There are no stationary axisymmetric star solutions in Hořava-Lifshitz gravity

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Introduction

- **Recently, Hořava proposed a power-counting renormalizable theory. This theory is expected to be a renormalizable and ghost-free theory.**

- **Our goal is to constrain Hořava gravity from astrophysical observations.**

- **There are no spherically symmetric and static star solutions with perfect fluid in Hořava gravity.**
  K. Izumi and S. Mukohyama, Phys. Rev. D 81, 044008(2010)

  $\Longrightarrow$ **How about rotating star with perfect fluid?**

Hořava-Lifshitz gravity

P. Hořava, Phys. Rev. D79, 084008 (2009).

P. Hořava, JHEP 0903, 020 (2009).

- **In the ultraviolet, the theory exhibits the Lifshitz-type anisotropic scaling $t \to b^z t,\ x^i \to b x^i$.** ($z$ is the dynamical critical exponent.)

- For $z = 3$ ($z > 3$), the theory is power-counting (super-)renormalizable.
- This theory has no general covariance.

foliation-preserving diffeomorphism

$$t \to \tilde{t}(t),\ x^i \to \tilde{x}^i(t, x)$$

$\Longrightarrow$ **Quantities on the constant-time hypersurfaces have to be regular.**

- **Arnowitt-Deser-Misner (ADM) form**

  $ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$
  g_{ij}: spatial metric tensor, N: lapse function, N^i: shift vector

- **Action**
  The action is constrained strongly from the view point of renormalization. (**detailed balance condition**)

$$I_g = \int dt d^3x \sqrt{g} N[\alpha(K^{ij}K_{ij} - \lambda K^2) + \beta C_{ij}C^{ij} + \gamma \varepsilon^{ijk} R_{il} D_j R^l_k + \zeta R_{ij}R^{ij} + \eta R^2 + \xi R + \sigma]$$

  where $\alpha, \beta, \gamma, \lambda, \zeta, \eta, \xi, \sigma$ are constant parameters,
  R_{ij} is the Ricci tensor of g_{ij},
  D_i is the covariant derivative compatible with g_{ij},
  K_{ij} is the extrinsic curvature of constant-time hypersurfaces and
  C_{ij} is the Cotton tensor.

- **In the infrared, we can get the same action as GR.**

Projectability condition and Hamiltonian constraint

- **Projectability condition** $\quad N = N(t)$

- **The variation of the action with respect to $N(t)$, we get the Hamiltonian constraint.**

- Projectable theory $\Longrightarrow$ **global Hamiltonian constraint.**

$$\int dx^3\sqrt{g}\Big[(\alpha K^{ij}K_{ij}-\lambda K^2)-\beta C_{ij}C^{ij}-\gamma\varepsilon^{ijk}R_{il}D_jR^l_k$$
$$-\zeta R_{ij}R^{ij}-\eta R^2-\xi R-\sigma\Big]+\int dx^3\sqrt{g}T_{\mu\nu}n^\mu n^\nu=0.$$

(Non-projectable theory $\Longrightarrow$ **local Hamiltonian constraint.**)

