# Bosenova collapse of axion cloud around a rotating black hole

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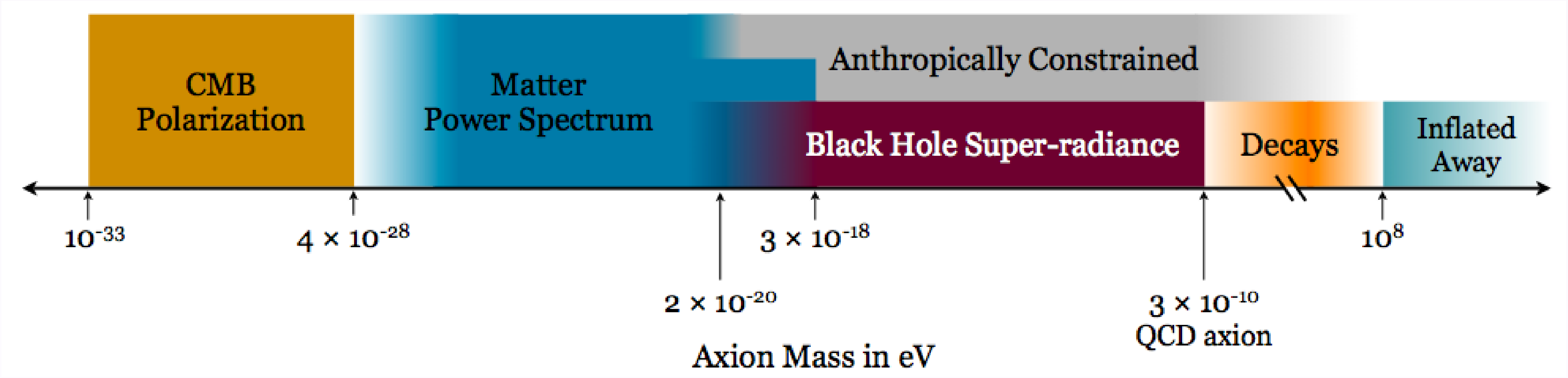
# Axions

- Candidates of massive scalar fields. $\nabla^2\Phi - \mu^2\Phi = 0$
  - QCD axion
    - QCD axion was introduced to solve the Strong CP problem.
    - It is one of the candidates of dark matter.
  - String axion

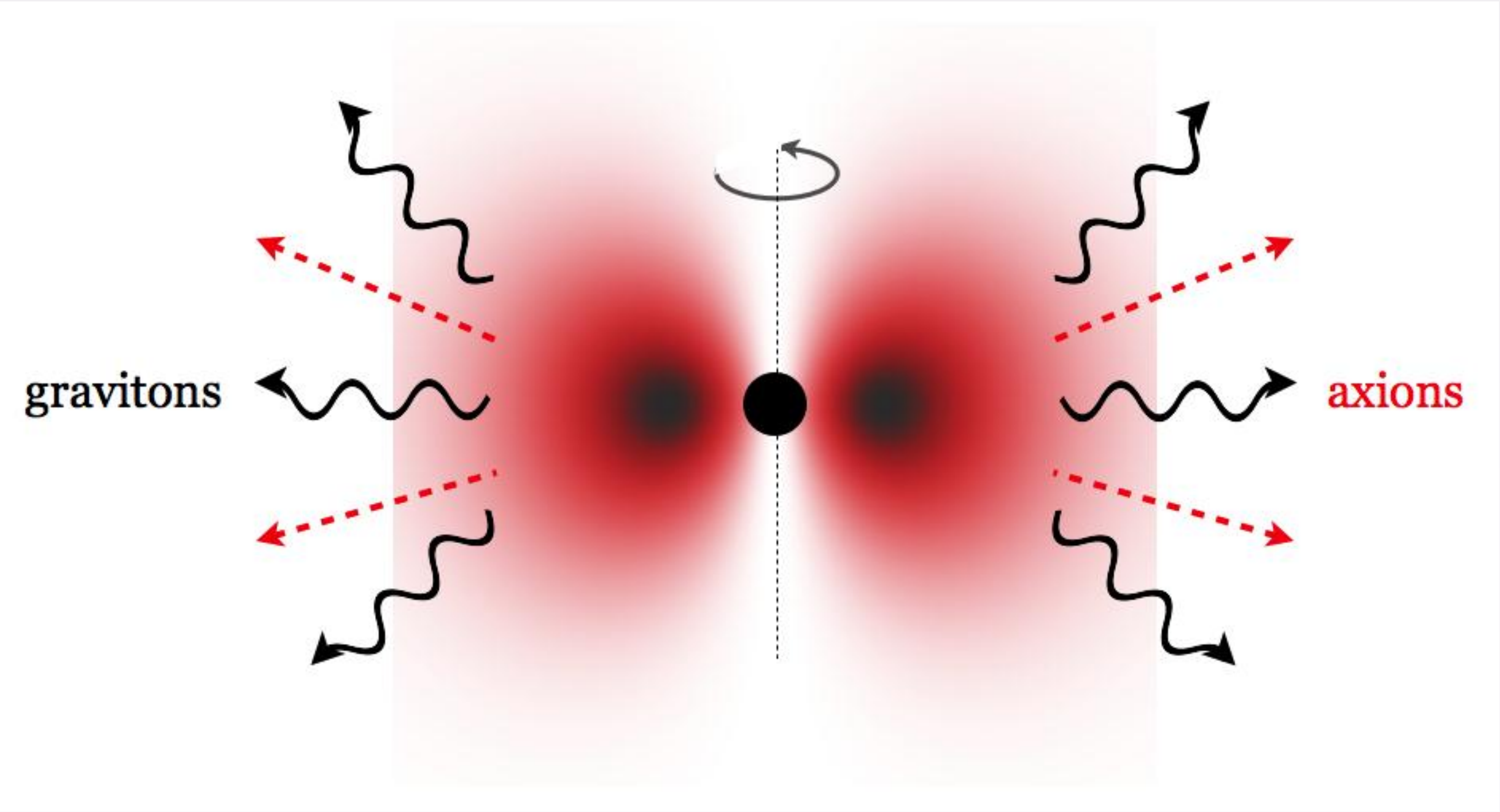
    Arvanitaki, Dimopoulos, Dubvosky, Kaloper, March-Russel, PRD81 (2010), 123530.

    - String theory predicts the existence of 10-100 axion-like massive scalar fields.
    - There are various expected phenomena of string axions.



# Axion field around a rotating black hole

- Axion field makes a bound state and causes the superradiant instability

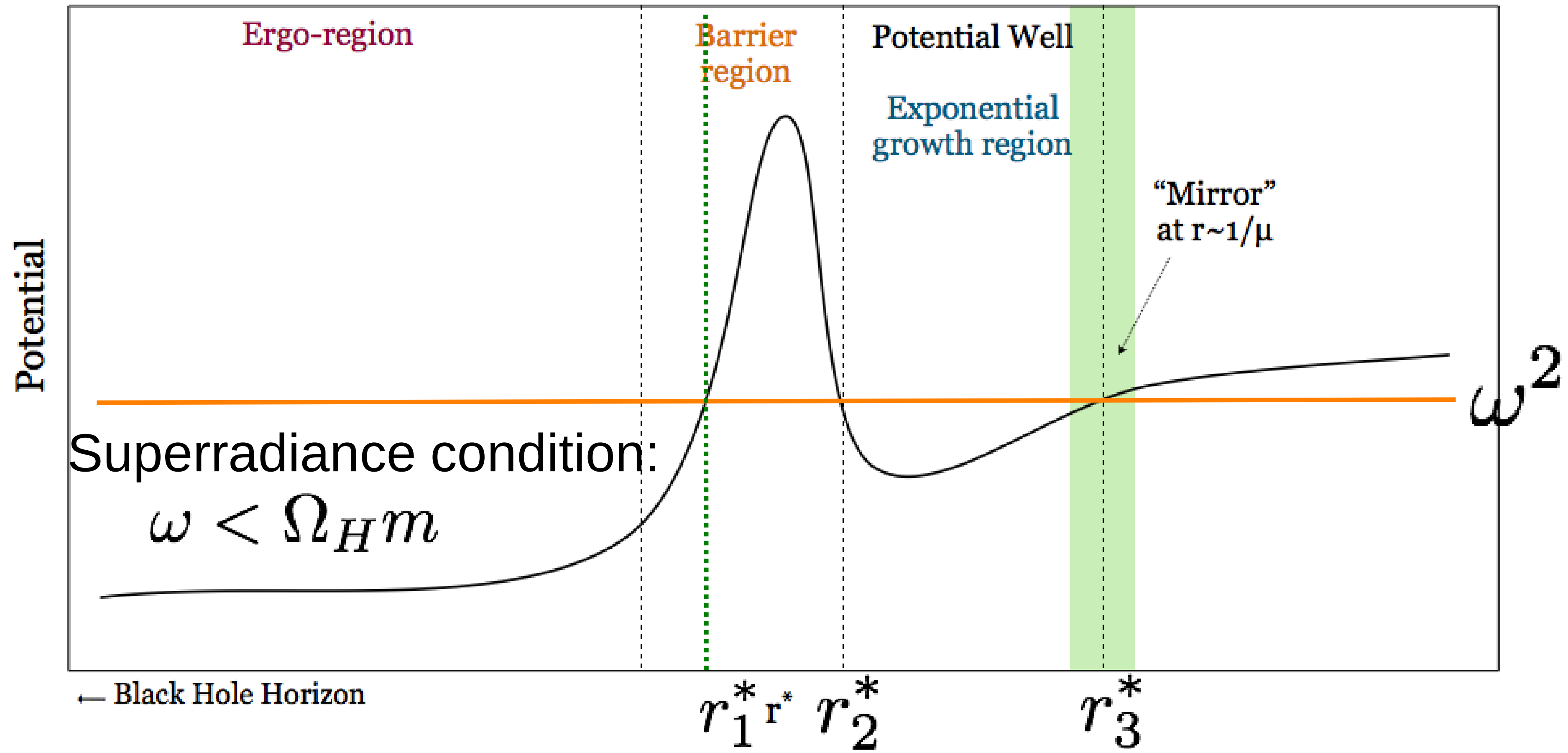



Detweiler, PRD22 (1980), 2323.

