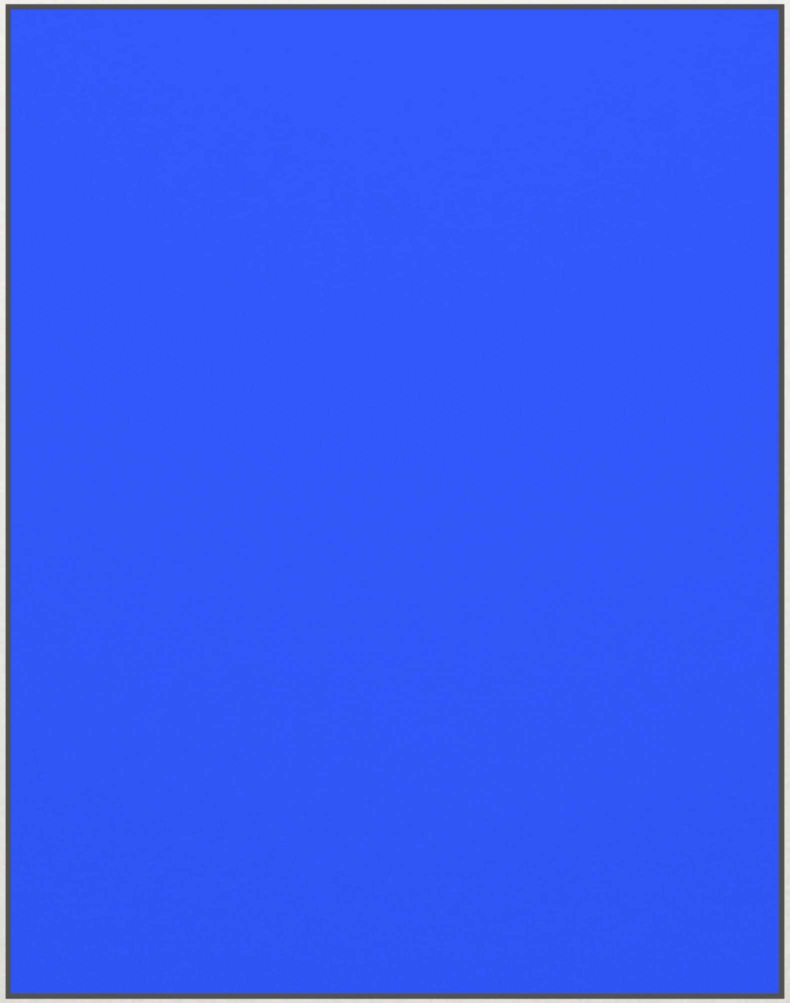
Braneworld black holes

x=1:
Brane

$$K_{ij} = \frac{1}{\ell}\,\gamma_{ij}\,,$$

$$\xi_x = 0\,, \quad F = 0\,, \quad \Rightarrow \quad \partial_x \xi_r = \frac{2}{\ell}\,\xi_r$$

r=0:
non-extremal horizon
(fictitious bounary)
Neumann



r=1:
AdS Poincare horizon.
Dirichlet

x=0: symmetry axis
(fictitious boundary)
Neumann

Braneworld black holes results

Ricci Flow

•Since gravity on the brane is dynamical, we find that black holes on branesworlds have one and only one negative mode.

