High-velocity collision of particles around a Kerr black hole

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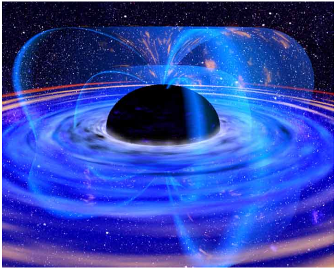
This talk is based on the collaboration with Masashi Kimura (YITP):

T. Harada and M. Kimura, Phys. Rev. D **83** (2011) 024002

T. Harada and M. Kimura, Phys. Rev. D **83** (2011) 084041

Astrophysical black holes (BHs)



- BH candidates
  - X-ray binary: Accretion disk around a BH of $\sim 10M_{\odot}$,
  - Supermassive BH @ galactic centres: $10^6 - 10^9 M_{\odot}$
- Towards "direct" observation
  - BH "shadow" in 1 mm EM waves
  - GWs from Extreme Mass-Ratio Inspirals (EMRIs)

Kerr BH

- Kerr metric

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar\sin^2\theta}{\rho^2}d\phi dt + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \left(r^2+a^2+\frac{2Mra^2\sin^2\theta}{\rho^2}\right)\sin^2\theta d\phi^2,$$

where $\rho^2 = r^2 + a^2\cos^2\theta$、 $\Delta = r^2 - 2Mr + a^2$。

- If $0 \le |a| \le M$, Δ vanishes at $r = r_\pm = M \pm \sqrt{M^2 - a^2}$. The horizon radius is given by $r_H = r_+$。
- The angular velocity of the horizon

$$\Omega_H = \frac{a}{r_H^2 + a^2} = \frac{a}{2M(M+\sqrt{M^2-a^2})}.$$

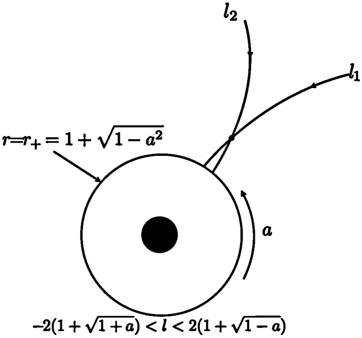
- Nondimensional Kerr parameter: $a_* = a/M$

Rotating BHs as particle accelerators

Bañados, Silk and West (2009)

1. Drop two particles at rest at infinity on the equatorial plane.
2. Consider the collision of the two particles near the horizon.
3. Take the maximum rotation limit and fine-tune the angular momentum of either particle, then the centre-of-mass energy can be arbitrarily high!

- Robust?
- General condition?



Centre-of-mass (CM) energy

- The CM energy of particles 1 and 2 at the same spacetime point is given by

$$E_{\rm cm}^2 = -(p_1 + p_2)^a (p_1 + p_2)_a = m_1^2 + m_2^2 - 2 g_{ab} p_1^a p_2^b.$$

- Coordinate-independent and in principle observable

General geodesic particles

- Conserved quamtities: m, E, L, Carter const. Q (Carter 1968)

$$\begin{aligned} \rho^2 \dot{t} &= -a(aE\sin^2\theta - L) + (r^2 + a^2)P/\Delta, \\ \rho^2 \dot{\phi} &= -\left(aE - L/\sin^2\theta\right) + aP/\Delta, \\ \rho^2 \dot{r} &= \sigma_r \sqrt{R}, \\ \rho^2 \dot{\theta} &= \sigma_\theta \sqrt{\Theta}, \end{aligned}$$

where $\cdot = d/d\lambda$, $\sigma_r = \pm 1$, $\sigma_\theta = \pm 1$,

$$\begin{aligned} R &= R(r) = P^2 - \Delta[m^2 r^2 + (L - aE)^2 + Q], \\ P &= P(r) = (r^2 + a^2)E - aL, \\ \Theta &= \Theta(\theta) = Q - \cos^2\theta\left[a^2(m^2 - E^2) + L^2/\sin^2\theta\right]. \end{aligned}$$

- ‘Forward-in-time’ condition: For $\dot{t} > 0$ near the horizon

$$L \le \Omega_H^{-1} E \equiv L_c.$$

Collision of two gedesic particles

- The CM energy is bounded except for near the horizon.
- The CM energy for the near-horizon collision is given by

$$
\begin{aligned}
E_{\rm cm}^2 &= m_1^2 + m_2^2 + \frac{1}{r_H^2 + a^2\cos^2\theta}\left[(m_1^2 r_H^2 + \mathcal{K}_1)\frac{E_2 - \Omega_H L_2}{E_1 - \Omega_H L_1}\right. \\
&\quad + (m_2^2 r_H^2 + \mathcal{K}_2)\frac{E_1 - \Omega_H L_1}{E_2 - \Omega_H L_2} \\
&\quad \left. - \frac{2(L_1 - a\sin^2\theta E_1)(L_2 - a\sin^2\theta E_2)}{\sin^2\theta} - 2\sigma_{1\theta}\sqrt{\Theta_1}\sigma_{2\theta}\sqrt{\Theta_2}\right],
\end{aligned}
$$

  where $\mathcal{K}_i \equiv Q_i + (L_i - aE_i)^2$.
- For $L_i \to \Omega_H^{-1} E_i$, $E_{\rm cm} \to \infty$.
- Particles with $L = \Omega_H^{-1} E$ are called *critical particles*.

