

Gravitational Waves from Binary Systems as Probes of the Universe

Nicolas Yunes
Assistant Professor
MSU

Sept 28th 2011,
JGRG 21 - Tohoku University

Standing on the Shoulders of...

Professors:

Clifford Will, Jim Gates, David Spergel, Stephon Alexander, Abhay Ashtekar, Sam Finn, Ben Owen, Bernd Bruegman, Pablo Laguna, Emanuele Berti, Alessandra Buonanno, Uli Sperhake, Dimitrios Psaltis, Avi Loeb, Scott Hughes, Carlos Sopuerta, Vitor Carodoso, Leonardo Gualtieri, **Takahiro Tanaka, Frans Pretorius, Neil Cornish, Cole Miller, Avi Loeb.**

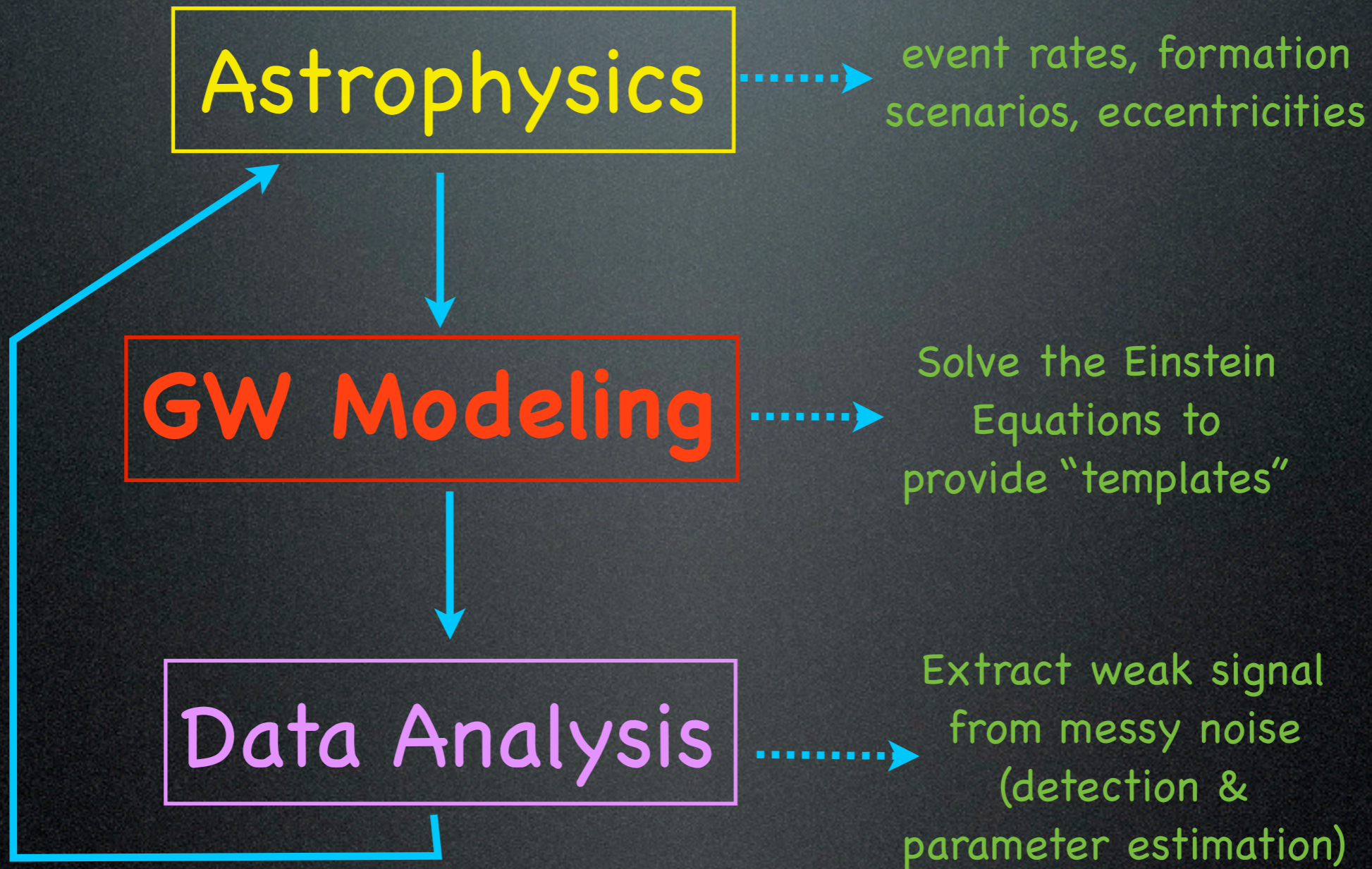
Post-Docs:

Victor Taveras, **Bence Kocsis**, Daniel Grumiller.

Graduate Students:

Laura Sampson, Leo Stein, Sarah Vigeland, Kent Yagi.

Multi-Messenger Astrophysics



GW Astrophysics: A two-way street

Astrophysical
Environment

Electromagnetic
Signal

Gravitational
Environment

Gravitational
Wave Event/
Signal

Detection on
Earth

GW Astrophysics: A two-way street



Example: Accretion Disk flare

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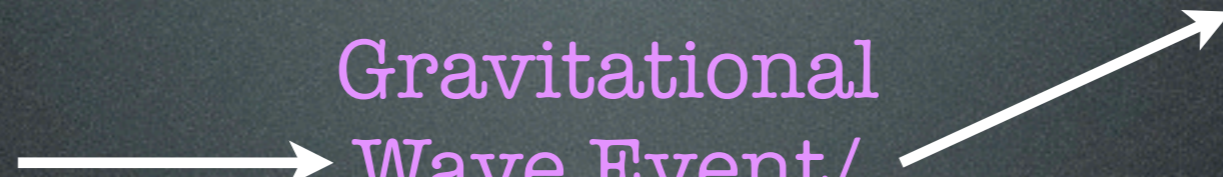
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Example: Accretion Disk flare

Example: BH-BH inspiral



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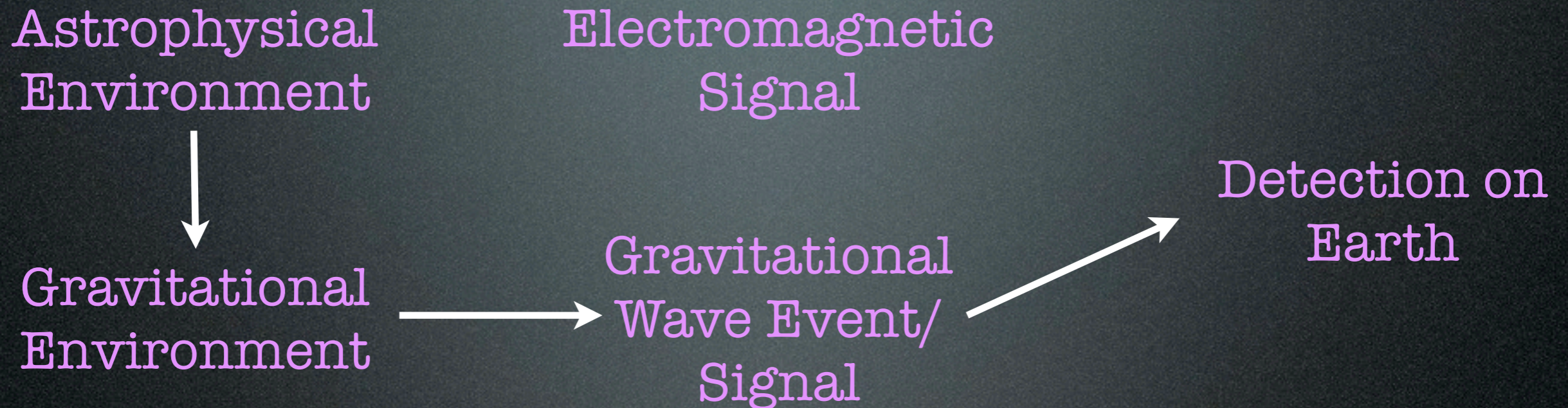
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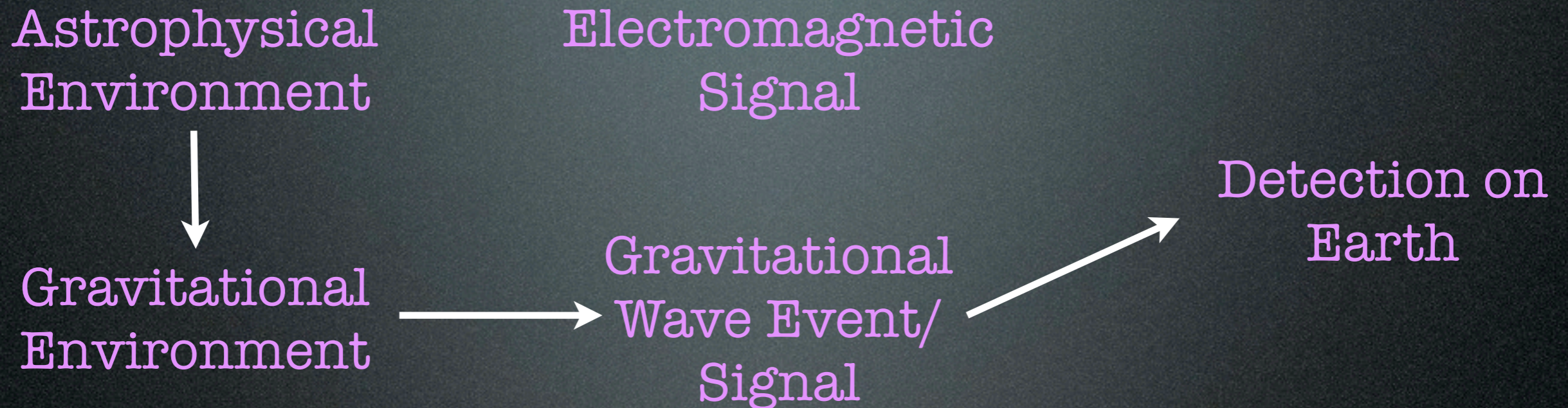
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Example: Accretion disk modifies the GW signal.

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Example: Accretion Disk flare

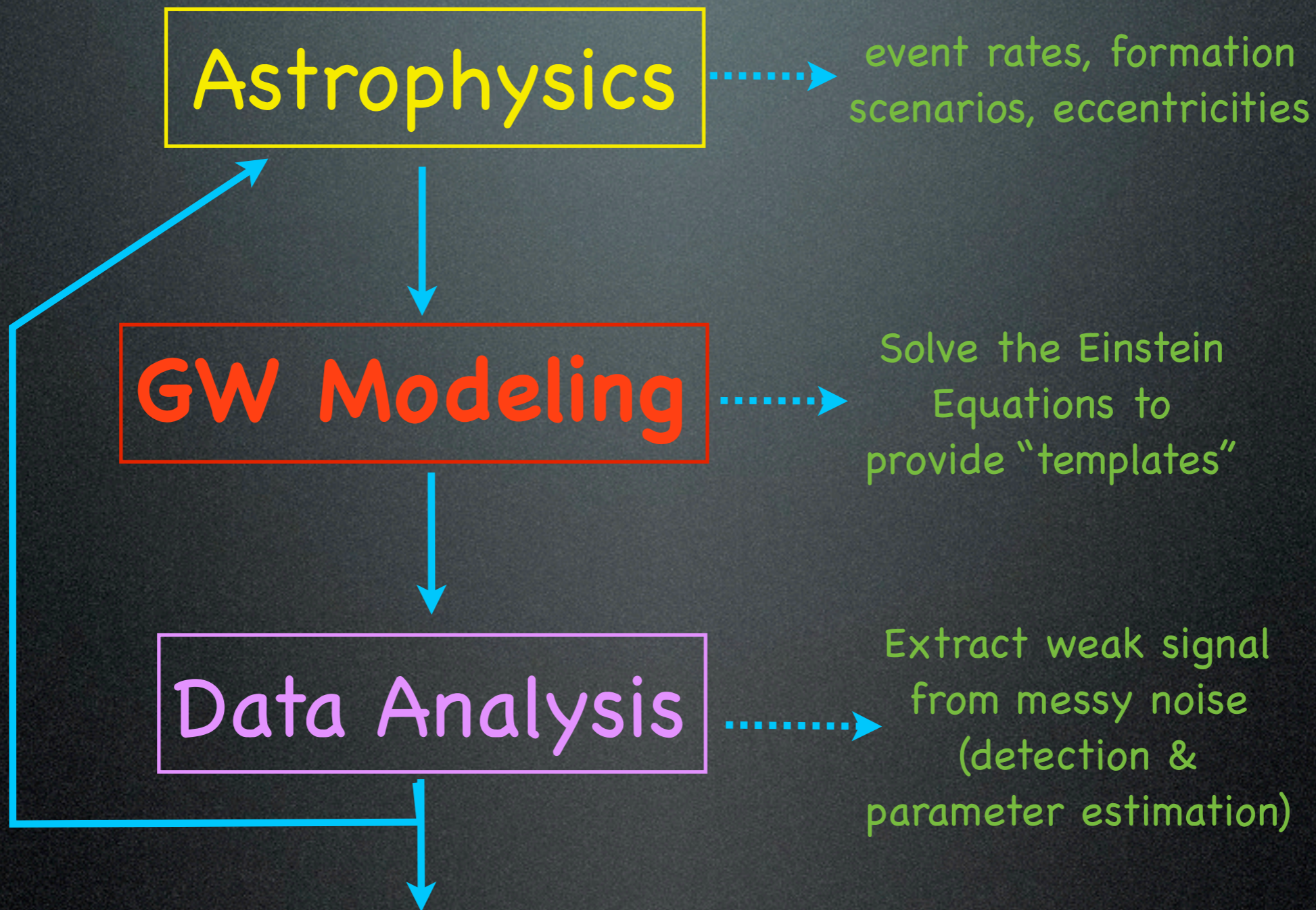
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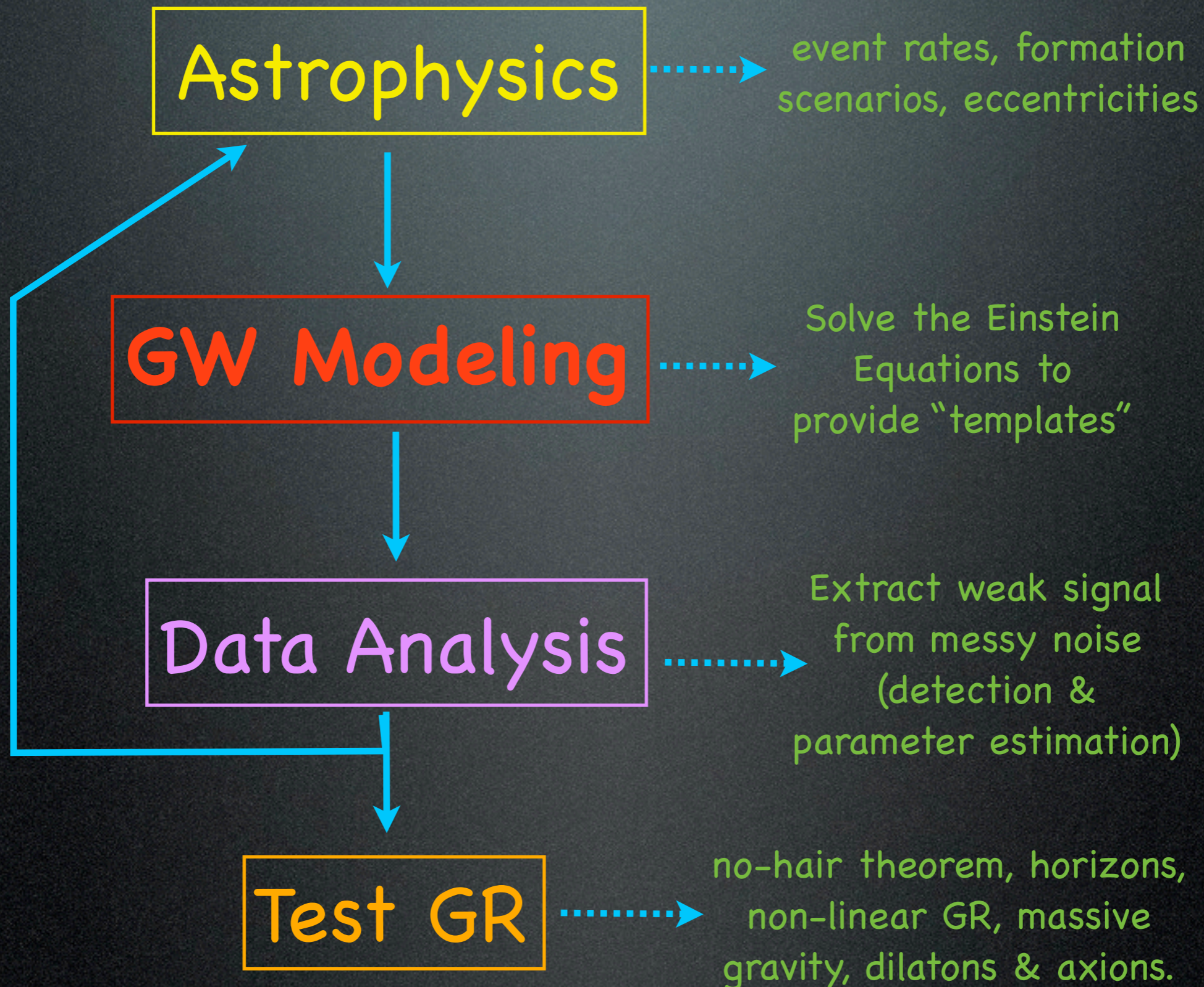
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Part of this talk is about how we can learn about astrophysics from gravitational wave detections.

Multi-Messenger Astrophysics



Multi-Messenger Astrophysics



GW Tests of GR

The non-linear, dynamical, strong-field regime of General Relativity remains completely unexplored.

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Eg. Brans-Dicke Theory.

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- Find generic features and construct a “meta”-theory to test. Eg. ppN, ppK, **ppE**

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- Find generic features and construct a “meta”-theory to test. Eg. ppN, ppK, **ppE**
- Search for model-independent deviations from GR in the strong-field.


Road Map

- I. Gravitational Wave Modeling
- II. Connection to Astrophysics
- III. Connection to Fundamental Theory

Part I: Gravitational Wave Modeling

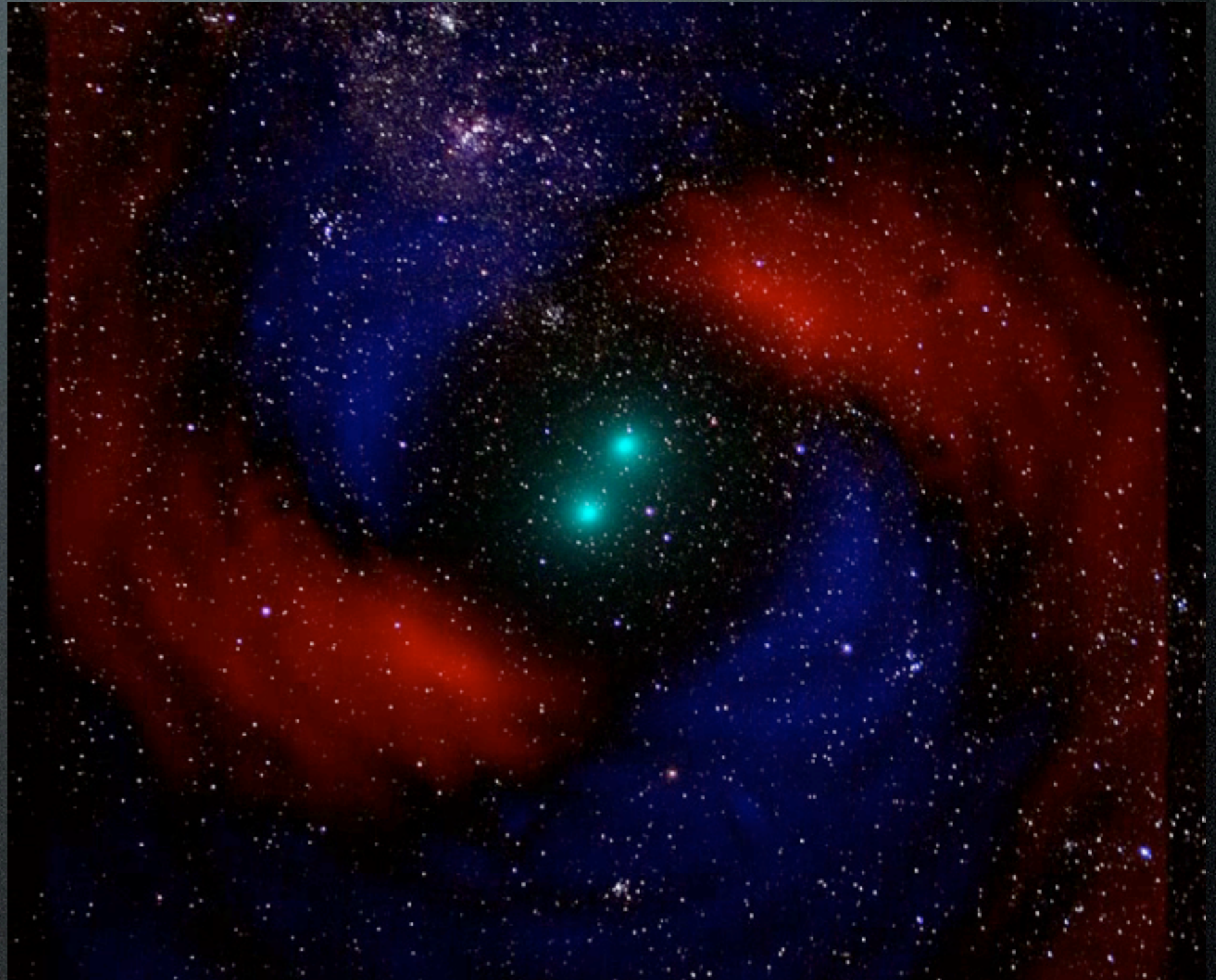
A Binary Black Hole Coalescence

$$m_1 = m_2 = 4e6 M_{\text{sun}}$$


$$10 M = 0.8 \text{ AU}$$

(courtesy,
I. Hinder)

