Dynamical Approaches for Secular Instabilities in Rotating Stars

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1. Introduction

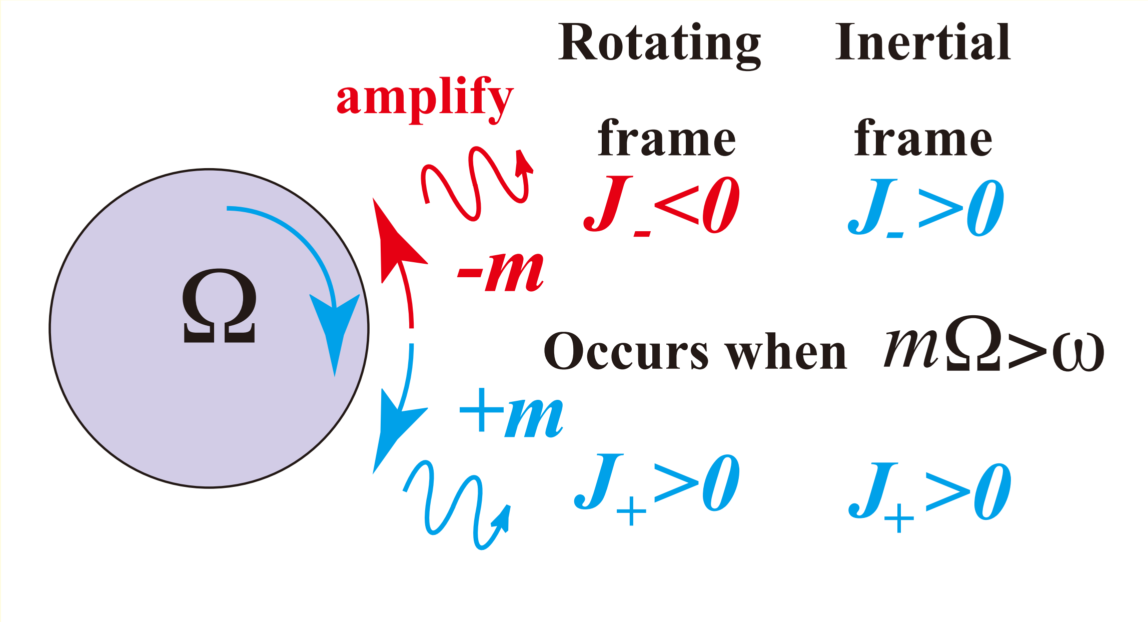
Various Instabilities in Secular Timescale

$e^{i(m\varphi-\omega t)}$

r-mode instability

(Andersson 98, Friedman & Morsink 98)

Instability occurs due to gravitational radiation



g-mode instability

- Fluid elements oscillate due to restoring force of buoyancy
- Instability occurs in nonadiabatic evolution or in convective unstable cases

bar-mode instability

- Instability occurs when T/W exceeds some critical value
- Dissipative effects such as gravitational radiation or viscosity plays a crucial role

Kelvin-Helmholtz instability

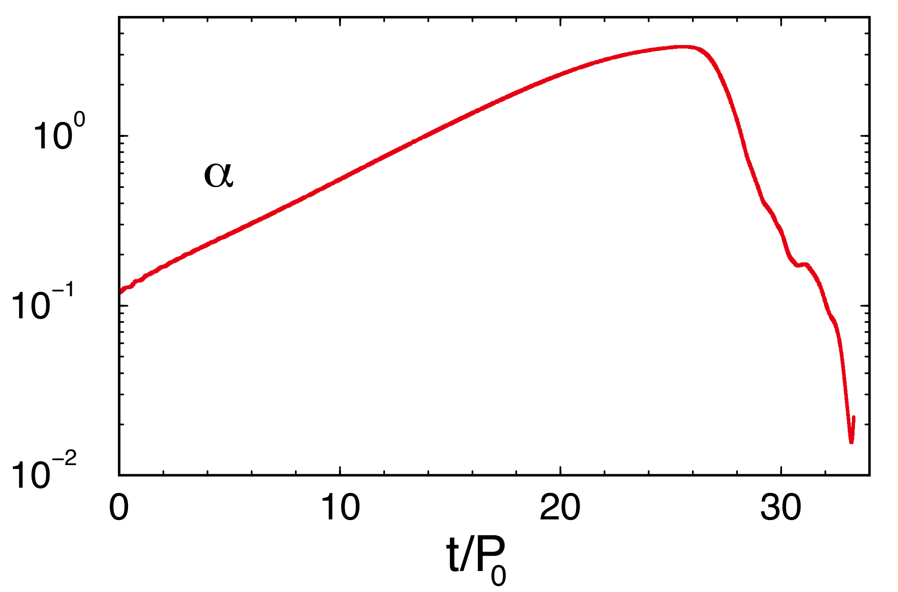
- Instability occurs when the deviation of the velocity between the different fluid layers exceeds some critical value

Dynamics of r-mode instabilities

Saturation amplitude of r-mode instability

3D simulation

- Saturation amplitude of o(1)
- Imposing large amplitude of radiation reaction potential in the system to control secular timescale with dynamics (Lindblom et al. 00)



