Cosmic censorship in overcharging a charged black hole with a charged particle

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Yukawa Institute for Theoretical Physics

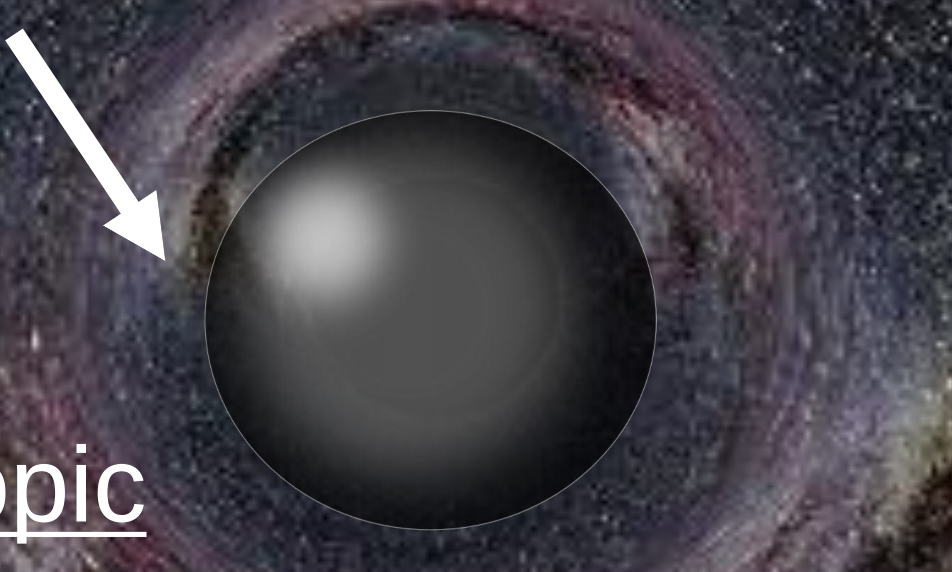
(Kyoto University)

The Next Generation of Physics, Spun from Universality & Emergence
Developing Independent Researchers to Explore New Frontiers

Soichiro Isoyama

Works with **Norichika Sago** and **Takahiro Tanaka**

A **singularity** (kills predictability of GR) cannot be seen from a distant observer.



Today's topic

「Cosmic censorship」

Today's problem and solution

「Challenge a “counter example”」

Overcharging a charged black hole

(V.Hubeny Phys. Rev. **D66** 024016 (2002))

Q. Can a radially falling charged particle "saturate" the charge of a (4-dim) Reisner-Nordström (RN) black hole and turn it to be a naked singularity ?

The RN **black hole** : $M \geq |Q|$

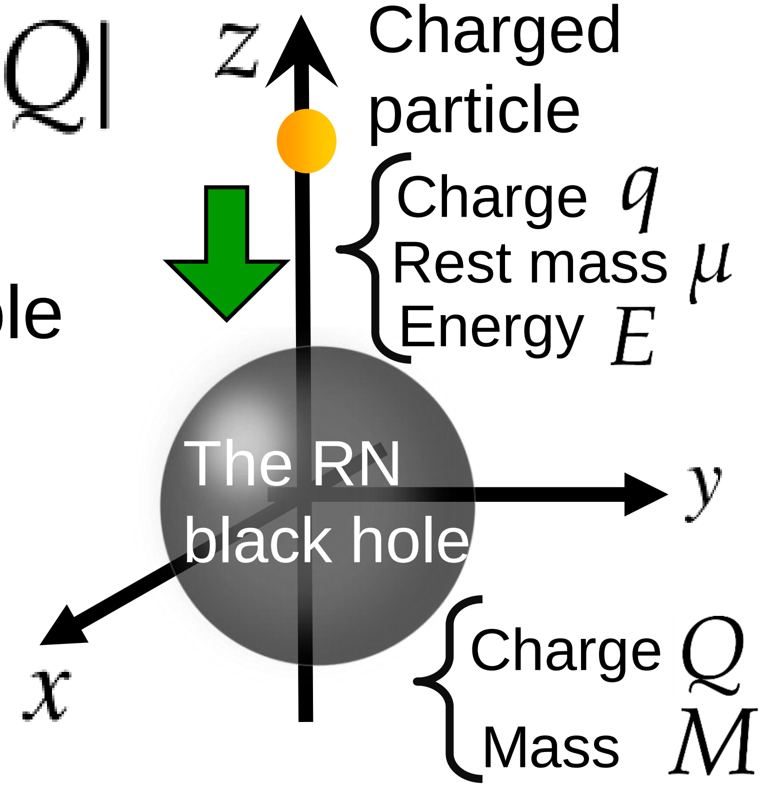
① Absorption condition :