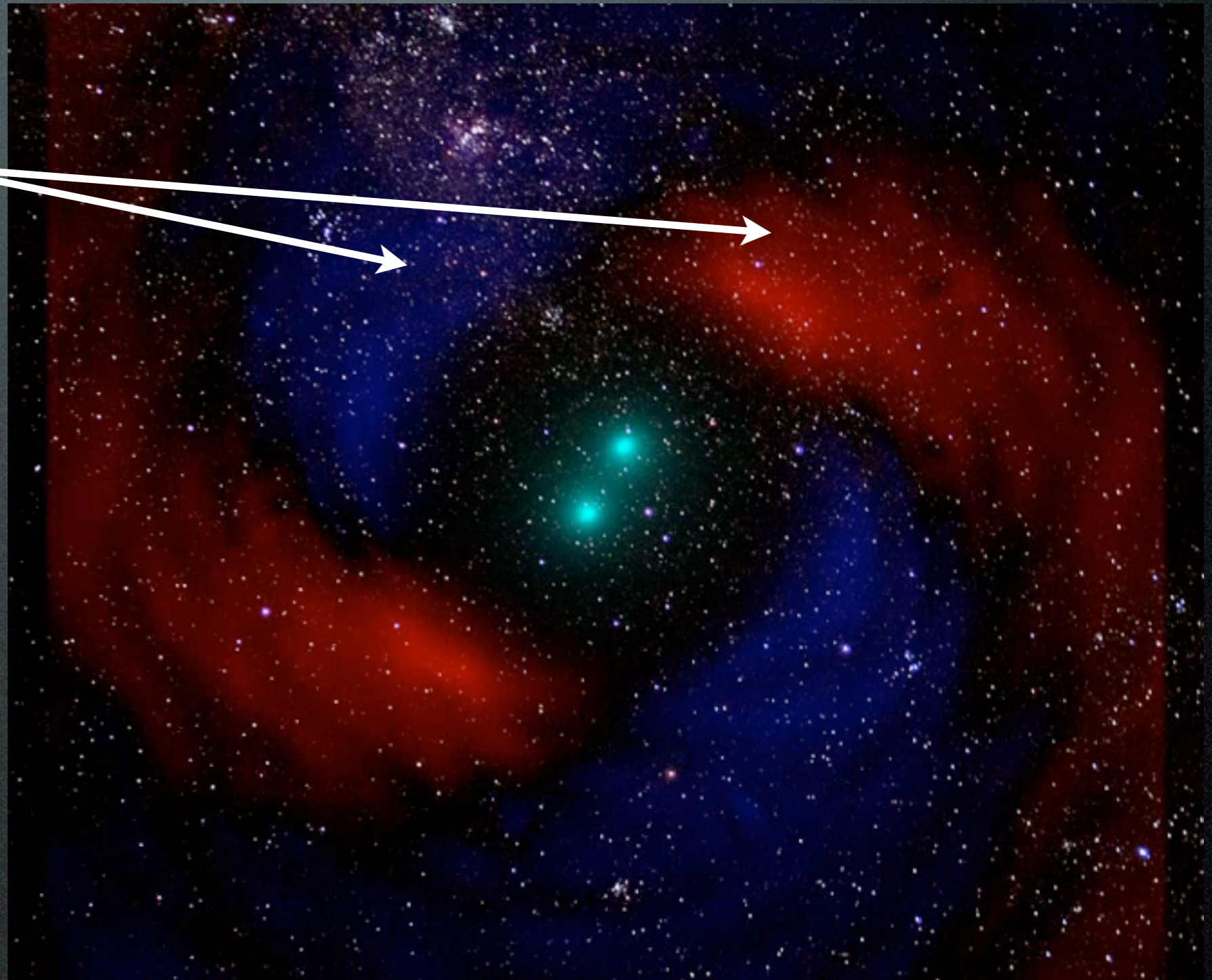
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Red/blue shows
grav. waves :=
ripples in space and
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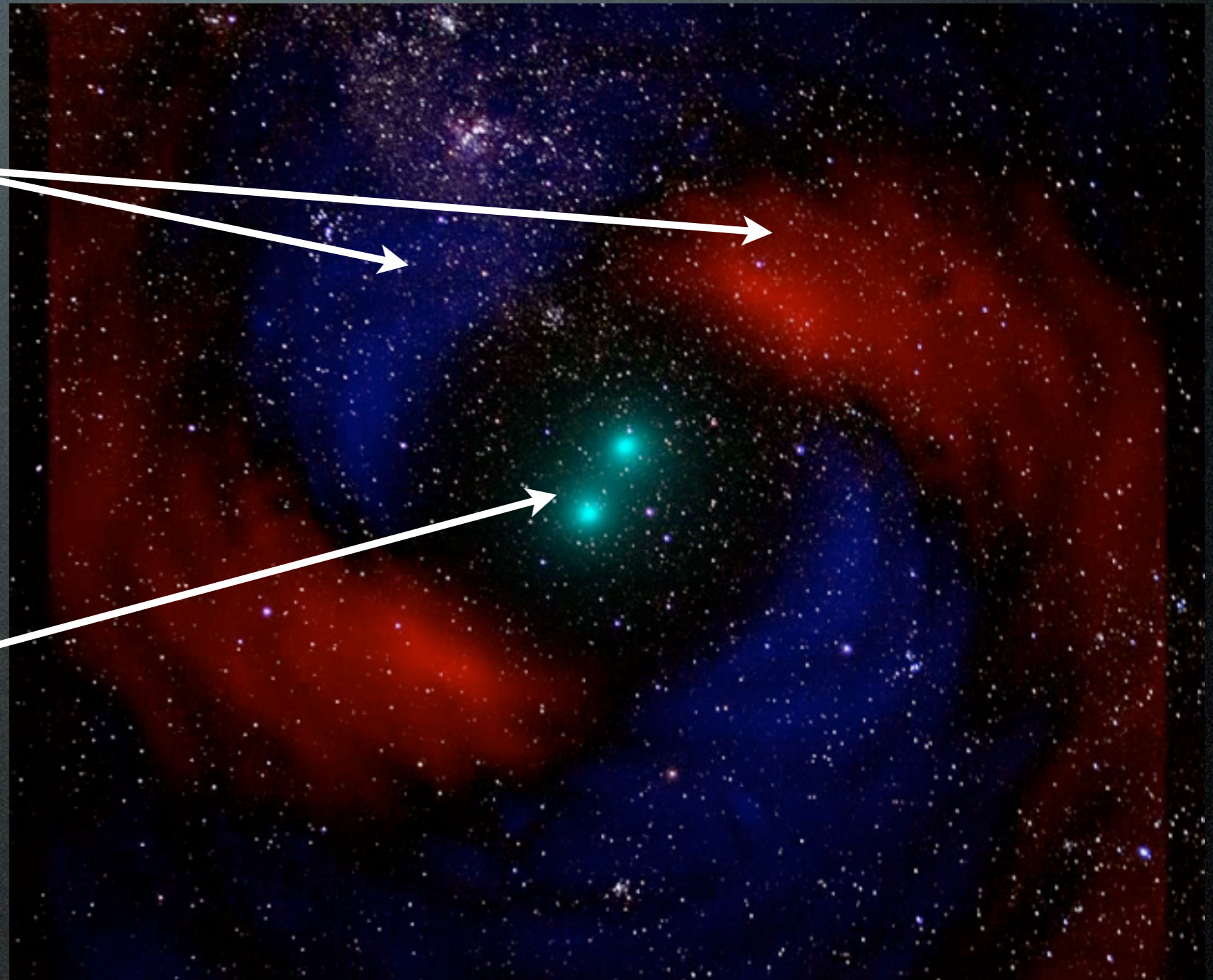


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A Binary Black Hole Coalescence

Red/blue shows
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light cyan shows the
strength of the grav.
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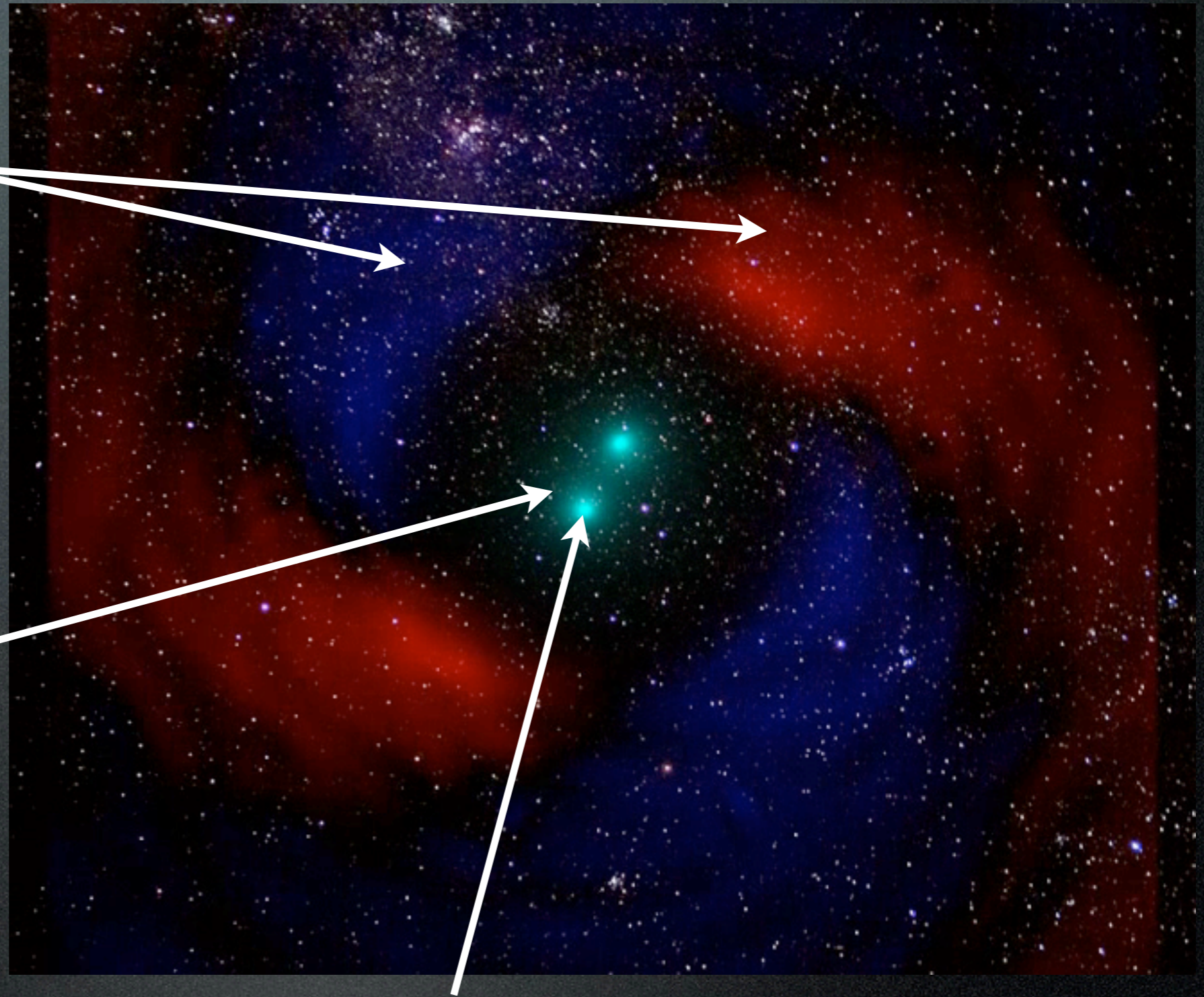
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solid cyan shows (roughly) the location of
the event horizon (BH surface)

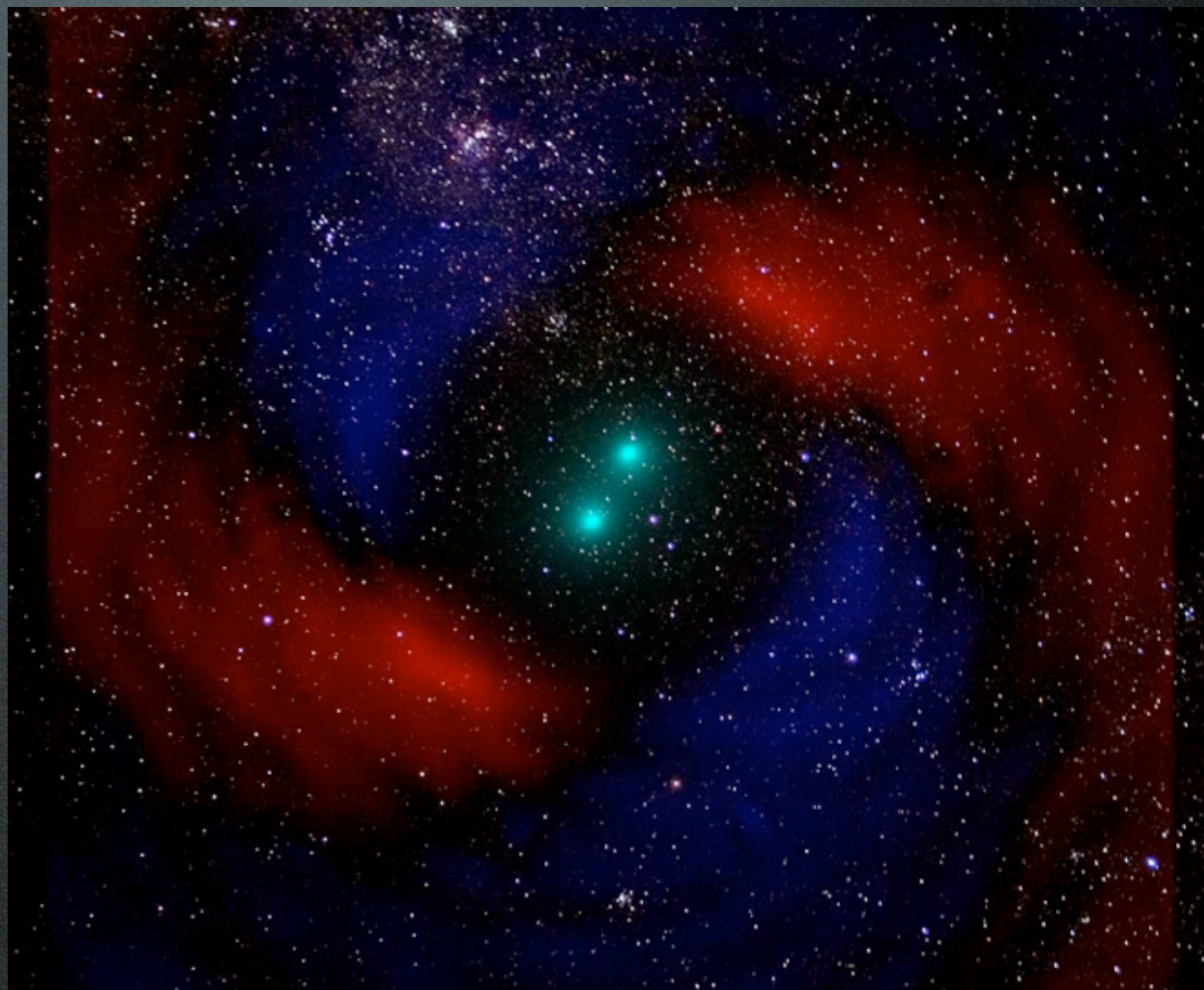
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Post-Newtonian Compact Binaries in GR

Metric Perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

(Futamase,
Will, Damour,
Blanchet, etc.)

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Metric Perturbation

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↑
metric
tensor

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Post-Newtonian Compact Binaries in GR

Metric Perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

metric
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flat

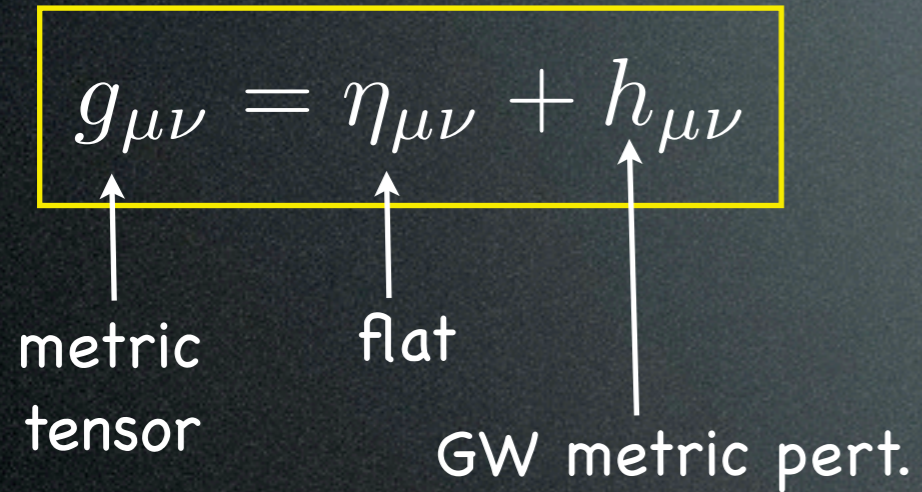
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metric tensor flat GW metric pert.



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$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

metric tensor flat GW metric pert.

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Einstein tensor

The diagram consists of two yellow-bordered boxes. The left box contains the equation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Below this box, three labels are positioned: 'metric tensor' under $g_{\mu\nu}$, 'flat' under $\eta_{\mu\nu}$, and 'GW metric pert.' under $h_{\mu\nu}$. White arrows point from each label to its corresponding term in the equation. The right box contains the equation $G_{\mu\nu} = 8\pi T_{\mu\nu}$. Below this box, the label 'Einstein tensor' is positioned under $G_{\mu\nu}$, with a white arrow pointing from the label to the term in the equation.

(Futamase,
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Post-Newtonian Compact Binaries in GR

Metric Perturbation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

metric tensor flat GW metric pert.

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Einstein tensor Stress-Energy tensor

The diagram consists of two equations enclosed in yellow boxes. The first equation is $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Below it, three labels are positioned: 'metric tensor' under $g_{\mu\nu}$, 'flat' under $\eta_{\mu\nu}$, and 'GW metric pert.' under $h_{\mu\nu}$. White arrows point from each label to its corresponding term in the equation. The second equation is $G_{\mu\nu} = 8\pi T_{\mu\nu}$. Below it, two labels are positioned: 'Einstein tensor' under $G_{\mu\nu}$ and 'Stress-Energy tensor' under $T_{\mu\nu}$. White arrows point from each label to its corresponding term in the equation.

(Futamase,
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Metric Perturbation

$$\boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}} \quad \boxed{G_{\mu\nu} = 8\pi T_{\mu\nu}} \quad \rightarrow \quad \boxed{\square_{\eta} h_{\mu\nu} = \tau_{\mu\nu}[h^2]}$$

metric tensor flat GW metric pert.

Einstein tensor Stress-Energy tensor

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Metric Perturbation

$$\begin{array}{ccc} \boxed{g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}} & \boxed{G_{\mu\nu} = 8\pi T_{\mu\nu}} & \rightarrow \boxed{\square_{\eta} h_{\mu\nu} = \tau_{\mu\nu} [h^2]} \\ \begin{array}{c} \uparrow \\ \text{metric} \\ \text{tensor} \end{array} & \begin{array}{c} \uparrow \\ \text{Einstein} \\ \text{tensor} \end{array} & \begin{array}{c} \uparrow \\ \text{Flat-space,} \\ \text{diff. wave op.} \end{array} \\ \begin{array}{c} \uparrow \\ \text{flat} \\ \text{GW metric pert.} \end{array} & \begin{array}{c} \uparrow \\ \text{Stress-Energy} \\ \text{tensor} \end{array} & \end{array}$$

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad G_{\mu\nu} = 8\pi T_{\mu\nu} \quad \rightarrow \quad \square_{\eta} h_{\mu\nu} = \tau_{\mu\nu}[h^2]$$

metric tensor flat GW metric pert. Einstein tensor Stress-Energy tensor Flat-space, diff. wave op. Annoying non-linearities

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Acceleration

$$\nabla^{\mu} (G_{\mu\nu} - 8\pi T_{\mu\nu}) = 0$$

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Solve Perturbatively, assuming

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$$\nabla^{\mu} (G_{\mu\nu} - 8\pi T_{\mu\nu}) = 0$$

Solve Perturbatively, assuming

Perfect Fluid

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu}$$

(Futamase, Will, Damour, Blanchet, etc.)

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Four-Velocity

(Futamase, Will, Damour, Blanchet, etc.)

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Four-Velocity Pressure

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Density Four-Velocity Pressure

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Density Four-Velocity Pressure

Point Particles

$$\rho = m \delta^3(x^i - y^i)$$

(Futamase, Will, Damour, Blanchet, etc.)

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Mass

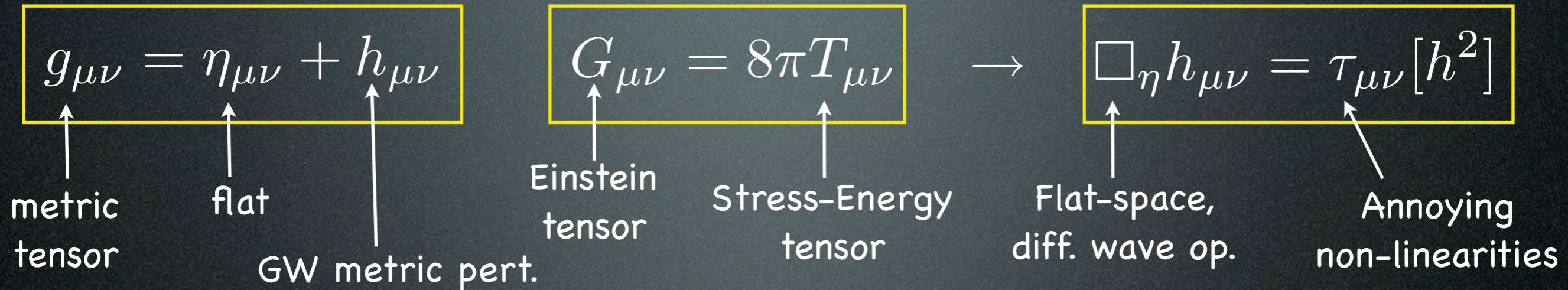
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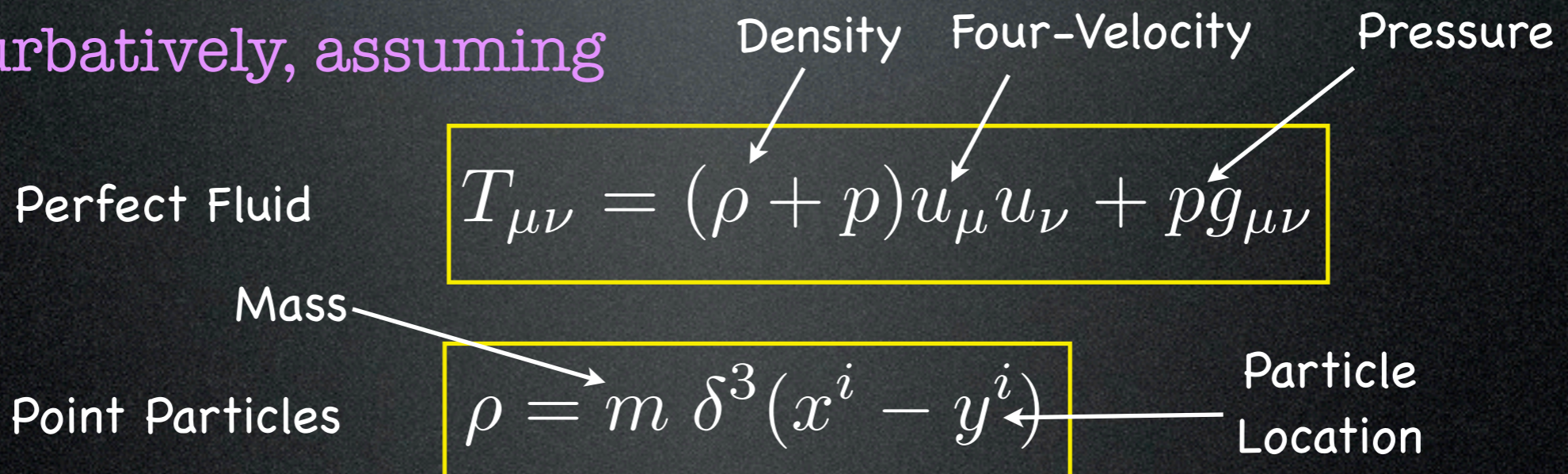
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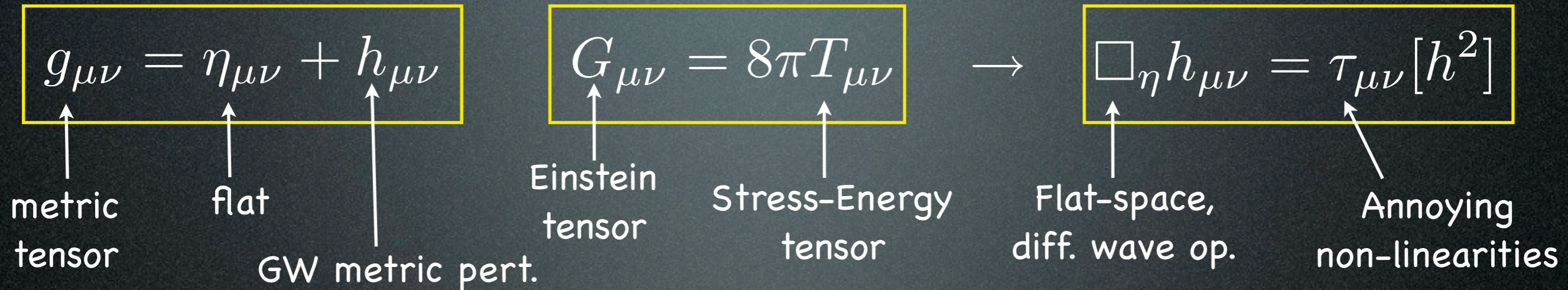
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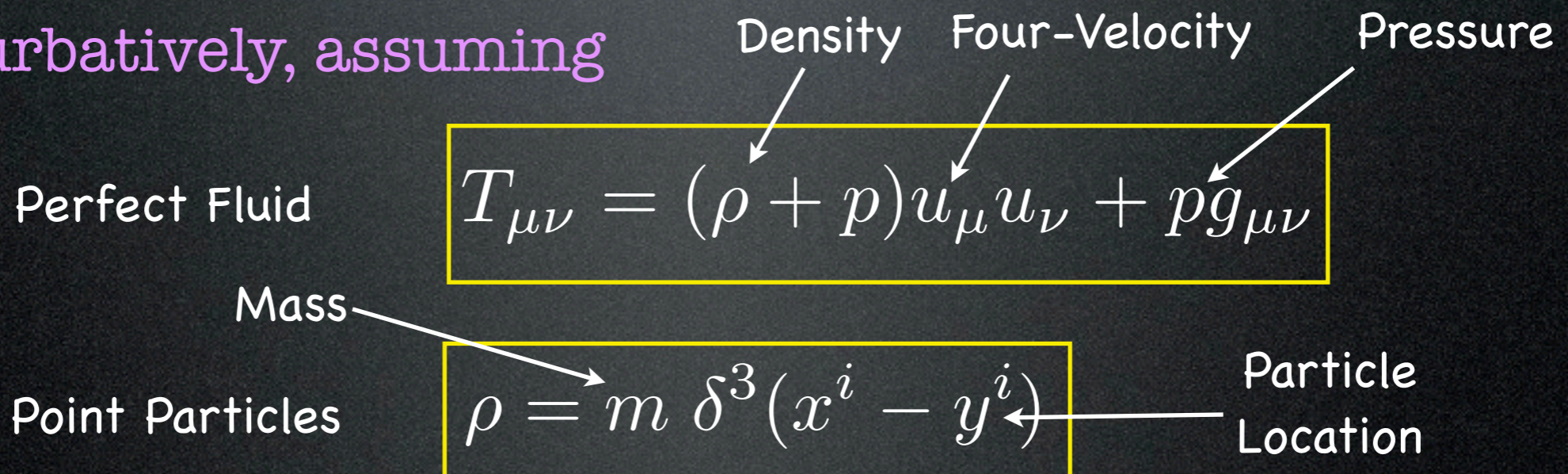
Metric Perturbation



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Solve Perturbatively, assuming



orbital velocity $\rightarrow v$ total mass m
 speed of light $\rightarrow c$ orbital separation r

$$\epsilon := \frac{v}{c} = \sqrt{\frac{m}{r}} \ll 1$$

(Futamase, Will, Damour, Blanchet, etc.)