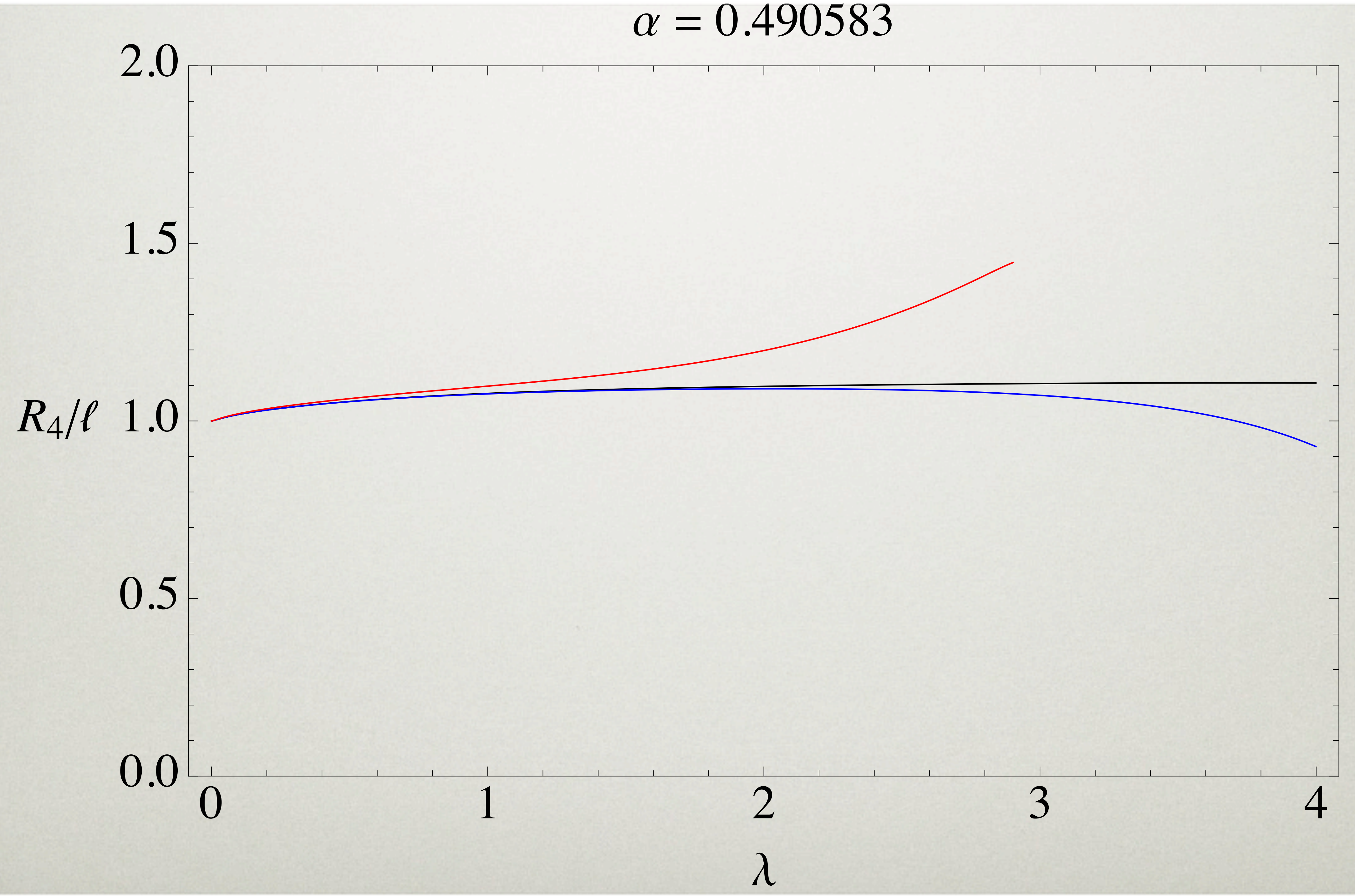
➡ Ricci flow does not work in a straightforward manner: We need a one parameter family of initial data to tune away the negative mode.

$$\{g_{tt},\, g_{rr}\} \to k(r)\{g_{tt},\, g_{rr}\}\,, \qquad k(r) = 1 - \alpha(1-r^2)^2$$

➡ Depending on the value of the parameter we observe two different flows.
➡ Existence of solutions: there should exist a fixed point for one particular value of this parameter.