Zouros and Eardley, Ann. Phys. 118 (1979), 139.

# Bound state

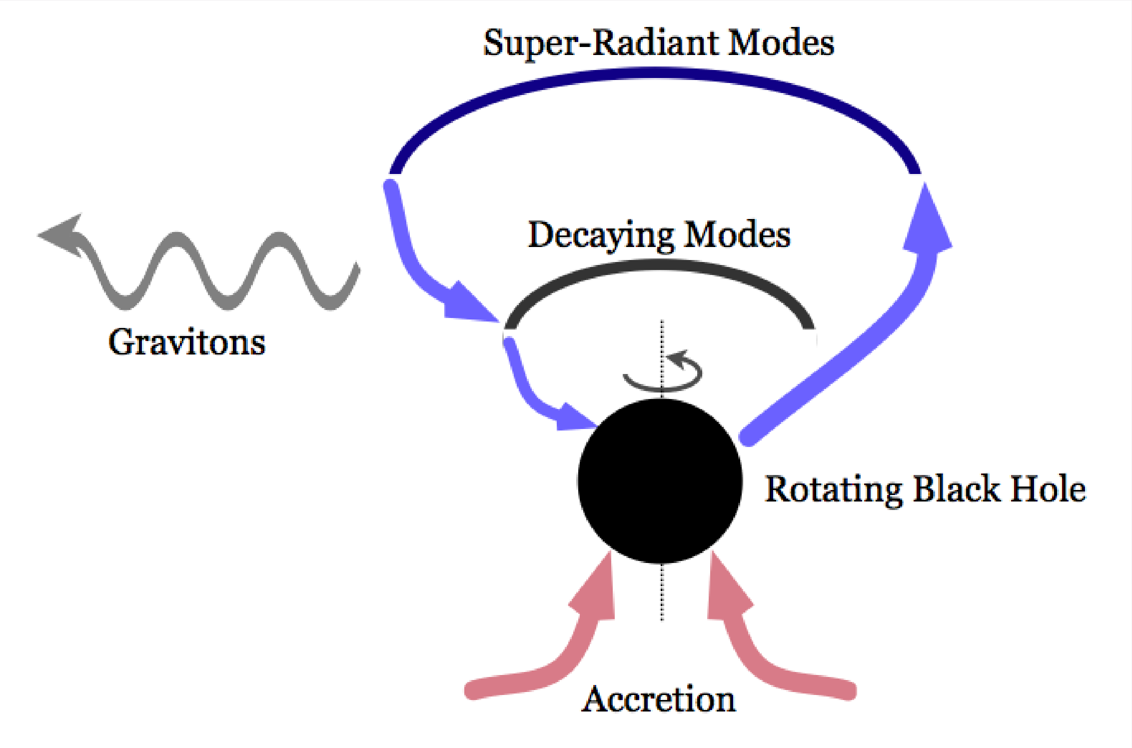
Zouros and Eardley, Ann. Phys. 118 (1979), 139.




$$\Phi = e^{-i\omega t} R(r) S(\theta) e^{im\phi}$$

$$R = \frac{u}{\sqrt{r^2 + a^2}} \quad \Rightarrow \quad \frac{d^2 u}{dr_*^2} + \left[\omega^2 - V(\omega)\right] u = 0$$

# BH-axion system




Arvanitaki and Dubovsky, PRD83 (2011), 044026.

- Superradiant instability
  - Emission of gravitational waves
  - Pair annihilation of axions
- Effects of nonlinear self-interaction
  - Bosenova
  - Mode mixing

# Nonlinear effect

- QCD axion

U(1)PQ symmetry $\xrightarrow{\text{break}}$ Z(N) discrete symmetry

$V(\Phi)$ becomes periodic. $\Delta\Phi = 2\pi v_a/N = 2\pi f_a$

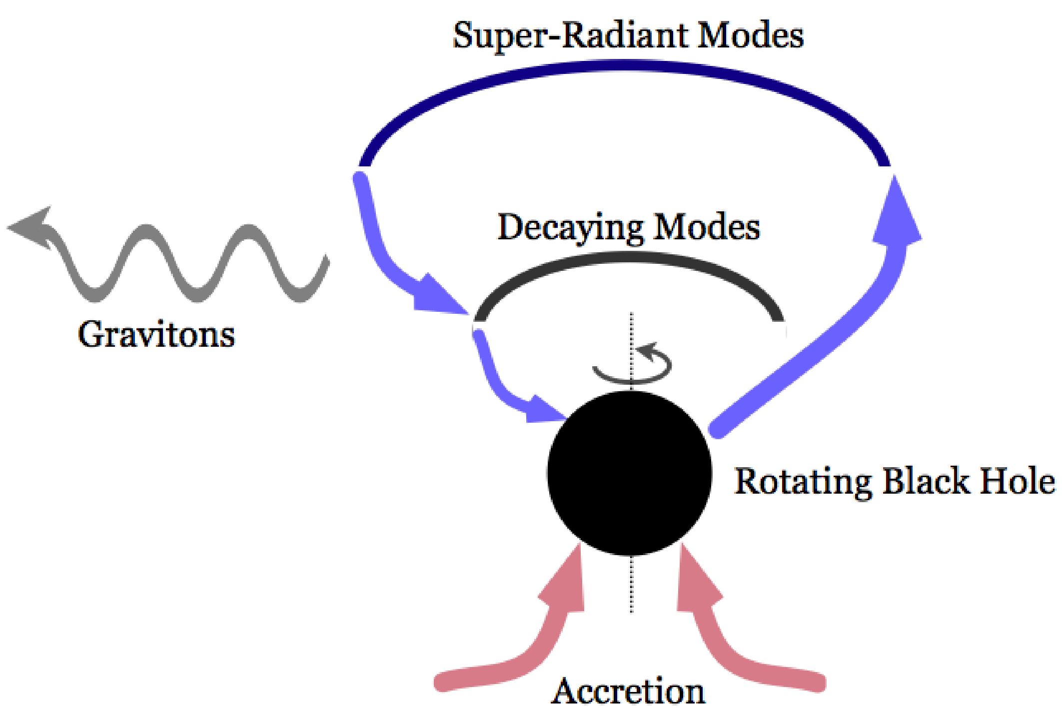
$$V = f_a^2 \mu^2 [1 - \cos(\Phi/f_a)]$$

$$\Rightarrow \boxed{\nabla^2 \Phi - \mu^2 f_a \sin\left(\frac{\Phi}{f_a}\right) = 0}$$

$$\nabla^2 \varphi - \mu^2 \sin\varphi = 0 \qquad \varphi \equiv \frac{\Phi}{f_a}$$

- The similar statement holds also for string axions.