Pass the horizon of the black hole

② Overcharging condition :

The final RN spacetime says…

$$M_{\mathrm{B.H.}} + E < Q_{\mathrm{B.H.}} + q$$



If all back reaction effects are ***negligible,*** the near-exremal RN black hole **can be overcharged** via charged particle absorption.

With a small parameter: $\epsilon \ll 1$

Charged particle: $E := O(\epsilon),\ \mu := O(\epsilon),\ q := O(\epsilon),$

A near extremal RN black hole: $M := 1 + O(\epsilon^2),\ Q := 1,$

The excess of the extremality:

$$(Q_{\mathrm{B.H.}} + q) - (M_{\mathrm{B.H.}} + E) \approx O(\epsilon^2)$$

However···

?

Back reaction effects are negligible

with $O(\epsilon^2)$?

Back reaction effects

Ignoring the back reaction effects on both the particle's motion and the system's energy **due to the particle existence is not valid**;

[e.g. the electromagnetic self-field : $f_{\mu\nu} \sim O(\epsilon)$]

① Absorption condition
(Particle's motion):

$$F_{\rm EM}^{\rm self} : q\,\nabla f \sim O(\epsilon^2)$$

② Overcharging condition
(System's total energy):

$$E_{\rm EM}^{\rm rad} : O(f^2) \sim O(\epsilon^2)$$

The same scaling as the proposed process **!**

Back reaction effects

Ignoring the back reaction effects on both the particle's motion and the system's energy **due to the particle existence is not valid**;

Back reaction effects on system's total energy

- Particle's self-field energy (static)
- Energy loss via radiation (dynamical)

$$f_{\mu\nu} \sim O(\epsilon)$$

② Overcharging condition
(System's total energy):

$$E_{\mathrm{EM}}^{\mathrm{rad}} : O(f^2) \sim O(\epsilon^2)$$

The same scaling as the proposed process **!**

Question

With the precision of $O(\epsilon^2)$, the back reaction effects make the total energy of the final RN spacetime be always greater than its total charge ?

(The RN black hole can be saved from overcharged ?)

