New features of black hole solutions in *f(R, G)* theories

Sergey Pavluchenko

Special Astrophysical Observatory of the Russian Academy of Sciences

General *f(R, G)* field equations - 1

Let us consider theory with an action

$$S = \int d^4x \sqrt{-g} f(R, \mathcal{G})$$

$$\mathcal{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

By varying this action w.r.t. the metric components one can retrieve the field equations

$$-\frac{1}{2}f g_{\mu\nu} + f_R R_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu}\Box f_R + 2f_{\mathcal{G}}\left(R_\mu{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma} + 2R_{\mu\alpha\beta\nu}R^{\alpha\beta} - \right.$$
$$-2R_{\alpha\mu}R^\alpha{}_\nu + RR_{\mu\nu}) - 4\left(R_{\mu\alpha\beta\nu} + R_{\mu\nu}g_{\alpha\beta} + R_{\alpha\beta}g_{\mu\nu} - R_{\alpha\mu}g_{\beta\nu} - R_{\alpha\nu}g_{\beta\mu} + \right.$$
$$\left. + \frac{1}{2}R\left(g_{\alpha\mu}g_{\beta\nu} - g_{\mu\nu}g_{\alpha\beta}\right)\right)\nabla^\alpha\nabla^\beta f_{\mathcal{G}} = 0.$$

In (3+1) the second Lovelock tensor is always zero

$$2\left(R_\mu{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma} + 2R_{\mu\alpha\beta\nu}R^{\alpha\beta} - 2R_{\alpha\mu}R^\alpha{}_\nu + RR_{\mu\nu}\right) - \frac{1}{2}\mathcal{G}g_{\mu\nu} = 0$$

General *f(R, G)* field equations - 2

$$-\frac{1}{2}f g_{\mu\nu} + f_R R_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu}\Box f_R + \frac{1}{2} f_{\mathcal{G}} \mathcal{G} g_{\mu\nu} - 4\left(R_{\mu\alpha\beta\nu} + R_{\mu\nu} g_{\alpha\beta} + \right.$$

$$\left. + R_{\alpha\beta} g_{\mu\nu} - R_{\alpha\mu} g_{\beta\nu} - R_{\alpha\nu} g_{\beta\mu} + \frac{1}{2} R \left(g_{\alpha\mu} g_{\beta\nu} - g_{\mu\nu} g_{\alpha\beta}\right)\right) \nabla^\alpha \nabla^\beta f_{\mathcal{G}} = 0.$$

Spherically-symmetric metric

$$g_{\mu\nu} = \mathrm{diag}\{-A(r), B(r), r^2, r^2 \sin\theta\}$$

00-equation

$$\frac{1}{2}Af - f_R\left(\frac{1}{4}\frac{\dot{A}^2}{AB} - \frac{1}{2}\frac{\ddot{A}}{B} + \frac{1}{4}\frac{\dot{A}\dot{B}}{B^2} - \frac{\dot{A}}{Br}\right) + f_{\mathcal{G}}\left(\frac{\dot{A}^2}{AB^2r^2} + \frac{3\dot{A}\dot{B}}{B^3r^2} - \frac{\dot{A}\dot{B}}{B^2r^2} + \frac{2\ddot{A}}{Br^2} - \frac{2\ddot{A}}{B^2r^2} - \right.$$

$$\left. - \frac{\dot{A}^2}{ABr^2}\right) - \frac{A}{B}\ddot{f}_R - \dot{f}_R\left(\frac{2A}{Br} + \frac{1}{2}\frac{\dot{A}}{B} - \frac{1}{2}\frac{A\dot{B}}{B^2} + \frac{1}{2}\dot{A}\right) + \frac{4A}{B^2r^2}(1-B)\ddot{f}_{\mathcal{G}} + \frac{2\dot{B}A}{B^2r^2}(1+B)\dot{f}_{\mathcal{G}}$$

General $f(R, G)$ field equations - 3

11-equation

$$-\frac{1}{2}Bf + f_R\left(\frac{1}{4}\frac{\dot{A}^2}{A^2} - \frac{1}{2}\frac{\ddot{A}}{A} + \frac{1}{4}\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}}{Br}\right) - f_{\mathcal{G}}\left(\frac{\dot{A}^2}{A^2Br^2} + \frac{3\dot{A}\dot{B}}{AB^2r^2} - \frac{\dot{A}\dot{B}}{ABr^2} + \frac{2\ddot{A}}{Ar^2} - \right.$$
$$\left. -\frac{2\ddot{A}}{ABr^2} - \frac{\dot{A}^2}{A^2r^2}\right) + \dot{f}_R\left(\frac{2}{r} + \frac{1}{2}\frac{\dot{A}}{A} - \frac{1}{2}\frac{\dot{B}}{B} + \frac{1}{2}\dot{B}\right) - \frac{2\dot{A}}{Ar^2}\left(B-3\right)\dot{f}_{\mathcal{G}}.$$