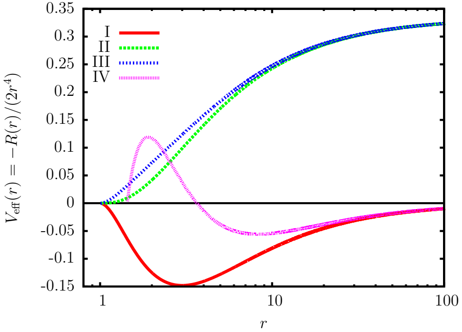
Classification of critical particles

- Effective potential

$$\frac{1}{2}\dot{r}^2 + \frac{r^4}{\rho^4}V(r) = 0,$$

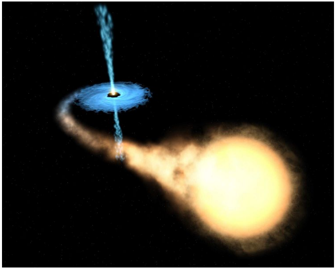
$$V(r) = -\frac{R(r)}{2r^4}.$$

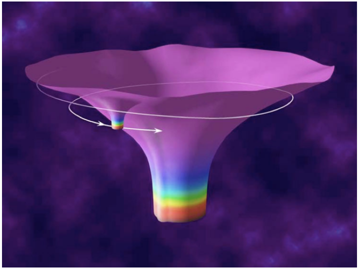
- Classes I and II are physically relevant.



Class	$V(r)$ at $r = r_H$	BH spin	Scenario
I	$V = V' = 0, V'' < 0$	$\lvert a\rvert = M$	Direct collision
II	$V = V' = V'' = 0$	$\lvert a\rvert = M$	LSO (ISCO) collision
III	$V = V' = 0, V'' > 0$	$\lvert a\rvert = M$	Multiple scattering
IV	$V = 0, V' > 0$	$0 < \lvert a\rvert < M$	Mutiple scattering

ISCO and LSO

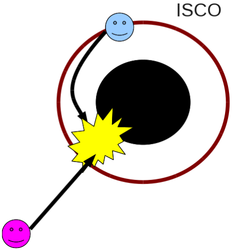




- Innermost stable circular orbit = ISCO
- The inner edge of the standard disk is given by the ISCO.
- The transition of quasicircular orbit to plunge at the ISCO in the EMRI
- The inclined counterpart of an ISCO is a *last stable orbit (LSO)*.
- The fine-tuning is naturally realised by an ISCO particle:
  as $a_* \to 1$, $r_{\rm ISCO} \to r_H$ and $L \to \Omega_H^{-1} E$.

Collision of an ISCO particle

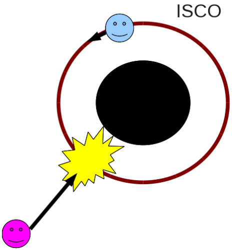
Near-horizon collision



$$\frac{E_{\rm cm}}{2m_0} \approx \frac{1}{2^{1/2}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[4]{1-a_*^2}}$$

for $a_* \approx 1.$

On-ISCO collision

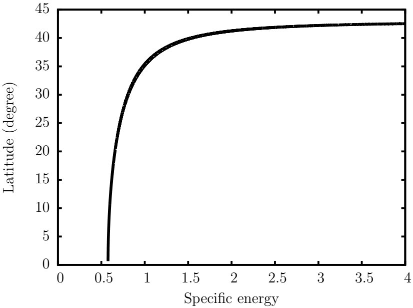


$$\frac{E_{\rm cm}}{2m_0} \approx \frac{1}{2^{1/6}3^{1/4}} \frac{\sqrt{2e_2 - l_2}}{\sqrt[6]{1-a_*^2}}$$

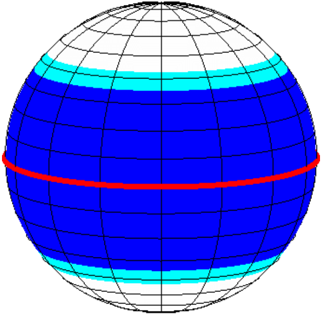
for $a_* \approx 1.$

High-velocity belt

- Does the collision with arbitrarily high CM energy occur apart from the equatorial plane?
- Suppose $V = V' = 0$ and $V'' \leq 0$ at $r = r_H$, then the BH must be maximally rotating and θ is restricted.



Highest latitude



High-velocity colision between $\pm 43°$.

Summary

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- Future prospects
  - The effects of gravitational radiation reaction and self-force
  - Theoretical predictions to astrophysical observation