A quasi-radial stability criterion for rotating relativistic stars

(MNRAS, 416, L1, 2011)

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The 21st workshop on General Relativity and Gravitation in Japan

Outline

Introduction

Stability of a Relativistic Star

A lot of oscillations are excited by many processes.

- Starquakes by secular spin-down of a pulsar.
- Phase transition of a neutron star. (e.g. Cheng et al. 1998)
- Core collapse by a supernova explosion. (e.g. Mönchmeyer et al. 1991)
- Merging binary neutron star. (e.g. Shibata et al. 2000, Baiotti et al. 2008)
- and so on ...

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These oscillation and stability give us important physical information such as equation of state of high dense matter, general relativistic effects, and so on.

Stability of a Relativistic Star

non-rotating stars(TOV)

- secular-instability point
- quasi-radial dynamical-instability point (neutral point)

$$\sigma^2 = 0$$

- turning point

$$\frac{\partial M}{\partial \rho_{\rm c}} = 0$$

where

$$\begin{array}{ccl} \sigma & : & \text{eigenvalue for F mode} \\ M & : & \text{total gravitational mass} \\ \rho_{\rm c} & : & \text{central density} \end{array}$$

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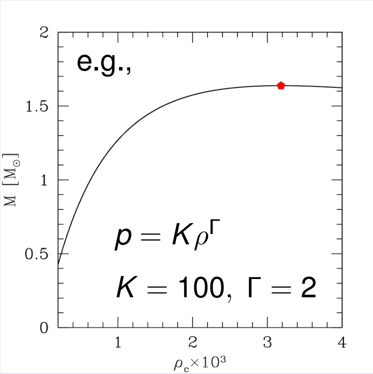
These three points are theoretically agreement for a barotropic star.



Stability of a Relativistic Star

rotating stars

- secular-instability point
  $\Rightarrow$ We **don't know** yet.
- quasi-radial dynamical-instability point (neutral point)
  $\Rightarrow$ We **don't know** yet, although the condition is $\sigma^2 = 0$.
- turning point
  $\Rightarrow$ We **know**, because the condition is $\left.\frac{\partial M}{\partial \rho_c}\right|_{J=\mathrm{const.}} = 0$

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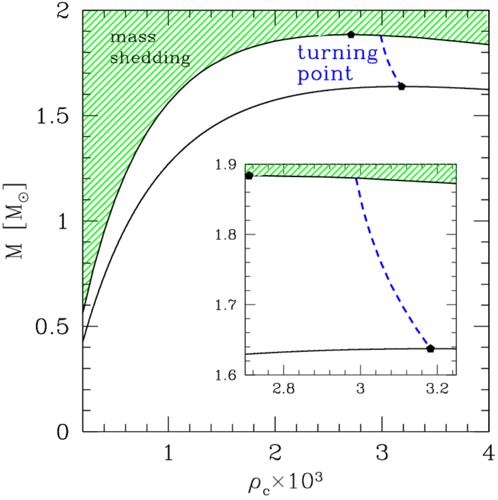
Friedmann, Ipser & Sorkin(1988) proved this is a sufficient condition for secular instability.

They suggested turning point coincide with secular-instability point from the assumption that viscosity leads to uniform rotation.

Stability of a Relativistic Star

That is to say, ...



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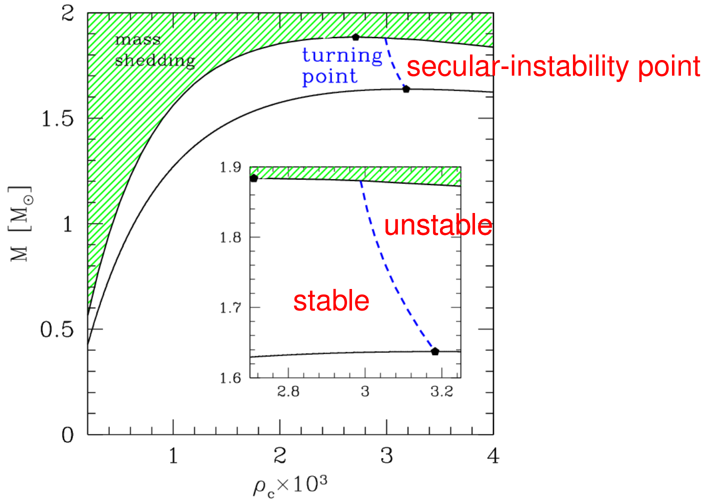