✕ $(Q+q)-(M+E)=O(\epsilon^2)>0$

⇓

? $(Q+q)-(M+E+\underbrace{E_{\mathrm{Back}}}_{\approx O(\epsilon^2)})<0$

Assumption: Particle's motion

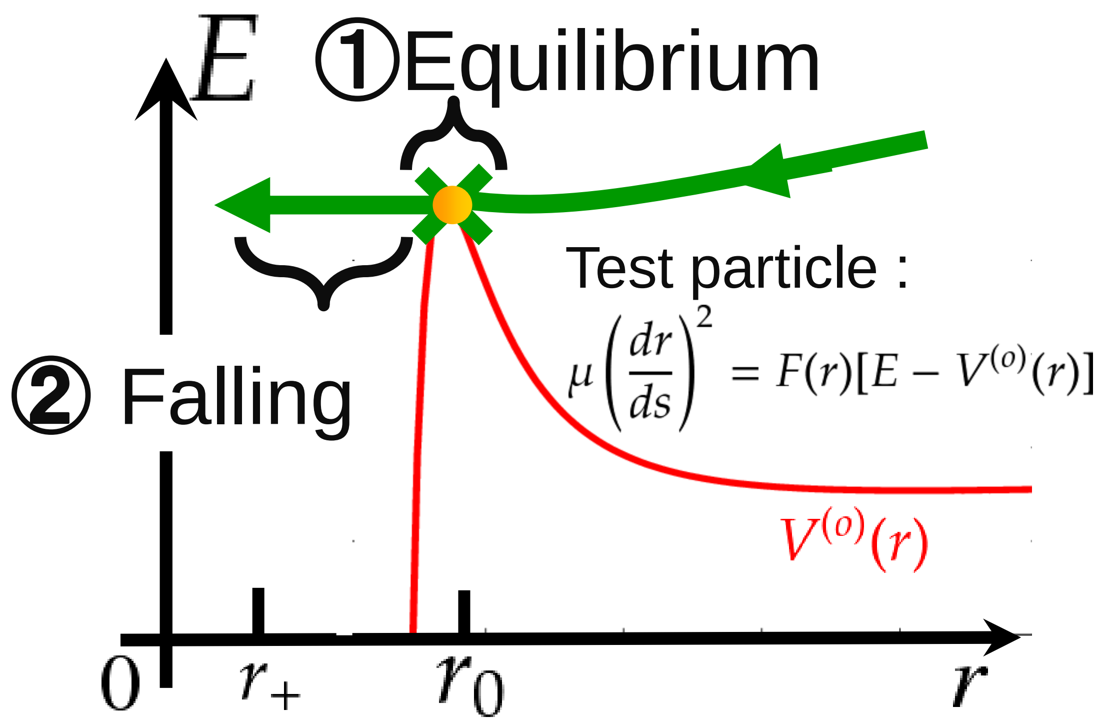
There always exists a (radial) **marginal orbit**:
A separatrix between plunge and bounce orbits

Enjoys the (unstable) **equilibrium configuration**

Evaluate back reaction effects on the **system's total energy**

① Particle's self-field energy (static)

② Energy loss via radiation (dynamical)



① Equilibrium

Need to know the particle's self-energy
but **not need to single it out** …

$$\boxed{E^{\mathrm{eq}}_{\mathrm{total}}} = M + E + E_{\mathrm{self}}$$

The double Reisner-Nordström solution

(V.S.Manko Phys. Rev. **D76** 124032 (2007)) (G.A.Alekseev and V.A. Belinski Phys. Rev. **D76** 021501 (2007))

An axisymmetric static **exact solution** of Einstein-Maxwell system : the equilibrium configuration between a charged particle and a RN black hole

RN Black Hole:	Charged particle:	Separation:
(m_2, e_2)	(m_1, e_1)	l

$$ds^2 = H(\rho, z)dt^2 - f(\rho, z)(d\rho^2 + dz^2) - \frac{\rho^2}{H(\rho, z)}d\phi^2$$

$$A_t = \Phi(\rho, z), \; A_\rho = A_z = A_\phi = 0,$$

- **the total mass :** $m_1 + m_2$
- **the total charge :** $e_1 + e_2$

RN Black Hole: $(m_2,\ e_2)$

Charged particle: $(m_1,\ e_1)$

Separation: l

These five parameters must satisfy the following **balance condition** to be an exact solution.

(“Electromagnetic repulsive force” = “Gravitational attractive force”)

$$e_1 e_2 = (m_1 - \gamma)(m_2 + \gamma)$$

$$\gamma := \gamma(e_1,\ e_2,\ m_1,\ m_2,\ l)$$

$$e_2 > e_1 > 0,\ m_2 > m_1 > 0, \ldots$$

Constraint

$$E^{\mathrm{eq}}_{\mathrm{total}} := m_1 + m_2 > e_1 + e_2$$

Always **Total mass > Total charge**

Conclusion: ①

Thanks to the back reaction effects, the total energy of the system is **always greater** than the total charge **at the equilibrium configuration**

✕ $M + E < Q + q$

Mapping with accuracy of $O(\epsilon^2)$

$$e_1 + e_2 = Q + q$$
$$E^{\mathrm{eq}}_{\mathrm{total}} = M + E + E_{\mathrm{self}}$$