# BH-axion system

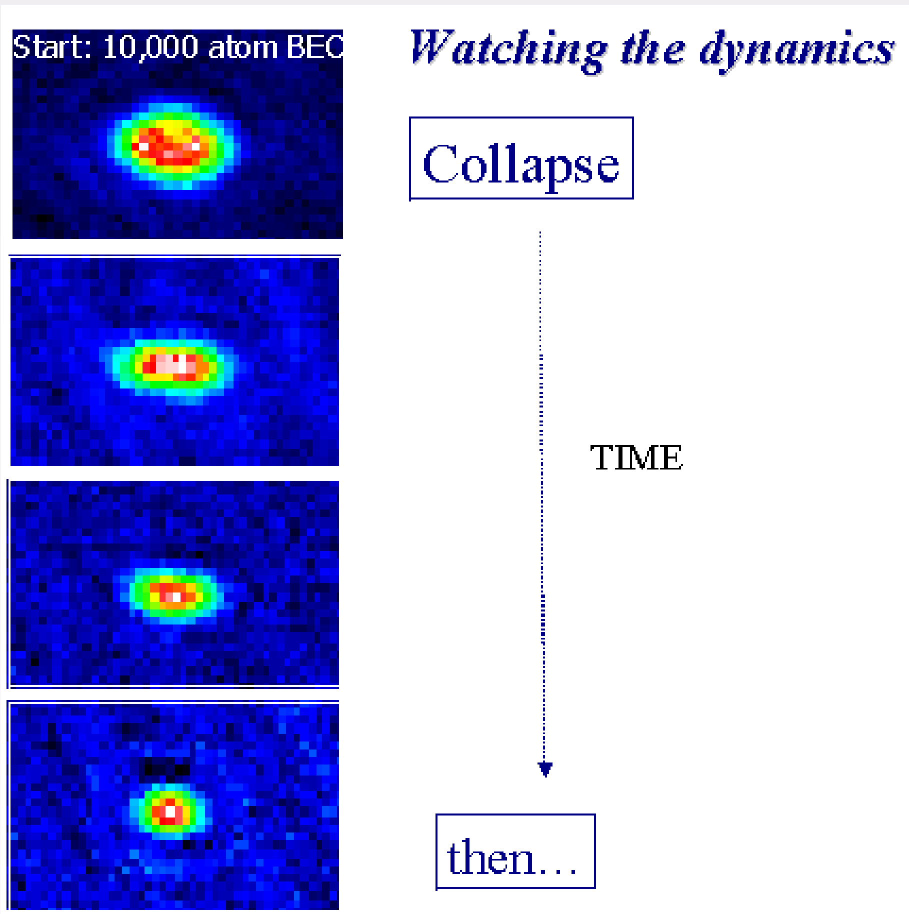


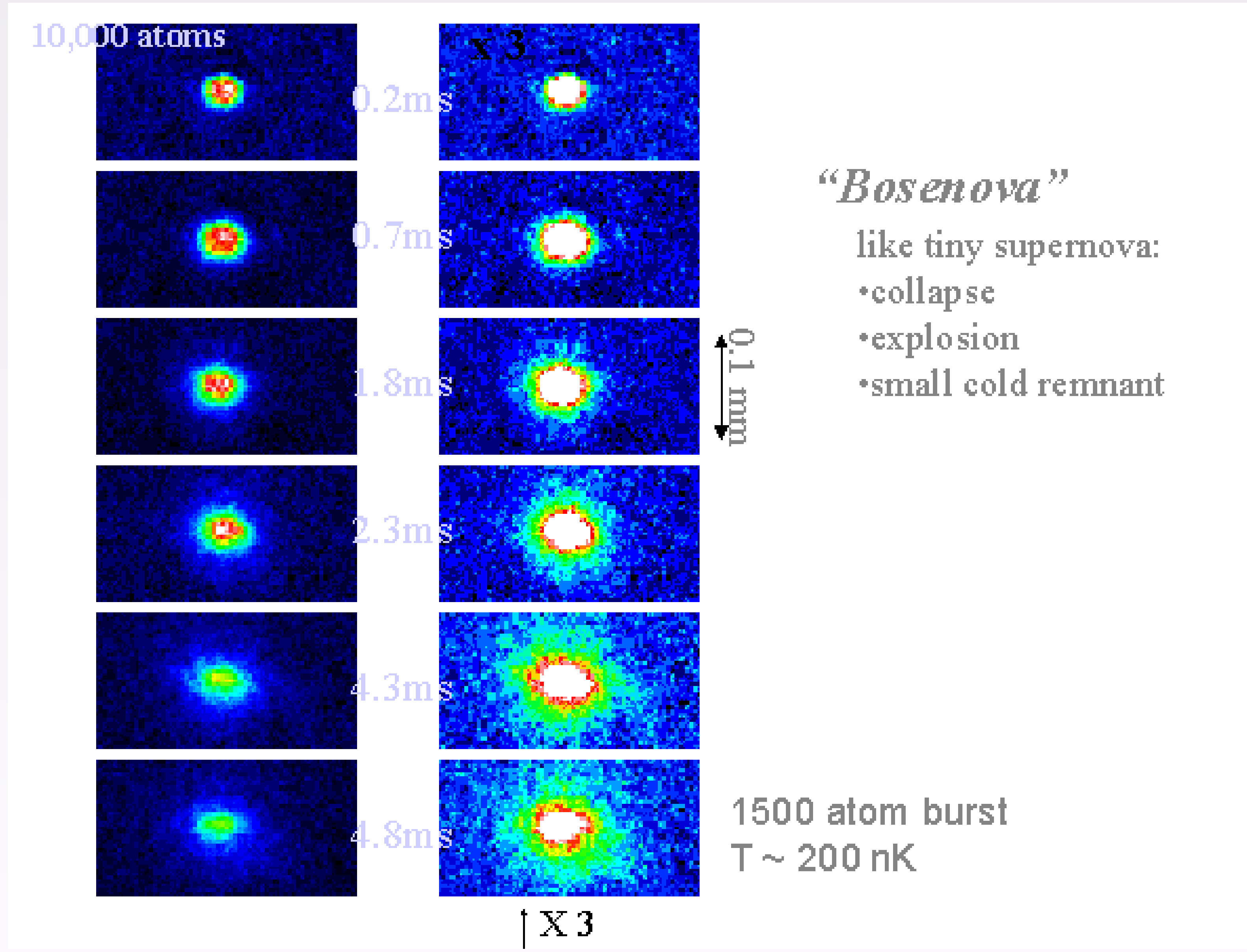

Arvanitaki and Dubovsky, PRD83 (2011), 044026.

- Superradiant instability
  - Emission of gravitational waves
  - Pair annihilation of axions
- Effects of nonlinear self-interaction
  - Bosenova
  - Mode mixing

# Bosenova in condensed matter physics

http://spot.colorado.edu/~cwieman/Bosenova.html




BEC state of Rb85（interaction can be controled）

Switch from repulsive interaction to attractive interaction

Wieman et al., Nature 412 (2001), 295

# What we would like to do

- We would like to study the phenomena caused by axion cloud generated by the superraciant instability around a rotating black hole.

- In particular, we study numerically whether “Bosenova” happens when the nonlinear interaction becomes important.

- We adopt the background spacetime as the Kerr spacetime, and solve the axion field as a test field.

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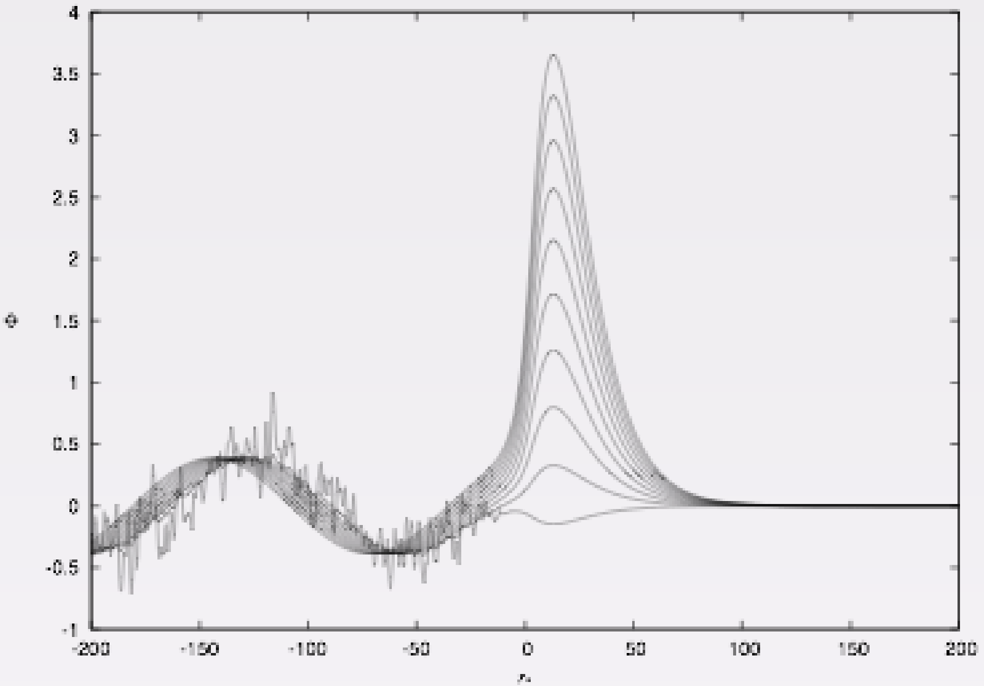
- Introduction
- Code
- Simulation
- Summary

# First difficulty

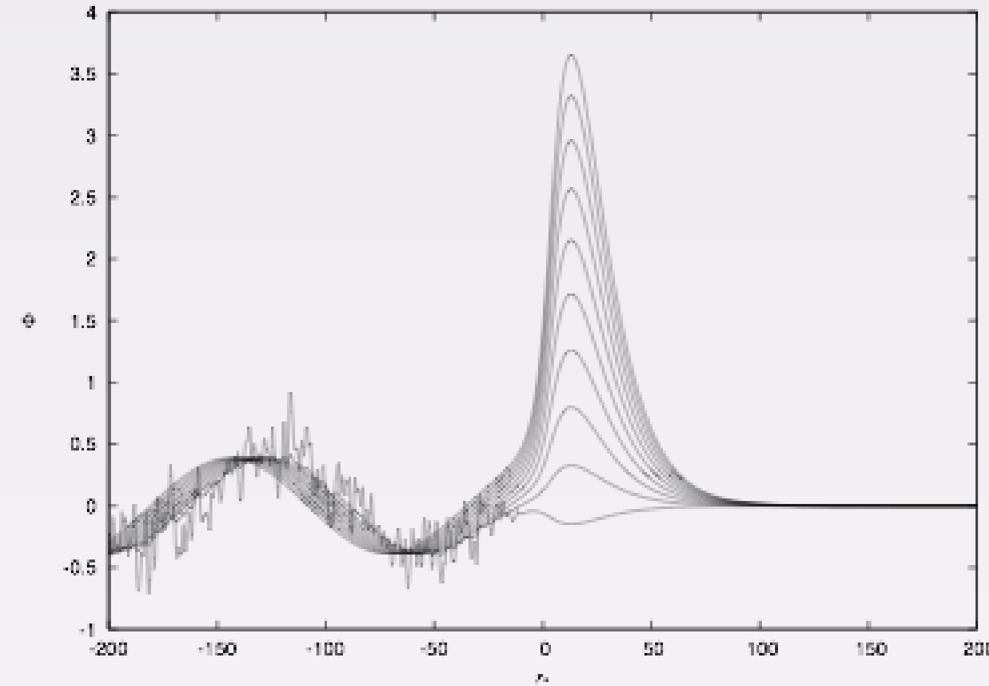
- Stable simulation cannot be realized in Boyer-Lindquist coordinates.