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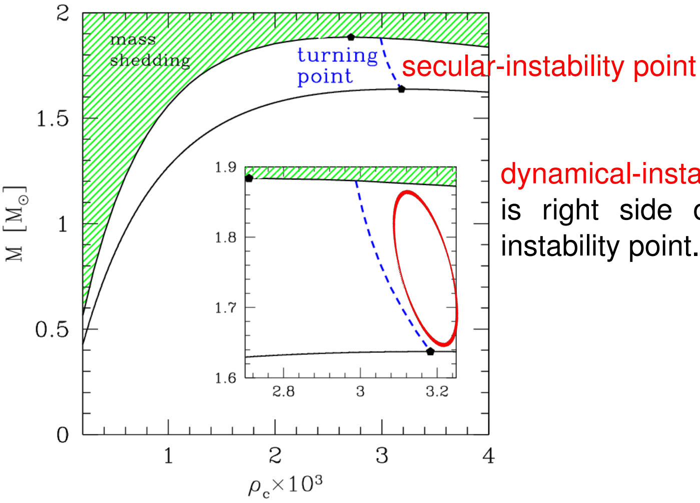

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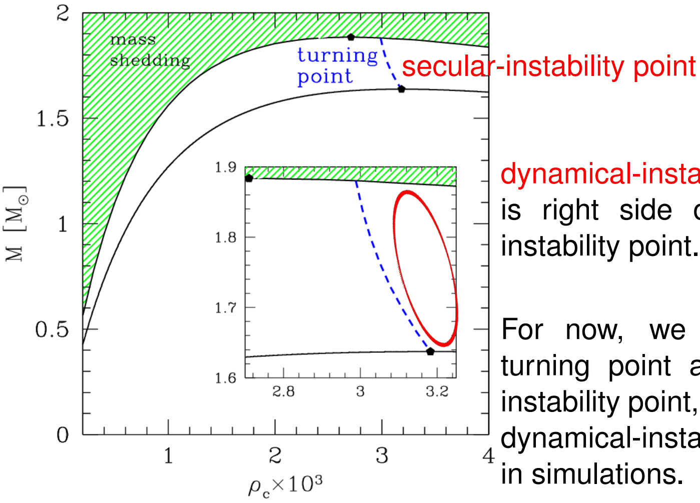

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For now, we are using turning point as secular-instability point, and to find dynamical-instability point in simulations.

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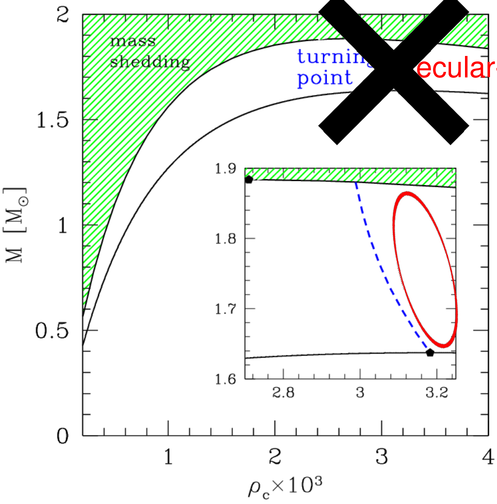

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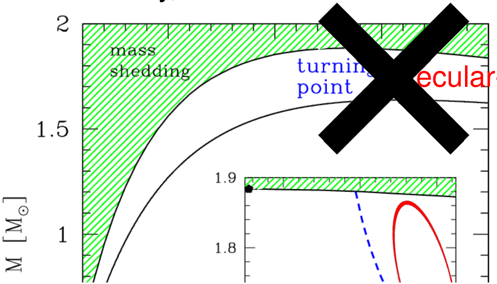

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We study dynamical-instability point ($\sigma^2 = 0$) for a fast rotating neutron star, including full general relativistic effects.