○ $E^{\mathrm{Eq}}_{\mathrm{total}} > Q + q$

②

Radiation

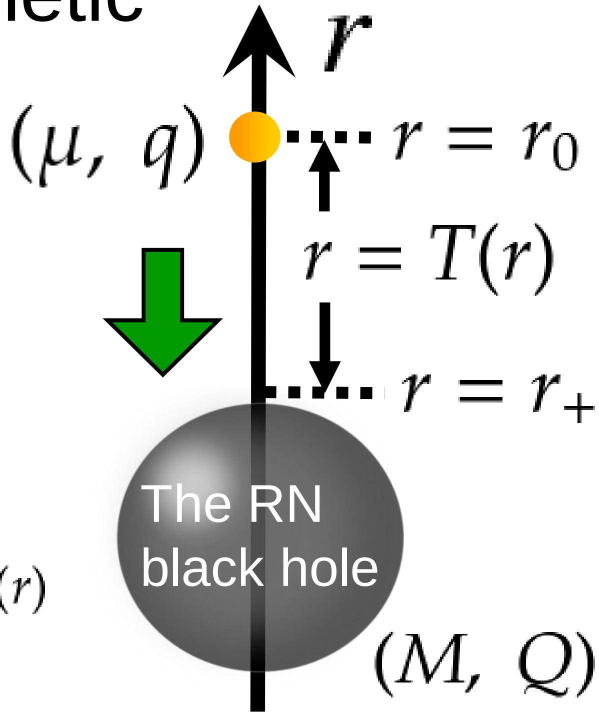
Energy loss from the equilibrium configuration to the horizon …

$$E_{\text{total}}^{\text{Eq}} - E_{\infty} \lesseqgtr Q + q$$?

Linear perturbation of the RN black hole

(e.g. Kodama and Ishibashi Prog. Theor. Phys **29** 021501(2004))

The emitted energy by electromagnetic and gravitational radiation from the charged particle is well handled by linear perturbation theory.



After decomposing perturbations and together with

$$X_\pm \propto K(q,\ \mu) \int_{r_+}^{r_0} I_\pm(r) e^{i\omega T(r)}$$

Energy loss via radiation (Decomposed by $e^{i\omega t},\ Y_{lm}(\theta,\ \phi)$)

$$E_\infty = \int_0^\infty d\omega \sum_l H(M,\ Q,\ l)\, \omega^{l+2} \left(|X_+|^2 + \frac{(l-1)(l+2)}{16} |X_-|^2 \right)$$

Radiated energy to the infinity

Near-extremality of the RN black hole assures **the slow motion** of the charged particle $\boxed{\epsilon \ll 1}$

$$\left(\frac{dr}{ds}\right)^2 = \frac{F(r)}{\mu}[E - V^{(0)}(r)] < O(\epsilon^2)$$

Using the identity and integration by parts

$$\mathcal{X}_\pm \propto O(\epsilon) \times \int_{r_+}^{r_0} I_\pm(r) e^{i\omega T(r)}$$

$$e^{i\omega T(r)} = \underbrace{\frac{1}{i\omega}\left(\frac{dT}{ds}\right)^{-1}}_{\sim O(1)} \underbrace{\left(\frac{dr}{ds}\right)}_{\sim O(\epsilon)} \left(\frac{\partial e^{i\omega T(r)}}{\partial r}\right)$$

$$E_\infty \propto \int_0^\infty d\omega \omega^{l+2} \sum_{l,\pm} |\mathcal{X}_\pm|^2 \lesssim O(\epsilon^4)$$

Radiation : **suppressed**

Conclusion: ②

The total energy of the final state can **never be reduced below the extremal bound** with precision of $O(\epsilon^2)$

$$E_{\rm total}^{\rm Eq} > Q + q$$

$$E_\infty \sim O(\epsilon^4)$$

$$E_{\rm total}^{\rm Eq} - E_\infty > Q + q$$

The final spacetime is **still the RN black hole !**

Summary

Neglecting the back reaction effects, there exist the particle's orbits that make the RN black hole be overcharged, which might be a **"counter example"** of the cosmic censorship conjecture.