Intimidation Slide

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$$a_1^i = -\frac{Gm_2n_{12}^i}{r_{12}^2}$$

Leading term: Newtonian gravity.

Intimidation Slide

$$a_1^i = -\frac{Gm_2 n_{12}^i}{r_{12}^2} + \frac{1}{c^2} \left\{ \left[\frac{5G^2 m_1 m_2}{r_{12}^3} + \frac{4G^2 m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2} \left(\frac{3}{2} (n_{12} v_2)^2 - v_1^2 + 4(v_1 v_2) - 2v_2^2 \right) \right] n_{12}^i + \frac{Gm_2}{r_{12}^2} (4(n_{12} v_1) - 3(n_{12} v_2)) v_{12}^i \right\}$$

Leading term: Newtonian gravity.

1 PN (Relativity corrections)

Intimidation Slide

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1 PN (Relativity corrections)

$$+ \frac{1}{c^4} \left\{ \left[-\frac{57G^3 m_1^2 m_2}{4r_{12}^4} - \frac{69G^3 m_1 m_2^2}{2r_{12}^4} - \frac{9G^3 m_2^3}{r_{12}^4} + \frac{Gm_2}{r_{12}^2} \left(-\frac{15}{8} (n_{12} v_2)^4 + \frac{3}{2} (n_{12} v_2)^2 v_1^2 - 6(n_{12} v_2)^2 (v_1 v_2) - 2(v_1 v_2)^2 + \frac{9}{2} (n_{12} v_2)^2 v_2^2 + 4(v_1 v_2) v_2^2 - 2v_2^4 \right) + \frac{G^2 m_1 m_2}{r_{12}^3} \left(\frac{39}{2} (n_{12} v_1)^2 - 39(n_{12} v_1)(n_{12} v_2) + \frac{17}{2} (n_{12} v_2)^2 - \frac{15}{4} v_1^2 - \frac{5}{2} (v_1 v_2) + \frac{5}{4} v_2^2 \right) + \frac{G^2 m_2^2}{r_{12}^3} (2(n_{12} v_1)^2 - 4(n_{12} v_1)(n_{12} v_2) - 6(n_{12} v_2)^2 - 8(v_1 v_2) + 4v_2^2) \right] n_{12}^i + \left[\frac{G^2 m_2^2}{r_{12}^3} (-2(n_{12} v_1) - 2(n_{12} v_2)) + \frac{G^2 m_1 m_2}{r_{12}^3} \left(-\frac{63}{4} (n_{12} v_1) + \frac{55}{4} (n_{12} v_2) \right) + \frac{Gm_2}{r_{12}^2} \left(-6(n_{12} v_1)(n_{12} v_2)^2 + \frac{9}{2} (n_{12} v_2)^3 + (n_{12} v_2) v_1^2 - 4(n_{12} v_1)(v_1 v_2) + 4(n_{12} v_2)(v_1 v_2) + 4(n_{12} v_1) v_2^2 - 5(n_{12} v_2) v_2^2 \right) \right] v_{12}^i \right\}$$

2 PN (more corrections ...)

$$+ \frac{1}{c^5} \left\{ \left[\frac{208G^3 m_1 m_2^2}{15r_{12}^4} (n_{12} v_{12}) - \frac{24G^3 m_1^2 m_2}{5r_{12}^4} (n_{12} v_{12}) + \frac{12G^2 m_1 m_2}{5r_{12}^3} (n_{12} v_{12}) v_{12}^2 \right] n_{12}^i + \left[\frac{8G^3 m_1^2 m_2}{5r_{12}^4} - \frac{32G^3 m_1 m_2^2}{5r_{12}^4} - \frac{4G^2 m_1 m_2}{5r_{12}^3} v_{12}^2 \right] v_{12}^i \right\}$$

and craziness ensues...

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3 PN (yet more corrections ...)

$$\begin{aligned}
 & + \frac{1}{c^6} \left\{ \left[\frac{Gm_2}{r_{12}^2} \left(\frac{35}{16}(n_{12}v_2)^6 - \frac{15}{8}(n_{12}v_2)^4 v_1^2 + \frac{15}{2}(n_{12}v_2)^4 (v_1 v_2) + 3(n_{12}v_2)^2 (v_1 v_2)^2 \right. \right. \right. \\
 & \quad - \frac{15}{2}(n_{12}v_2)^4 v_2^2 + \frac{3}{2}(n_{12}v_2)^2 v_1^2 v_2^2 - 12(n_{12}v_2)^2 (v_1 v_2) v_2^2 - 2(v_1 v_2)^2 v_2^2 \\
 & \quad \left. \left. \left. + \frac{15}{2}(n_{12}v_2)^2 v_2^4 + 4(v_1 v_2) v_2^4 - 2v_2^6 \right) \right. \right. \\
 & + \frac{G^2 m_1 m_2}{r_{12}^3} \left(-\frac{171}{8}(n_{12}v_1)^4 + \frac{171}{2}(n_{12}v_1)^3 (n_{12}v_2) - \frac{723}{4}(n_{12}v_1)^2 (n_{12}v_2)^2 \right. \\
 & \quad + \frac{383}{2}(n_{12}v_1)(n_{12}v_2)^3 - \frac{455}{8}(n_{12}v_2)^4 + \frac{229}{4}(n_{12}v_1)^2 v_1^2 \\
 & \quad - \frac{205}{2}(n_{12}v_1)(n_{12}v_2) v_1^2 + \frac{191}{4}(n_{12}v_2)^2 v_1^2 - \frac{91}{8}v_1^4 - \frac{229}{2}(n_{12}v_1)^2 (v_1 v_2) \\
 & \quad + 244(n_{12}v_1)(n_{12}v_2)(v_1 v_2) - \frac{225}{2}(n_{12}v_2)^2 (v_1 v_2) + \frac{91}{2}v_1^2 (v_1 v_2) \\
 & \quad - \frac{177}{4}(v_1 v_2)^2 + \frac{229}{4}(n_{12}v_1)^2 v_2^2 - \frac{283}{2}(n_{12}v_1)(n_{12}v_2) v_2^2 \\
 & \quad \left. \left. \left. + \frac{259}{4}(n_{12}v_2)^2 v_2^2 - \frac{91}{4}v_1^2 v_2^2 + 43(v_1 v_2) v_2^2 - \frac{81}{8}v_2^4 \right) \right. \right. \\
 & + \frac{G^2 m_2^2}{r_{12}^3} \left(-6(n_{12}v_1)^2 (n_{12}v_2)^2 + 12(n_{12}v_1)(n_{12}v_2)^3 + 6(n_{12}v_2)^4 \right. \\
 & \quad + 4(n_{12}v_1)(n_{12}v_2)(v_1 v_2) + 12(n_{12}v_2)^2 (v_1 v_2) + 4(v_1 v_2)^2 \\
 & \quad \left. \left. \left. - 4(n_{12}v_1)(n_{12}v_2) v_2^2 - 12(n_{12}v_2)^2 v_2^2 - 8(v_1 v_2) v_2^2 + 4v_2^4 \right) \right. \right. \\
 & + \frac{G^3 m_2^3}{r_{12}^4} \left(-(n_{12}v_1)^2 + 2(n_{12}v_1)(n_{12}v_2) + \frac{43}{2}(n_{12}v_2)^2 + 18(v_1 v_2) - 9v_2^2 \right) \\
 & + \frac{G^3 m_1 m_2^2}{r_{12}^4} \left(\frac{415}{8}(n_{12}v_1)^2 - \frac{375}{4}(n_{12}v_1)(n_{12}v_2) + \frac{1113}{8}(n_{12}v_2)^2 - \frac{615}{64}(n_{12}v_{12})^2 \pi^2 \right. \\
 & \quad \left. \left. \left. + 18v_1^2 + \frac{123}{64}\pi^2 v_{12}^2 + 33(v_1 v_2) - \frac{33}{2}v_2^2 \right) \right. \right. \\
 & + \frac{G^3 m_1^2 m_2}{r_{12}^4} \left(-\frac{45887}{168}(n_{12}v_1)^2 + \frac{24025}{42}(n_{12}v_1)(n_{12}v_2) - \frac{10469}{42}(n_{12}v_2)^2 + \frac{48197}{840}v_1^2 \right. \\
 & \quad - \frac{36227}{420}(v_1 v_2) + \frac{36227}{840}v_2^2 + 110(n_{12}v_{12})^2 \ln\left(\frac{r_{12}}{r_1'}\right) - 22v_{12}^2 \ln\left(\frac{r_{12}}{r_1'}\right) \\
 & \quad \left. \left. \left. + \frac{16G^4 m_2^4}{r_{12}^5} + \frac{G^4 m_1^2 m_2^2}{r_{12}^5} \left(175 - \frac{41}{16}\pi^2 \right) + \frac{G^4 m_1^3 m_2}{r_{12}^5} \left(-\frac{3187}{1260} + \frac{44}{3} \ln\left(\frac{r_{12}}{r_1'}\right) \right) \right. \right. \right. \\
 & \quad \left. \left. \left. + \frac{G^4 m_1 m_2^3}{r_{12}^5} \left(\frac{110741}{630} - \frac{41}{16}\pi^2 - \frac{44}{3} \ln\left(\frac{r_{12}}{r_2'}\right) \right) \right) \right] n_{12}^4 \right. \\
 & + \left[\frac{Gm_2}{r_{12}^2} \left(\frac{15}{2}(n_{12}v_1)(n_{12}v_2)^4 - \frac{45}{8}(n_{12}v_2)^5 - \frac{3}{2}(n_{12}v_2)^3 v_1^2 + 6(n_{12}v_1)(n_{12}v_2)^2 (v_1 v_2) \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \quad - 6(n_{12}v_2)^3 (v_1 v_2) - 2(n_{12}v_2)(v_1 v_2)^2 - 12(n_{12}v_1)(n_{12}v_2)^2 v_2^2 + 12(n_{12}v_2)^3 v_2^2 \\
 & \quad + (n_{12}v_2) v_1^2 v_2^2 - 4(n_{12}v_1)(v_1 v_2) v_2^2 + 8(n_{12}v_2)(v_1 v_2) v_2^2 + 4(n_{12}v_1) v_2^4 \\
 & \quad \left. \left. \left. - 7(n_{12}v_2) v_2^4 \right) \right. \right. \\
 & + \frac{G^2 m_2^2}{r_{12}^3} \left(-2(n_{12}v_1)^2 (n_{12}v_2) + 8(n_{12}v_1)(n_{12}v_2)^2 + 2(n_{12}v_2)^3 + 2(n_{12}v_1)(v_1 v_2) \right. \\
 & \quad \left. \left. \left. + 4(n_{12}v_2)(v_1 v_2) - 2(n_{12}v_1) v_2^2 - 4(n_{12}v_2) v_2^2 \right) \right. \right. \\
 & + \frac{G^2 m_1 m_2}{r_{12}^3} \left(-\frac{243}{4}(n_{12}v_1)^3 + \frac{565}{4}(n_{12}v_1)^2 (n_{12}v_2) - \frac{269}{4}(n_{12}v_1)(n_{12}v_2)^2 \right. \\
 & \quad - \frac{95}{12}(n_{12}v_2)^3 + \frac{207}{8}(n_{12}v_1) v_1^2 - \frac{137}{8}(n_{12}v_2) v_1^2 - 36(n_{12}v_1)(v_1 v_2) \\
 & \quad \left. \left. \left. + \frac{27}{4}(n_{12}v_2)(v_1 v_2) + \frac{81}{8}(n_{12}v_1) v_2^2 + \frac{83}{8}(n_{12}v_2) v_2^2 \right) \right. \right. \\
 & + \frac{G^3 m_2^3}{r_{12}^4} (4(n_{12}v_1) + 5(n_{12}v_2)) \\
 & + \frac{G^3 m_1 m_2^2}{r_{12}^4} \left(-\frac{307}{8}(n_{12}v_1) + \frac{479}{8}(n_{12}v_2) + \frac{123}{32}(n_{12}v_{12}) \pi^2 \right) \\
 & + \frac{G^3 m_1^2 m_2}{r_{12}^4} \left(\frac{31397}{420}(n_{12}v_1) - \frac{36227}{420}(n_{12}v_2) - 44(n_{12}v_{12}) \ln\left(\frac{r_{12}}{r_1'}\right) \right) \left. \right\} v_{12}^2 \\
 & + \frac{1}{c^7} \left\{ \left[\frac{G^4 m_1^3 m_2}{r_{12}^5} \left(\frac{3992}{105}(n_{12}v_1) - \frac{4328}{105}(n_{12}v_2) \right) \right. \right. \\
 & + \frac{G^4 m_1^2 m_2^2}{r_{12}^5} \left(-\frac{13576}{105}(n_{12}v_1) + \frac{2872}{21}(n_{12}v_2) \right) - \frac{3172 G^4 m_1 m_2^3}{21 r_{12}^5} (n_{12}v_{12}) \\
 & + \frac{G^4 m_1^2 m_2}{r_{12}^5} \left(48(n_{12}v_1)^3 - \frac{696}{5}(n_{12}v_1)^2 (n_{12}v_2) + \frac{744}{5}(n_{12}v_1)(n_{12}v_2)^2 - \frac{288}{5}(n_{12}v_2)^3 \right. \\
 & \quad - \frac{4888}{105}(n_{12}v_1) v_1^2 + \frac{5056}{105}(n_{12}v_2) v_1^2 + \frac{2056}{21}(n_{12}v_1)(v_1 v_2) \\
 & \quad - \frac{2224}{21}(n_{12}v_2)(v_1 v_2) - \frac{1028}{21}(n_{12}v_1) v_2^2 + \frac{5812}{105}(n_{12}v_2) v_2^2 \\
 & + \frac{G^4 m_1 m_2^3}{r_{12}^5} \left(-\frac{582}{5}(n_{12}v_1)^3 + \frac{1746}{5}(n_{12}v_1)^2 (n_{12}v_2) - \frac{1954}{5}(n_{12}v_1)(n_{12}v_2)^2 \right. \\
 & \quad + 158(n_{12}v_2)^3 + \frac{3568}{105}(n_{12}v_{12}) v_1^2 - \frac{2864}{35}(n_{12}v_1)(v_1 v_2) \\
 & \quad \left. \left. \left. + \frac{10048}{105}(n_{12}v_2)(v_1 v_2) + \frac{1432}{35}(n_{12}v_1) v_2^2 - \frac{5752}{105}(n_{12}v_2) v_2^2 \right) \right. \right. \\
 & + \frac{G^4 m_1 m_2}{r_{12}^5} \left(-56(n_{12}v_{12})^5 + 60(n_{12}v_1)^3 v_{12}^2 - 180(n_{12}v_1)^2 (n_{12}v_2) v_{12}^2 \right.
 \end{aligned}$$

$$\begin{aligned}
 & \quad \left. \left. \left. + 174(n_{12}v_1)(n_{12}v_2)^2 v_{12}^2 - 54(n_{12}v_2)^3 v_{12}^2 - \frac{246}{35}(n_{12}v_{12}) v_{12}^4 \right. \right. \right. \\
 & \quad + \frac{1068}{35}(n_{12}v_1) v_1^2 (v_1 v_2) - \frac{984}{35}(n_{12}v_2) v_1^2 (v_1 v_2) - \frac{1068}{35}(n_{12}v_1)(v_1 v_2)^2 \\
 & \quad + \frac{180}{7}(n_{12}v_2)(v_1 v_2)^2 - \frac{534}{35}(n_{12}v_1) v_1^2 v_2^2 + \frac{90}{7}(n_{12}v_2) v_1^2 v_2^2 \\
 & \quad + \frac{984}{35}(n_{12}v_1)(v_1 v_2) v_2^2 - \frac{732}{35}(n_{12}v_2)(v_1 v_2) v_2^2 - \frac{204}{35}(n_{12}v_1) v_2^4 \\
 & \quad \left. \left. \left. + \frac{24}{7}(n_{12}v_2) v_2^4 \right) \right] n_{12}^4 \right. \\
 & + \left[-\frac{184 G^4 m_1^3 m_2}{21 r_{12}^5} + \frac{6224 G^4 m_1^2 m_2^2}{105 r_{12}^5} + \frac{6388 G^4 m_1 m_2^3}{105 r_{12}^5} \right. \\
 & \quad + \frac{G^5 m_1^2 m_2}{r_{12}^5} \left(\frac{52}{15}(n_{12}v_1)^2 - \frac{56}{15}(n_{12}v_1)(n_{12}v_2) - \frac{44}{15}(n_{12}v_2)^2 - \frac{132}{35}v_1^2 + \frac{152}{35}(v_1 v_2) \right. \\
 & \quad \left. \left. \left. - \frac{48}{35}v_2^2 \right) \right. \right. \\
 & + \frac{G^5 m_1 m_2^2}{r_{12}^5} \left(\frac{454}{15}(n_{12}v_1)^2 - \frac{372}{5}(n_{12}v_1)(n_{12}v_2) + \frac{854}{15}(n_{12}v_2)^2 - \frac{152}{21}v_1^2 \right. \\
 & \quad \left. \left. \left. + \frac{2864}{105}(v_1 v_2) - \frac{1768}{105}v_2^2 \right) \right. \right. \\
 & + \frac{G^5 m_1 m_2}{r_{12}^5} \left(60(n_{12}v_{12})^4 - \frac{348}{5}(n_{12}v_1)^2 v_{12}^2 + \frac{684}{5}(n_{12}v_1)(n_{12}v_2) v_{12}^2 \right. \\
 & \quad - 66(n_{12}v_2)^2 v_{12}^2 + \frac{334}{35}v_1^4 - \frac{1336}{35}v_1^2 (v_1 v_2) + \frac{1308}{35}(v_1 v_2)^2 + \frac{654}{35}v_1^2 v_2^2 \\
 & \quad \left. \left. \left. - \frac{1252}{35}(v_1 v_2) v_2^2 + \frac{292}{35}v_2^4 \right) \right] v_{12}^2 \right\} \\
 & + \mathcal{O}\left(\frac{1}{c^8}\right).
 \end{aligned}$$

[Blanchet 2006, Liv Rev Rel 9, 4, Eq. (168)]

Compact Binaries in **Effective** Alternative Theories

$$G_{\mu\nu} + \zeta C_{\mu\nu} = 8\pi T_{\mu\nu}$$

Start with the modified field equations

(Yagi, Stein, Yunes and Tanaka '10)

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Expand about Minkowski

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and now you can use the same PN tools as always to solve the above wave equations (see eg. the DIRE approach or dim regularization).

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But be careful!!

The point-particle description of BHs works in GR (in part due to the Birkhoff theorem), but this need not be so in Alternative Theories. In fact, usually one must compensate for violations of this description.

(Yagi, Stein, Yunes and Tanaka '10)

Semi-Analytical Phase Modeling

If we know that the only thing modified is the Hamiltonian and the Radiation-Reaction force --> Modified Hamiltonian Evolution
(effective-one-body approach)

Damour & Buonanno '08,
Yunes, et al, PRL '11,
PRD 83 '11, PRL 104 '10.

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A modification to either of these components leads to a correction that we might observe.

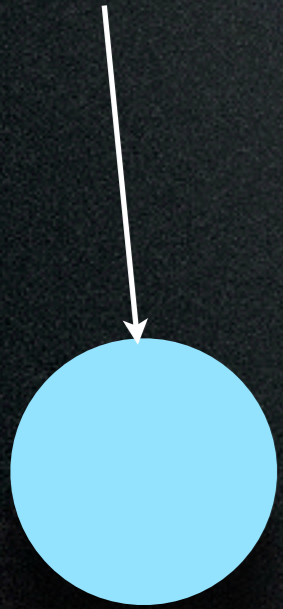
Part II: Connection to Astrophysics

EMRIs and Massive Perturbers

Yunes, Miller, Thornburg, PRD 83 (2010)

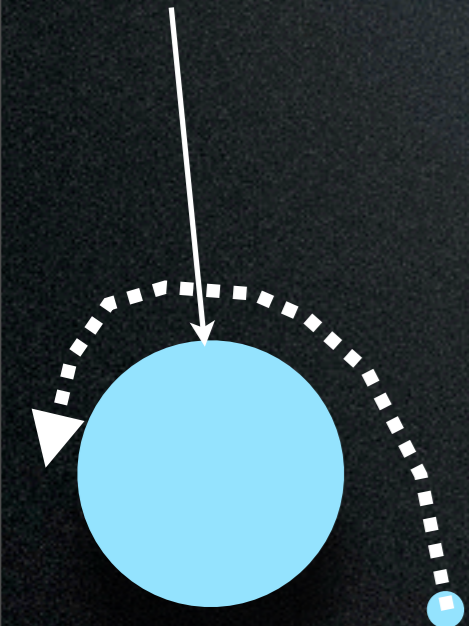
EMRIs and Massive Perturbers

SMBH w/mass M



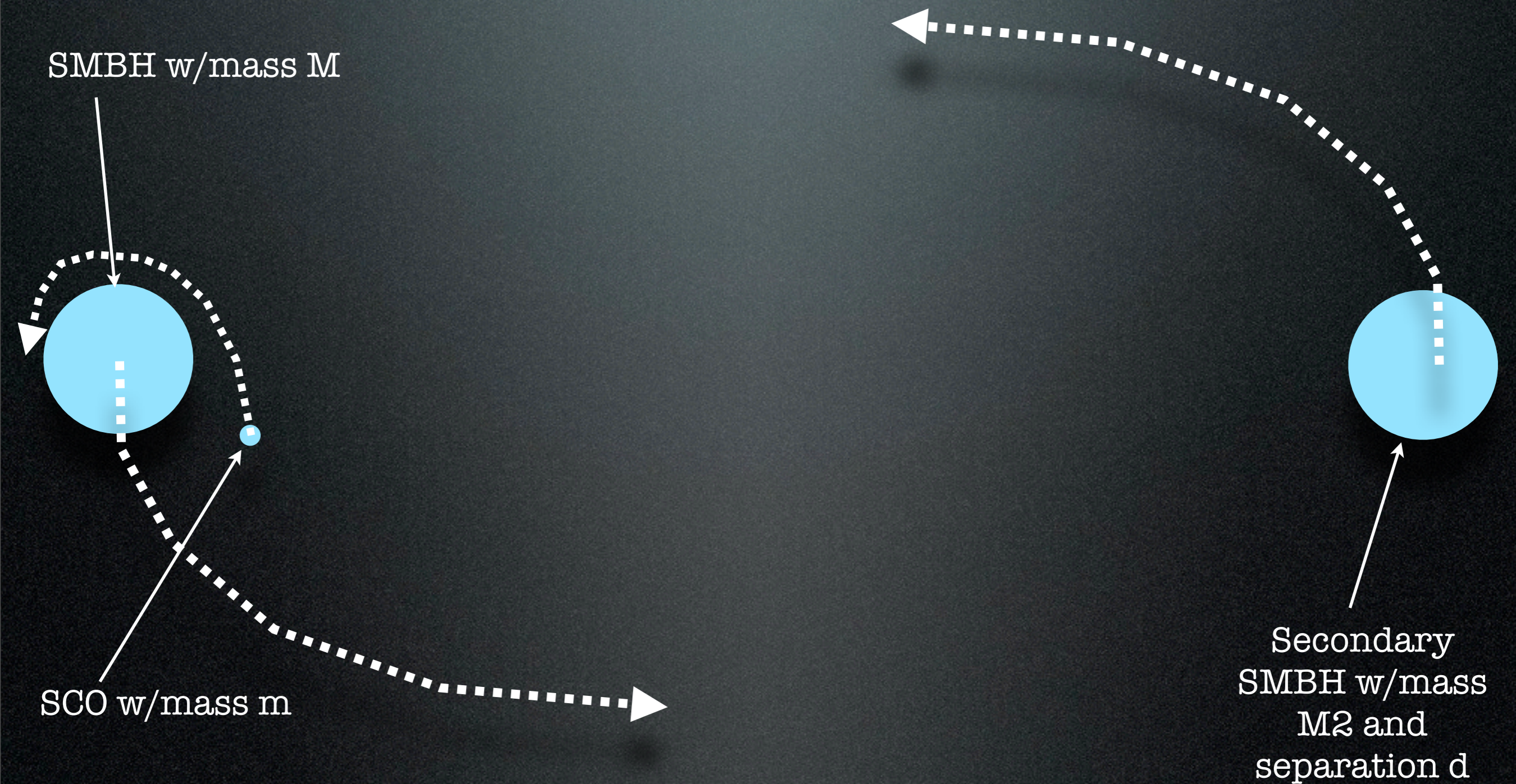
EMRIs and Massive Perturbers

SMBH w/mass M



SCO w/mass m

EMRIs and Massive Perturbers



Yunes, Miller, Thornburg, PRD 83 (2010)

EMRIs and Massive Perturbers

SMBH w/mass M

What must this separation and secondary mass be before we can see the imprint of the secondary on the EMRI gravitational waves?