Method

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   - Cowling approximation:

     Ignoring the perturbation of space-time.

     $\Rightarrow$ no good approximation for such as fundamental mode of quasi-radial oscillation.

Flowchart of our Method

1. Background star:
   RNS code (public code developed by Nikolaos Stergioulas.)
2. Perturbation:
   Mapping the eigenfunction(mass density/pressure/energy density) of non-rotational NS by computing linear perturbation theory to the equilibrium model.
3. To evolve the star using a numerical relativity.
4. To decide the frequency by DFT of the central mass density.

Detail of our Method

Space-time Part

- The code developed by AEI.
  - Cactus Computational Toolkit (Goodale et al. 2003)
  - BSSNOK formalism (Nakamura et al. 1987, Shibata et al. 1995, Baumgarte et al. 1998)
  - Cartesian coordinate
  - “cartoon” method (Alcubierre et al. 2001)

$\Rightarrow$ **Axi-symmetry**

Detail of our Method

Fluid Part

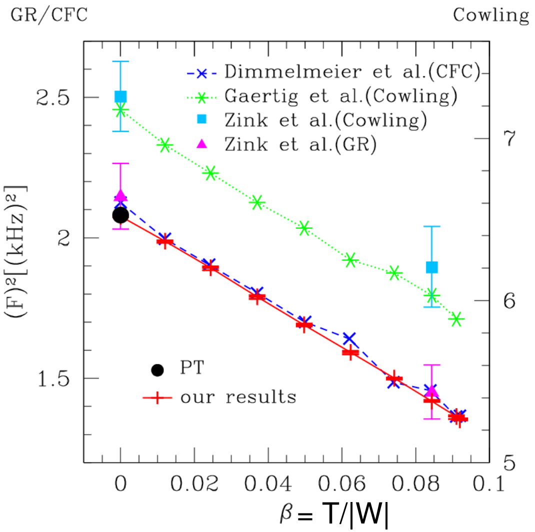
- Whisky2D code (Kellerman et al. 2008)
  - Cactus Computational Toolkit (Goodale et al. 2003)
  - Cartesian coordinate
  - Piecewise-Parabolic-Method (PPM) (Colella et al. 1984)
  - Harten-Lax-van Leer-Einfeldt (HLLE) solver
  - Method of Line with 3rd order Runge-Kutta

$\Rightarrow$ **Axi-symmetry**

- EOS: polytropic EOS ($p = K\rho^\Gamma$, $e = \rho + \frac{p}{\Gamma - 1}$)

Numerical Test

Comparison with our Results and Previous Works



- * uniform rotation
- * Polytropic EOS ($K = 100, \Gamma = 2$)
- * $\rho_c = 1.28 \times 10^3$

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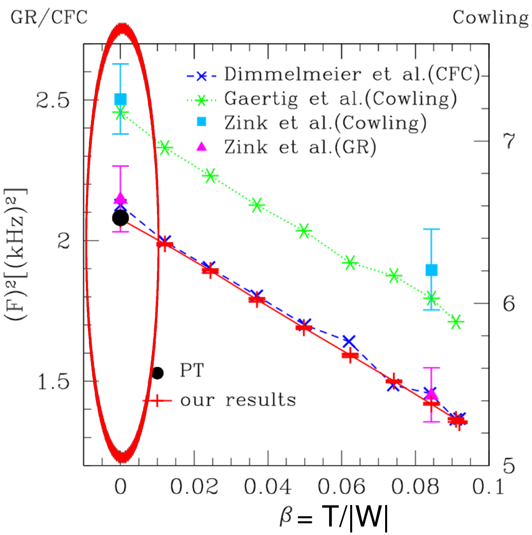