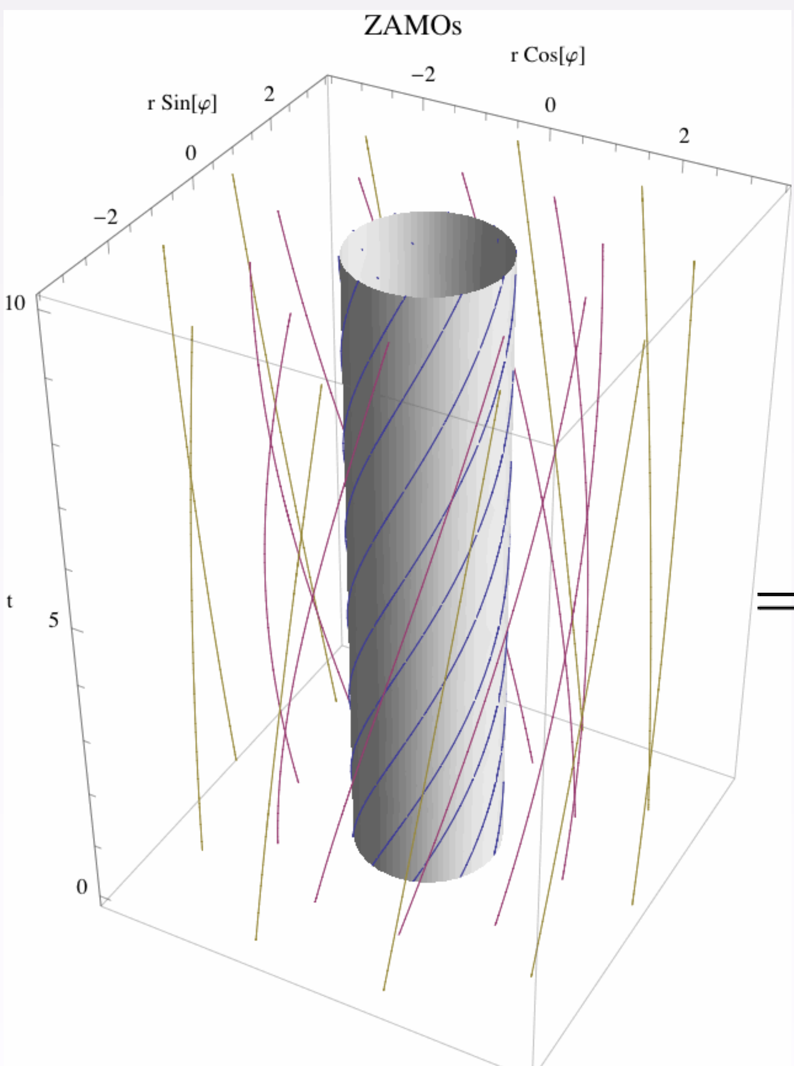


# First difficulty

- Stable simulation cannot be realized in Boyer-Lindquist coordinates.

⇨ We use ZAMO coordinates.



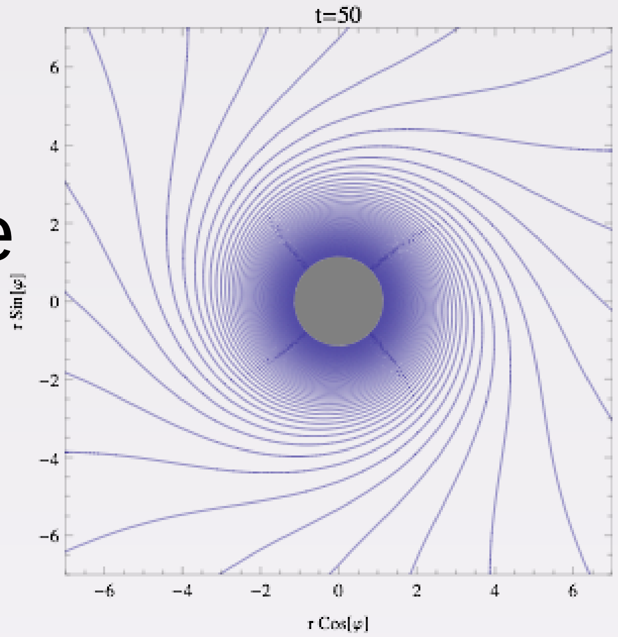




$$\Omega = \frac{d\phi}{dt} = \frac{u^\phi}{u^t} = -\frac{g_{t\phi}}{g_{\phi\phi}}$$

$$= \frac{2Mar}{(r^2+a^2)^2 - \Delta a^2 \sin^2\theta}$$

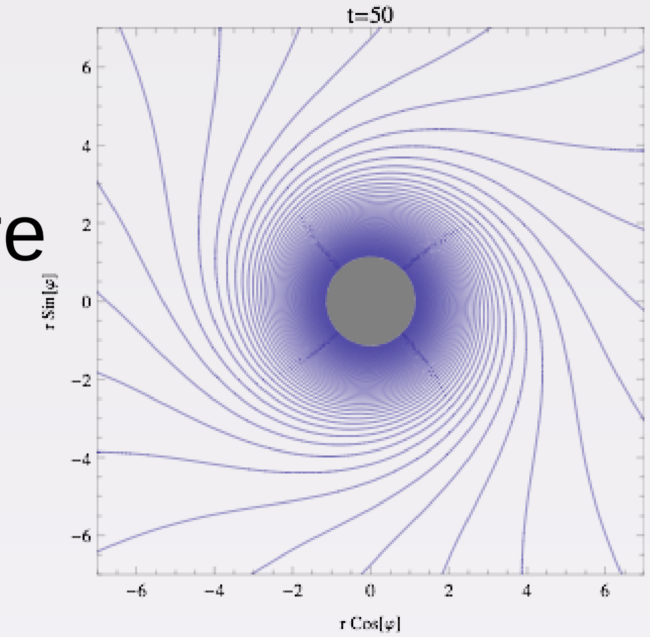
$$\tilde{t} = t,$$
$$\tilde{\phi} = \phi - \Omega(r,\theta)t,$$
$$\tilde{r} = r,$$
$$\tilde{\theta} = \theta,$$

# Second difficulty




- ZAMO coordinates becomes more and more distorted in the time evolution

# Second difficulty

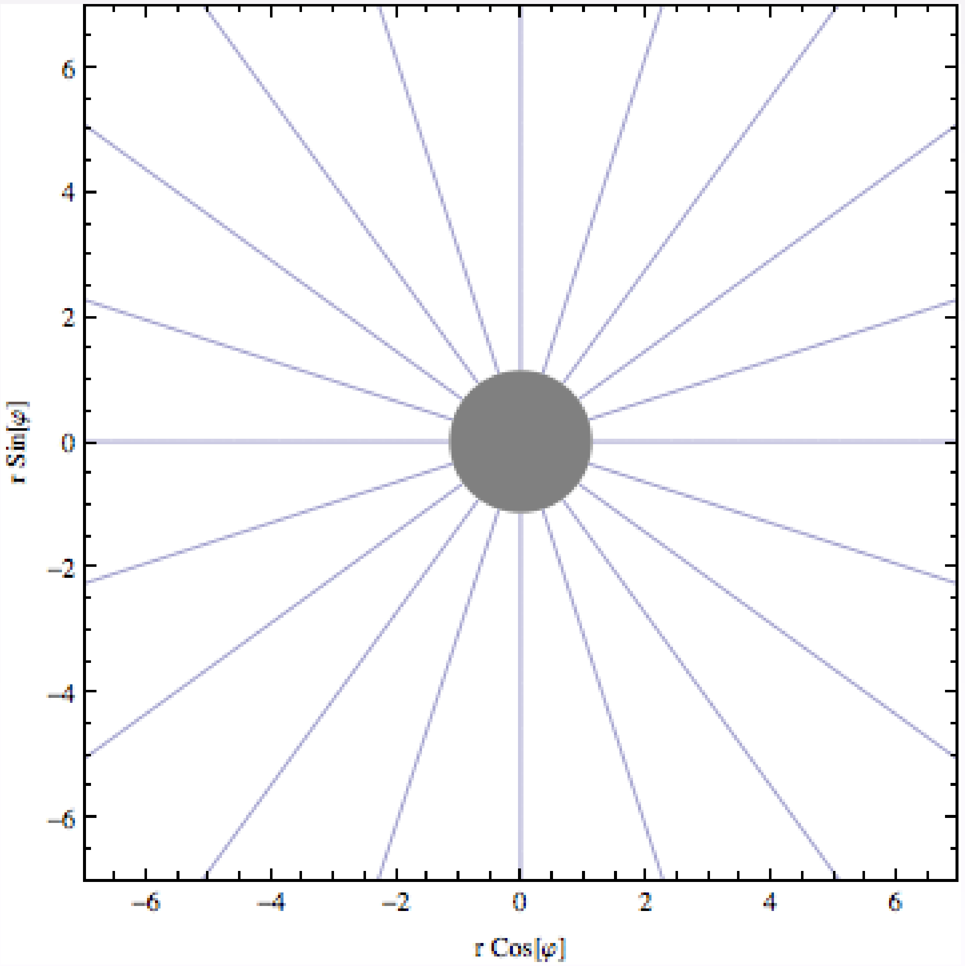



- ZAMO coordinates becomes more and more distorted in the time evolution

➢ We "pull back" the coordinates

$$nT_P \leq t \leq (n+1)T_P :$$

$$\begin{aligned} t^{(n)} &= t, \\ \phi^{(n)} &= \phi - \Omega(r,\theta)(t - nT_P), \\ r^{(n)} &= r, \\ \theta^{(n)} &= \theta. \end{aligned}$$