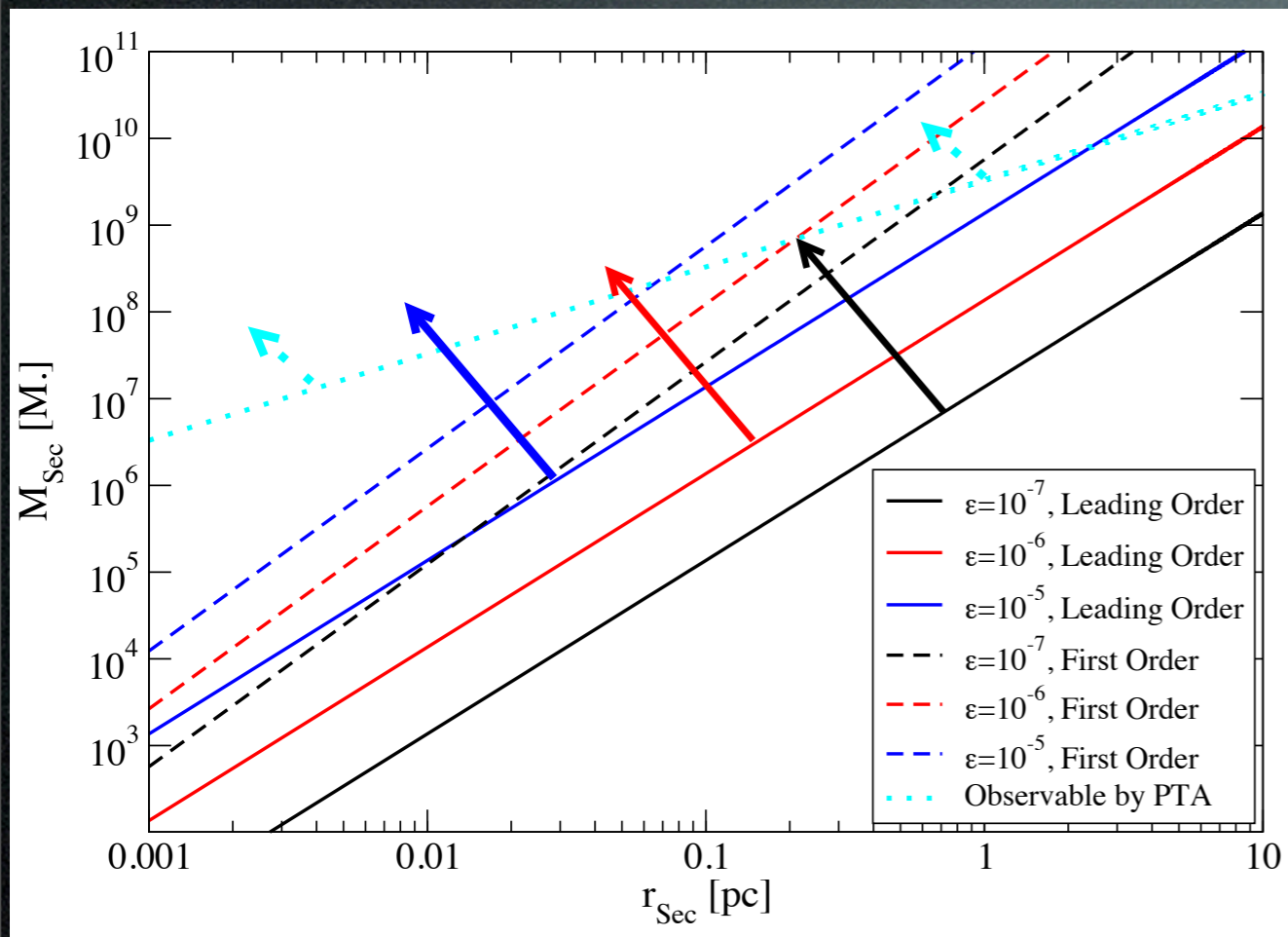
SCO w/mass m

Secondary SMBH w/mass M_2 and separation d

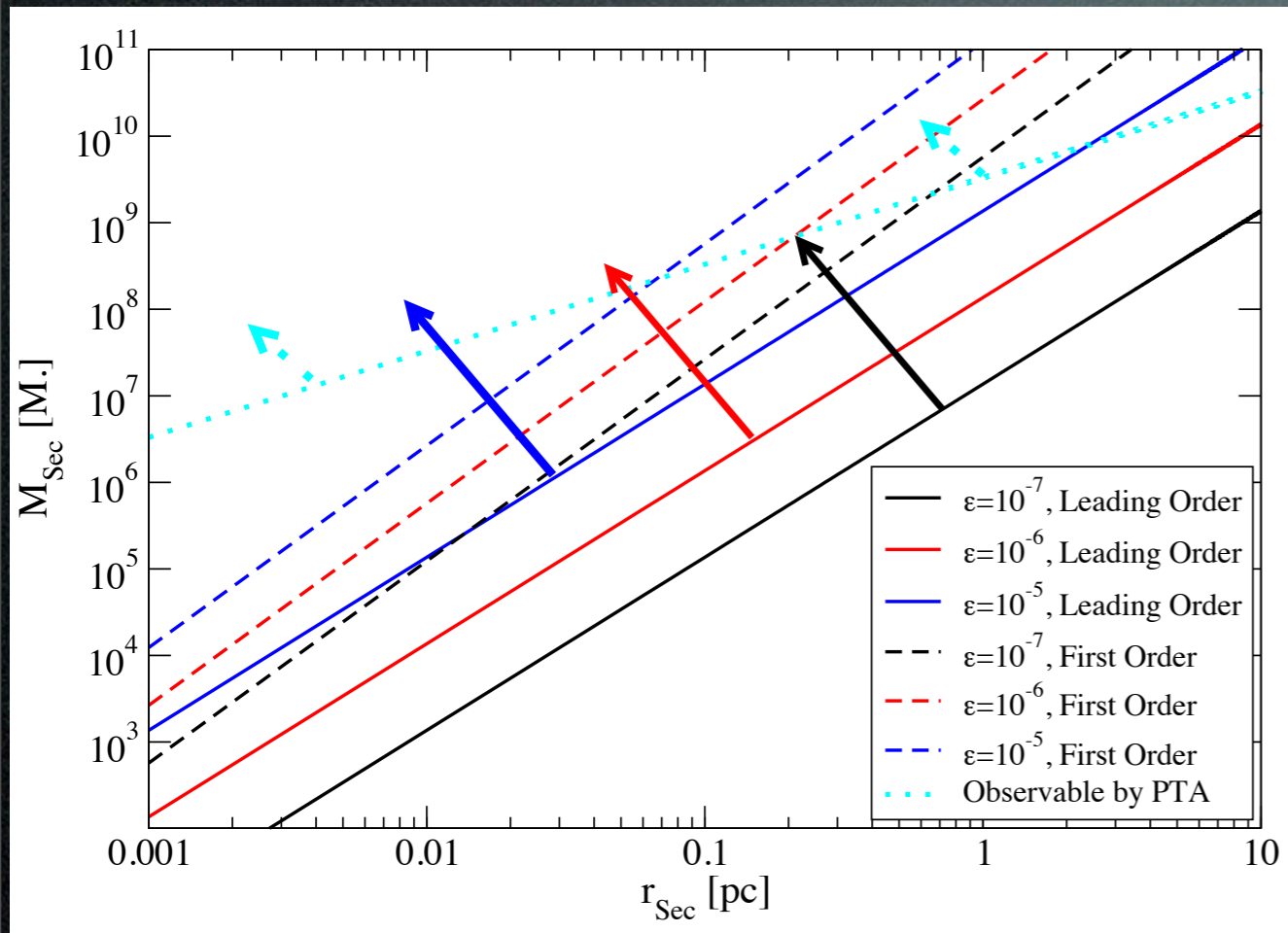
Yunes, Miller, Thornburg, PRD 83 (2010)

Detecting a Massive Companion

$$\Phi_{\text{GW}} = \int F(t) (1 + v_{\text{los}}) dt$$



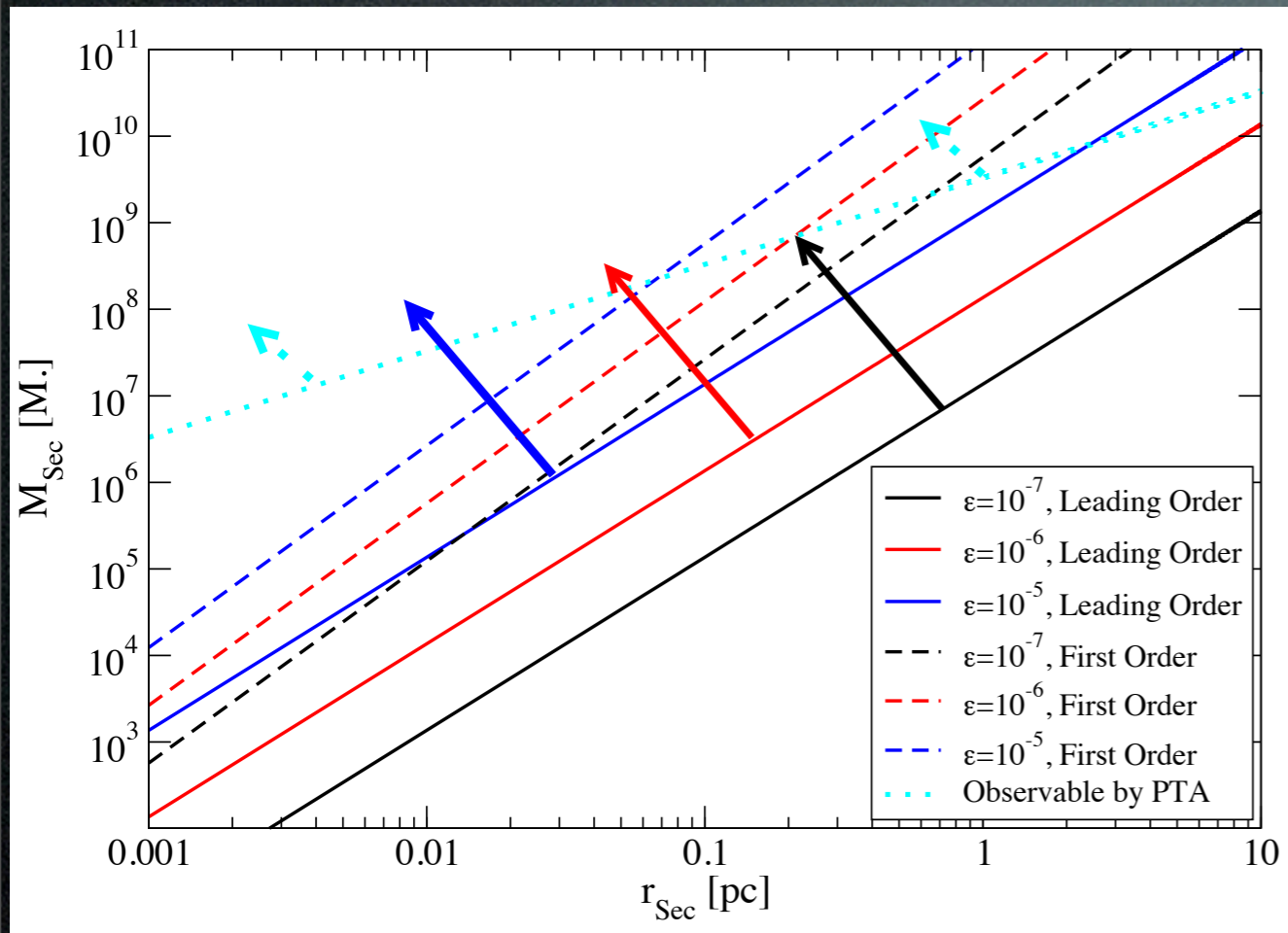
Detecting a Massive Companion



$$\Phi_{\text{GW}} = \int F(t) (1 + v_{\text{los}}) dt$$

Φ_{GW} (GW Phase) $F(t)$ (GW Frequency) v_{los} (Velocity along the line of sight)

Detecting a Massive Companion



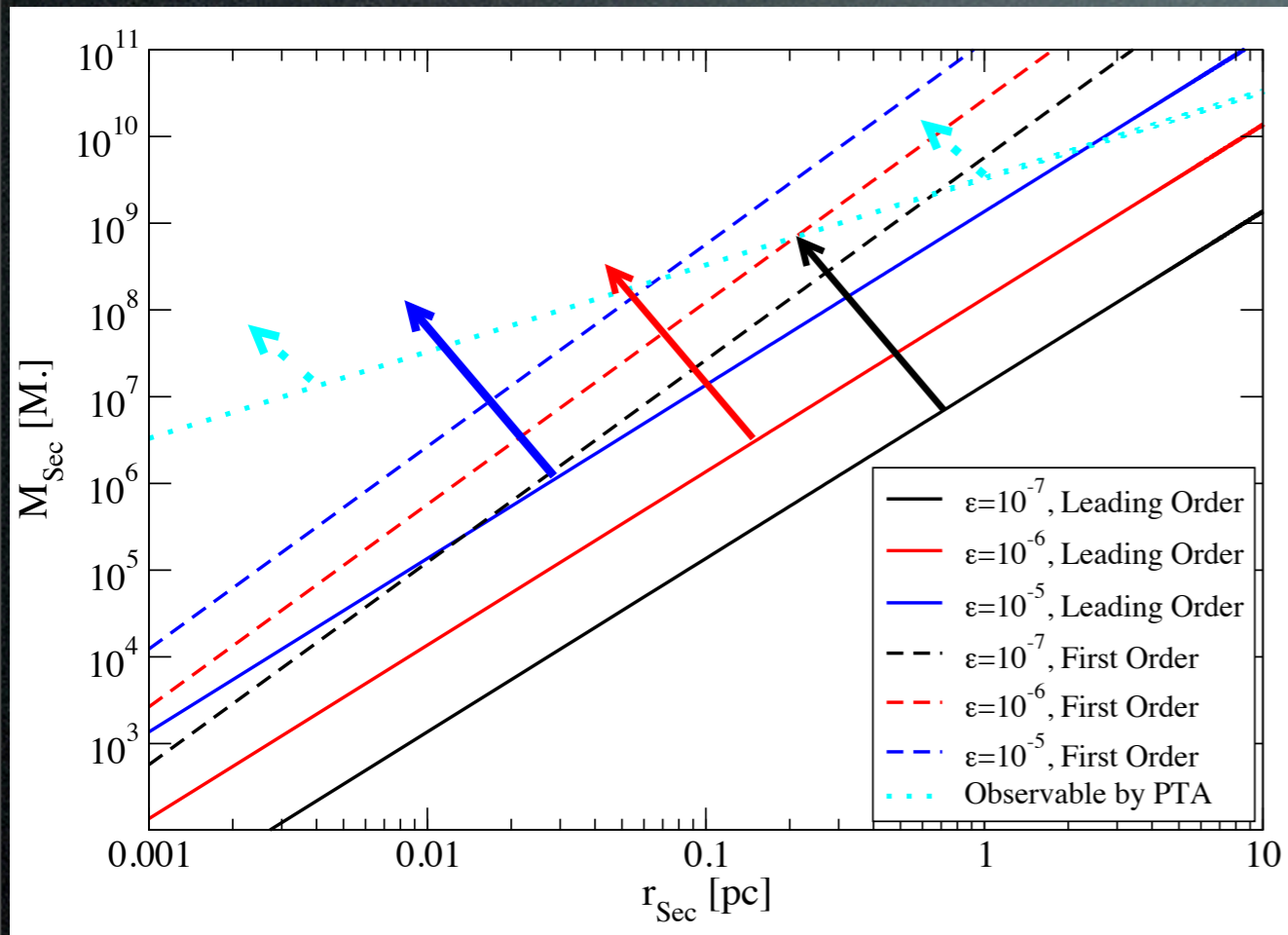
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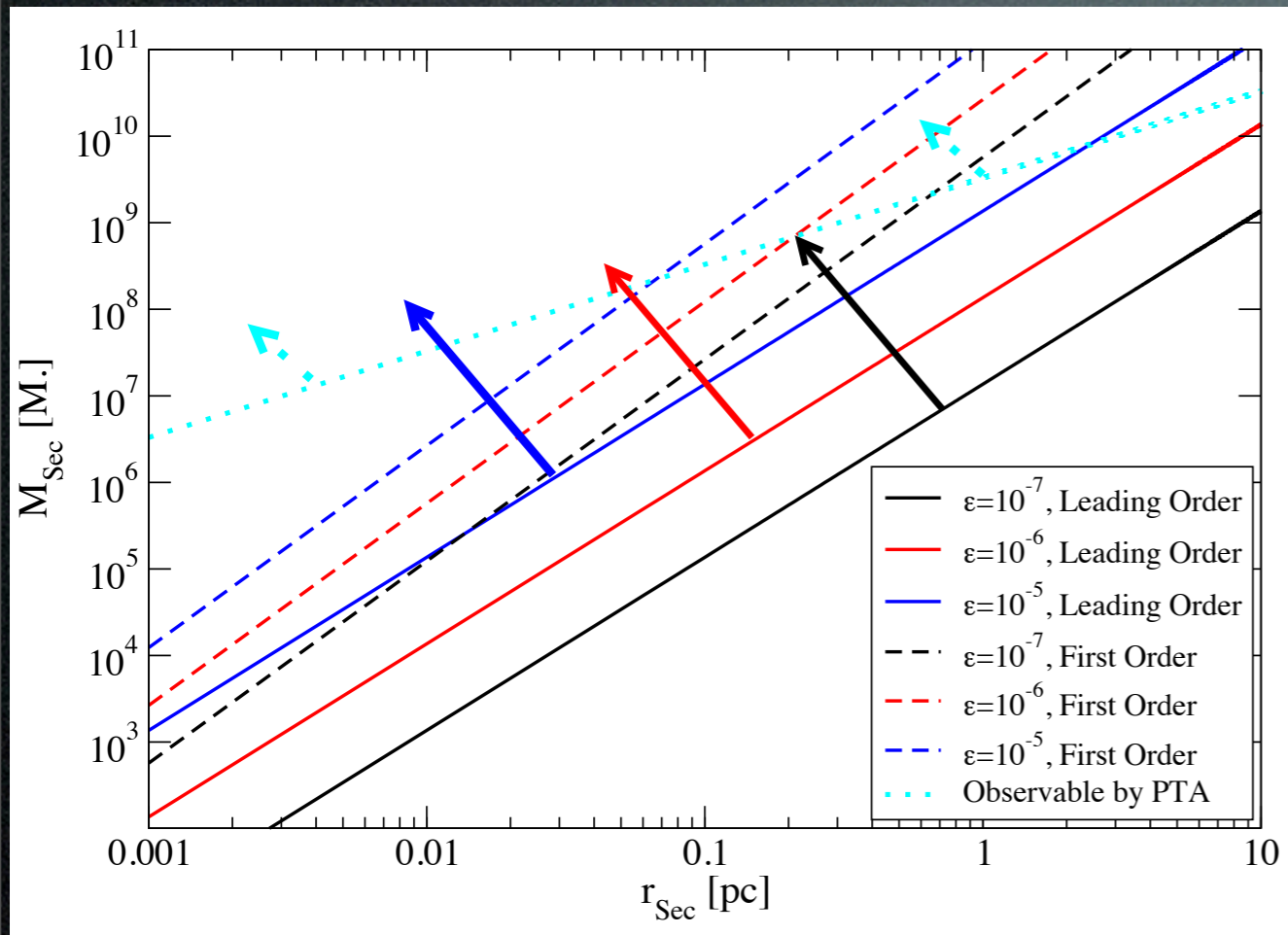
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\uparrow clear spectral signature

Yunes, Miller, Thornburg, PRD 83 (2010)

Accretion Disks and EMRIs

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Disk Effects Considered

Yunes, Kocsis, Haiman, Loeb (2011), Kocsis, Yunes, Loeb (2011)

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- Migration: Disk Spiral arms carry L away from the SCO, forcing it to inspiral into SMBH faster -> changes \dot{E} . Two types: I (no gap forms) and II (a gap forms, gas pile up).
 $\delta\phi \in (1, 10^3)$ rads

Part III: Connection to Fundamental Theory

Waveforms in Alternative Theories

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(i) Scalar-Tensor theories:

(Will '94, Scharre & Will '02, Will & Yunes '04, Berti, Buonanno & Will '05, Yagi & Tanaka '09)

$$\tilde{h} = \tilde{h}_{\text{GR}} e^i \beta_{\text{BD}} \eta^{2/5} u^{-7/3}$$

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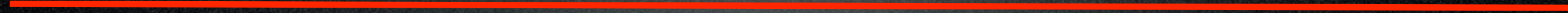
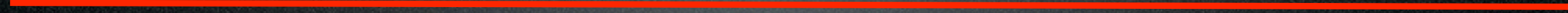
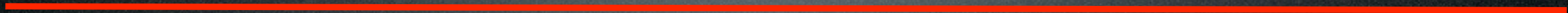
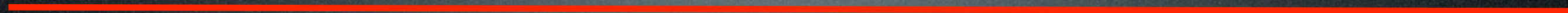
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Search GW Data with these templates and let the data decide what the parameters should be.

