Binary Inspiral in Quadratic Gravity

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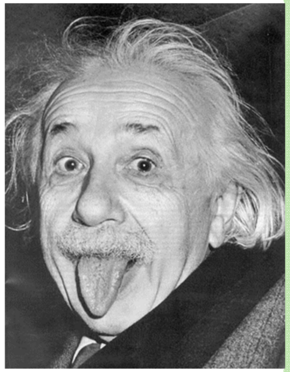
§ 1
INTRODUCTION

- GR test in the weak field regime

  Solar system experiment [Will (2006)]

  (Deviation from GR) < 10^{-5} (Shapiro time delay)



Aim: GR test in the strong field regime using binary GWs

- **Quadratic curvature correction** in the action

  ⇒ Correction is larger when the field is stronger

- Quite **generic quadratic curvature theory**, coupled to a scalar field, that includes some known theories.

⇓

Quadratic Gravity

§ 2
ABC of
Quadratic Gravity

Action:

[Yunes & Stein (2011)]

$$c = G = 1$$

$$\kappa \equiv \frac{1}{16\pi}$$

$$S \equiv \int \sqrt{-g} \{ \kappa R + \alpha_1 f_1(\vartheta) R^2 + \alpha_2 f_2(\vartheta) R_{ab} R^{ab} + \alpha_3 f_3(\vartheta) R_{abcd} R^{abcd} + \alpha_4 f_4(\vartheta) R_{abcd} {}^* R^{abcd} - \frac{\beta}{2} [\nabla_a \vartheta \nabla^a \vartheta + 2V(\vartheta)] + \mathcal{L}_{\rm mat} \} ,$$

e.g. (i) Dilatonic Einstein-Gauss-Bonnet (**DEGB**)

$$\alpha_1 = -\frac{\alpha_2}{4} = \alpha_3 = \alpha, \ \alpha_4 = 0, \ \beta = 1, \ V = 0,$$

$$f_1 = f_2 = f_3 = e^{-\gamma\theta}$$

(ii) Dynamical Chern-Simons (**DCS**)

$$\alpha_1 = \alpha_2 = \alpha_3 = 0, \ f_4 = \theta$$

- expanding $f_i(\vartheta)$ around $\vartheta = 0$. $f_i(\vartheta) = f_i(0) + f_i'(0)\vartheta + \cdots .$

$f_i(0)$: GB & CS: Field equations unmodified.

$f_i'(0)$: Absorbed to α_i.

$\Rightarrow$ $f_i(\vartheta) = \vartheta.$

$$\zeta_i \equiv \frac{\alpha_i^2}{\beta \kappa m^4} \ll 1 \qquad \mathcal{O}(\zeta_i)$$

(Corrction to GR is small)

Modified Einstein Eq.

$$G_{ab} + \frac{\alpha_1}{\kappa}\mathcal{H}_{ab}^{(\vartheta)} + \frac{\alpha_2}{\kappa}\mathcal{I}_{ab}^{(\vartheta)} + \frac{\alpha_3}{\kappa}\mathcal{J}_{ab}^{(\vartheta)} + \frac{\alpha_4}{\kappa}\mathcal{K}_{ab}^{(\vartheta)} = \frac{1}{2\kappa}\left(T_{ab}^{\rm mat} + T_{ab}^{(\vartheta)}\right)$$

$$\mathcal{H}_{ab}^{(\vartheta)} \equiv -4v_{(a}\nabla_{b)}R - 2R\nabla_{(a}v_{b)} + g_{ab}(2R\nabla^c v_c + 4v^c\nabla_c R) + \vartheta\left[2R_{ab}R - 2\nabla_{ab}R - \tfrac{1}{2}g_{ab}(R^2 - 4\Box R)\right],$$

$$\mathcal{I}_{ab}^{(\vartheta)} \equiv -v_{(a}\nabla_{b)}R - 2v^c(\nabla_{(a}R_{b)c} - \nabla_c R_{ab}) + R_{ab}\nabla_c v^c - 2R_{c(a}\nabla^c v_{b)} + g_{ab}(v^c\nabla_c R + R^{cd}\nabla_c v_d) + \vartheta\left[2R^{cd}R_{acbd} - \nabla_{ab}R + \Box R_{ab} + \tfrac{1}{2}g_{ab}(\Box R - R_{cd}R^{cd})\right],$$