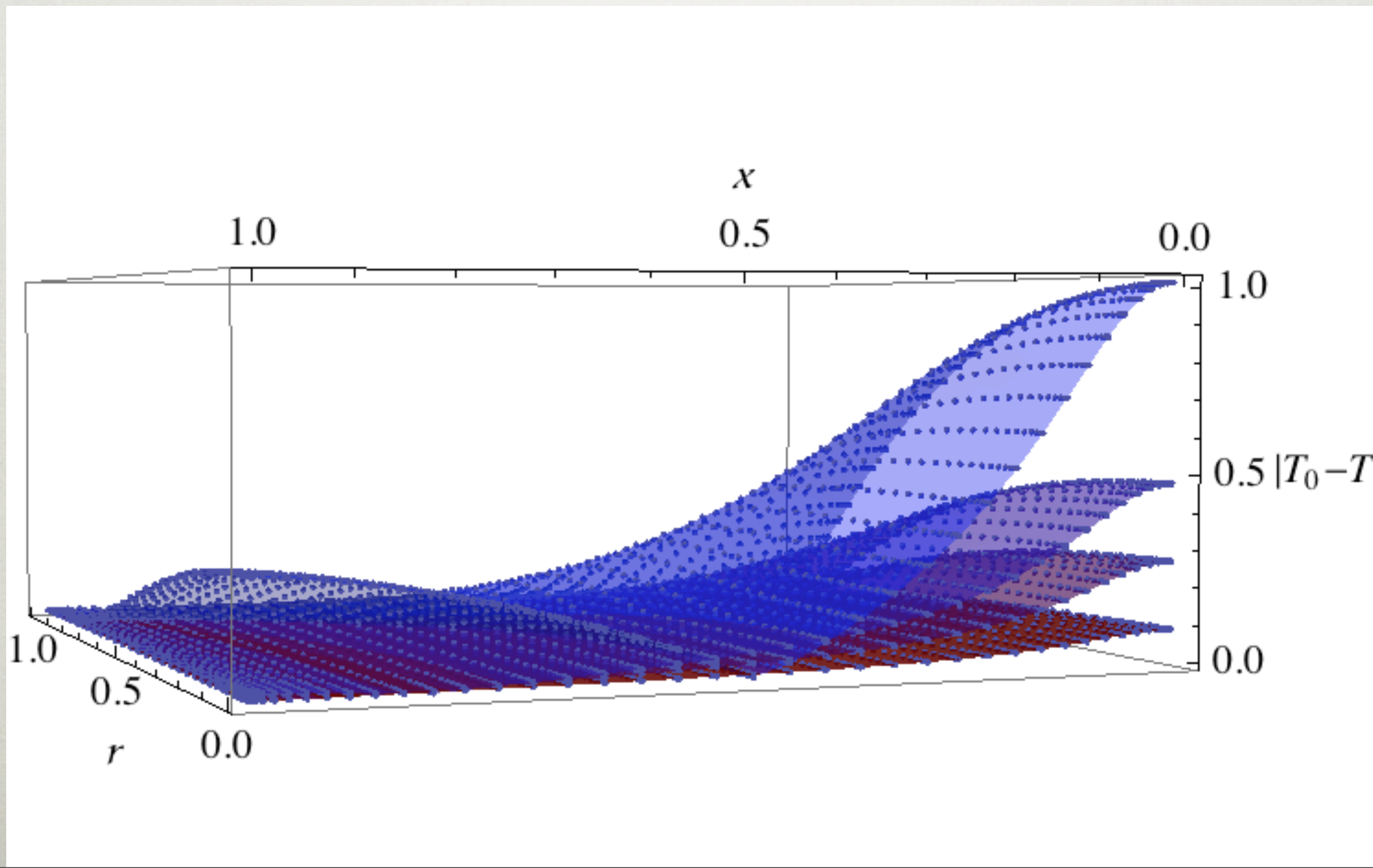
Braneworld black holes results

Braneworld black holes results



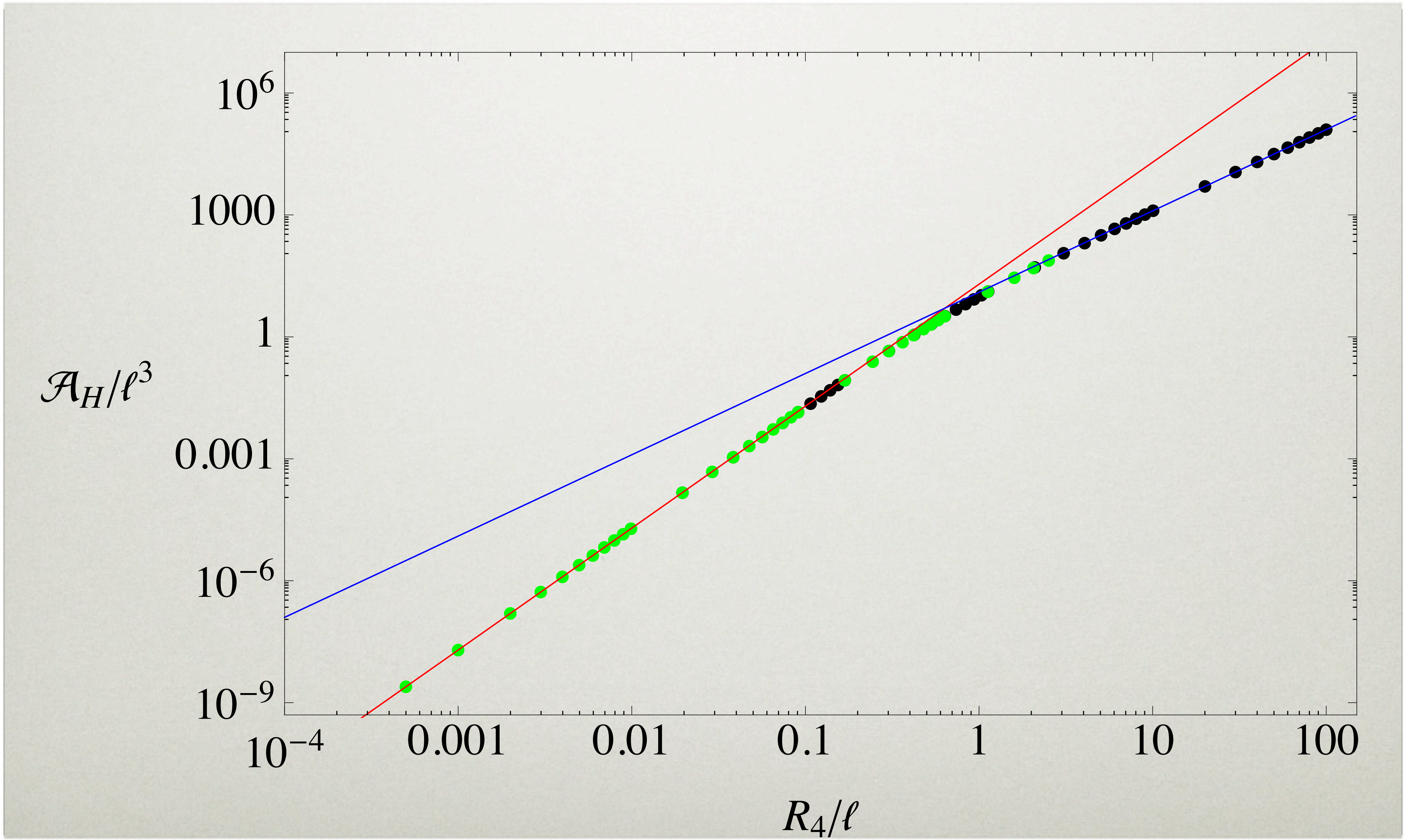
Braneworld black holes results

- Using Newton's method we can find black holes with $5 \times 10^{-3} \lesssim R_4/\ell \lesssim 100$
- Even though we cannot prove that solitons do not exist, we do NOT find any.
- Large braneworld black holes are "close" to the AdS/CFT solution.