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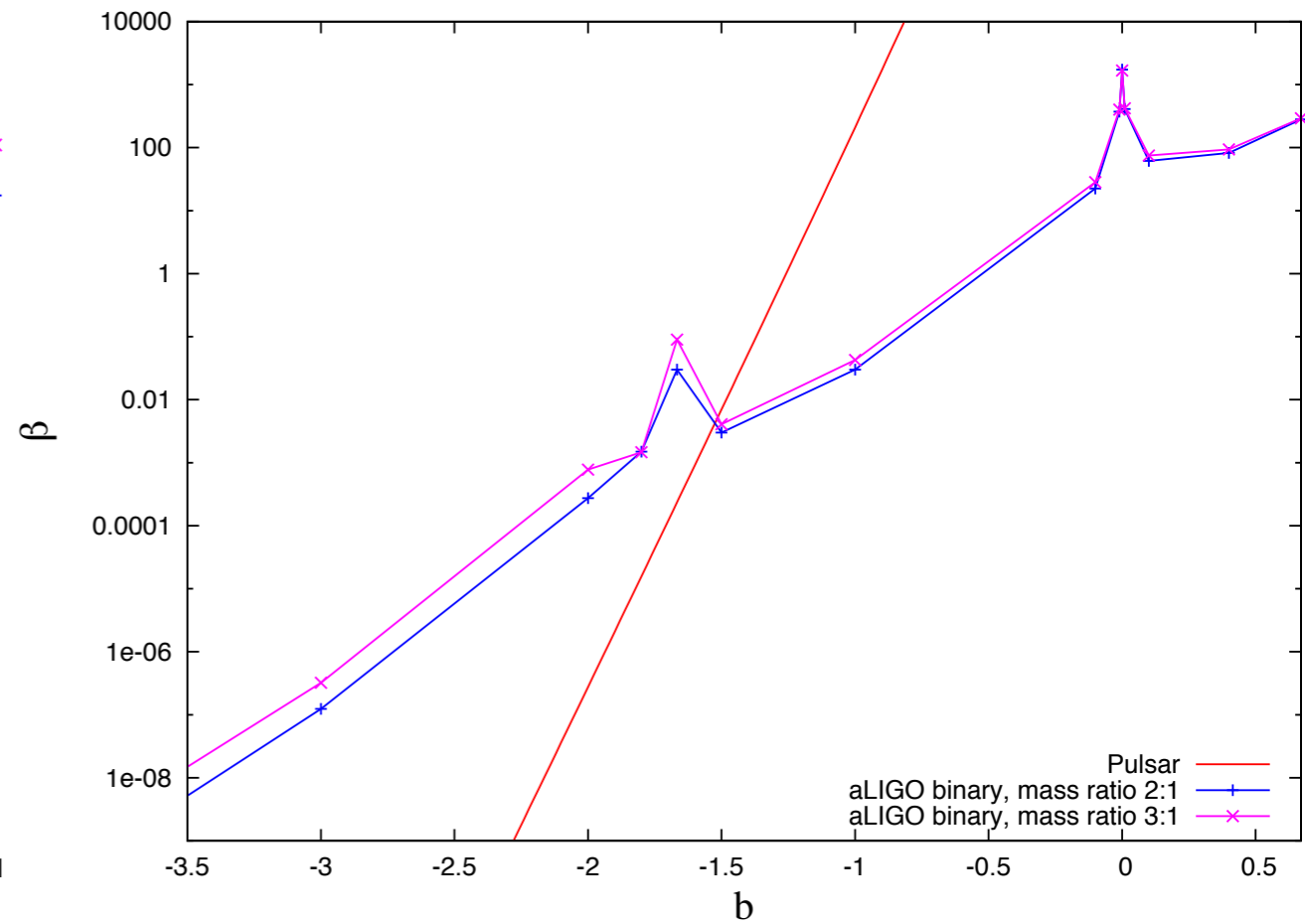
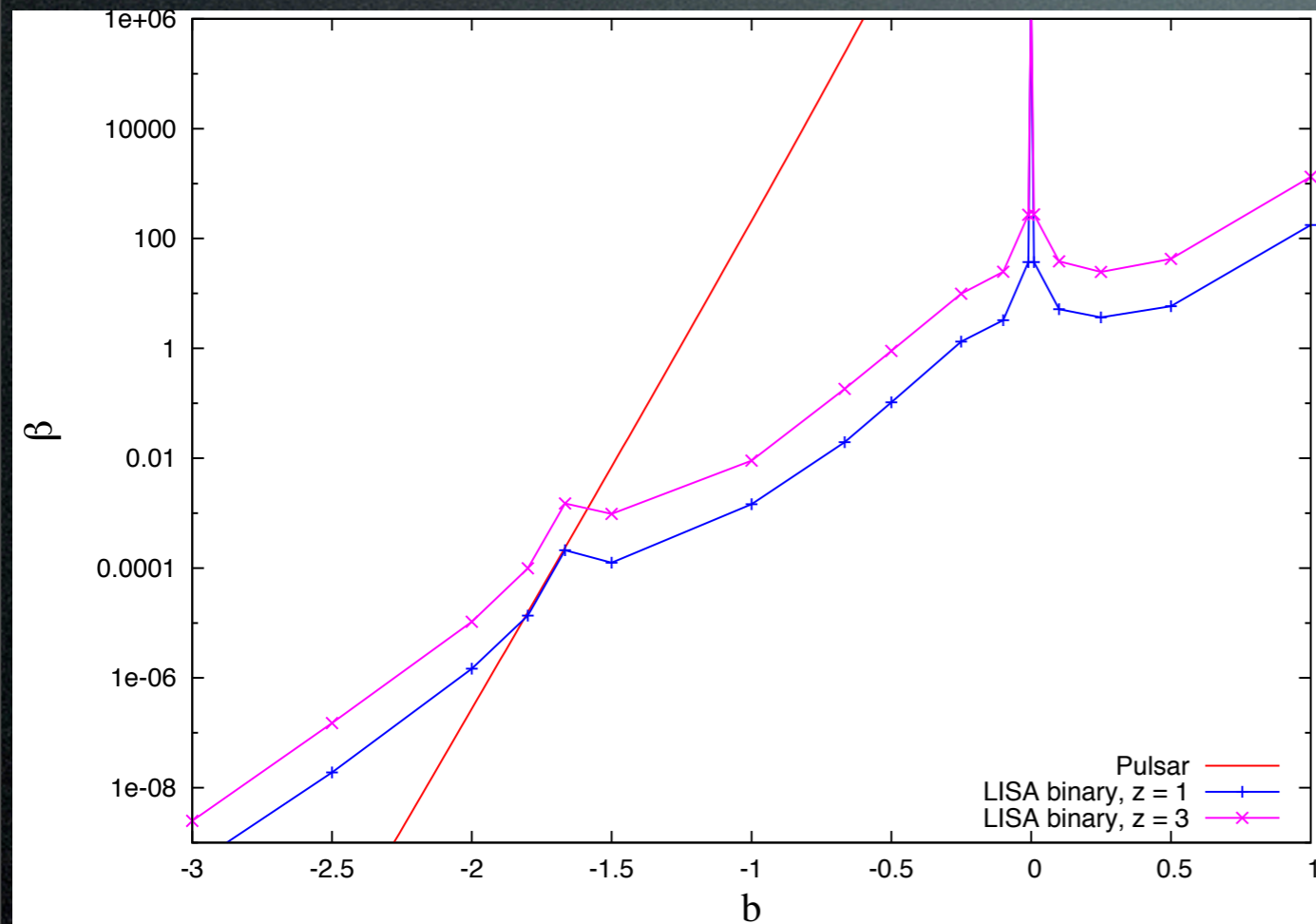
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Templates/ Theories	GR	ppE
GR	Business as usual	Quantify the likelihood of GR being the underlying theory describing the detected event, within the class of alt. theories captured by ppE
Not GR	Understand the bias that could be introduced filtering non-GR events with GR templates	Measure deviations from GR characterized by non-GR ppE parameters.

Constraining GR Deviations

GR Signal/ppE Templates, 3-sigma constraints, SNR = 20



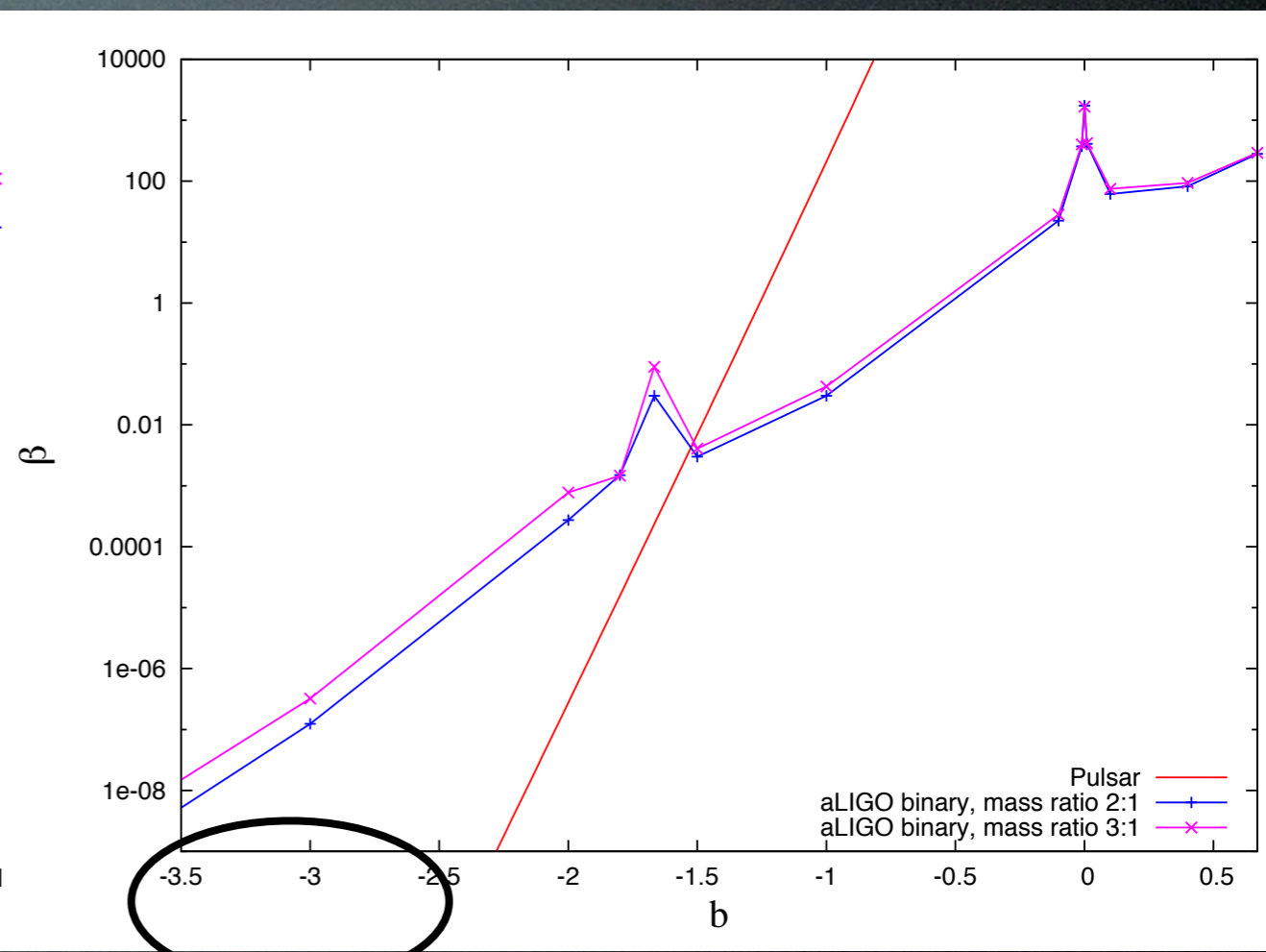
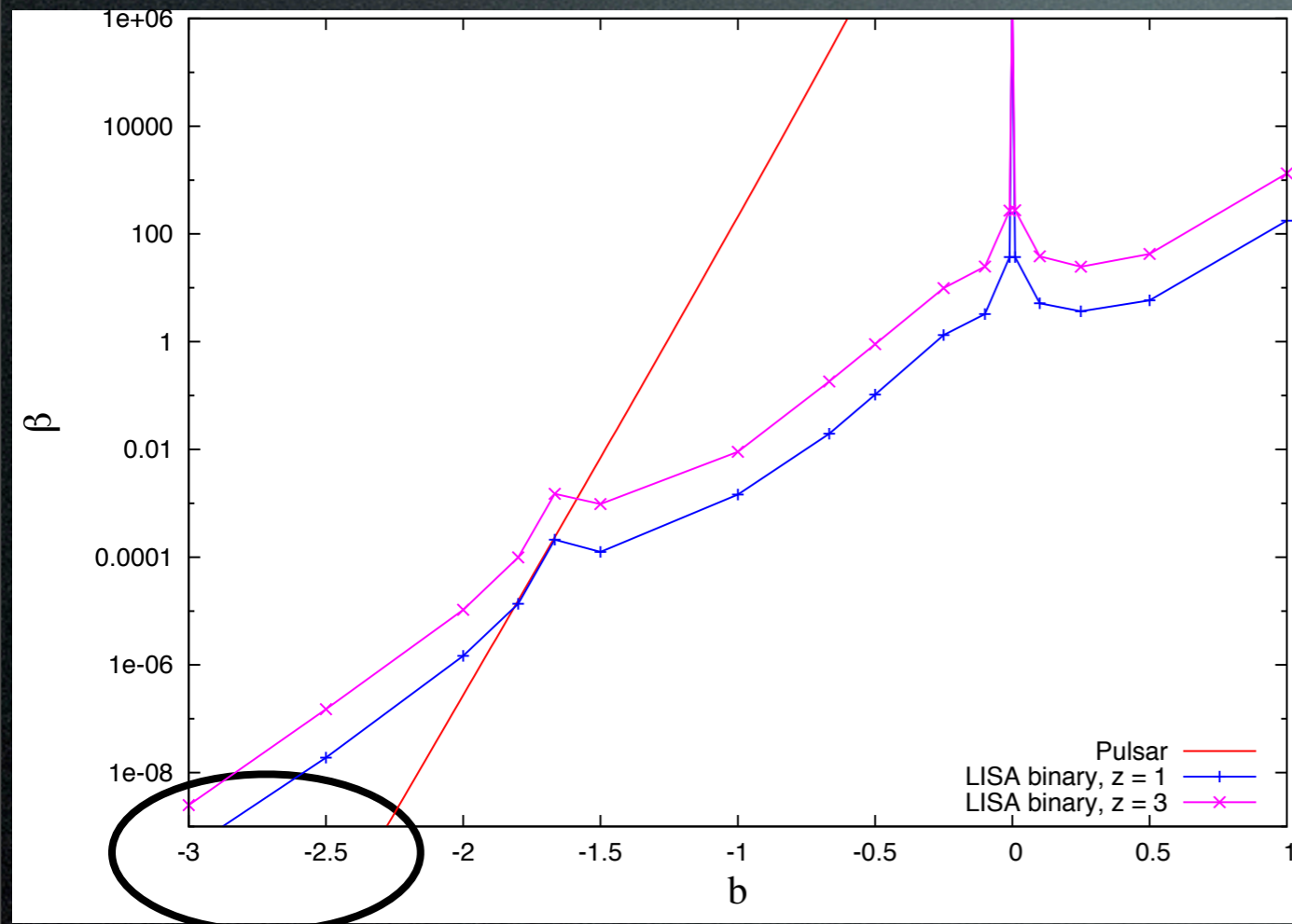
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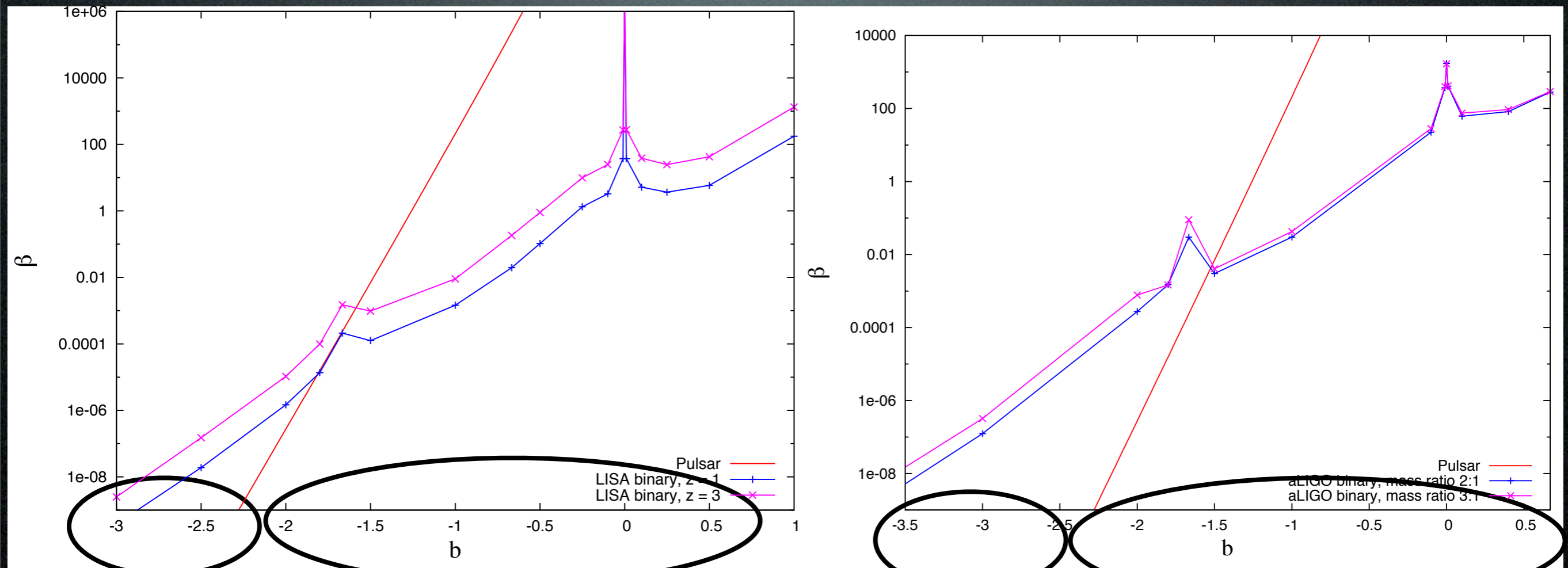
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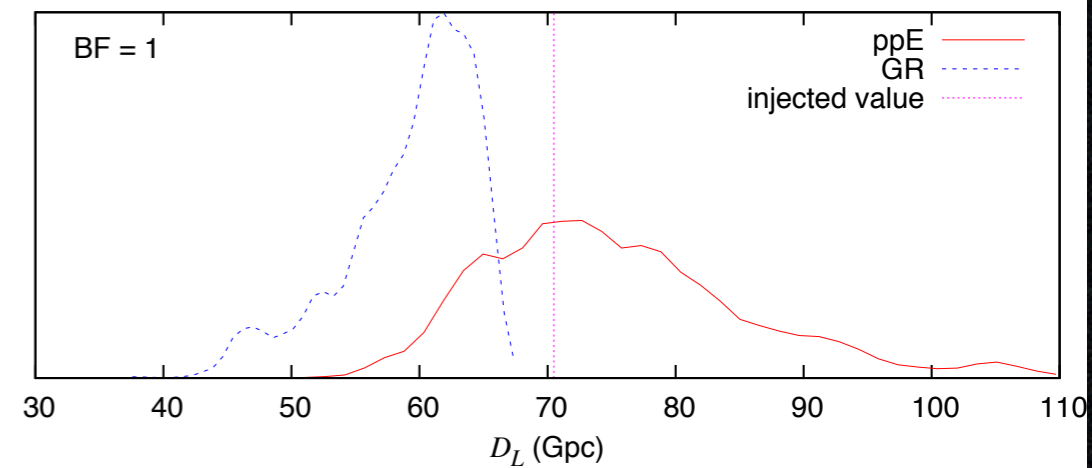
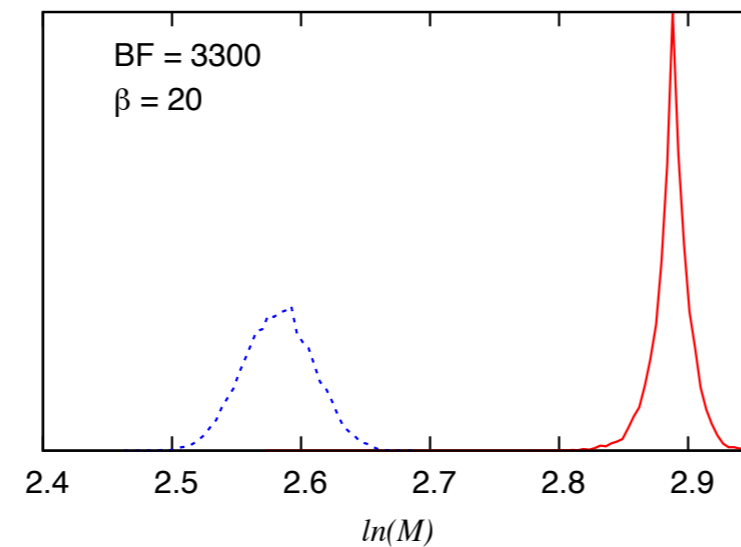
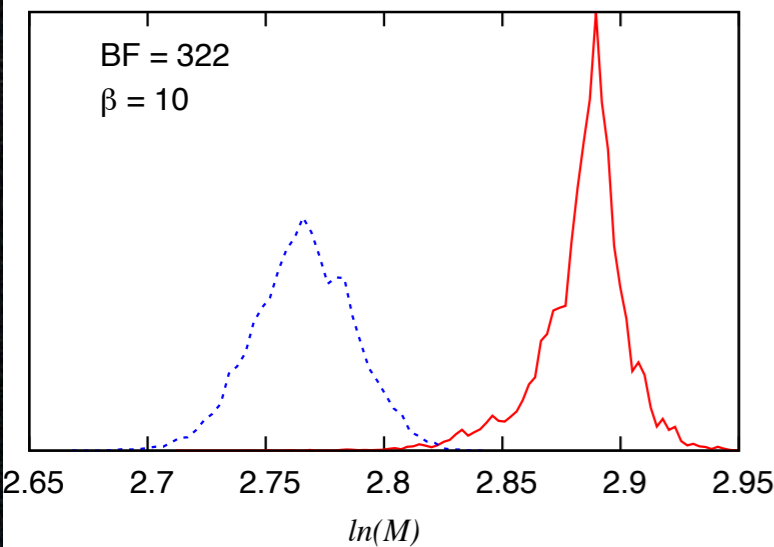
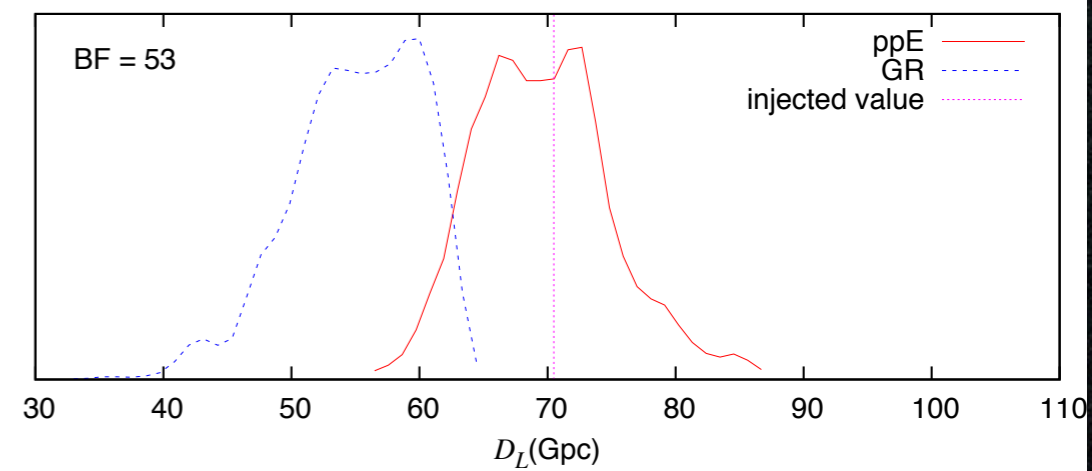
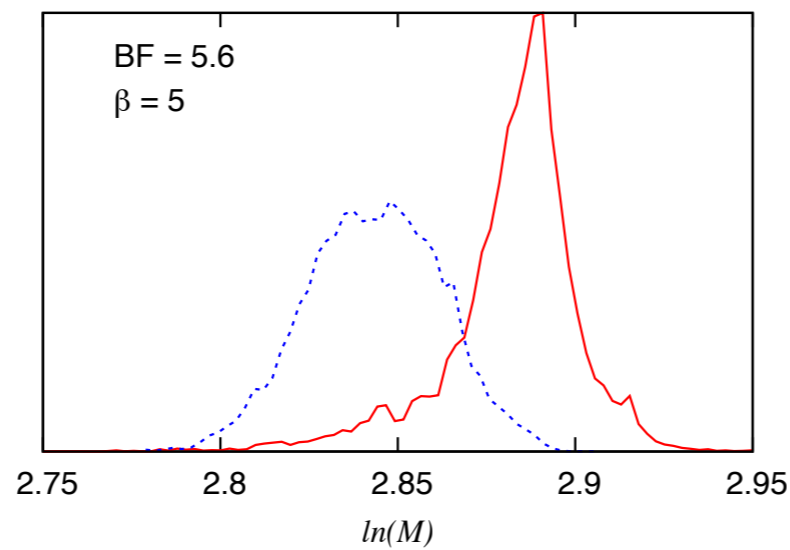
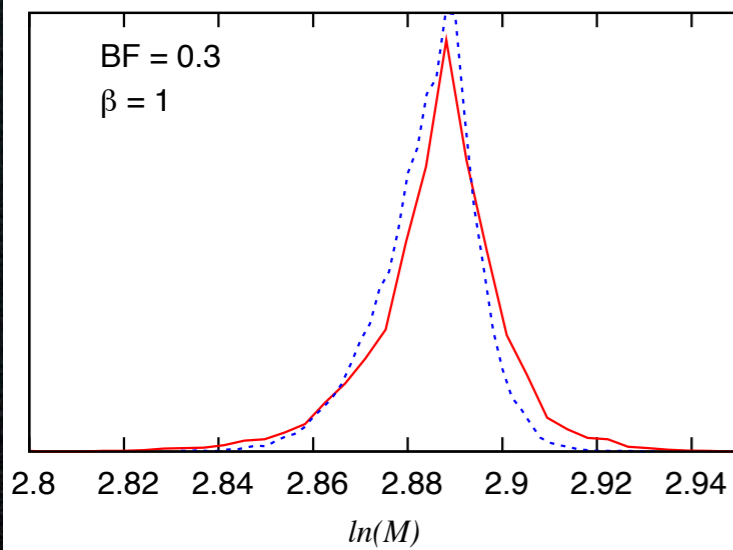
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Parameter Bias

Non-GR Signal/GR Templates, SNR = 20

Non GR injection, extracted with GR templates (blue) and ppE templates (red). GR template extraction is “wrong” by much more than the systematic (statistical) error. “Fundamental Bias”