$$v_a \equiv \nabla_a\vartheta$$

$$\mathcal{J}_{ab}^{(\vartheta)} \equiv -8v^c(\nabla_{(a}R_{b)c} - \nabla_c R_{ab}) + 4R_{acbd}\nabla^c v^d - \vartheta[2(R_{ab}R - 4R^{cd}R_{acbd} + \nabla_{ab}R - 2\Box R_{ab}) - \tfrac{1}{2}g_{ab}(R^2 - 4R_{cd}R^{cd})],$$

$$\mathcal{K}_{ab}^{(\vartheta)} \equiv 4v^c\epsilon_c{}^d{}_{e(a}\nabla^e R_{b)d} + 4\nabla_d v_c {}^*R_{(a}{}^c{}_{b)}{}^d,$$

$$T_{ab}^{(\vartheta)} = \frac{\beta}{2}\left[\nabla_a\vartheta\nabla_b\vartheta - \frac{1}{2}g_{ab}\left(\nabla_c\vartheta\nabla^c\vartheta - 2V(\vartheta)\right)\right]$$

Scalar Wave Eq.

$$\beta\Box\vartheta - \beta\frac{dV}{d\vartheta} = -\alpha_1 R^2 - \alpha_2 R_{ab}R^{ab} - \alpha_3 R_{abcd}R^{abcd} - \alpha_4 R_{abcd}{}^*R^{abcd}$$

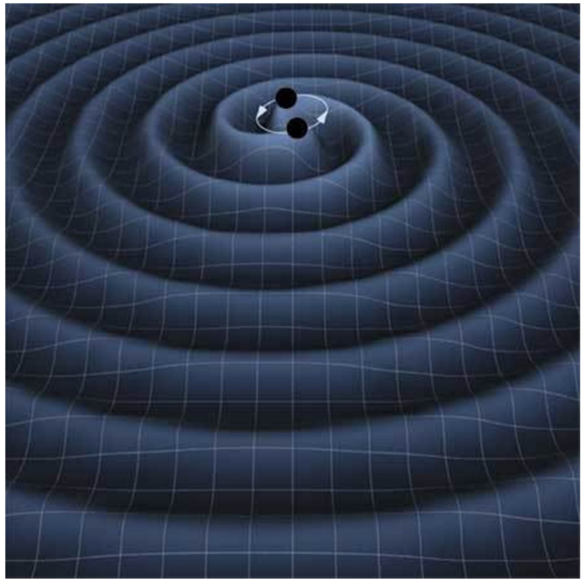
- For simplicity, we consider $V(\vartheta) = 0$.
- We solve this wave equation with the **Post-Newtonian (PN) approach**.