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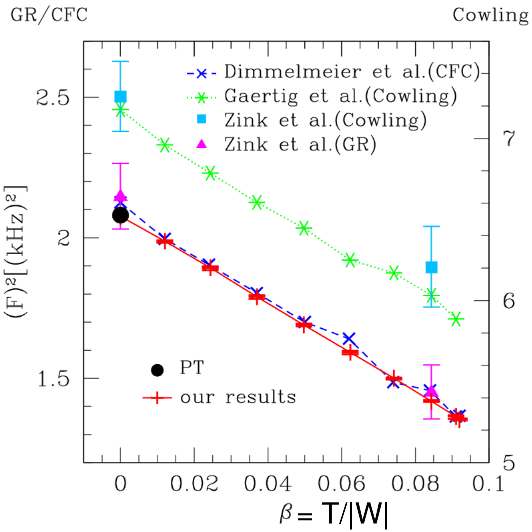

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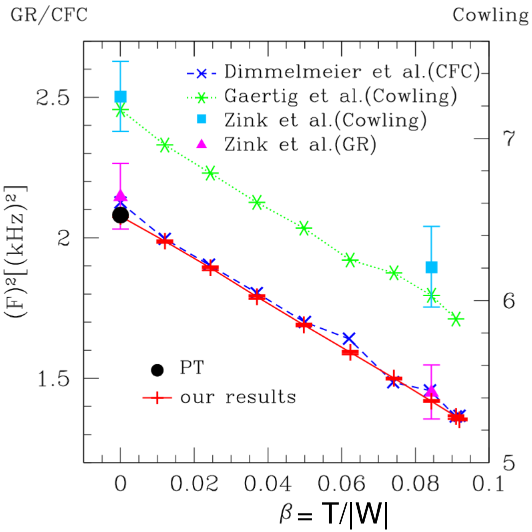

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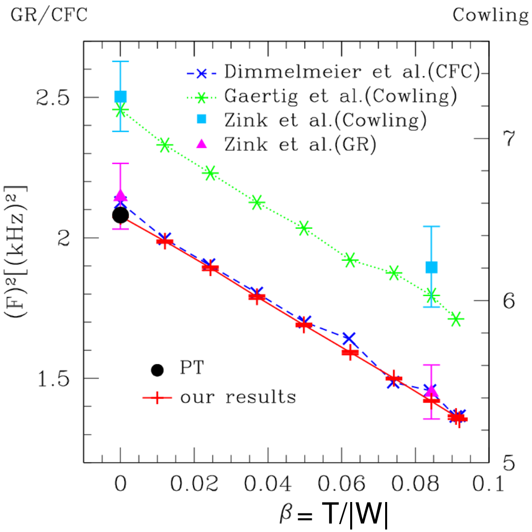

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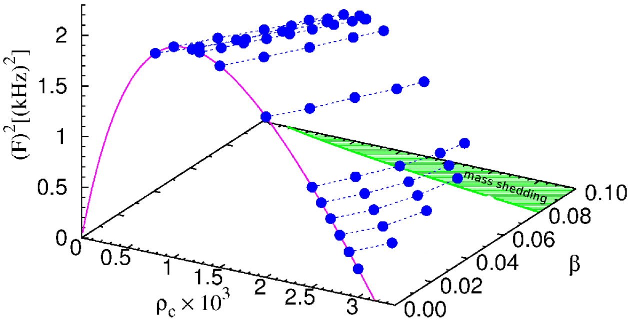
- Very small error bars.

Results

F-mode Frequencies for Wide Range of ρ_c and $T/|W|$



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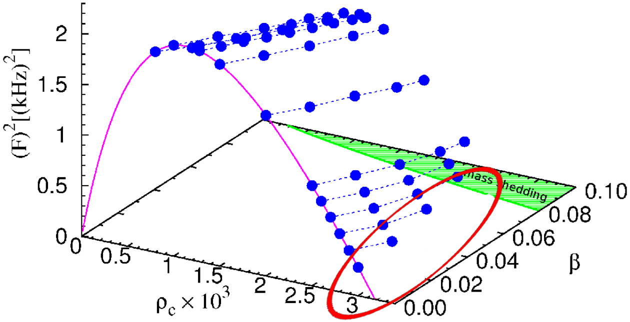

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- Oscillation time-scale tends to become extremely large.
- Models are artificially induced to collapse by the accumulation of the truncation error.