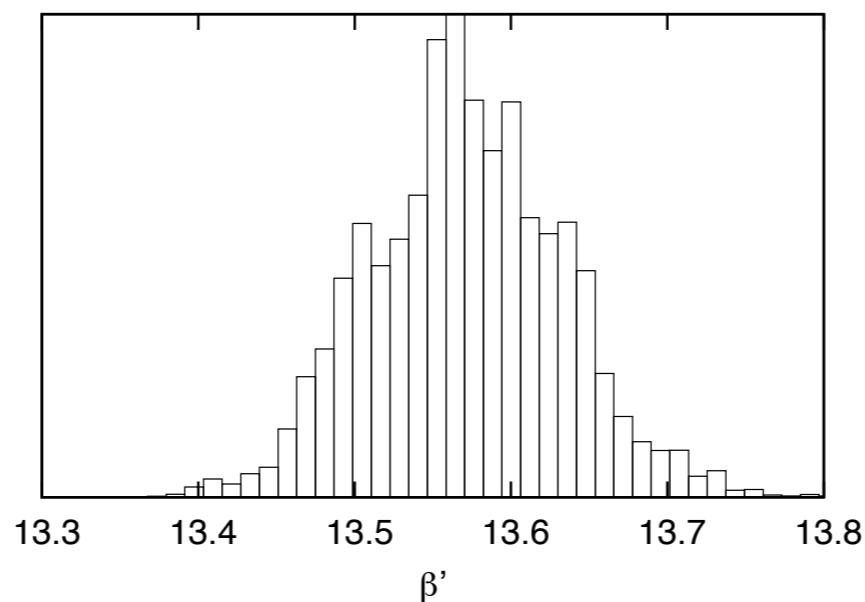
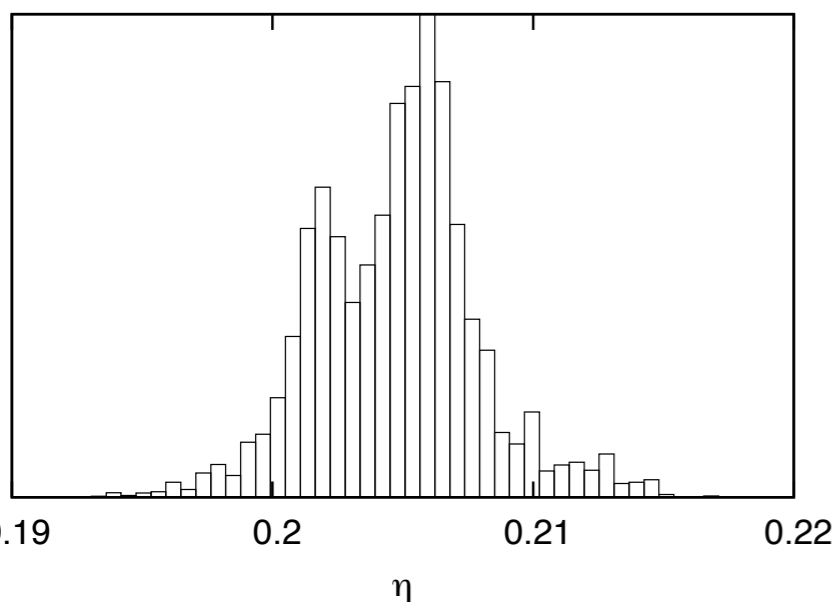
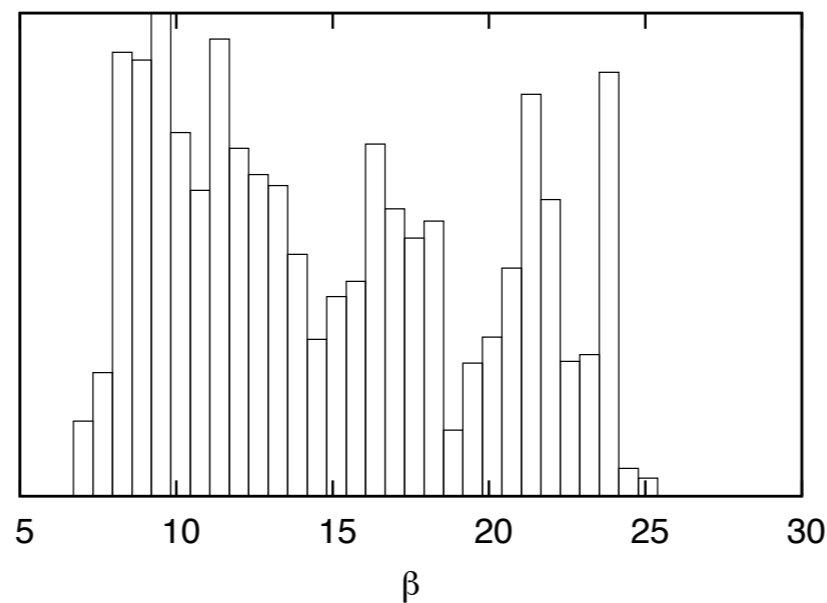
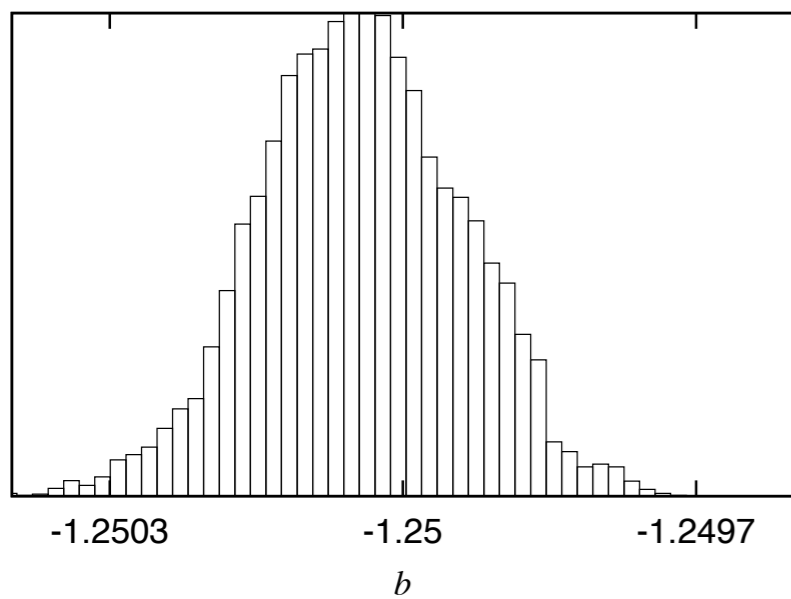


(Cornish, Sampson, Yunes & Pretorius, 2011.)

Identifying GR Deviations

Non-GR Signal/ppE Templates, Ad.LIGO, SNR = 20

Filter an injected ppE signal $(a, \alpha, b, \beta) = (-0.5, 4.0, 1.25, 10.0)$ with a ppE template family. The marginalized posterior for beta clearly shows a preference away from GR. LIGO (non-equal mass)



You can also compute the Bayes factor as a function of (b, β) . You would find a strong preference ($BF > 100$) for $\beta > 2/10$

(Cornish, Sampson, Yunes & Pretorius, 2011.)

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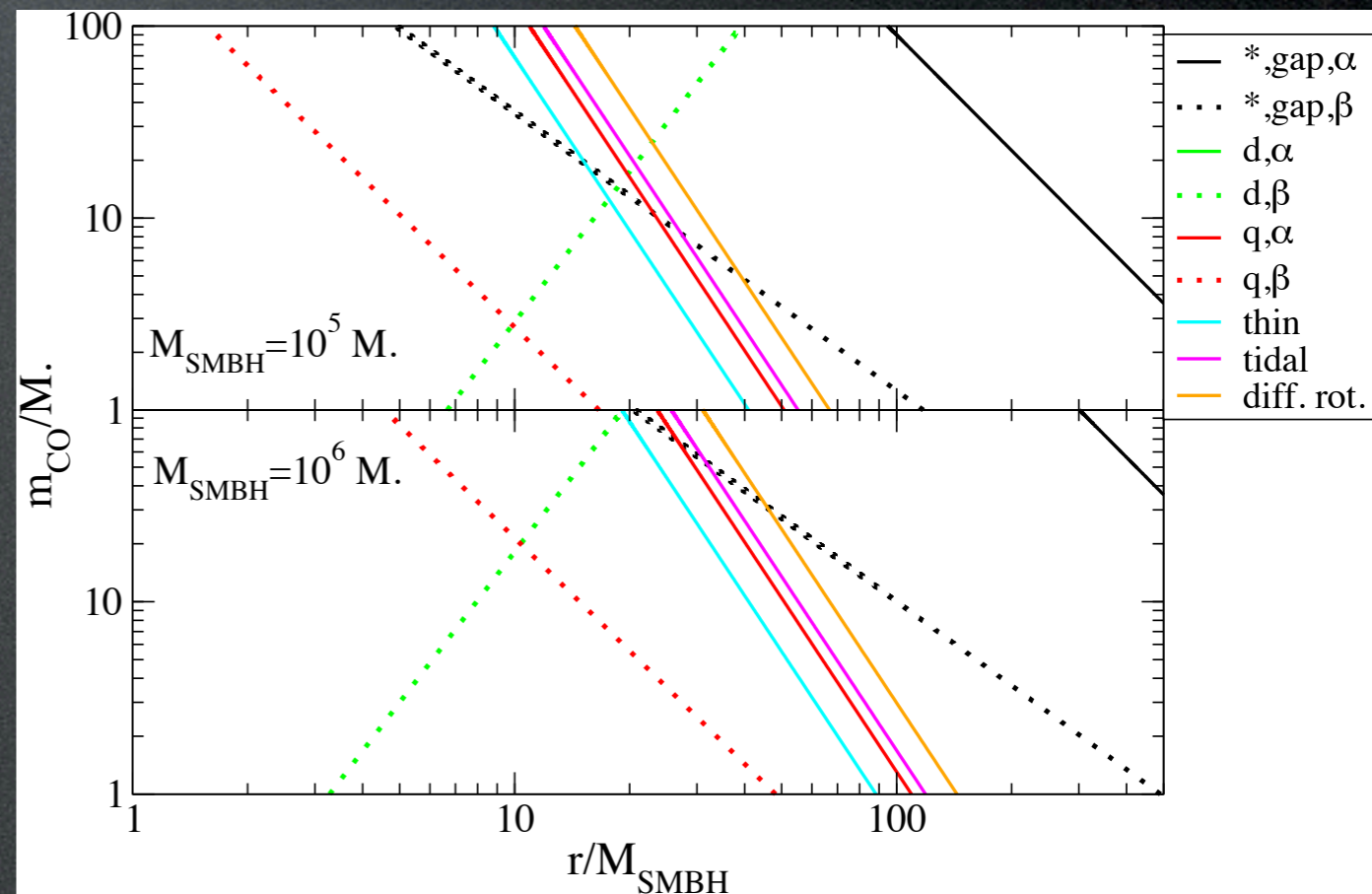
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The Era of Precision GW Astrophysics is at our doorstep...

Quenching of Disk Effects

Yunes, Kocsis, Haiman, Loeb (2011), Kocsis, Yunes, Loeb (2011)

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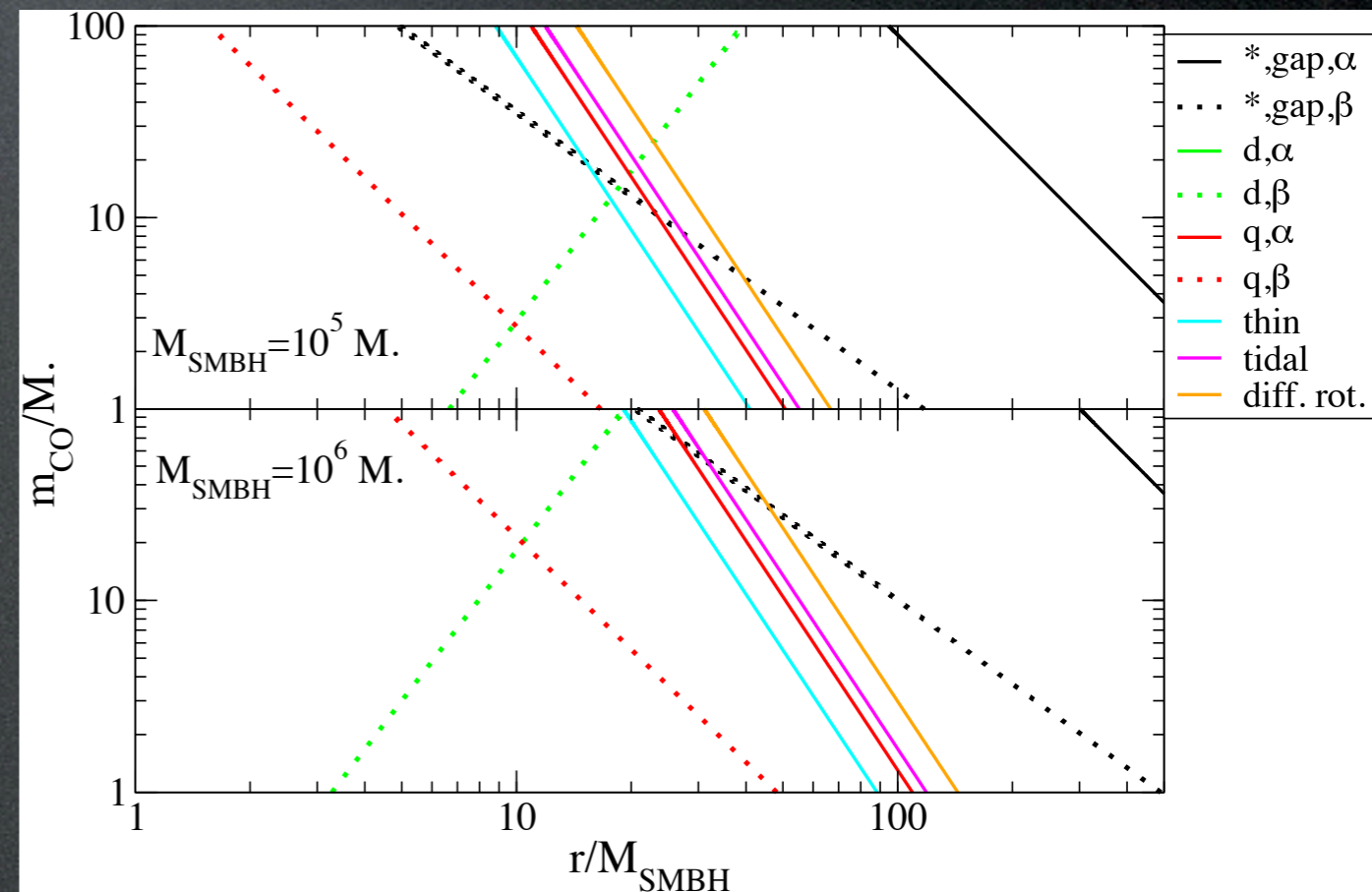


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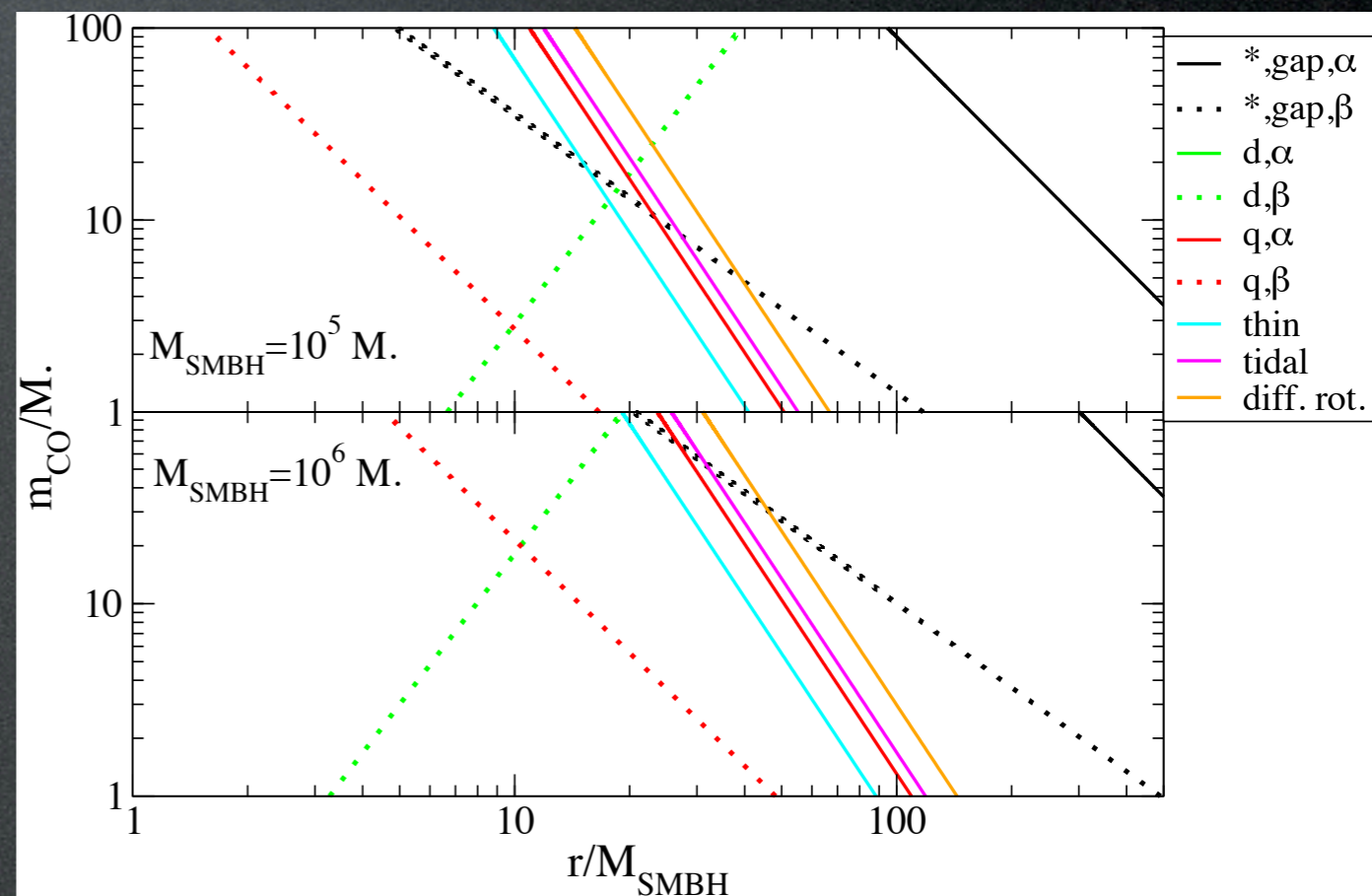


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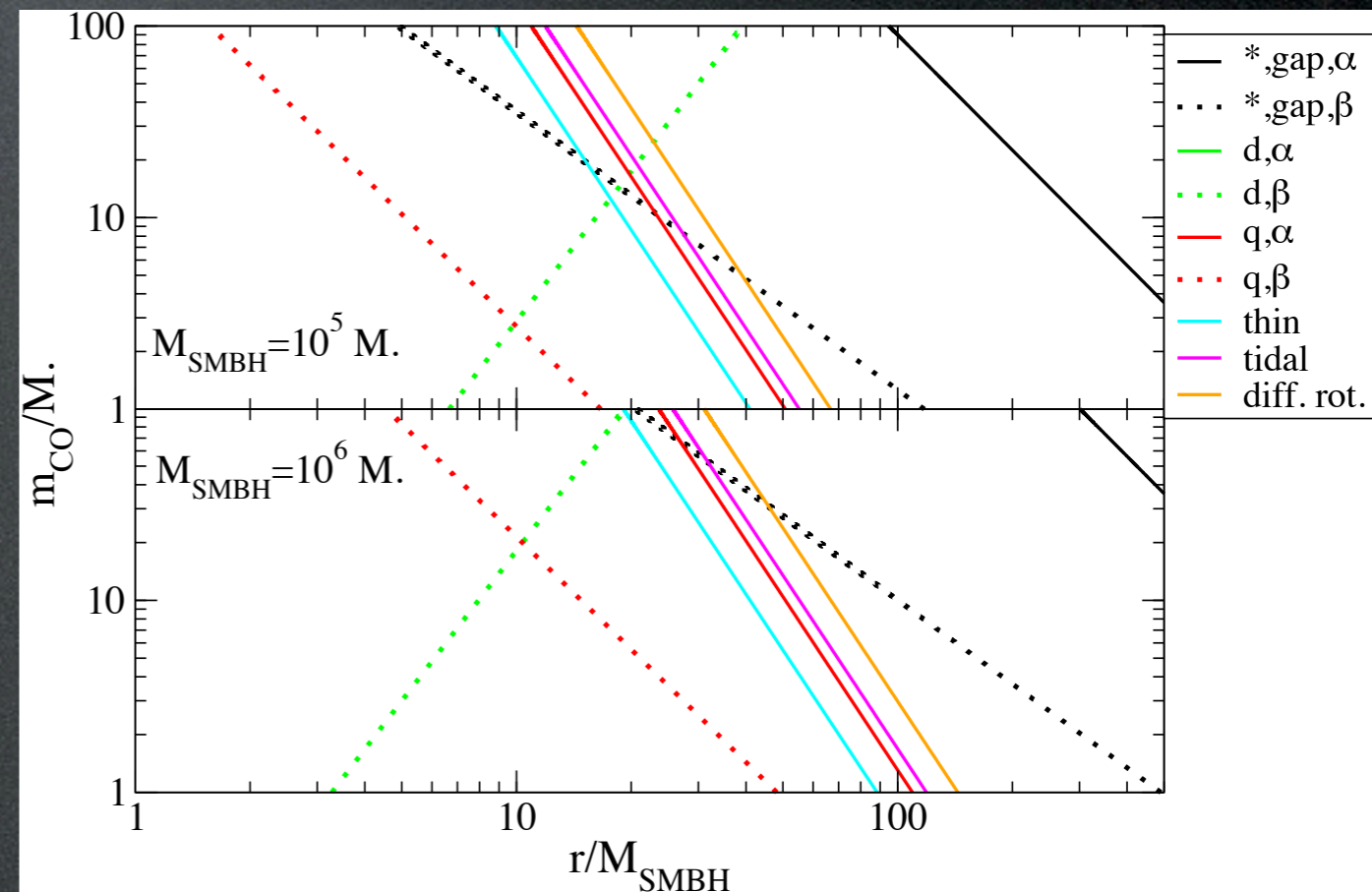