1D evolution with partially included 3 wave interaction

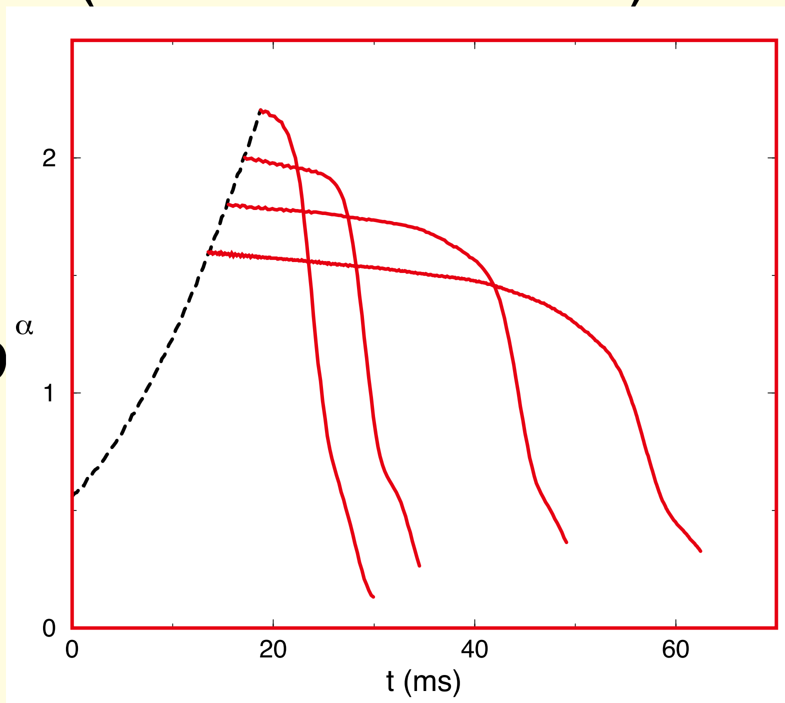
- Saturation amplitude of ~ o(0.001), which depends on interaction term

(Schenk et al. 2001)

Final fate of r-mode instability

3D simulation

- Evolution starting from the amplitude o(1)
- Imposing large amplitude of radiation reaction potential
- Energy dissipation of r-mode catastrophically decays to differentially rotating configuration in dynamical timescale



1D evolution including mode couplings network

- After reaching the saturation amplitude ~o(0.001), Kolmogorov-type cascade occurs
- Destruction timescale is secular

(Gressman et al. 02, Lin & Suen 06)

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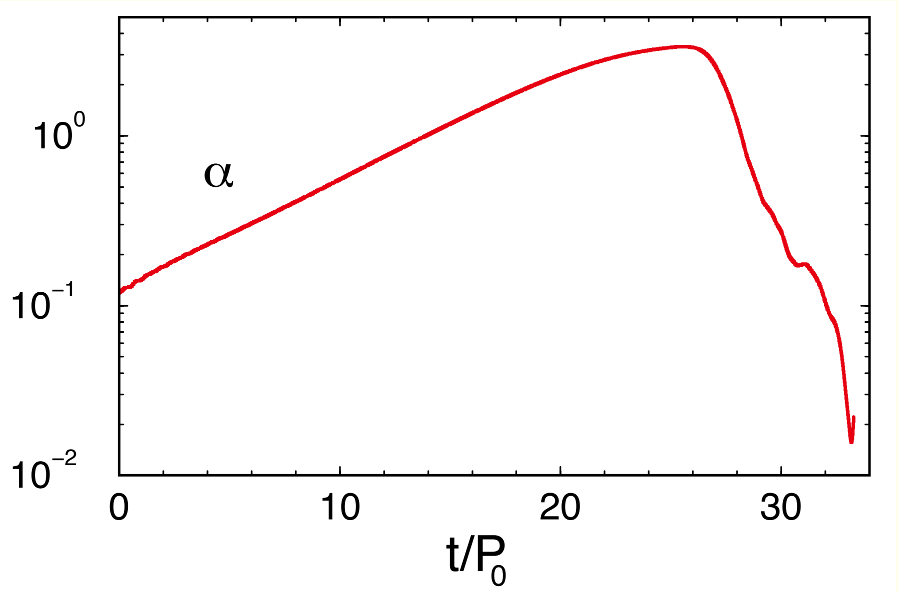

1D evo

- Satu ... m

Final f ... . 2001)