``mixed'' equation

$$\ddot{f}_R + \dot{f}_R\left(\frac{\dot{A}B}{2A} - \frac{1}{2}\dot{B}\right) - f_R\left(\frac{\dot{B}}{Br} + \frac{\dot{A}}{Ar}\right) = \frac{4}{Br^2}\left(1-B\right)\ddot{f}_{\mathcal{G}} + \frac{2}{r^2}\left(3\frac{\dot{A}}{A} - \frac{\dot{A}B}{A} + \frac{\dot{B}}{B} + \dot{B}\right)\dot{f}_{\mathcal{G}}$$

General notes

Ricci scalar

$$R = \frac{1}{2B}\left(-\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} - \frac{4\dot{A}}{Ar} + \frac{4\dot{B}}{Br} - \frac{2}{r^2}\right) + \frac{2}{r^2}$$

Gauss-Bonnet term

$$\mathcal{G} = \frac{1}{B^2}\left(-\frac{2\dot{A}^2}{A^2r^2} - \frac{6\dot{A}\dot{B}}{ABr^2} + \frac{4\ddot{A}}{Ar^2}\right) + \frac{1}{B}\left(\frac{2\dot{A}\dot{B}}{ABr^2} - \frac{4\ddot{A}}{Ar^2} + \frac{2\dot{A}^2}{A^2r^2}\right)$$

Horizon: $(A \to 0,\ B \to \infty)$

$$R \to 2/r^2 \text{ and } \mathcal{G} \to 0$$

$$\dot{R} \to -4/r^3 \qquad \dot{\mathcal{G}} \to 0$$

Horizon existence - 1

``Mixed'' equation under horizon conditions one can rewrite as

$$\frac{2\dot{f}_{\mathcal{G}}\dot{B}}{r^2} - \frac{2B\dot{A}\dot{f}_{\mathcal{G}}}{Ar^2} - \frac{1}{2}\frac{B\dot{A}\dot{f}_R}{A} + \frac{1}{2}\dot{f}_R\dot{B} = 0$$

$$\left(\frac{2\dot{f}_{\mathcal{G}}}{r^2} + \frac{\dot{f}_R}{2}\right)\left(\dot{B} - \frac{B\dot{A}}{A}\right) = 0$$

Second parenthesis imply $A = \mathrm{const} \times B$

$$\dot{f}_R = -\frac{4\dot{f}_{\mathcal{G}}}{r^2} \qquad \ddot{f}_R = -\frac{4\ddot{f}_{\mathcal{G}}}{r^2} + \frac{8\dot{f}_{\mathcal{G}}}{r^3}$$

Horizon existence - 2

For analitic functions

$$\dot{f}_R = \dot{R} f_{RR} + \dot{\mathcal{G}} f_{R\mathcal{G}}; \quad \dot{f}_{\mathcal{G}} = \dot{R} f_{R\mathcal{G}} + \dot{\mathcal{G}} f_{\mathcal{G}\mathcal{G}}$$

Applying horizon conditions one gets

$$f_{RR} = -\frac{4}{r^2} f_{R\mathcal{G}}$$

If there are no cross-terms in *f(R, G)*, i.e $f(R, \mathcal{G}) = f_1(R) + f_2(\mathcal{G})$ this imply

$$f_{RR} = 0 \text{ or } \dot{R} = 0$$

Similarly for the singularity, if there are no cross-terms $f_{\mathcal{G}\mathcal{G}} = 0$ or $\dot{\mathcal{G}} = 0$

$R + f(R)$ case

``Mixed'' equation takes form

$$\frac{1}{r}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)(1+F_R)=0$$

The solution

$$A(r)=\frac{C_0}{B(r)}$$

After applying everything, 11-equation takes form

$$-\frac{\dot{B}}{Br}-\frac{B}{r^2}+\frac{1}{r^2}-\frac{1}{2}BF-\frac{\dot{B}^2F_R}{B^2}+\frac{\ddot{B}F_R}{2B}+\frac{\dot{B}F_R}{Br}=0$$

Solving it altogether with $R=R_0$

$$B(r) = \frac{6r(1+F_R)}{r^3(F - R_0 F_R) + 6r(1+F_R) + 6C_1(1+F_R)}$$

$$F_R = -1/2,\ C_1 = -2m \text{ and } F = \Lambda - R_0/2$$

$$B(r) = \frac{3r}{\Lambda r^3 + 3r - 6m}$$

A. de la Cruz-Dombris *et al.* (2011)   AB=const, R=const

$R + f(\mathcal{G})$ case

Similarly to the previous case

$$\frac{1}{r}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0$$

$$B(r) = \frac{6r}{r^3(F - \mathcal{G}_0 f_{\mathcal{G}}) + 6r + 6C_1}$$

$$C_1 = -2m \text{ and } F = 2\Lambda + \mathcal{G}_0 f_{\mathcal{G}}$$

$R + f(R) + F(\mathcal{G})$ case

$$B(r) = \frac{12r}{r^3(f + F - \mathcal{G}_0 f_{\mathcal{G}} - R_0 f_R) + 12r + 12C_1}$$

$$C_1 = -2m \text{ and } f + F - \mathcal{G}_0 F_{\mathcal{G}} - R_0 f_R = 4\Lambda$$

Results

If there are no cross-terms in f(R,G), the theory from the considered point of view cannot be distinguished from GR.

The work is in progress, and there are hints pointing that if there are cross-terms, there might be solutions different from Schw-dS.