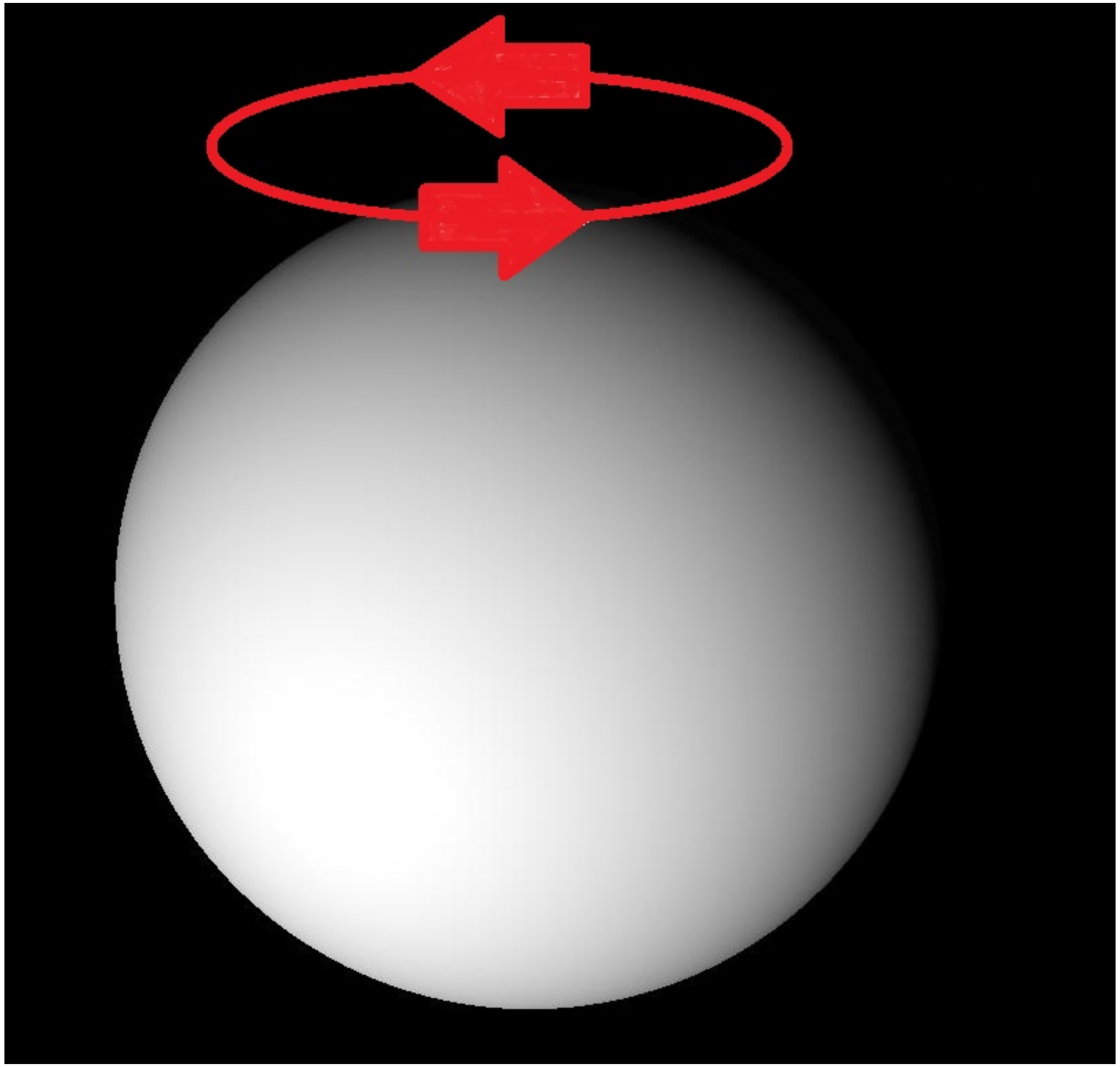
There are no spherically symmetric and static star solutions with perfect fluid.

K. Izumi and S. Mukohyama, Phys. Rev. D 81, 044008(2010).

$\Longrightarrow$ **But stars rotate more or less.**

$\Longrightarrow$ **So we will investigate stationary axisymmetric stars.**

Stationary Axisymmetric Stars



Assumptions

- Stationary and axisymmetric spacetime
  $t^\mu \partial_\mu = \partial_t, \phi^\mu \partial_\mu = \partial_\phi$
- Perfect fluid

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

- The four-velocity

$$u^\mu \partial_\mu = \frac{1}{D}(t^\mu + \omega \phi^\mu)\partial_\mu .$$

  $D \equiv (N^2 - N_i N^i - 2\omega N_\phi - \omega^2 g_{\phi\phi})^{\frac{1}{2}},$
- $\rho(P) \geqq 0$.
- ρ is a piecewise-continuous function.
- $P_c \equiv P(r = 0) > 0$.
- Reflection symmetry about the equatorial plane

Spatial Line Element

- As a part of gauge conditions, we take

$$g_{r\theta} = g_{r\phi} = 0.$$

- Under this gauge condition, we can take the spatial line element generally,

$$dl^2 = \psi^4[A^2 dr^2 + \frac{r^2}{B^2} d\theta^2 + r^2 B^2 (\sin\theta d\phi + \xi d\theta)^2],$$

ψ, A, B, and ξ are functions of r and θ for the stationary and the axisymmetric spacetime.

Momentum conservation

By the invariance of the matter action I_m under the infinitesimal transformation $\delta x^i = \zeta^i(t,x)$, we get the r component of the momentum conservation

$$0 = -\frac{1}{N} N^j D_j (T_{r\mu} n^\mu) + K T_{r\mu} n^\mu - \frac{1}{N} T_{j\mu} n^\mu D_r N^j - D^j T_{rj}.$$