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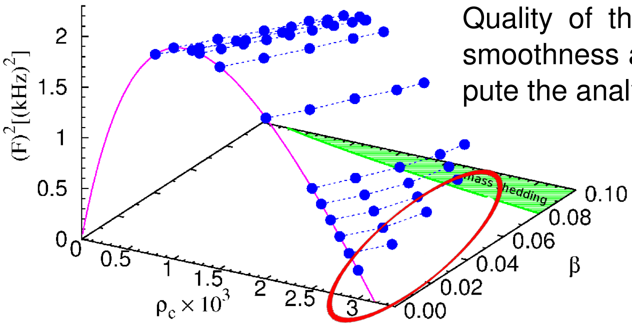

Quality of the data and the smoothness allow us to compute the analytic fit.

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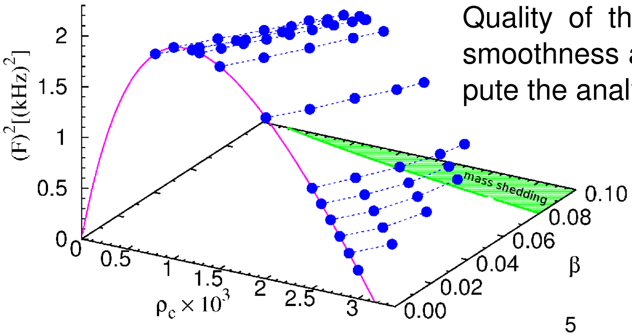

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$$
\begin{aligned}
(F)^2_{\text{fit}}(\rho_c, \beta) &= (F)^2_{\text{fit}}(\rho_c, 0) + \beta \sum_{n=0}^{5} b_n (\rho_c)^n \\
&= \sum_{n=0}^{5} a_n (\rho_c)^n + \beta \sum_{n=0}^{5} b_n (\rho_c)^n
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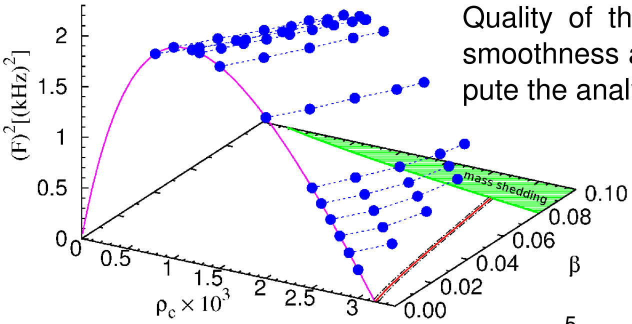

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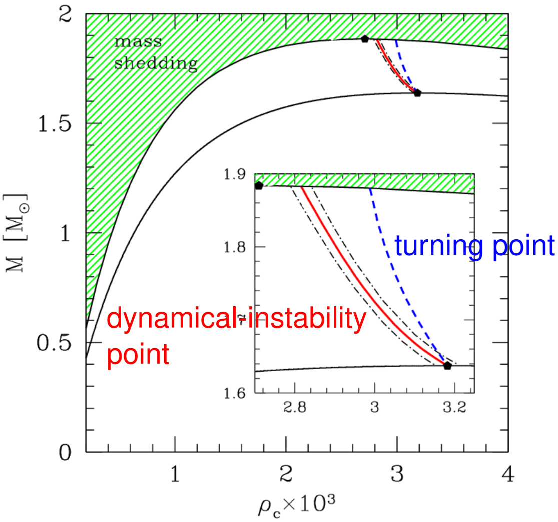
Dynamical-instability Points ($F = 0$)