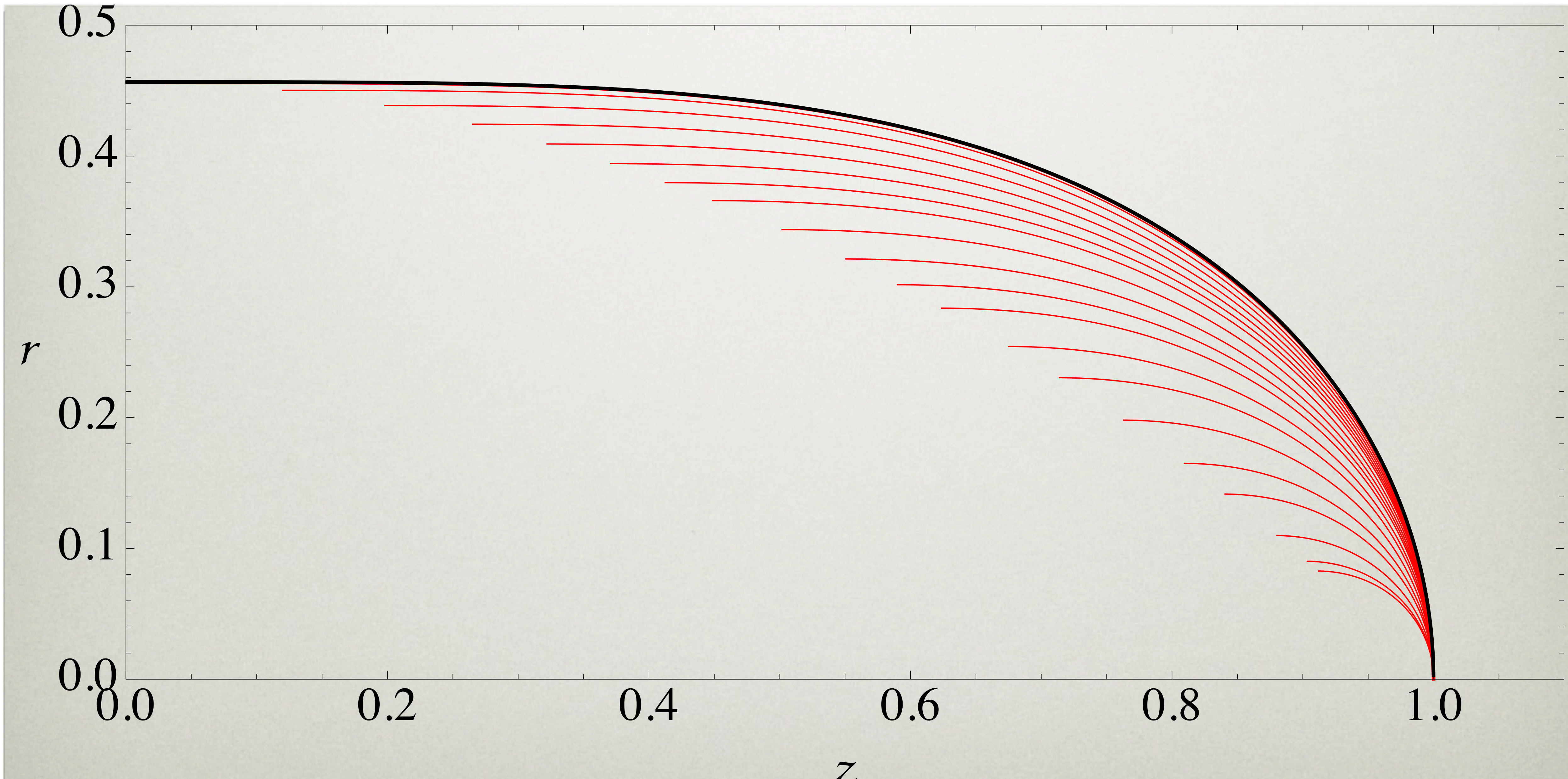
Braneworld black holes results

- 5d/4d behaviour



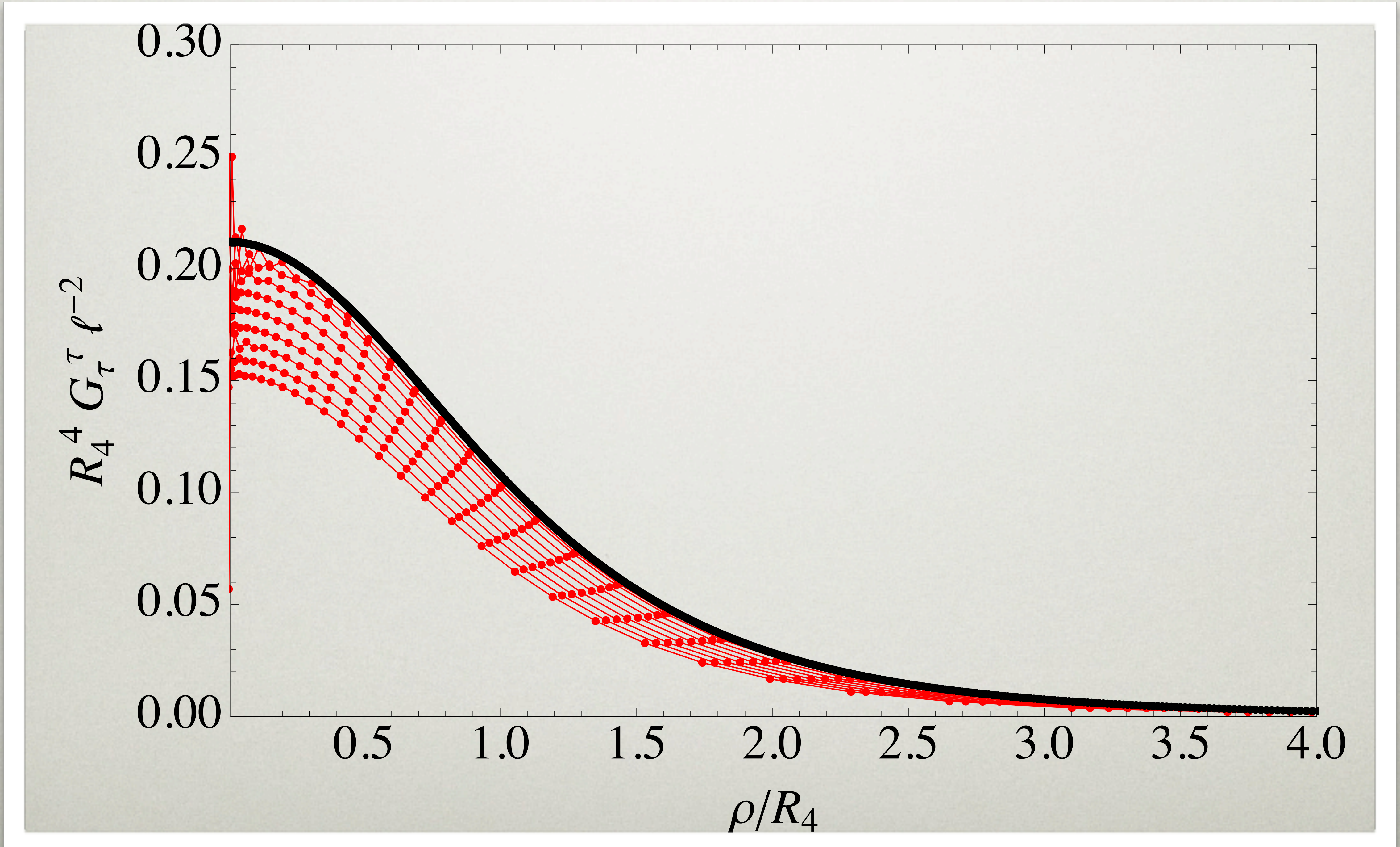
Braneworld black holes results

- Embeddings of the horizon geometry into $\mathbb{H}^4$



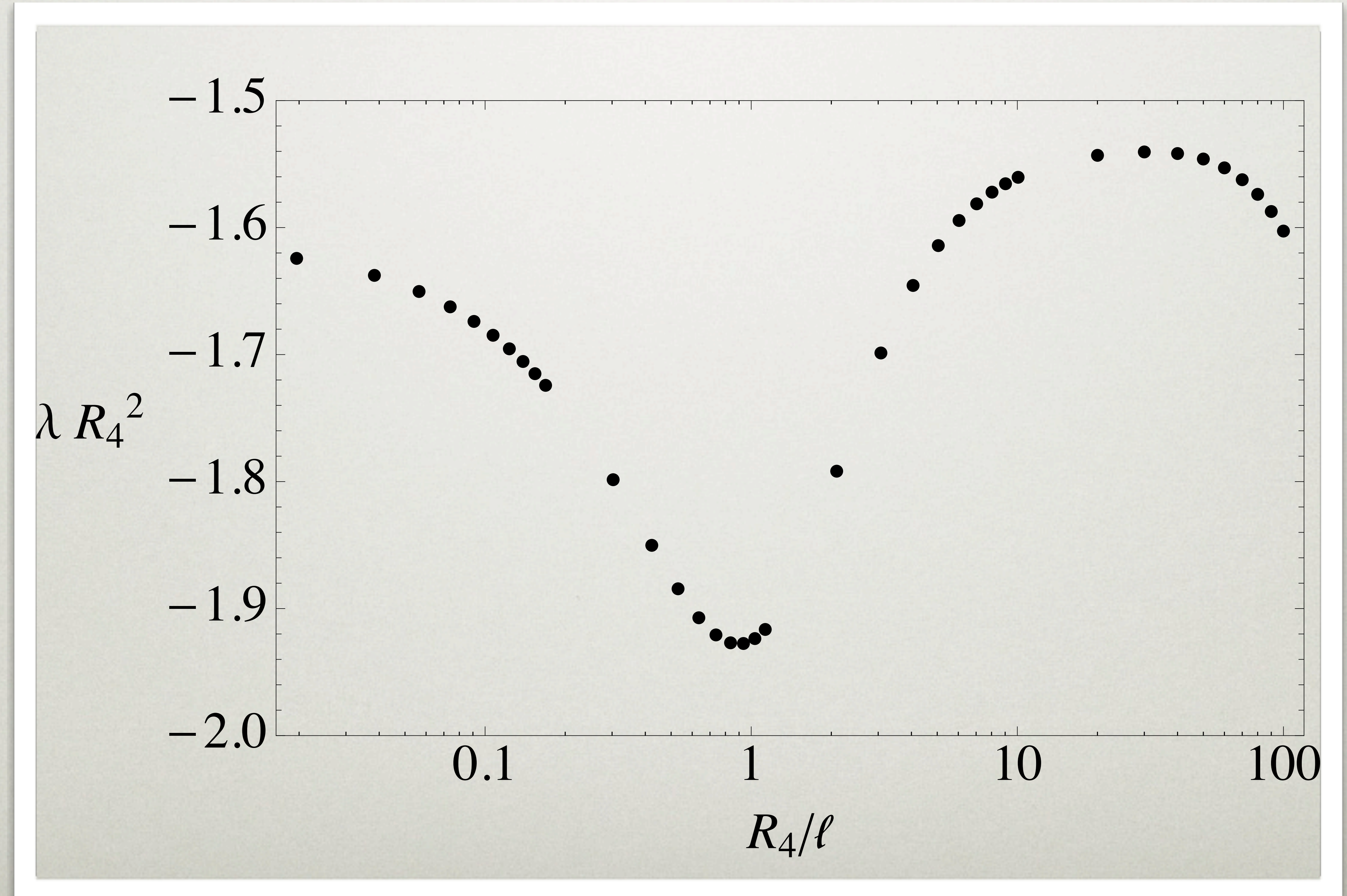
Braneworld black holes results

- Einstein tensor on the brane



Braneworld black holes results

- Spectrum of Δ_L



Summary

• The method that we have used is based on a characteristic formulation of the Einstein equations:

- Numerical stability.
- Fully covariant: allows for dependence on any number of coordinates.

• We have found a solution in AdS/CFT which corresponds to $\mathcal{N}$=4 SYM in the background of Schwarzschild in the Unruh vacuum.

• The AdS/CFT solution with 4d Schwarzschild boundary metric allows to understand the existence of large braneworld black holes.

• We have found static non-extremal braneworld black holes of any size.

• Braneworld black holes are likely to be stable.

• 4d gravity is recovered on the brane for large black holes

Thank you!!!!