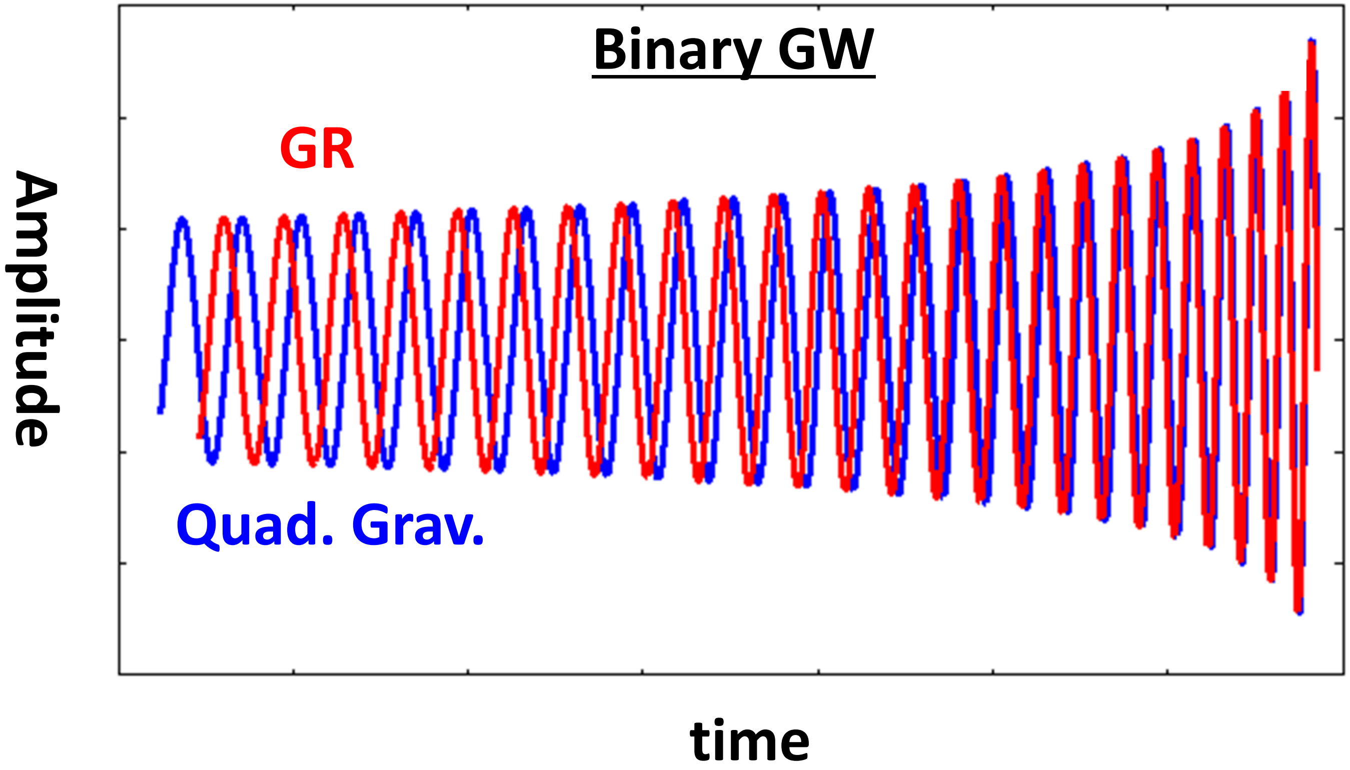
Scalar radiation

⇒ Changes the radiated energy flux

⇒ Changes binary's **orbital evolution**

⇒ Correction in GW





§ 3
PN approach of Scalar radiation

Solving the scalar wave equation with PN approach

Slow motion, weak field expansion

Background metric: **flat**

Problems of using PN in modified gravity

Point particle approximation?
Proper regularization scheme? Strong field?

PN near zone (**NZ**) solution

Matching — Taking **the strong field effect** into account

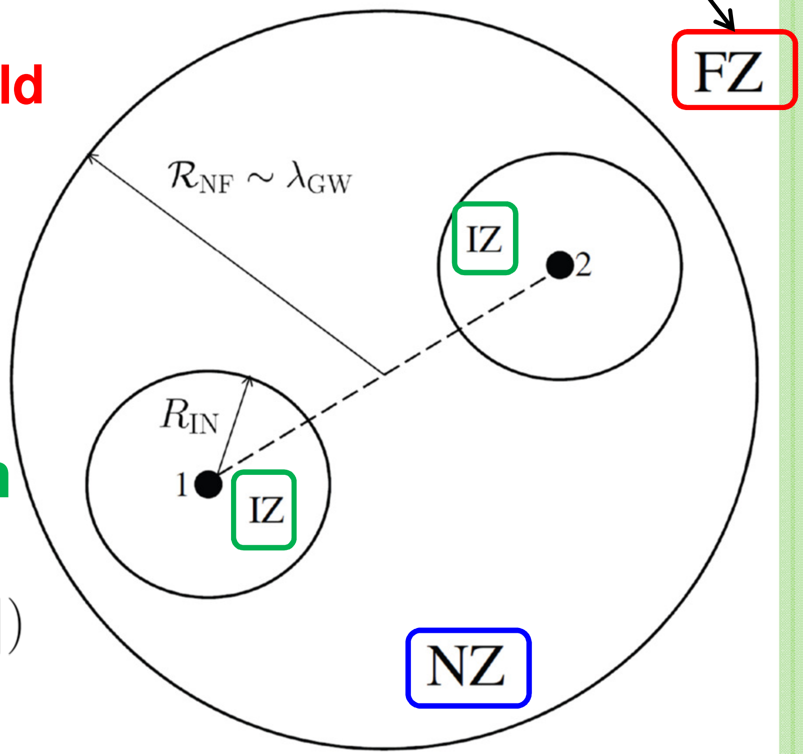
(known) inner zone (**IZ**) strong field solution