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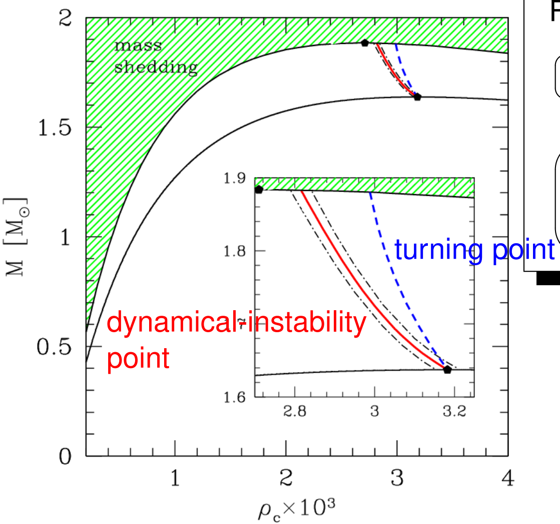

For rotating stars,

turning points

$\not\Leftrightarrow$

secular-instability points

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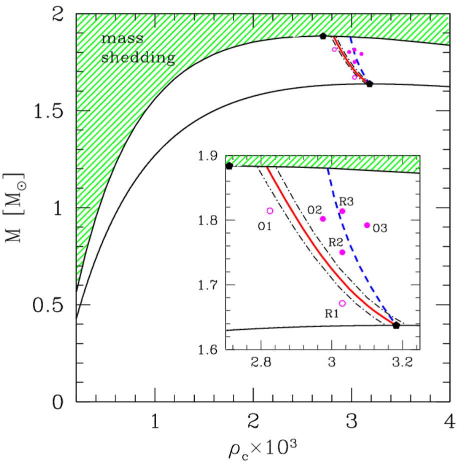

$O1$-$O3$: constant Ω

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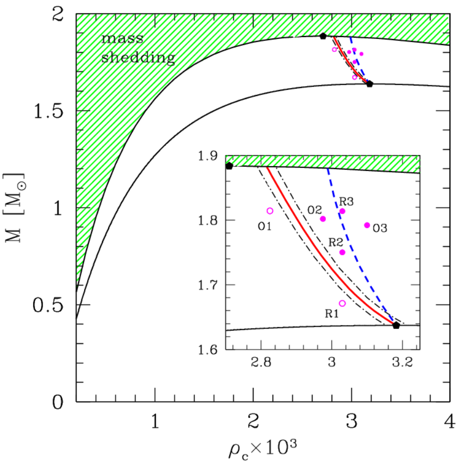

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Friedmann et al. expect...
$O1, O2, R1, R2$: stable

Our results expect...
$O1, R1$: stable

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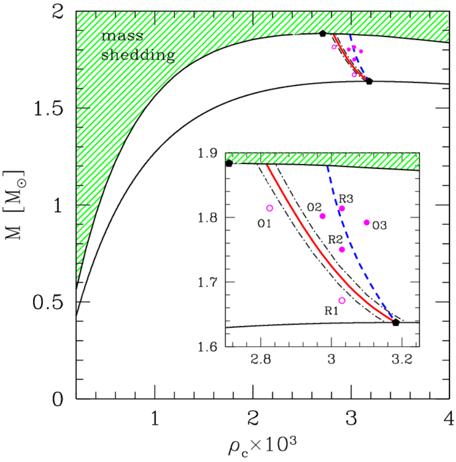

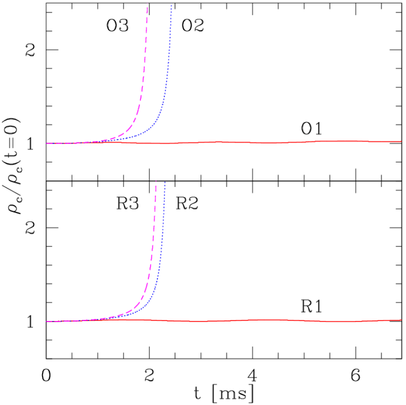

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