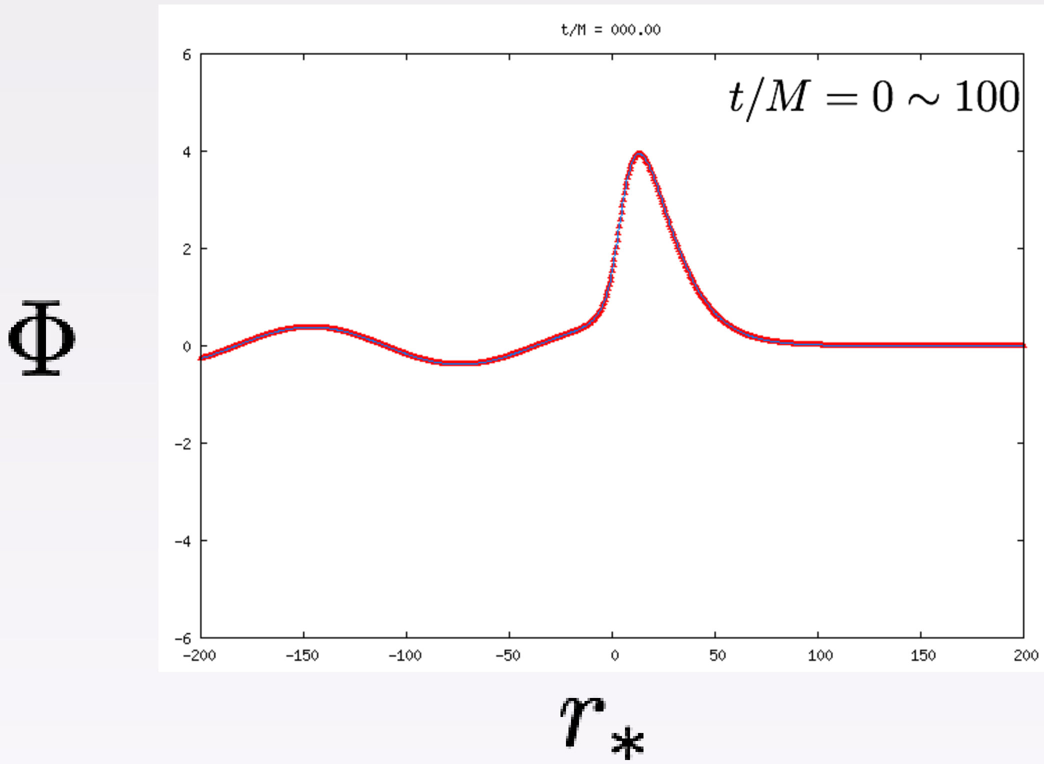


# Our 3D code

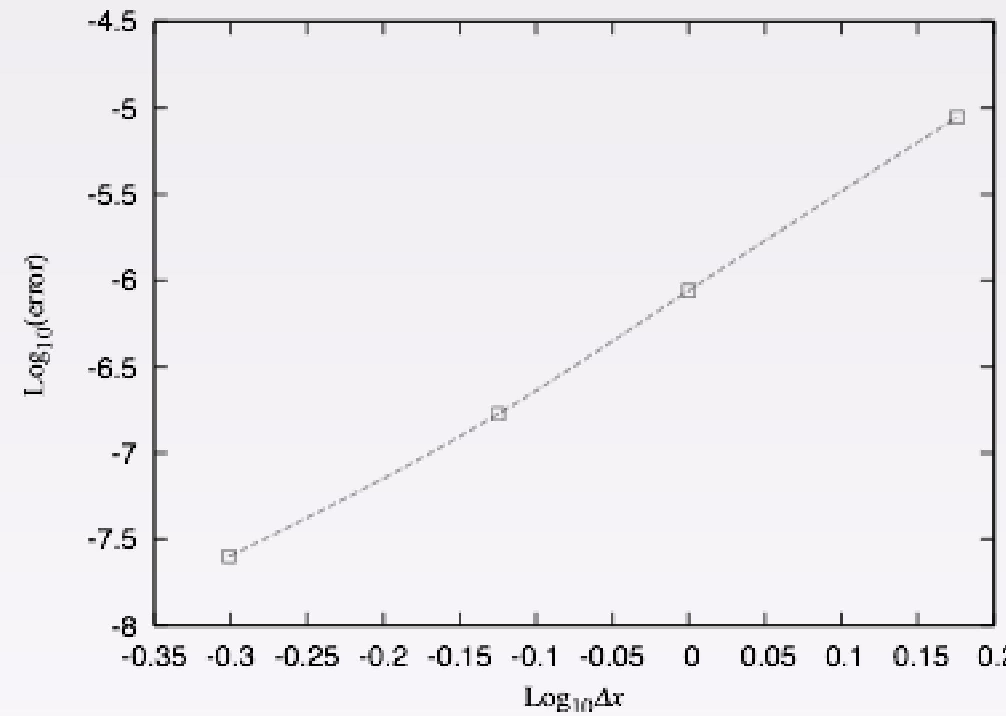
- Space direction : 6th-order finite discretization
- Time direction : 4th-order Runge-Kutta
- Grid size: $$\Delta r_* = 0.5 \quad (M = 1)$$ $$\Delta\theta = \Delta\phi = \pi/30$$
- Courant number: $$C = \frac{\Delta t}{\Delta r_*} = \frac{1}{20}$$
- Pure ingoing BC at the inner boundary,
  Fixed BC at the outer boundary
- Pullback: 7th-order Lagrange interpolation

# Code check

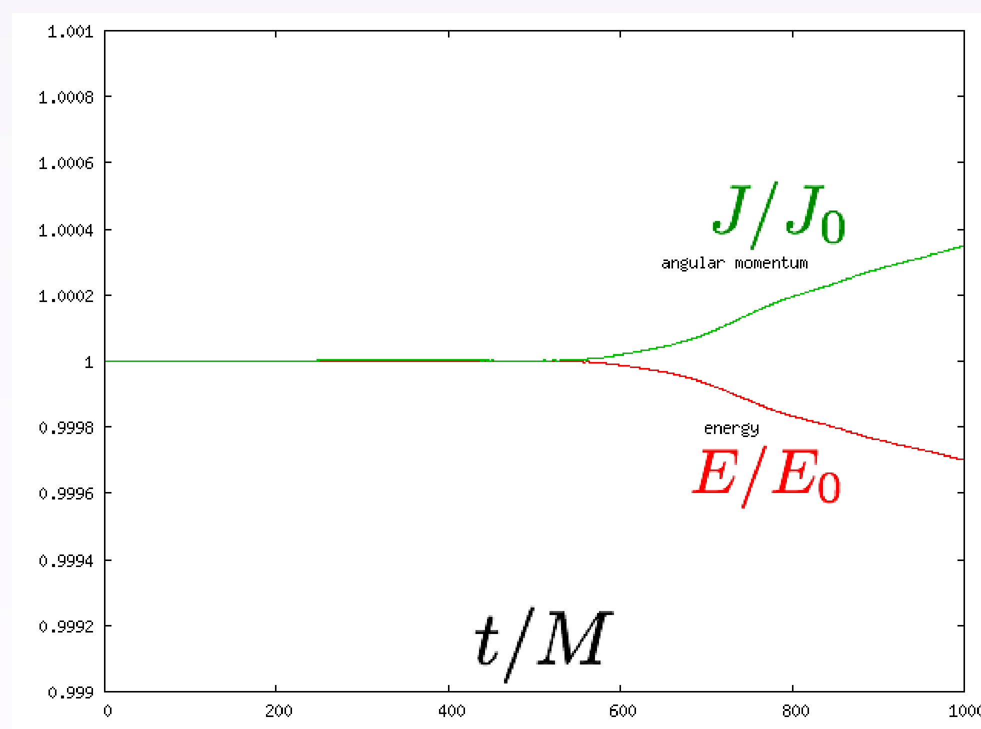
- Comparison with semianalytic solution in the Klein-Gordon case