(isolated) BH background solution

$$\vartheta^{\rm IZ} = \frac{2\alpha_3}{\beta}\frac{1}{m_A r_A} + \mathcal{O}\left(\frac{1}{r_A^2}\right) \qquad (r_A = |\boldsymbol{x} - \boldsymbol{x}_A|)$$

[Yunes & Stein (2011)]

We are interested in the FZ solution



FZ solution: $\Box\vartheta \approx -4\pi q_1 \delta^3(\boldsymbol{x}-\boldsymbol{x}_1) + (1 \leftrightarrow 2)$

$$q_A \equiv \frac{2\alpha_3}{\beta m_A}$$ ← **BH scalar charge**

(This source reproduces the BH background solution in the NZ.)

$$\vartheta^{\rm FZ} = \frac{1}{r}\sum_m \frac{1}{m!}\frac{\partial^m}{\partial t^m}\int q_1\delta^3(\boldsymbol{x}-\boldsymbol{x}_1)(\boldsymbol{n}\cdot\boldsymbol{x})^m d^3x + (1 \leftrightarrow 2)$$

[Pati & Will (2002)]

$$= \frac{1}{r}\left(q_1\frac{m_2}{m} - q_2\frac{m_1}{m}\right) v_{12i} n^i + \mathcal{O}\left(v^2\right)$$

$v_{12i} = v_{1i} - v_{2i}$

r : Distance to the source

n^i : Unit vector to the source

Correction in energy flux: $\dot{E}^{(\vartheta)} = \lim_{r\to\infty}\int_{S_r^2} r^2 d\Omega\ \beta\left(\partial_r\vartheta_{\rm FZ}\right)\left(\partial_t\vartheta_{\rm FZ}\right)$

$$\frac{\dot{E}^{(\vartheta)}}{\dot{E}^{\rm GR}} = \frac{5}{96}\zeta_3\eta^{-4}\frac{\delta m^2}{m^2}v^{-2}$$

$\eta \equiv \frac{m_1 m_2}{m^2}$ $\delta m \equiv m_1 - m_2$

-1PN correction

(**scalar dipole radiation**)

§ 4
Constraints with GW observations

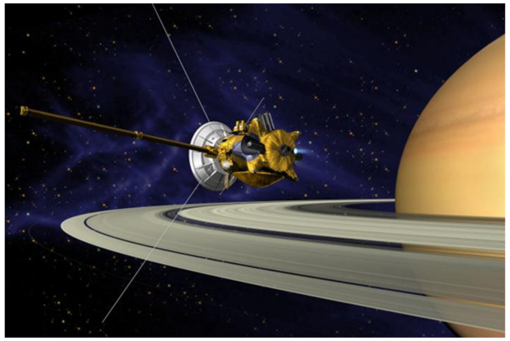
Current Constraints from Solar System Experiment

Amendola *et al.* (2007)

- Shapiro time delay measurement by Cassini

(DEGB)

$$|\alpha_3| < 1.6 \times 10^{24}\mathrm{cm}^2 \qquad (\beta = 1)$$



Mapping to the Parametrized Post-Einsteinian (PPE)

Yunes & Pretorius (2009)

- ppE waveform (in Fourier domain):

$$\tilde{h}(f) = \tilde{h}_{\mathrm{GR}}(f)\left[1 + \alpha_{\mathrm{ppE}} u^{a_{\mathrm{ppE}}}\right] \exp\left(i \beta_{\mathrm{ppE}} u^{b_{\mathrm{ppE}}}\right)$$

$$\begin{cases} u \equiv \pi \mathcal{M} f \propto v^3 \\ \mathcal{M} \equiv m \eta^{3/5} \\ \eta \equiv \dfrac{m_1 m_2}{m^2} \end{cases}$$