After some calculation, we obtain

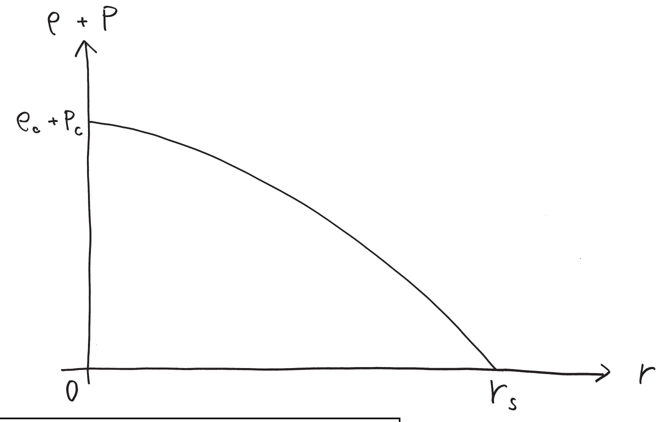
$$\begin{aligned} 0 &= -P_{,r} + \frac{\rho + P}{D^2} \left\{ \frac{1}{2} (N_i N^i)_{,r} + \omega N_{\phi,r} + \frac{1}{2} \omega^2 g_{\phi\phi,r} + \frac{N_{,r}}{N} N^r N_r + \frac{N_{,\theta}}{N} N^\theta N_r \right\} \\ &= -P_{,r} + \frac{\rho + P}{D^2} \left\{ \frac{1}{2} (-N^2 + N_i N^i)_{,r} + \omega N_{\phi,r} + \frac{1}{2} \omega^2 g_{\phi\phi,r} \right\}, \end{aligned}$$

In the second line, we used the projectability condition $N = N(t)$.
Here, we concentrate on the rotation axis $\theta = 0$.
$g_{\phi\phi} = \psi^4 r^2 B^2 \sin^2\theta$ and the regularity of the triad component of the shift vector $N_{(3)} = \frac{N_\phi}{\psi^2 r B \sin\theta}$. implies $N_{\phi,r} = g_{\phi\phi,r} = 0$.

$$\left\{\log\left(N^2 - N_i N^i\right)\right\}_{,r} = -2\frac{P_{,r}}{\rho + P}.$$

Under the assumption $\rho = \rho(P)$, we can transform

$$-2\frac{P_{,r}}{\rho + P} = -2\left(\int \frac{dP}{\rho + P}\right)_{,r}.$$



Integrating over $0 \le r < r_s$,

$$\log\left(N^2 - N_i N^i\right)\Big|_{r=r_s} - \log\left(N^2 - N_i N^i\right)\Big|_{r=0} = -2\int_{P_c}^{P_s} \frac{dP}{\rho + P},$$

The regularity of $N_{(i)}$ and $K_{(i)(j)}$ implies $P(r)$ is continuous.

From $\rho(P) \geqq 0$, $P(r)$ and r_s, we obtain $P_s < 0 \Longrightarrow P_s \le 0 < P_c$.

This implies that

the right-hand side of momentum conservation is positive.

The left-hand side of momentum conservation

We use locally Cartesian coordinates near the origin. From Axisymmetry, we obtain

$$N^i{}_{,j}\phi^j - \phi^i{}_{,j}N^j = 0.$$

The general regular solution of these equations is

$$\frac{N^r}{r} = \sin^2\theta F_1 + \frac{1}{r}\cos\theta F_3, \qquad \frac{N^\theta}{\sin\theta} = \cos\theta F_1 - \frac{F_3}{r}, \qquad N^\phi = F_2.$$

where each F_n is an independent and regular function.

- Reflection symmetry about the equatorial plane implies F_1, F_2 are even functions of z, and F_3 is an odd function of z.
  $\Longrightarrow$ $N^r|_{r=0} = N^\theta\Big|_{r=0} = 0,$

Thus

$$N_iN^i\Big|_{r=0} = 0,$$

The left-hand side of momentum conservation is non-positive.

$$\log\left(N^2 - N_iN^i\right)\Big|_{r=r_s} - \log\left(N^2 - N_iN^i\right)\Big|_{r=0} \le 0.$$

Contradiction of momentum conservation

The r component of the momentum conservation on $\theta = 0$ is

$$\log\left(N^2 - N_i N^i\right)\Big|_{r=r_s} - \log\left(N^2 - N_i N^i\right)\Big|_{r=0} = -2\int_{P_c}^{P_s} \frac{dP}{\rho + P}.$$

Its left-hand side is **non-positive**, while the right-hand side is **positive**.

$\Longrightarrow$ **There are no stationary axisymmetric star solutions in Hořava-Lifshitz gravity**.

Summary

In Hořava-Lifshitz gravity

Projectability condition $N = N(t)$

$+$

Regularity of N^i,$K_{(i)(j)}$

due to foliation-preserving diffeomorphism

$\downarrow$

No stationary axisymmetric star solutions