- Convergence




- Conserved quantities



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# Numerical simulation

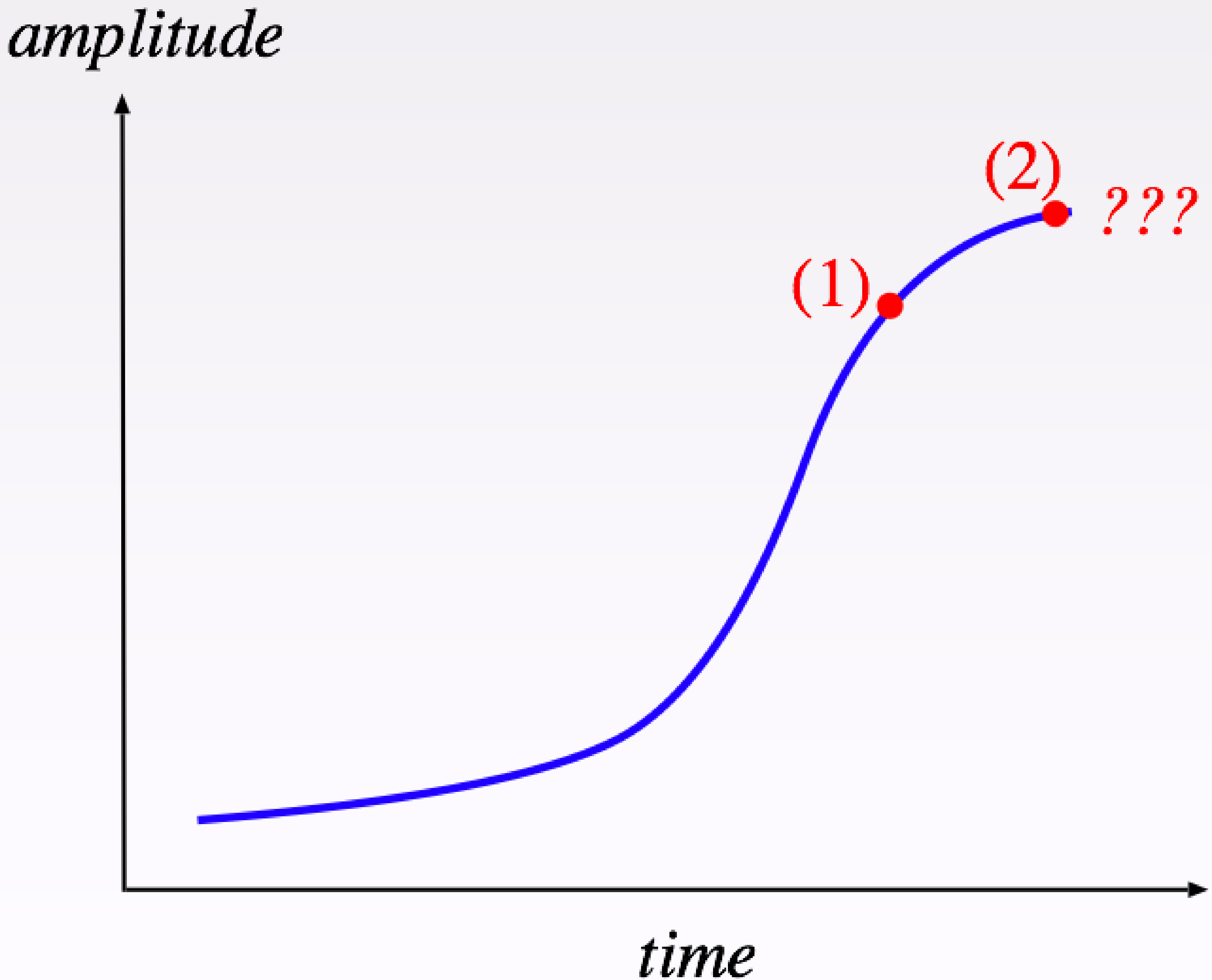
- Sine-Gordon equation

$$\nabla^2\varphi - \mu^2 \sin\varphi = 0$$

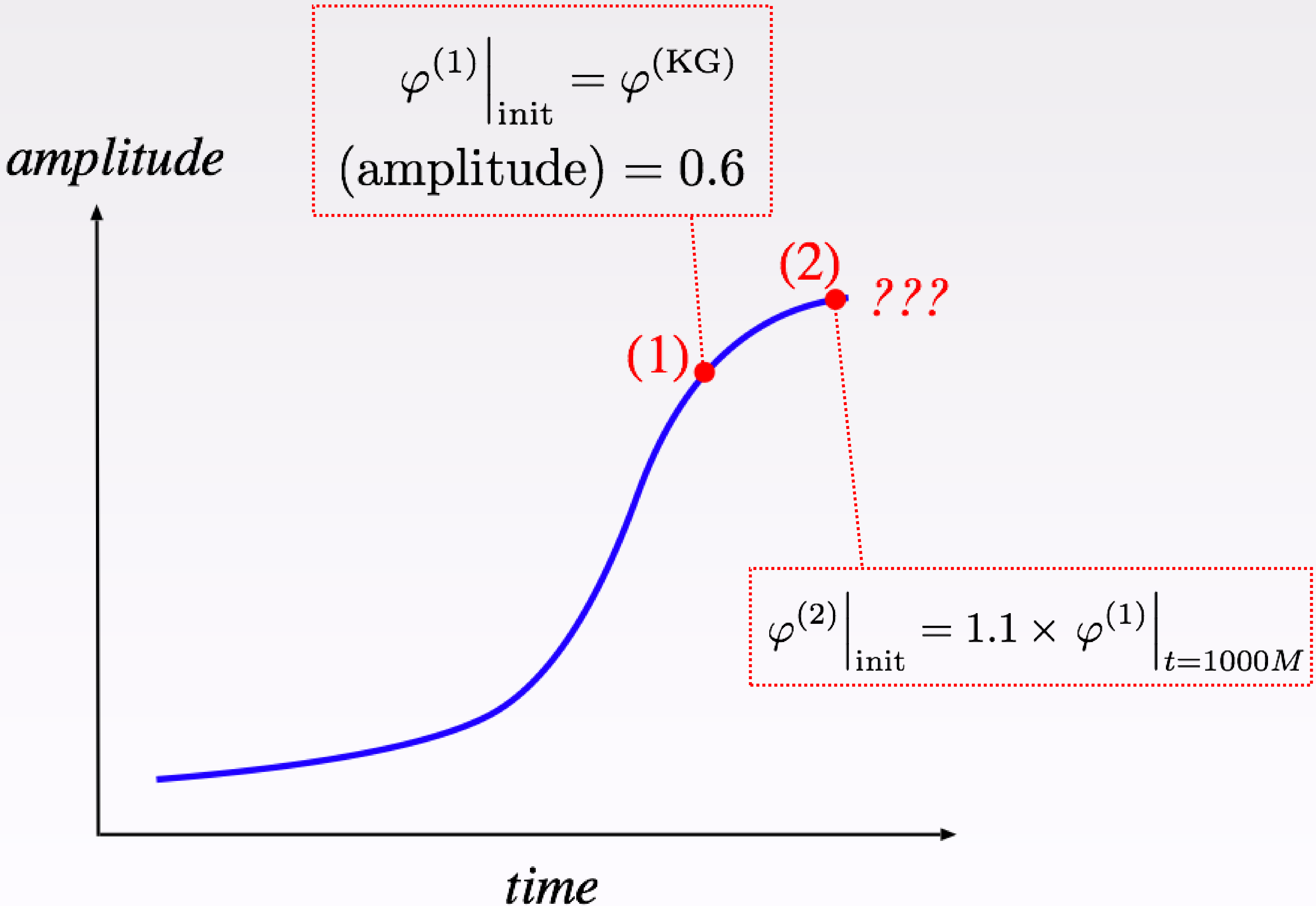
- Setup $a/M = 0.99, \quad M\mu = 0.4$

- We choose the state of axion cloud that has grown by superradiant instability of $l = m = 1$ mode.