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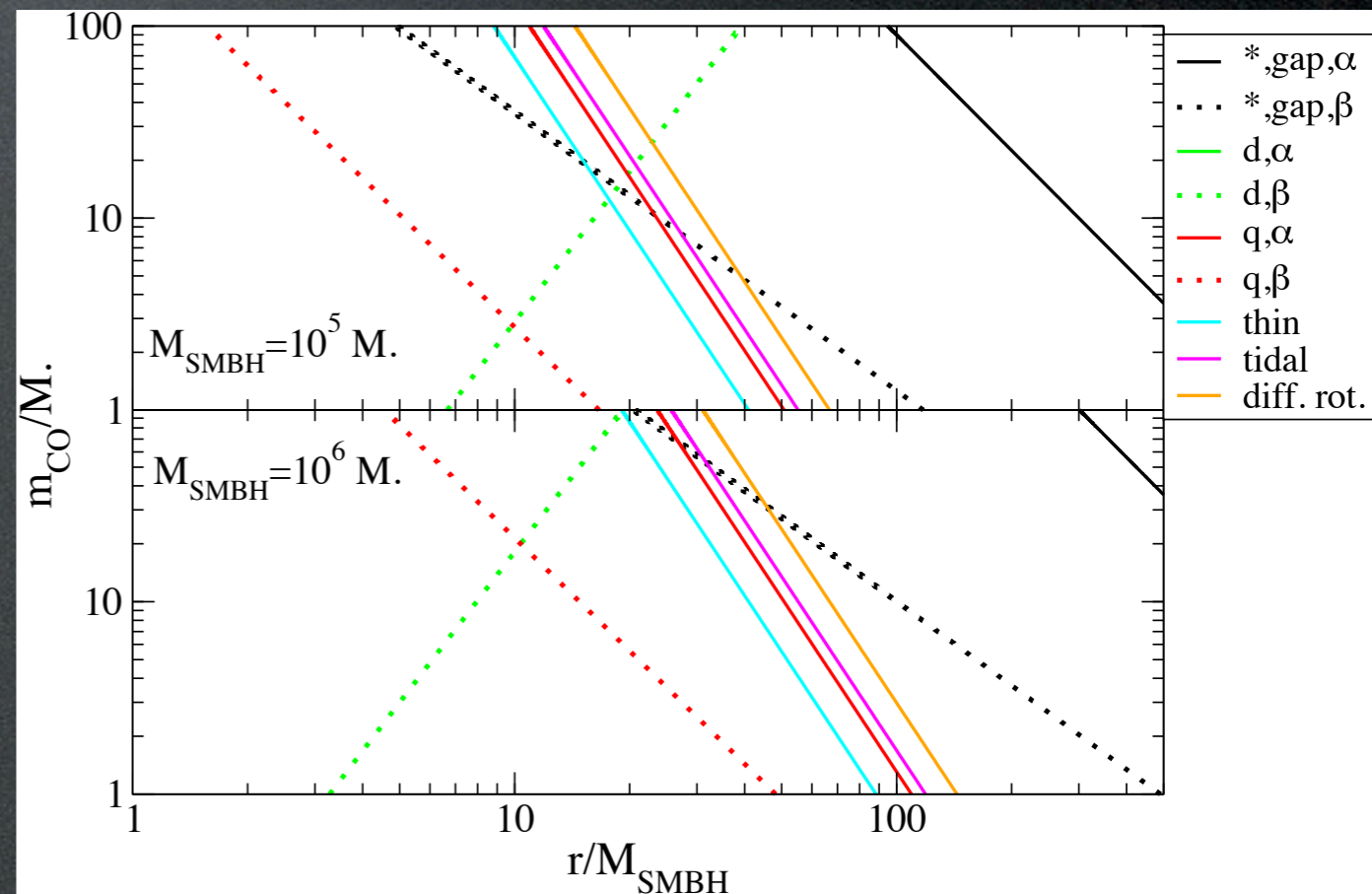
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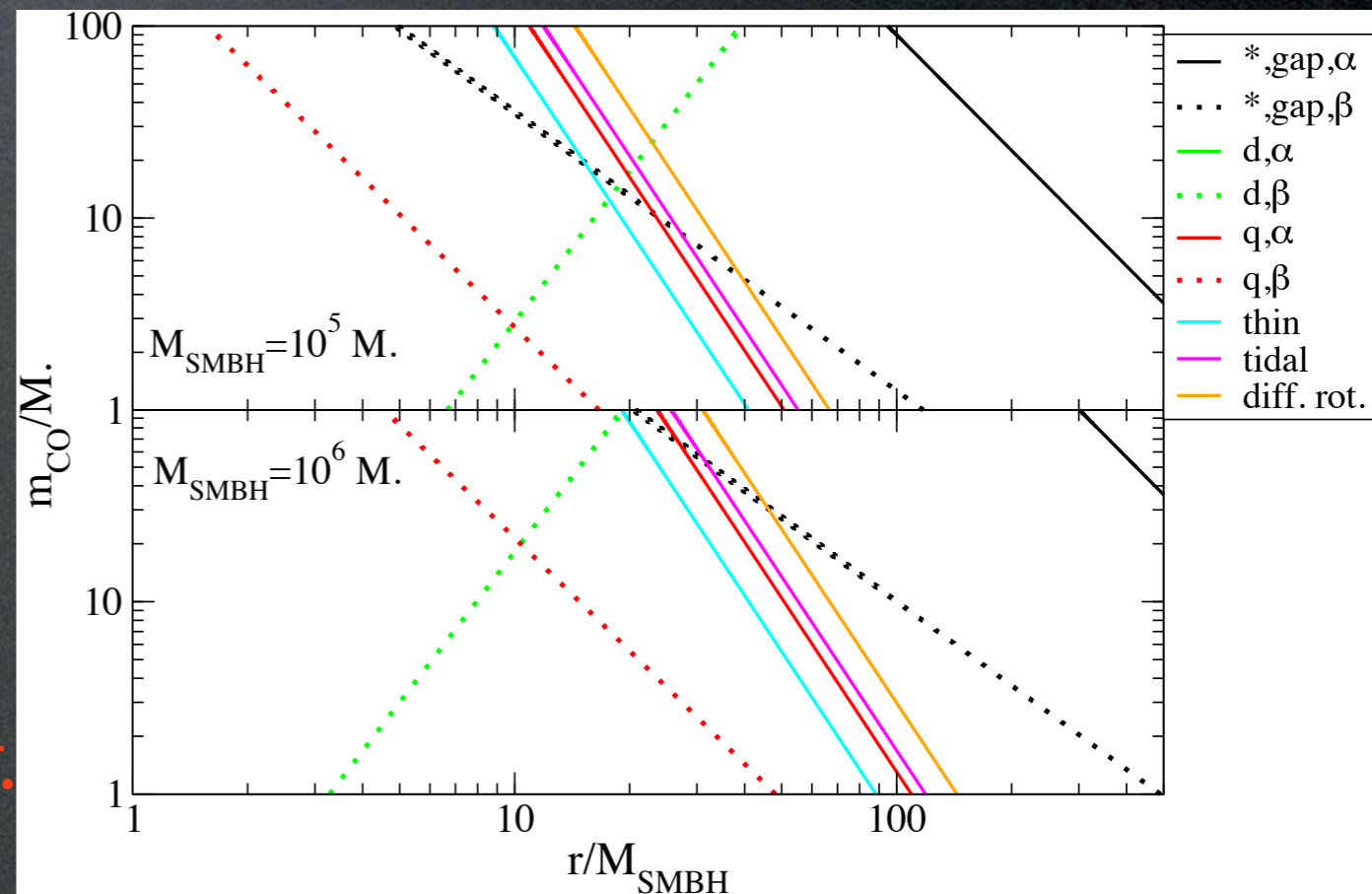
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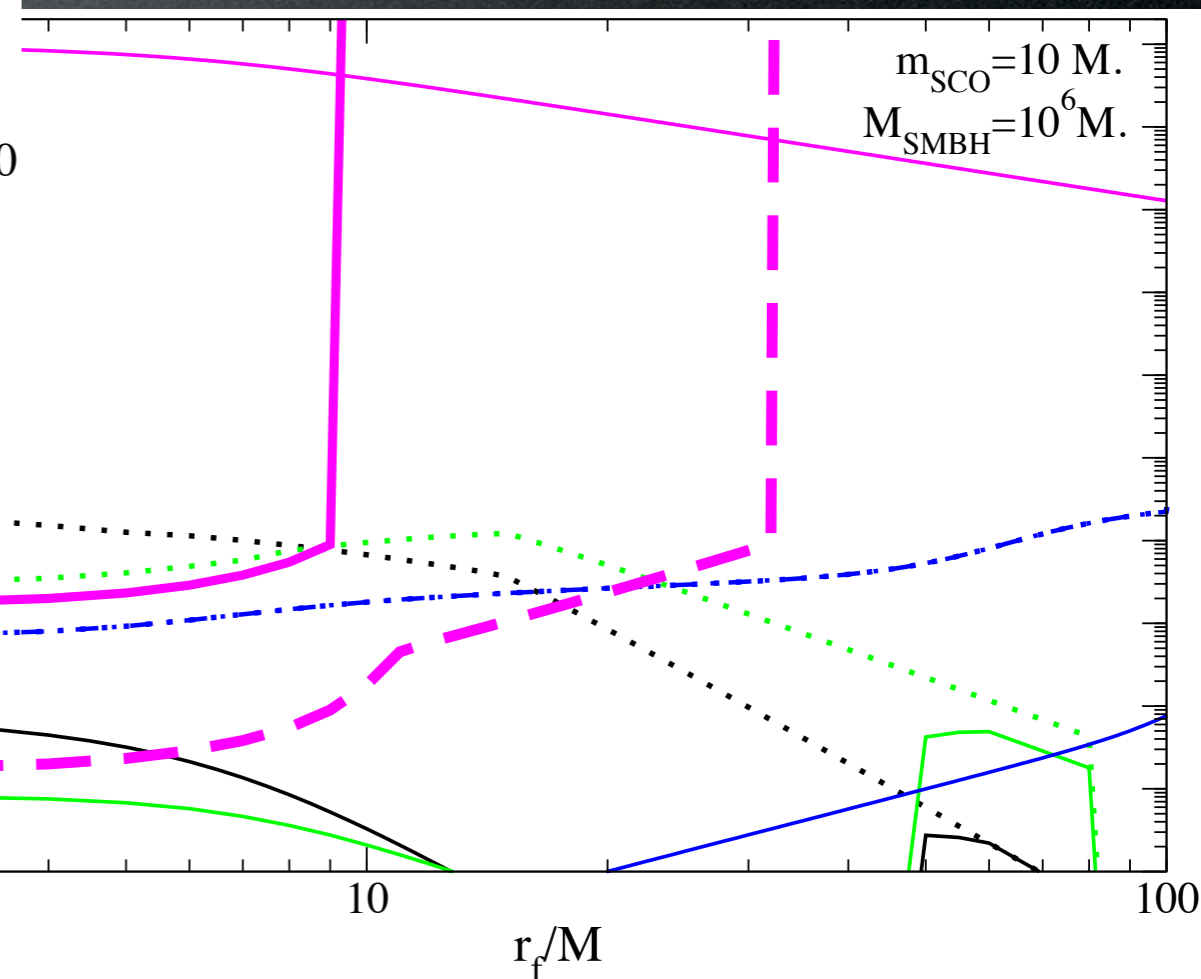
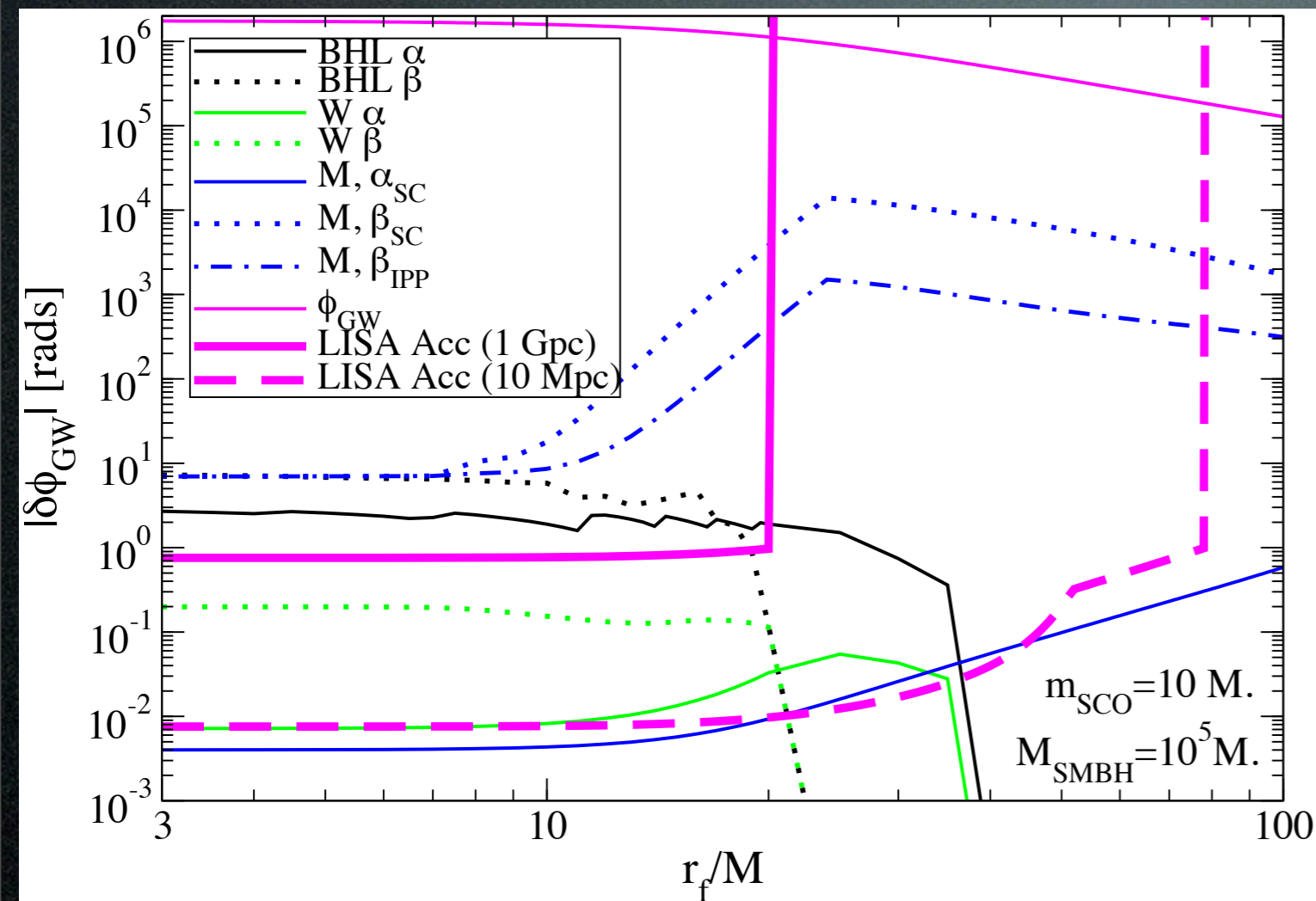
- Limited Gas Supply: If Bondi accretion rate $>$ gas influx, then accretion limited by gas supply.



Accretion Disk-Induced Dephasings

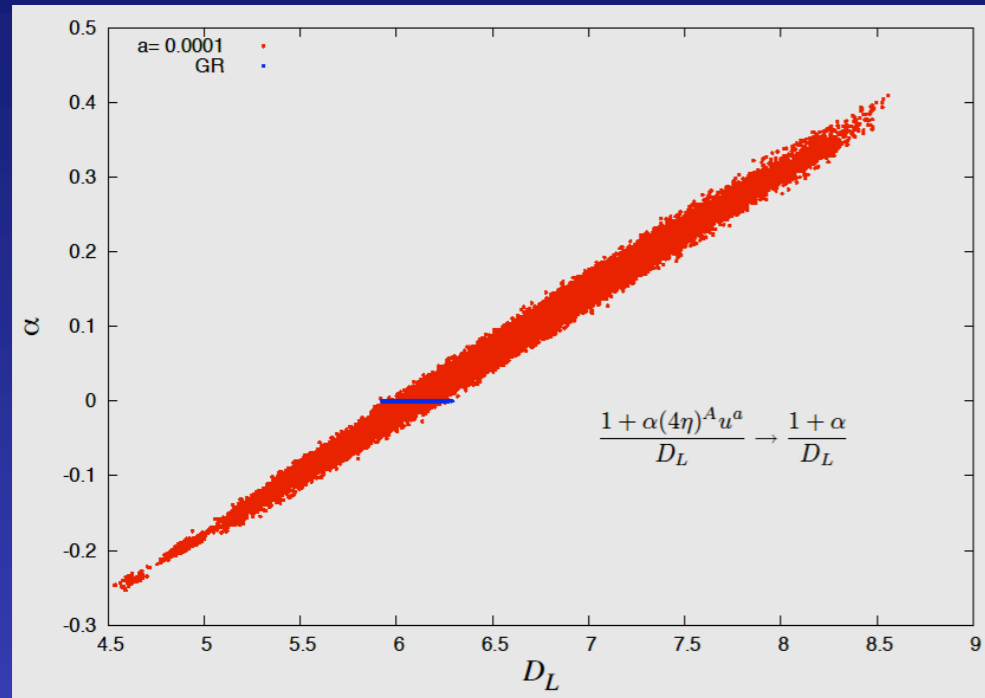
Yunes, Kocsis, Haiman, Loeb (2011),
Kocsis, Yunes, Loeb (2011)

Curves above the thick magenta line can be detected by LISA. Thin magenta line is the vacuum GW phase.

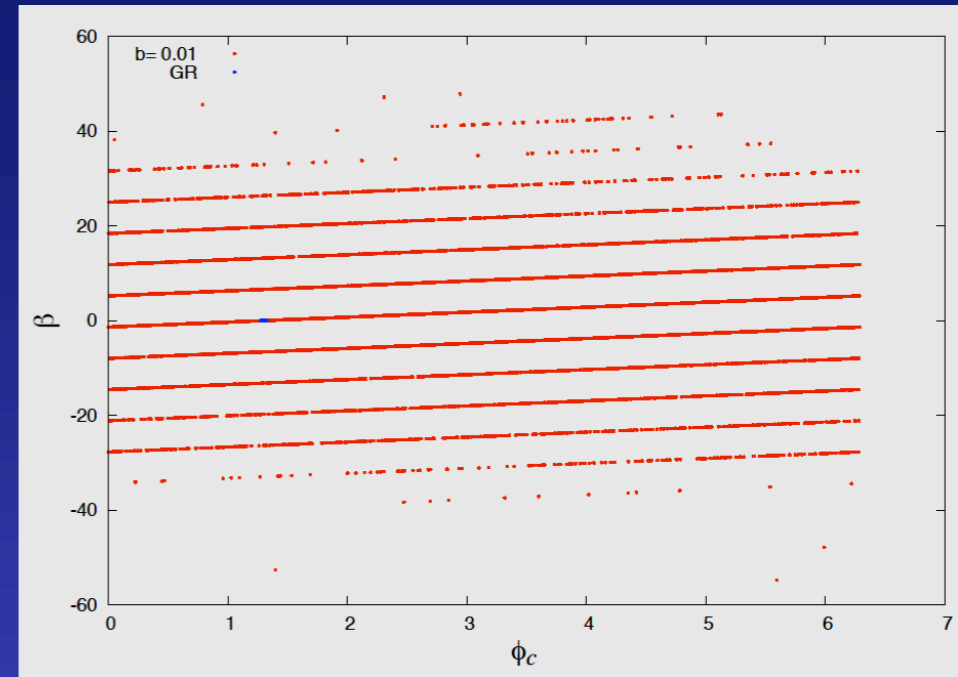


Each point in the figures is the dephasing after a 1-year inspiral, terminating at r_f . Solid (dotted) lines are for alpha (beta) disks, diff. colors are for different disk effects.

Correlations

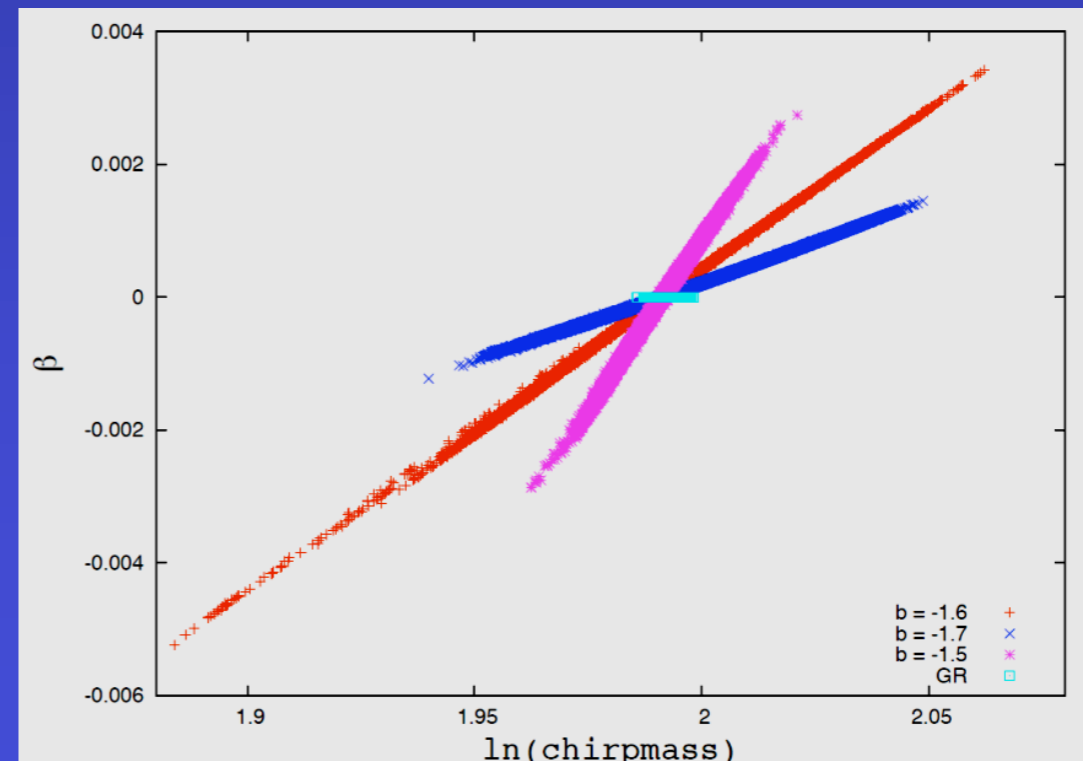


peak at $a=0$ is degeneracy between luminosity distance and effective α (LISA example)

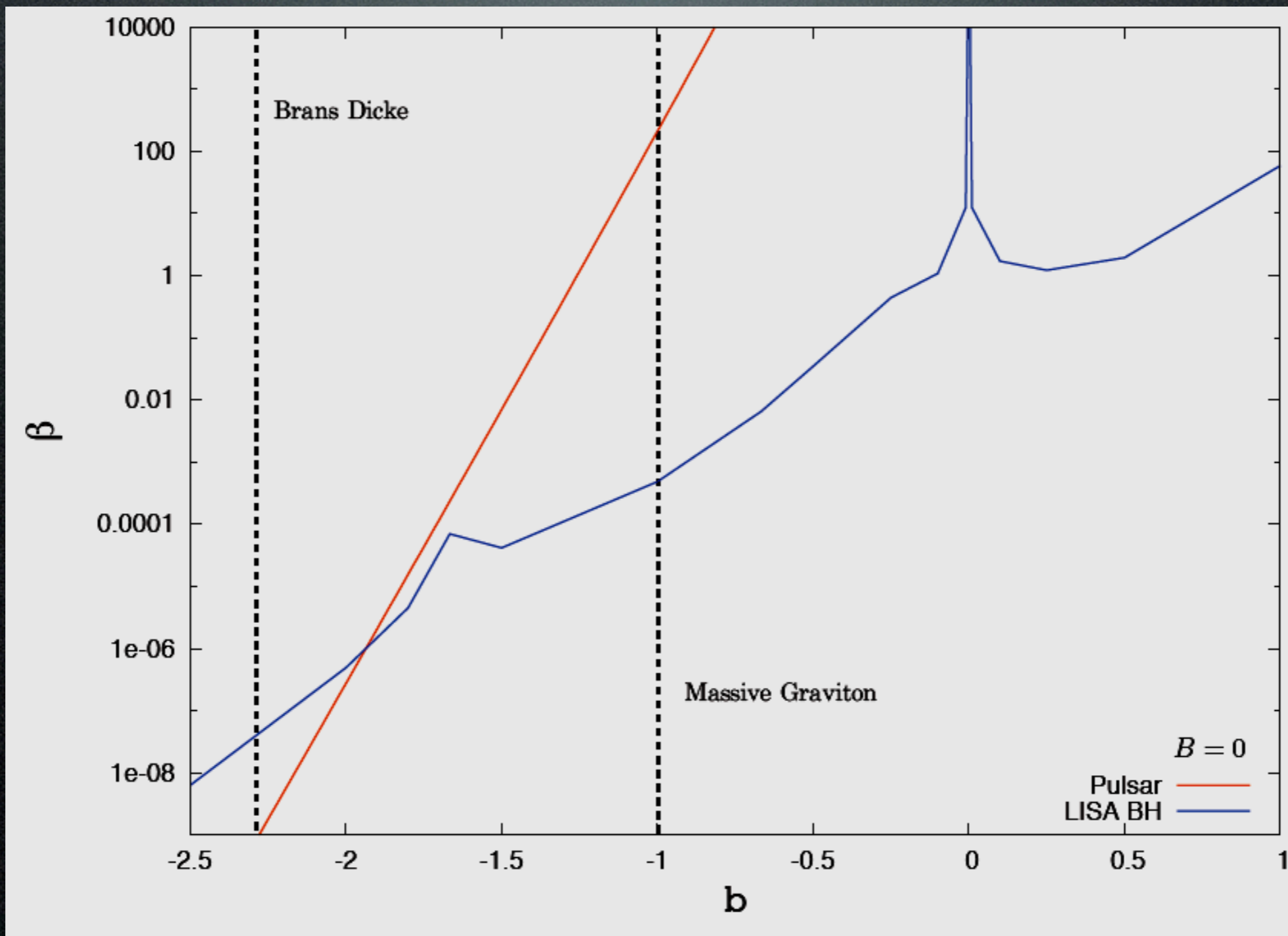


peak at $b=0$ is degeneracy between phase of coalescence and β (LISA example)

bump at $b=-5/3$ (PN value) is a partial degeneracy between chirp mass and β (LIGO example)