Quadratic Gravity:

$$\frac{\dot{E}^{(\vartheta)}}{\dot{E}^{\mathrm{GR}}} = \frac{5}{96} \zeta_3 \eta^{-4} \frac{\delta m^2}{m^2} v^{-2}$$

$$\alpha_{\mathrm{ppE}} = a_{\mathrm{ppE}} = 0$$

$$\beta_{\mathrm{ppE}} = -\frac{5}{7168} \zeta_3 \eta^{-18/5} \frac{\delta m^2}{m^2} \qquad b_{\mathrm{ppE}} = -\frac{7}{3}$$

$$\beta_{\rm ppE} = -\frac{5}{7168}\zeta_3 \eta^{-18/5}\frac{\delta m^2}{m^2} \qquad b_{\rm ppE} = -\frac{7}{3} \qquad \zeta_3 = \frac{\alpha_3^2}{\beta \kappa m^4}$$

Constraints using adv. LIGO

$m_1 = 6M_\odot, \ m_2 = 12M_\odot$ SNR=20

➡ $|\beta_{\rm ppE}| < 1.8 \times 10^{-5}$

➡ $|\alpha_3| < 3.3 \times 10^{10}{\rm cm}^2$

Validity of the constraint

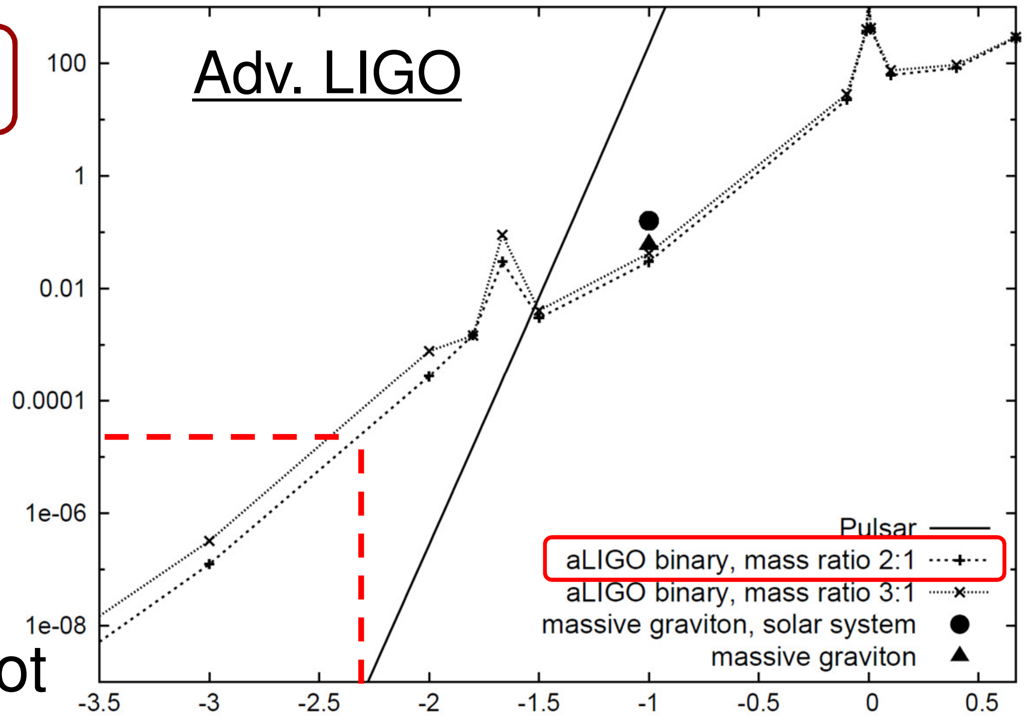
$$\zeta_3 = \frac{\alpha_3^2}{\beta \kappa m^4} \ll 1$$

➡ $|\alpha_3| \ll 10^{12}{\rm cm}^2$

The constraint above cannot be realized by the solar system experiment!!