① Absorption condition :

② Overcharging condition :
The total energy of the final spacetime

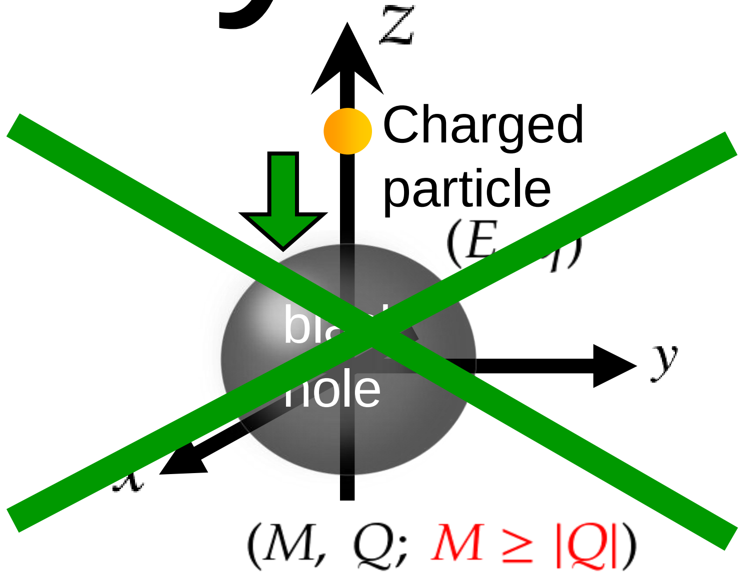
Back reaction effects on system's total energy

① self-field's energy
② Radiation

The total energy of the final spacetime is **always greater than** its total charge.

Summary

Q. Is the RN black hole overcharged via charged particle absorption ?



***A. No.* The back reaction effects do prevent** the Reisner-Nordström black hole from being overcharged, and **save the cosmic censorship.**

Thank You !

Soichiro Isoyama,
Norichika Sago and Takahiro Tanaka

Back up

Coordinate transformation with limit $\rho \longrightarrow +\infty,\ z \longrightarrow +\infty$

$$A_t = \Phi(\rho, z), \quad ds^2 = H(\rho, z)dt^2 - f(\rho, z)(d\rho^2 + dz^2) - \frac{\rho^2}{H(\rho, z)}d\phi^2$$

