Spherical collapse of dust cloud in Lovelock theory

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1. Introduction

- Motivation
- Lovelock Gravity

1.1 Motivation

Our Interest

Cosmic Censorship in higher dimensions

- Cosmic Censorship Conjecture

  The singularities formed in gravitational collapse can not be naked

- Why higher dimensions ?

  High energy near the singularity. Quantum effects come into play

  One of the most promising theory for quantum gravity is Superstring theory

  Superstring theory is formulated in higher dimensions

1.2 Gravity in higher dimensions

- How to generalize gravitational theory into HD

  String theory predicts special combination of higher curvature corrections to Einstein theory

  ⇒ Lovelock Gravity is natural extension of Einstein gravity into HD including higher curvature term

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Lovelock Gravity

1. Field eqs are symmetric rank-2 tensor ($\mathcal{G}_{\mu\nu} = \mathcal{G}_{\nu\mu}$)
2. Divergence free ($\nabla_\nu \mathcal{G}^\nu_\mu = 0$)
3. Field eqs are second order

1.3 Lovelock Theory

Lagrangian of the theory

$$\mathcal{L} = \sum_{m=1}^{k} \frac{a_m}{m} \mathcal{L}_m,$$

$$\mathcal{L}_m = \frac{1}{2^m} \delta^{\nu_1 \nu_2 \dots \nu_{2m-1} \nu_{2m}}_{\mu_1 \mu_2 \dots \mu_{2m-1} \mu_{2m}} R_{\nu_1 \nu_2}{}^{\mu_1 \mu_2} \dots R_{\nu_{2m-1} \nu_{2m}}{}^{\mu_{2m-1} \mu_{2m}}.$$

$k = [\frac{D-1}{2}]$ (D is spacetime dimensions)

a_m: arbitrary constants (We assume positive)

$$\delta^{\nu_1 \dots \nu_n}_{\mu_1 \dots \mu_n} = \det \begin{pmatrix} \delta^{\nu_1}_{\mu_1} & \dots & \delta^{\nu_1}_{\mu_n} \\ \dots & \dots & \dots \\ \delta^{\nu_n}_{\mu_1} & \dots & \delta^{\nu_n}_{\mu_n} \end{pmatrix}$$

2. Model: Spherical Dust Collapse

- Gravitational collapse of matter
- Cosmic Censorship in Lovelock gravity

2.1 Model

Assumptions

Spacetime is Spherically symmetric ($Dim = n + 2$)

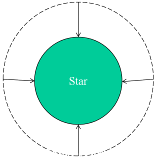
$$ds^2 = -A^2(t,r)dt^2 + B^2(t,r)dr^2 + R^2(t,r)d\Omega_n^2$$

$A(t,r), B(t,r), R(t,r)$: arbitrary functions of t, r

Collapsing matter is Dust cloud ($P = 0$)

$$T_\mu^\nu = -\epsilon(t,r)\delta_\mu^t\delta_t^\nu$$

$\epsilon(t,r)$ energy density of dust



2.2 Field equations

Solving equations

$$ds^2 = -dt^2 + \frac{R'^2}{W^2(r)}dr^2 + R^2 d\Omega_n^2$$

$$\sum_{m=1}^{k} c_m \left(\dot{R}^2 + 1 - W^2\right)^m \frac{1}{R^{2m}} = \frac{F(r)}{R^{n+1}}.$$

$F(r) \equiv \frac{2}{n} \int \epsilon R' R^n dr \propto$ Miser-Sharp mass

$W(r)$: arbitrary function of r

$c_l \equiv \frac{a_l}{l} \prod_{p=1}^{2l-2}(n-p)$

Examine the dynamics of the spacetime

2.3 Singularity and Apparent horizon

List of what we will look at

- Does Singularity occur ?
- Whether singularities are wrapped by apparent horizon or not ?

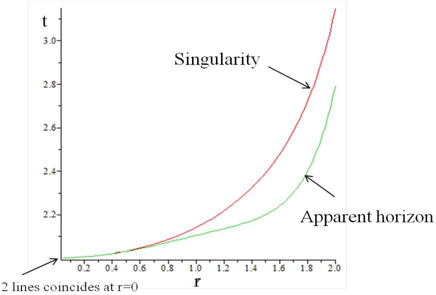
  No light ray can emerge form the region inside the apparent horizon
- Is Singularity naked ?

  Can light ray emerge from Singularity
- Strength of singularity

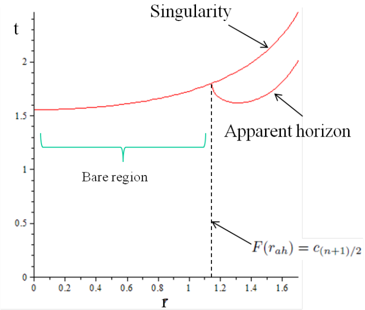
  Divergent behavior of singularity

Difference between Even and Odd

Even



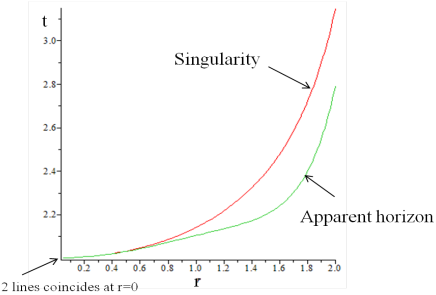
Odd



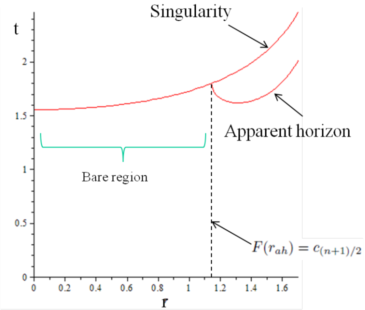
2.4 Singularity vs Apparent horizon

Difference between Even and Odd

Even



Odd



1. Singularities are formed
2. Formation of apparent horizon depends on the spacetime dimension
3. In even dim Singularity at $r = 0$ can be naked. In odd dim $0 \leq r < r_{ah}$ can be naked.

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2.5 Strength of singularity

$$\psi = R_{\mu\nu} k^{\mu} k^{\nu}$$

k^{μ}: tangent vector of null geodesics emanating from singularity
Strength of singularity: Divergent behavior of ψ with respect to affine parameter λ of null geodesic

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Strength of singularity: Divergent behavior of ψ with respect to affine parameter λ of null geodesic

- Even dim ($r = 0$): $\psi \sim (\lambda - \lambda_0)^{-\frac{4(n+1)}{3n+1}}$ Depend on dimension
- Odd dim ($r = 0$): $\psi \sim (\lambda - \lambda_0)^{-\frac{4}{3}}$ NOT depend on dimension
- Odd dim ($r \neq 0$): $\psi \sim (\lambda - \lambda_0)^{-1}$ NOT depend on dimension

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e.g.(Gauss-Bonnet vs Lovelock)

	$D = 7$	$D = 8$
Lovelock	$-\frac{4}{3} \sim -1.33$	$-\frac{28}{19} \sim -1.47$
Gauss-Bonnet	$-\frac{12}{7} \sim -1.71$	$-\frac{28}{15} \sim -1.86$

$\Rightarrow$ Higher curvature terms weaken the strength of singularity

3. Summary

Summary

- Gravitational collapse of dust cloud in Lovelock gravity
- Singularities are formed and can be naked
- Formation of apparent horizon crucially depends on the spacetime dimensions (Odd/Even)
- Higher curvature terms weaken the strength of singularity