Upper bounds on $|\beta_{\rm ppE}|$

$|\beta_{\rm ppE}|$



$b_{\rm ppE}$

Cornish *et al*. (2011)

§ 5
Dynamical Chern-Simons Case

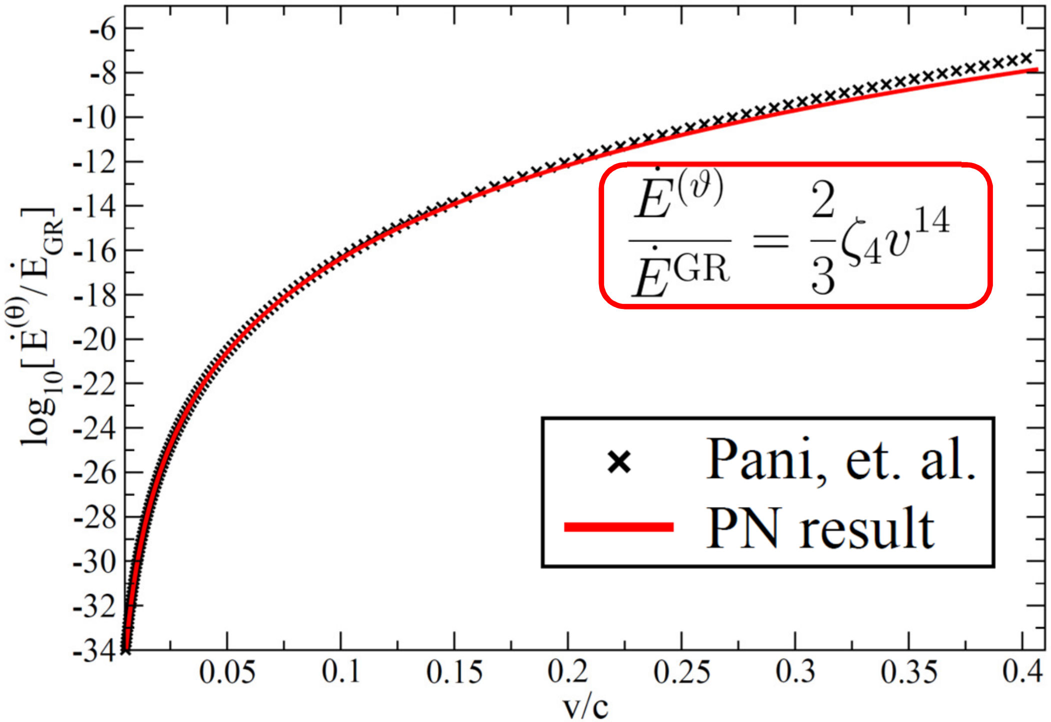
$$(\alpha_1 = \alpha_2 = \alpha_3 = 0)$$

(I) Non-spinning BH

Pani *et al*. (2010): **Non-spinning** BH perturbation in DCS

Scalar radiation ⇒ 7PN correction

Our result shows **good agreement** with theirs!



$$\zeta_4 \equiv \frac{16\pi}{m^4}\frac{\alpha_4^2}{\beta}$$

Metric Perturbation ⇒ 6PN [Pani et al. (2010)]

(our analysis: only PN order counting)

(II) Spinning BH 3.5PN (O(a)), 2PN (O(a^2)), (only PN order counting)

§ 6 Summary

Probing **Quad. Grav.** in the **strong field regime** with BH/BH GWs.

Matching PN & BH solution ⇒ taking the strong field effect into account.

(I) Even parity sector:

Scalar dipole radiation ⇒ **-1PN** correction

adv. LIGO ⇒ **Unique constraint on the quadratic graivty** compared to the solar system experiment.

(II) Odd parity sector:

(i) non-spinning BH:

Scalar radiation ⇒ 7PN correction, **consistent with BH perturbation**.

Correction in the metric ⇒ 6PN correction (PN order counting)

(ii) Spinning BH:

3.5PN (O(a)), 2PN ($O(a^2)$) (PN order counting)

(iii) **Future works**:

① Isolated BH solution at $O(a^2)$
② BH deformation by the other BH
③ Non-dissipative corrections (e.g. binding energy)