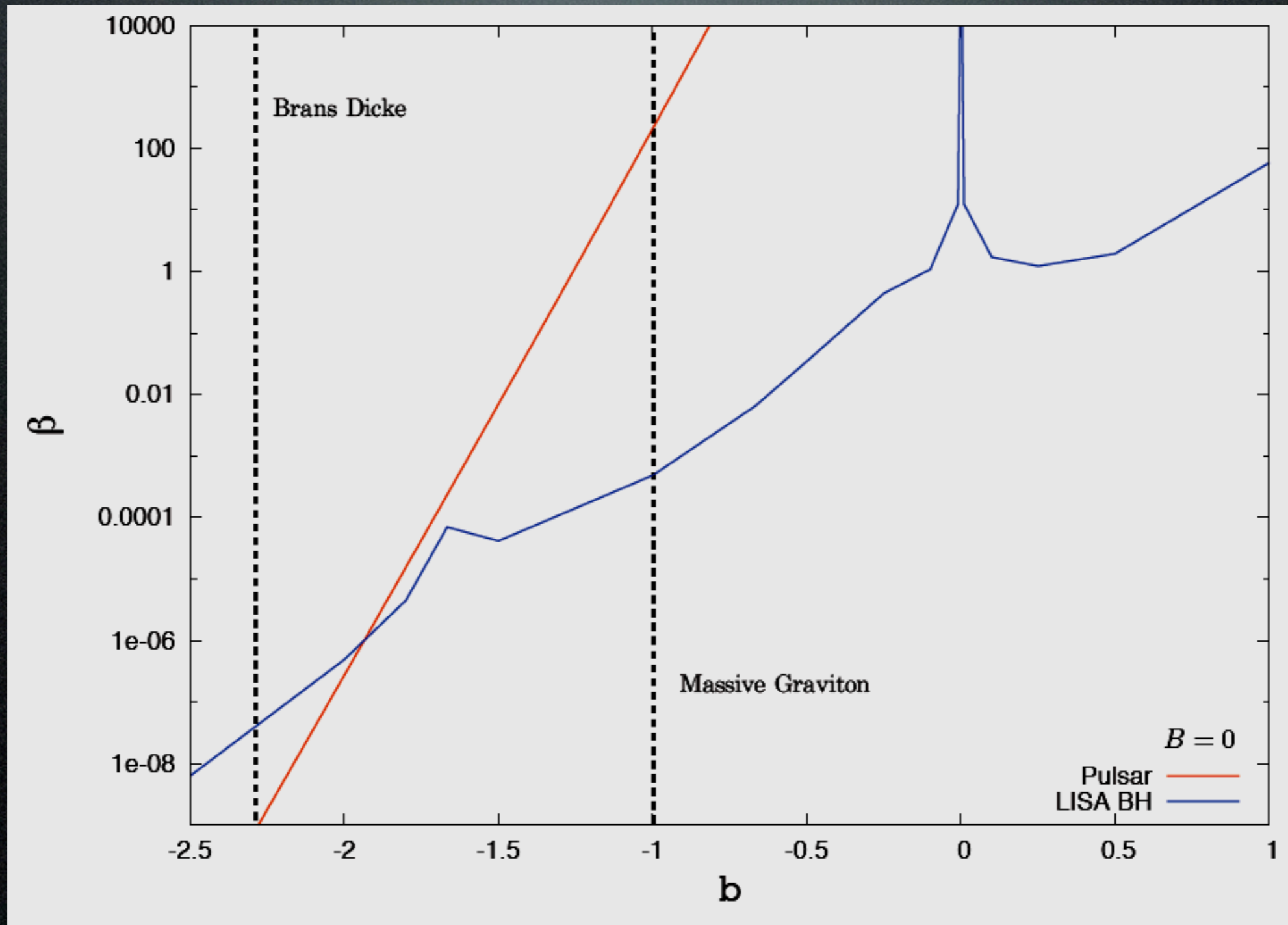


Resonances

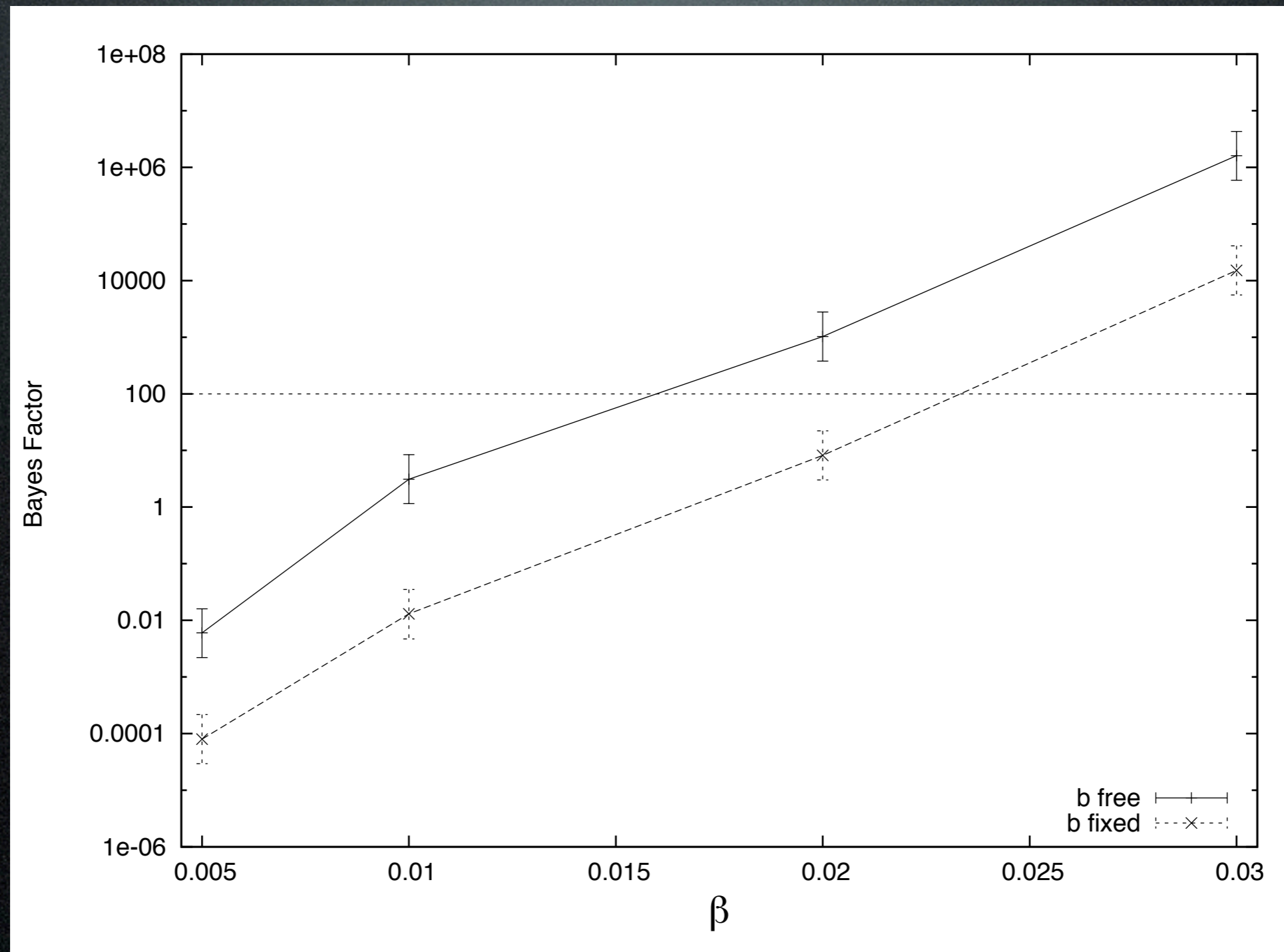


Resonances

At certain “resonant” exponents, you cannot distinguish between GR and an alternative theory modification (spikes).
(degeneracies not sampled in the previous plot)

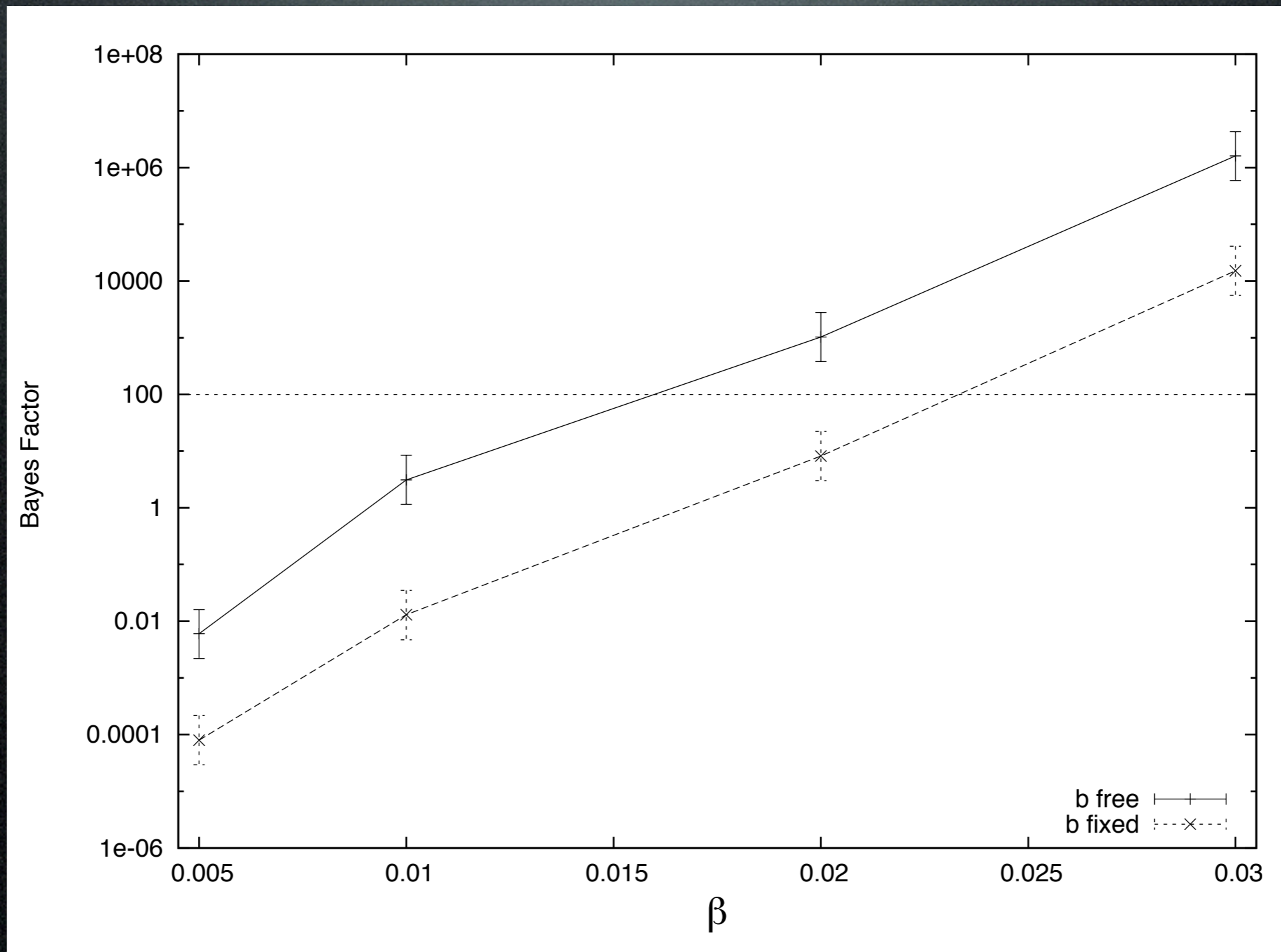


Bayes Factor

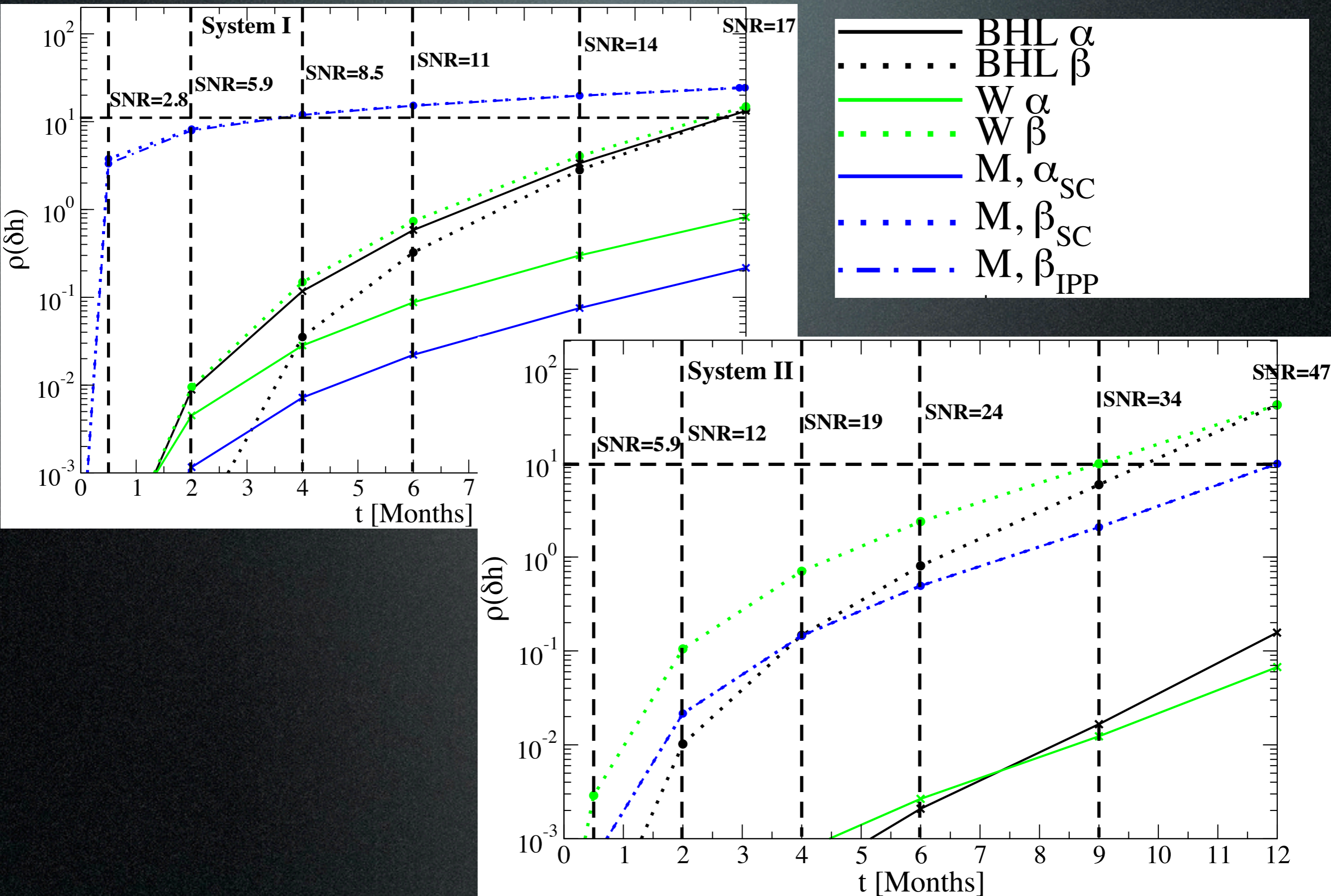


Bayes Factor

Odds ratio for ppE signal injection at different values of beta and $(a, \alpha, b) = (0, 0, -1.25)$. Extraction with ppE template. Suggests $\beta > 2/10$ can easily be observed.



SNR effects



Parameter Estimation Degeneracies

	\tilde{A}_1	\tilde{B}_1	\tilde{a}_1	\tilde{a}_2	\tilde{a}_3	\tilde{a}_4	\tilde{a}_5
BH α	-1 (-8)	-1 (-7)	-1	-5	1	4	-20/3
BH β	-2 (-5)	-1 (-4)	-4/5	-17/5	6/5	79/25	-79/15
W α	2 (-17)	1 (-16)	-1	-3	1	16/5	-16/3
W β	9 (-12)	3 (-11)	-4/5	-7/5	6/5	59/25	-59/15
MI α	7 (-10)	4 (-9)	-1	0	-2	1/5	-16/3
MI β	7 (-7)	3 (-6)	-4/5	0	-1/5	24/25	-59/15
MIIa α	8 (-6)	2 (-5)	0	1	1	-2/5	-8/3
MIIa β	8 (-3)	2 (-2)	1/2	5/8	1/4	-1/8	-25/12
MIIb β	7 (-4)	2 (-3)	2/7	11/14	4/7	-17/70	-7/3

$$u \equiv \pi \mathcal{M} f$$

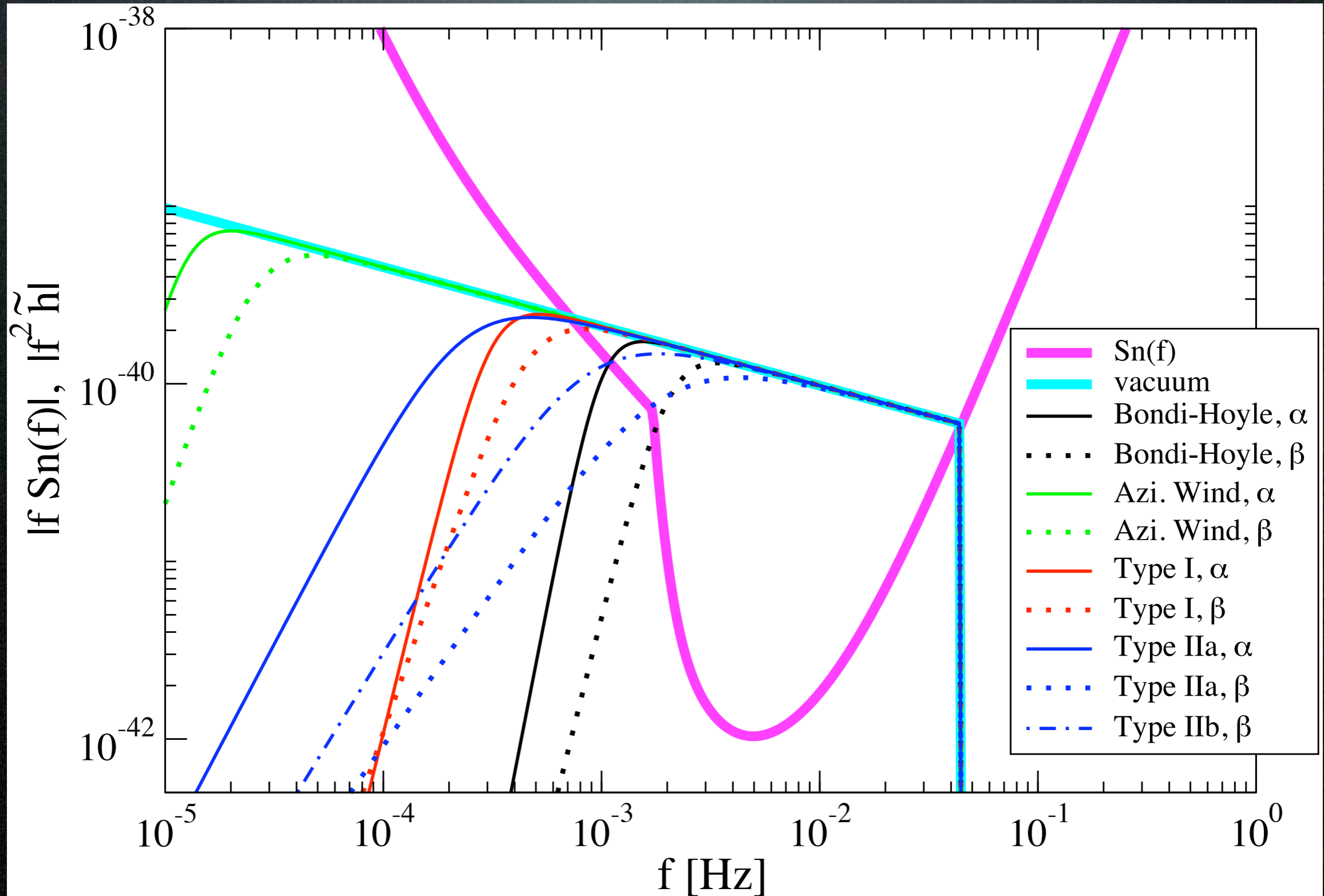
$$\psi/\psi_{\text{vac}} = 1 - \tilde{A}_1 \alpha_1^{\tilde{a}_1} \dot{m}_{\bullet 1}^{\tilde{a}_2} M_{\bullet 5}^{\tilde{a}_3} q_0^{\tilde{a}_4} u_0^{\tilde{a}_5}$$

$$\frac{3}{128} u^{-5/3}$$

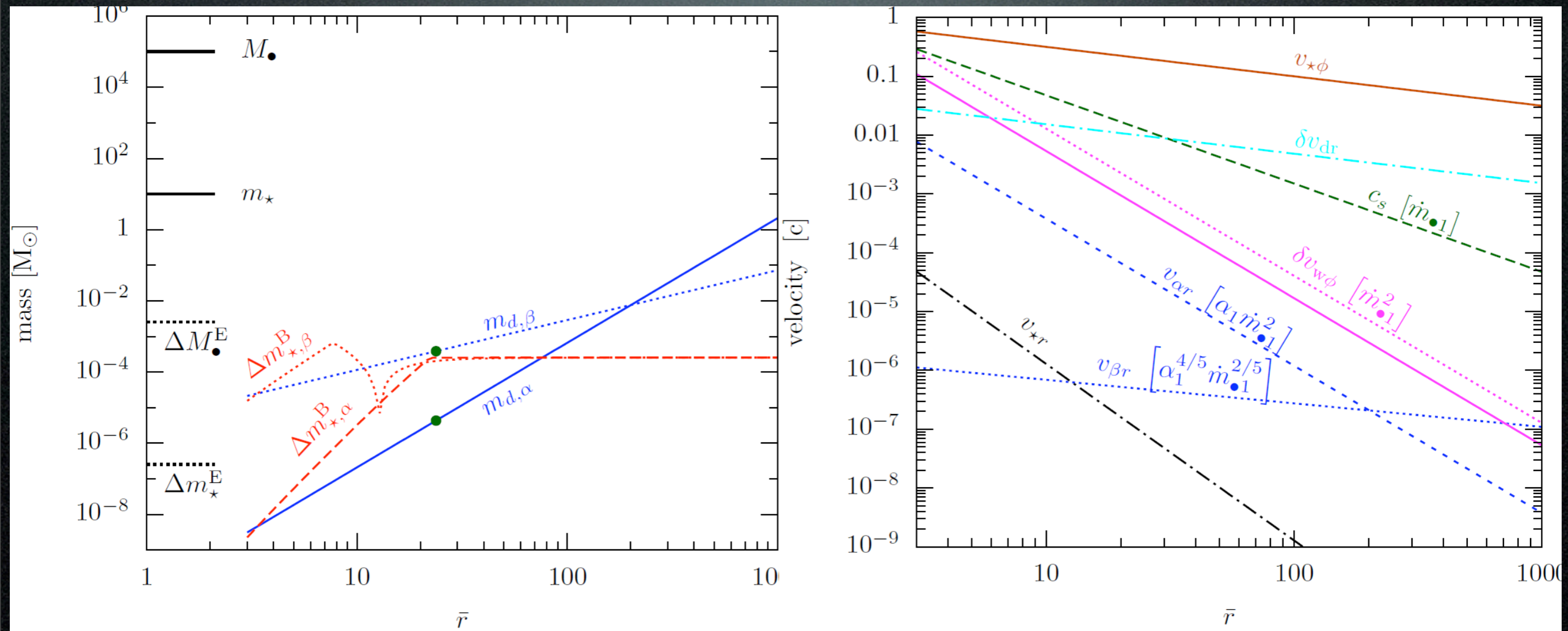
$$|\tilde{h}|/|\tilde{h}|_{\text{vac}} = 1 - \tilde{B}_1 \alpha_1^{\tilde{a}_1} \dot{m}_{\bullet 1}^{\tilde{a}_2} M_{\bullet 5}^{\tilde{a}_3} q_0^{\tilde{a}_4} u_0^{\tilde{a}_5}$$

$$\frac{\mathcal{M}^{5/6}}{\pi^{2/3} \sqrt{30} D_L} f^{-7/6}$$

Amplitude Effects



Mass Scales



But What Theory Do We Pick?

A Minimal (?) Set of Criteria:

1. Weak-Field Consistency (existence and stability of physical solutions, satisfaction of precision tests).
2. Strong-Field Inconsistency (deviations only where experiments cannot currently rule out modifications)

Other Nice Criteria:

3. Well motivated from fundamental physics.
4. Well-posed theory ?? This is hard to do...

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It's not easy to fool Mother Nature! (Wald)

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Top-Down Accuracy Studies

	Will	Scharre, Will	Will, Yunes	Berti, Buonanno , Will	Arun, Will	Stravridis, Will	Yagi, Tanaka	Ajith, Keppel	Solar System
Binary Mass	x	1.4:1E3	1.4:1E3	1.4:1E3	x	x	1.4:1E3	x	x
BD Coupling Par. (e4)	x	24	20	10	x	x	0.7	x	4
Binary Mass	1E7: 1E6	x	1E6:1E6	1E6:1E6	2E6:1E7	1E6:1E6	1E7:1E6	5E7:5E7	x
Graviton Compton Wavelgth (e21 cm)	6.9	x	3.1	1.33	5	4	3.1	52	0.00028
Details	First MG study, no spin	First ST study, no spin	As a func. of Det.	non-prec., spinning	amp. corr.	spin + prec	spin + prec + ecc	IMR	Cassini, 3rd Law Solar Sys

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- One samples the “Strong-Field” regime of GR -> where gravity is strong and velocities close to c .

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Simplifications

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Dynamical Chern-Simons Theory

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Einstein-Dilaton-Gauss-Bonnet Theory

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