# Expected evolution

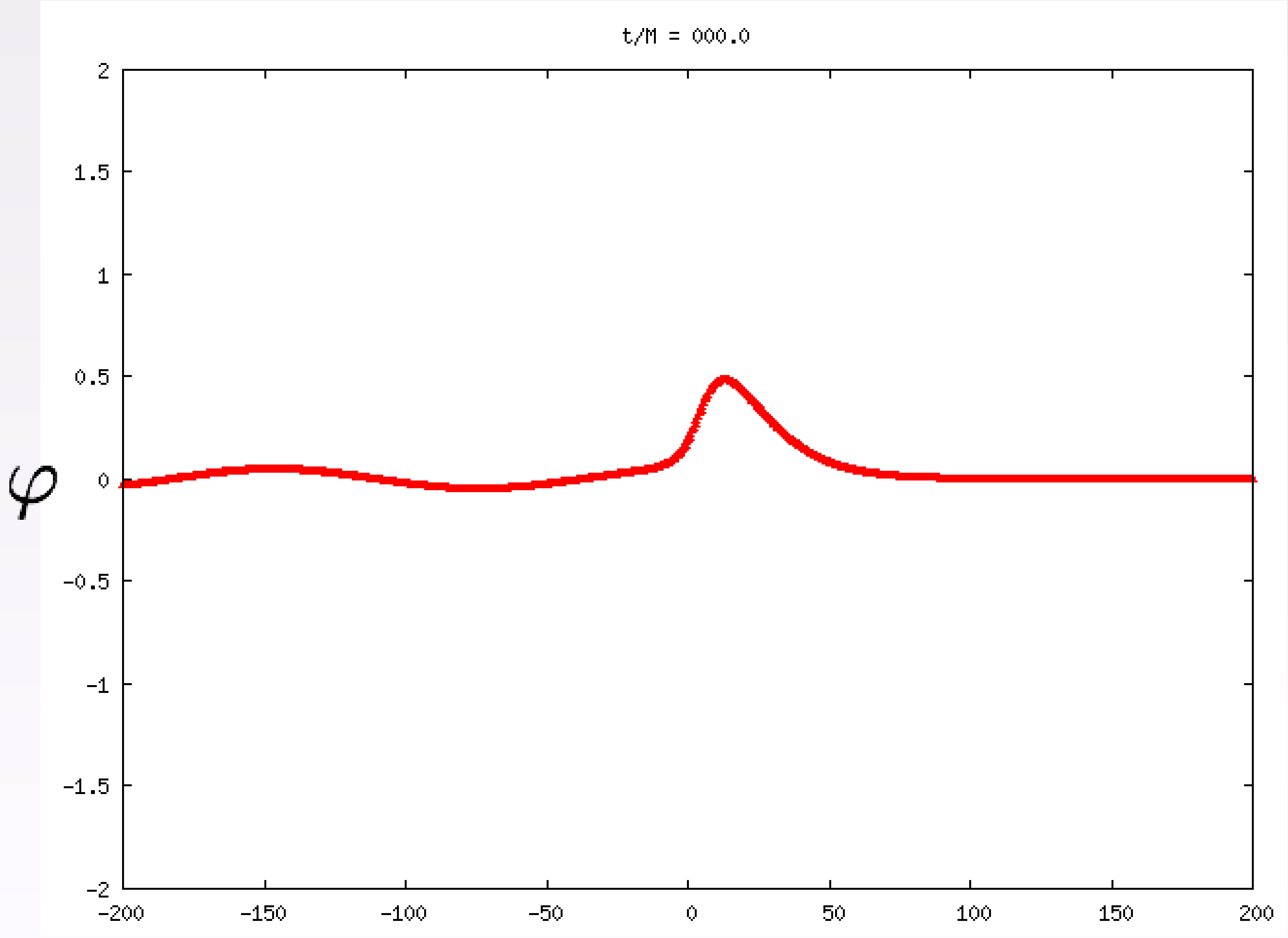


# Expected evolution



# Simulation (1)

- Axion field on equatorial plane $(\phi = 0)$



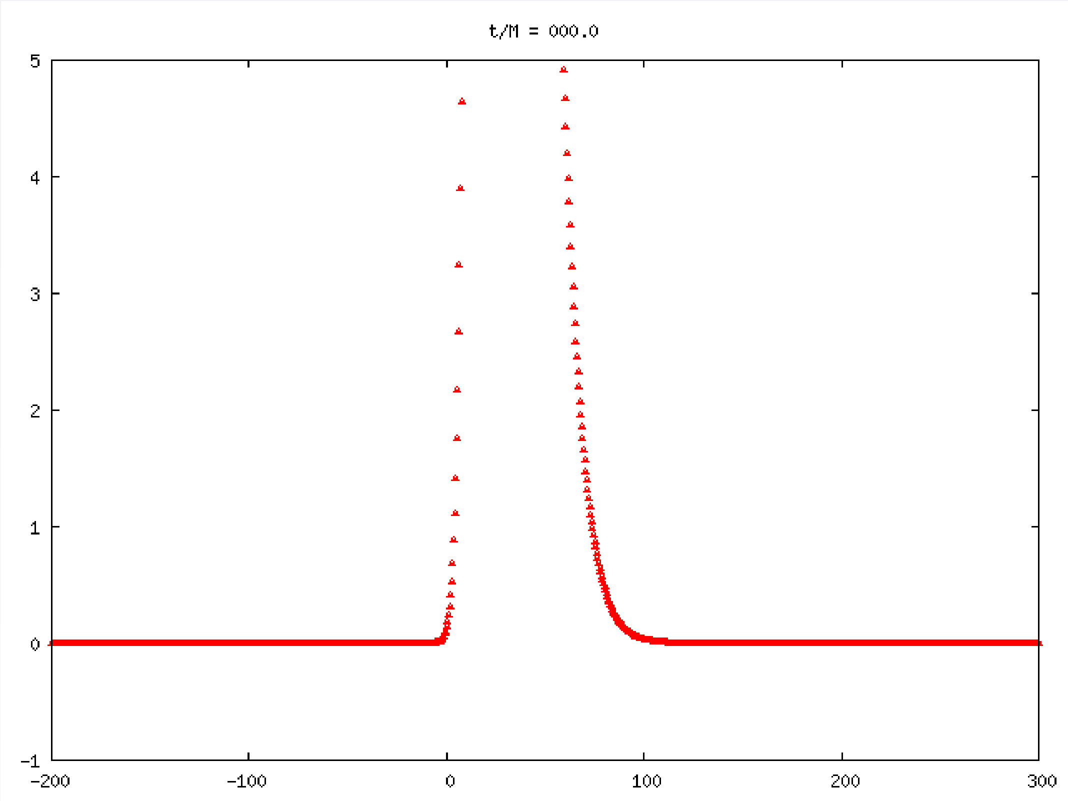

$$-200 \leq r_*/M \leq 200$$

# Simulation (1)

- Energy density with respect to the tortoise coordinate

$$t/M = 0 \qquad\qquad t/M = 1000$$

$$\frac{dE}{dr_*}$$

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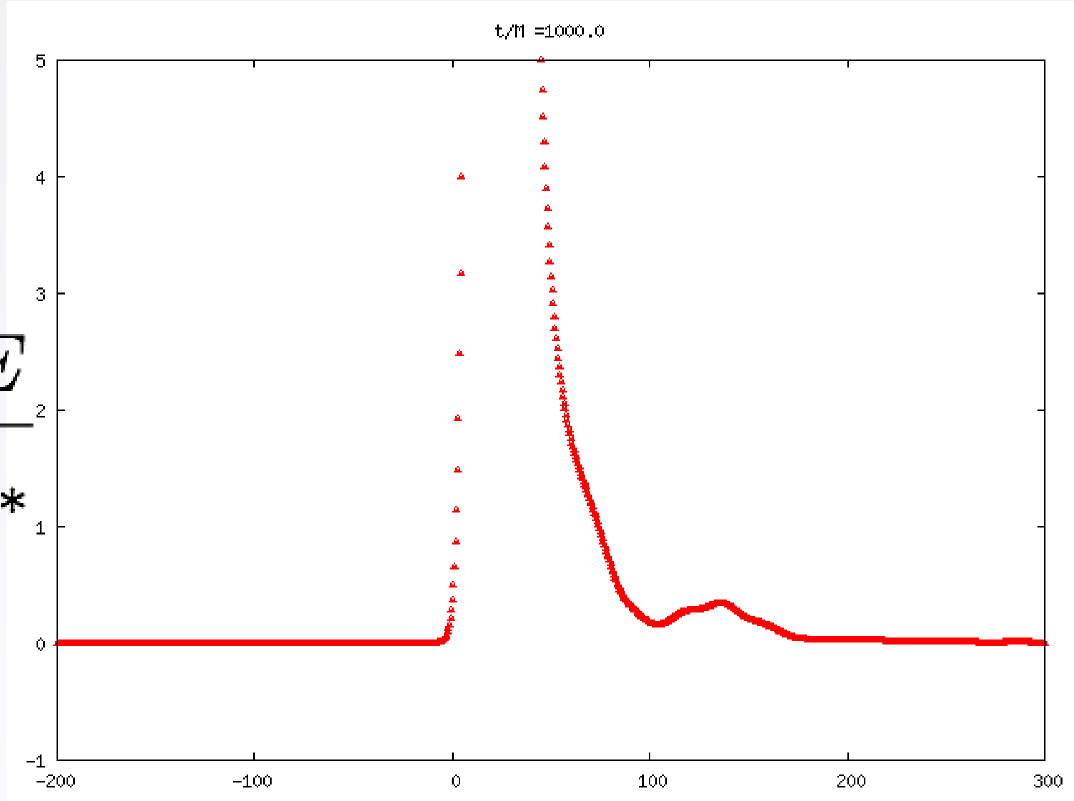
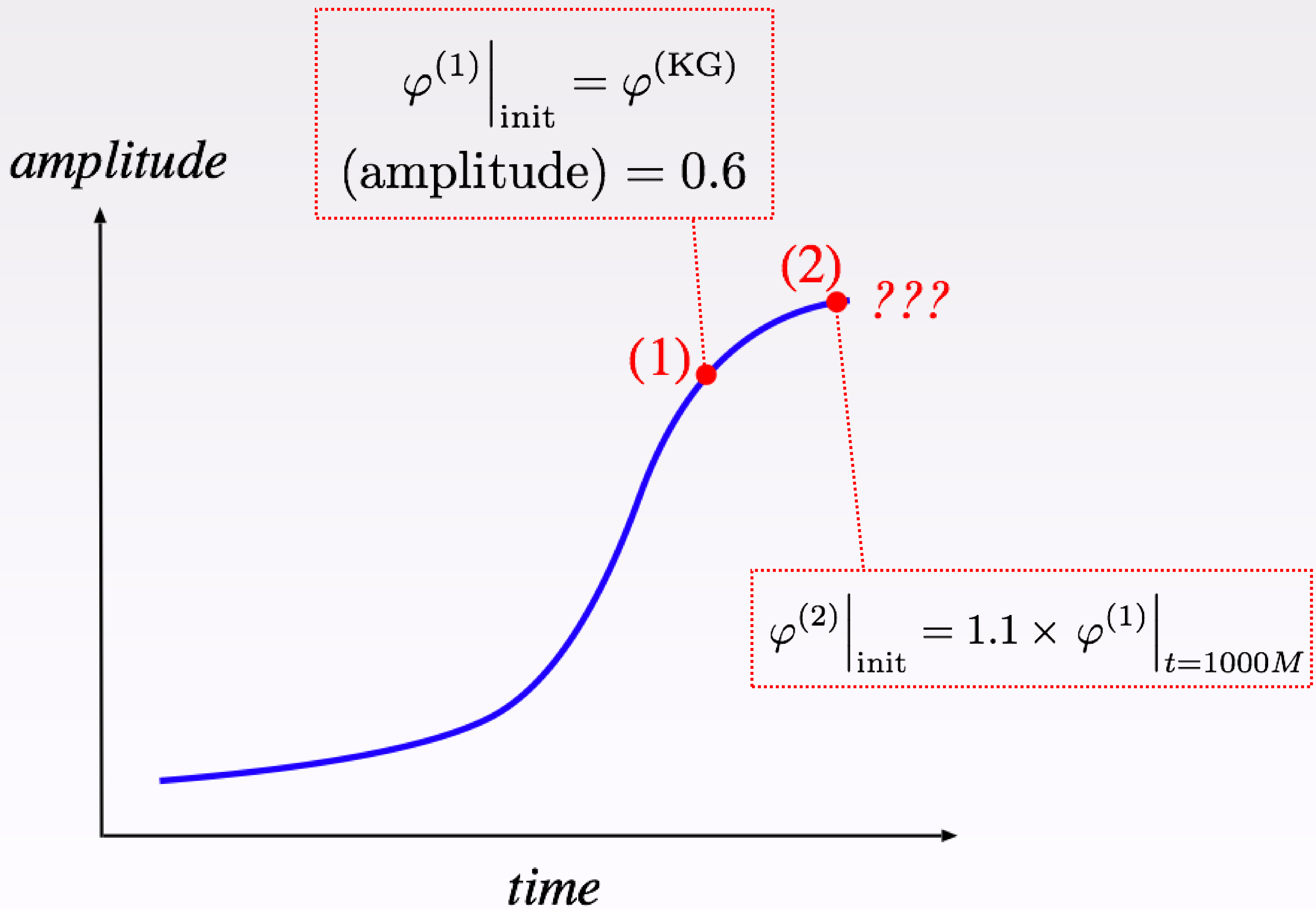