$$\begin{cases} \rho = r\sin\theta \\ z = r\cos\theta \end{cases}$$

Polar coordinate

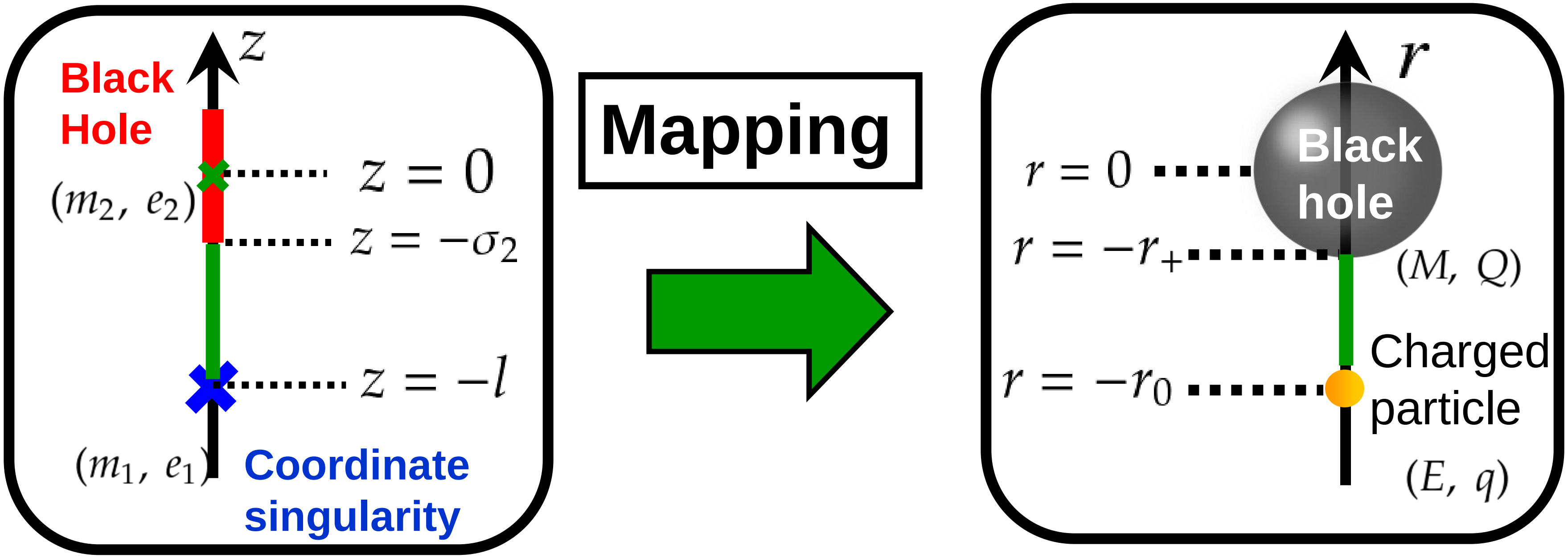
$$H = f^{-1} = 1 - \frac{m_1 + m_2}{r} + O\left(\frac{1}{r^2}\right)$$

$$\Phi = \frac{e_1 + e_2}{r} + O\left(\frac{1}{r^2}\right)$$

Same as the metric and the vector potential of the RN geometry with

total mass: $m_1 + m_2$

total charge: $e_1 + e_2$



Key point: **Covariant quantities** (Proper distance)

Double RN geometry

$$ds^2 = H(\rho, z)dt^2 - f(\rho, z)(d\rho^2 + dz^2) - \frac{\rho^2}{H(\rho, z)}d\phi^2 \rightarrow \delta L = \int_{-l}^{-\sigma_2} \sqrt{f(z)}dz$$

Particle + RN geometry

$$ds^2 = -g(r)dt^2 + g(r)^{-1}dr^2 + d\Omega^2_{(2)} \rightarrow \delta L = \int_{-r_0}^{-r_+} \left(\sqrt{g(r)}\right)^{-1} dr$$

Back reaction effects

Conditions: $(M \gg \mu \sim O(\epsilon),\ Q \gg q \sim O(\epsilon))$

Self fields can be treated as the small perturbations on the background geometry.

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{RN}} + \epsilon\, h_{\mu\nu}$$

$$F_{\mu\nu} = F_{\mu\nu}^{\mathrm{RN}} + \epsilon\, f_{\mu\nu}$$

$\Longleftrightarrow$

$$G_{\mu\nu}[g^{\mathrm{RN}},\ \epsilon\, h] = \epsilon T_{\mu\nu}^{\mathrm{Particle}}$$

$$\nabla^{\mu}\left(F_{\mu\nu} + \epsilon f_{\mu\nu}\right) = \epsilon J_{\nu}^{\mathrm{Particle}}$$

Back ground: RN black hole

Self fields of a charged particle

$$\boxed{\epsilon \ll 1}$$

Violation of cosmic censorship

Q. Can the spin / charge of a (4-dim) black hole be "saturated" by throwing a particle into the black hole ?

No. for **an extremal** Kerr-Newman black hole

[e.g. Wald (1974)]

Yes?

- Overspinning / overcharging a **near-**extremal black hole (Today's topic)

[Hod (2002); Jacobson and Sotiriou (2009); Saa and Santarelli (2011)]

The double RN solution is **uniquely** mapped to the charged particle + the RN black hole system in equilibrium with precision up to $O(\epsilon^2)$