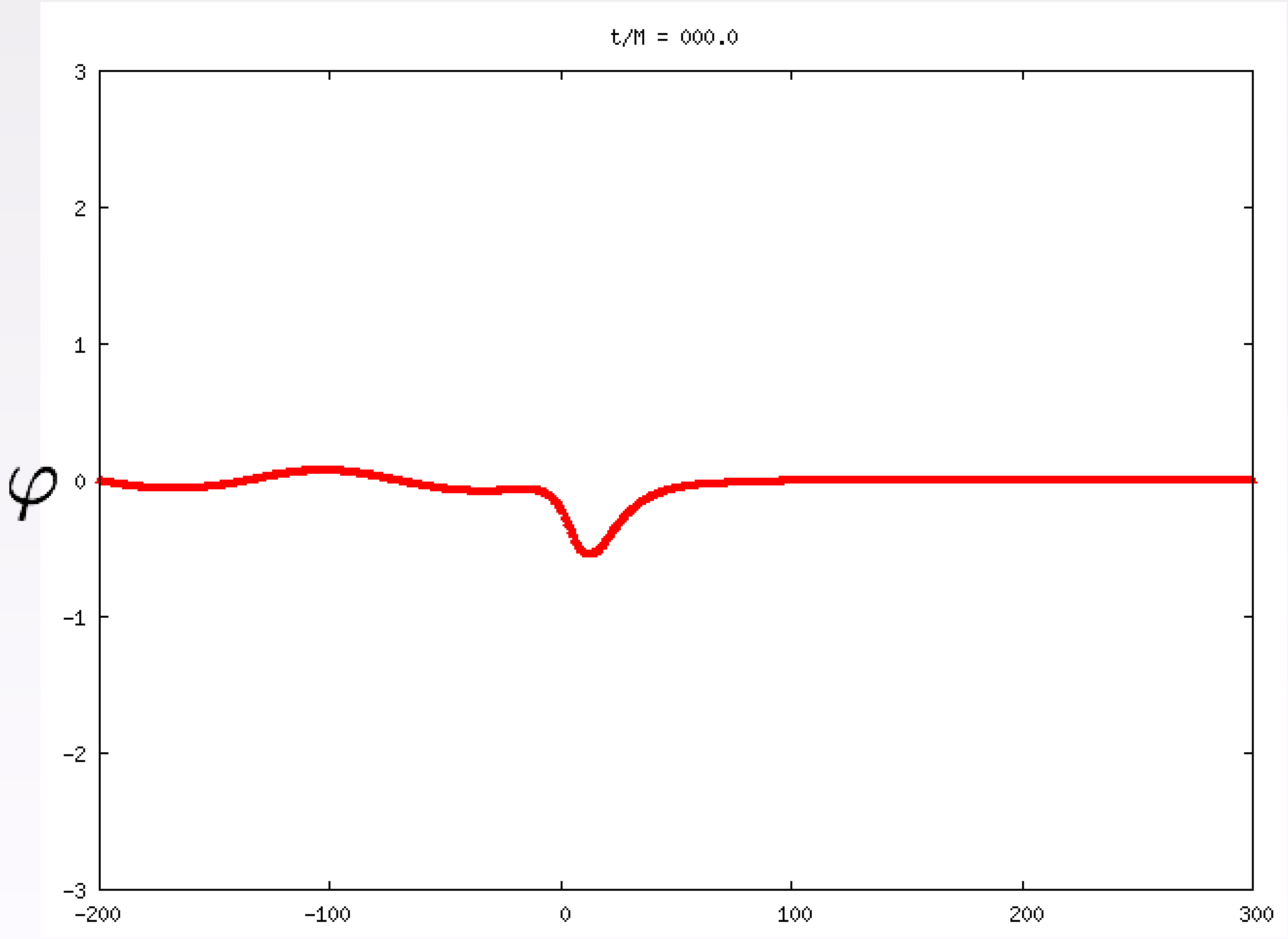

$$-200 \leq r_*/M \leq 300 \qquad\qquad -200 \leq r_*/M \leq 300$$

# Expected evolution



# Simulation (2)

- Axion field on equatorial plane $(\phi = 0)$

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# Simulation (2)

- Energy density with respect to the tortoise coordinate

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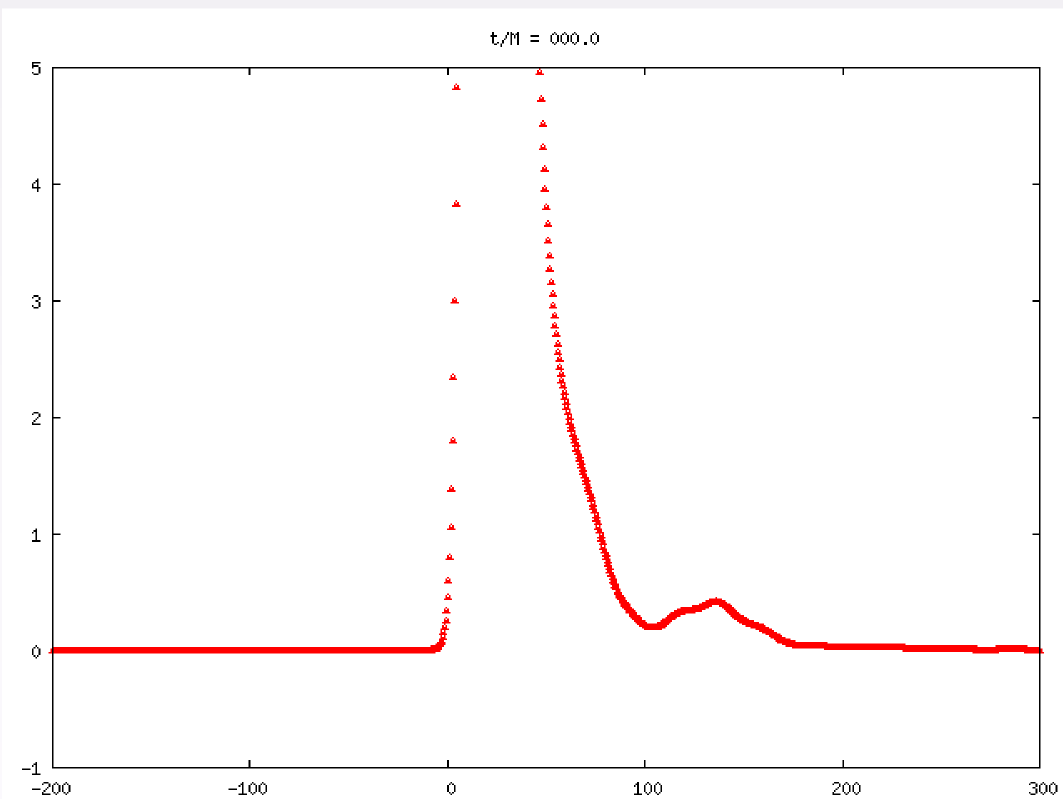
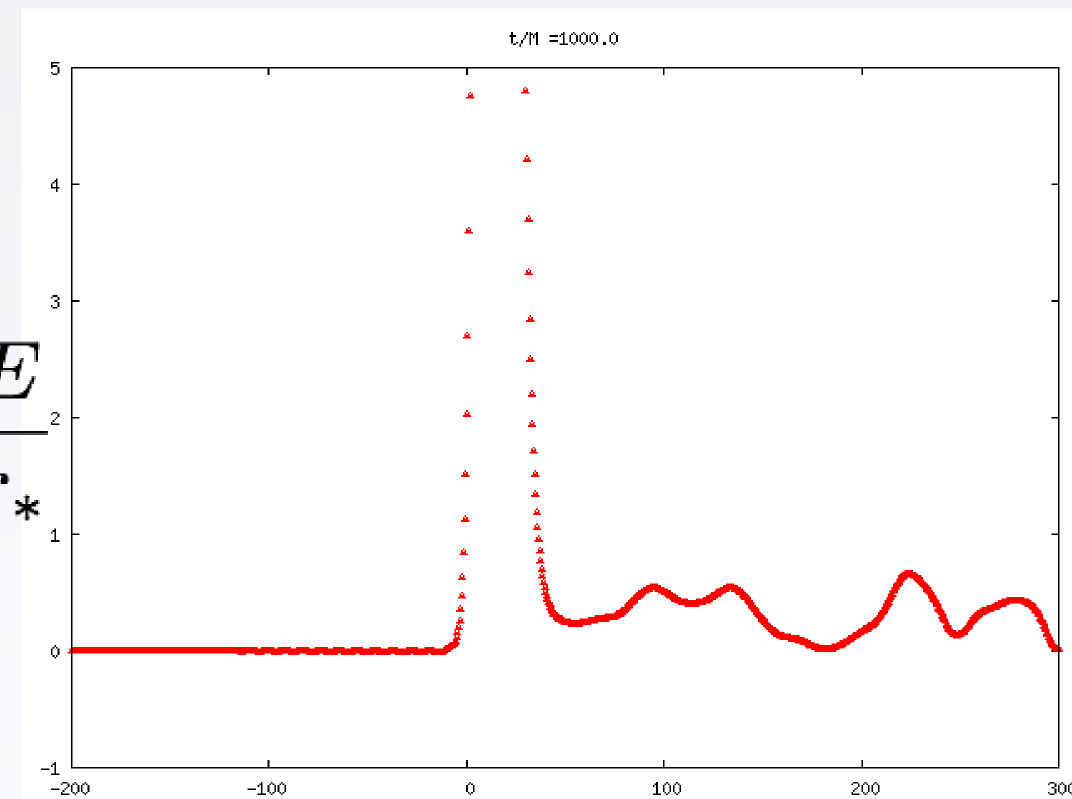



$$-200 \leq r_*/M \leq 300$$

$$t/M = 1000$$

$$\frac{dE}{dr_*}$$




$$-200 \leq r_*/M \leq 300$$

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# Summary

- We developed a reliable code and numerically studied the behaviour of axion field around a rotating black hole.

- When the nonlinear self-interaction becomes relevant, the “bosenova collapse” can be seen, but not very violent.

- The final state of superradiant instability would be a quasi-stationary state.

# Issues for future

- Calculation of the gravitational waves emitted in bosenova.

- The case where axions couple to magnetic fields.