3D si

- Evolu
- Imposing large amplitude of radiation reaction potential
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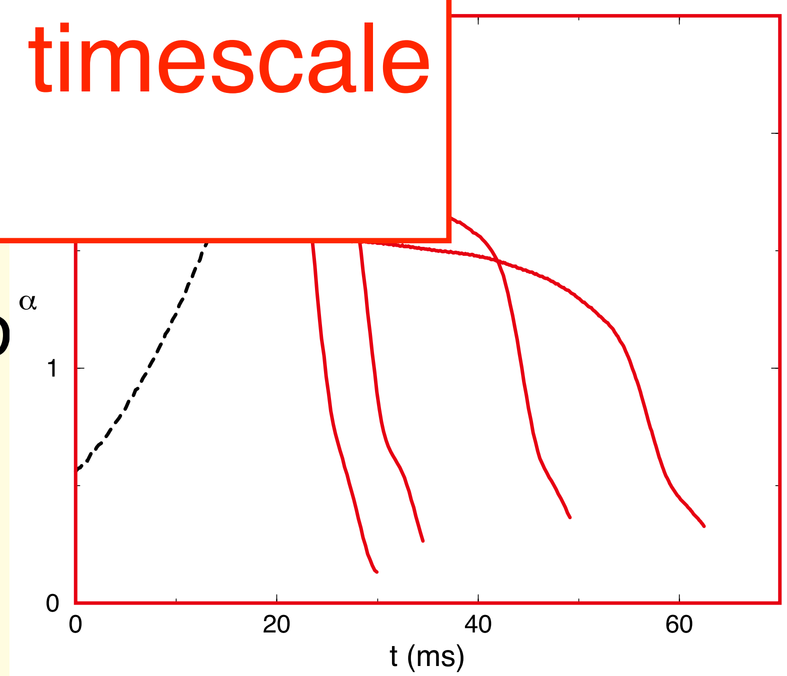

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Alternative approaches

- **From linear regime to nonlinear regime**
- **From dynamical timescale to secular timescale**

are necessary!

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Timescales in Neutron Stars

Acoustic time	Duration of acoustic waves travelling across the diameter	0.2ms
Rotation period		~2ms
Ekman #	Ratio between the viscous term and the Coriolis term	$E \sim \nu/(\rho\Omega R^2) \lesssim 10^{-7}$
Rossby #	Ratio between the nonlinear and the Coriolis force	$Ro \sim W/(R\Omega)$
Reynolds #	Ratio between the nonlinear term and the viscous term	$Re \sim \rho W R/\nu$
Chandra #	Ratio between the radiation reaction force and the viscous term	$Ch \sim n^2 G R^9 \Omega^6/(c^7 \nu)$

Gravitational Waves

- r-mode instability ... gravitational wave source for the ground based detectors

  upper limit for the youngest known neutron stars ~o(0.0001)

  (LIGO 10)
- Frequency band 100 Hz - few kHz
- Seek for the interior structure of neutron stars through unstable modes

2. Anelastic Approximation in Linearised Regime

Basic Equations

Perfect Fluid in Newton gravity

Continuity Equation $\frac{\partial \rho}{\partial t} + \nabla_i(\rho v^i) = 0$

Energy Equation $\frac{\partial \varepsilon}{\partial t} = -\frac{P}{\rho^2}\nabla_i(\rho v^i)$

Euler Equations $\frac{\partial(\rho v^j)}{\partial t} + \nabla_i(\rho v^i v^j) = -\nabla^j P - \rho \nabla^j \Phi$

Equation of State $P = P(\rho, \varepsilon)$

Same procedure is available for conformally flat spacetime (relativistic gravitation)

Linear Perturbation

(Villain & Bonazzola 02)

$$\rho = \rho_{\rm eq} + \delta\rho$$
$$v^j = v^j_{\rm eq} + \delta v^j$$
$$h = h_{\rm eq} + \delta h$$

$+ \quad P = P(\rho)$

(Villain et al. 05)

$$P = (\Gamma - 1)\rho\varepsilon$$
$$h = \varepsilon + \frac{P}{\rho} = \Gamma\varepsilon$$

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)\delta v^i = S^i_v$$

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)\delta h = -\delta v^j \nabla_j h_{\rm eq} - (\Gamma - 1) h_{\rm eq} \nabla_j \delta v^j$$ =0

Killing the acoustic wave of δh in rotating frame

$$\Rightarrow \quad \triangle \delta h = S_h(\delta h, \delta v^i, {\rm b.g.})$$

Boundary condition: $\delta h = 0$

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3. Anelastic Approximation in Nonlinear Regime

Deviation from the equilibrium state

$$\rho = \rho_{\rm eq} + \Delta\rho$$
$$v^j = v^j_{\rm eq} + \Delta v^j \quad \mathbf{+} \quad P = P(\rho)$$
$$h = h_{\rm eq} + \Delta h$$

$$P = (\Gamma - 1)\rho\varepsilon$$
$$h = \varepsilon + \frac{P}{\rho} = \Gamma\varepsilon$$

Free evolution scheme

Same procedure is available for conformally flat spacetime (relativistic gravitation)

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)\rho + \nabla_j(\rho\Delta v^j) = 0$$

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)(\rho\Delta v^i) + \nabla_j(\rho\Delta v^j\Delta v^i) = S^i_{\rho v}$$

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)\Delta h = -\Delta v^j\nabla_j h - (\Gamma - 1)h\nabla_j\Delta v^j$$ =0 !

→ Numerically unstable !

$\nabla^j P$ term

Due to numerical fluctuation, propagation of Δh is not completely killed.

Control acoustic wave at each timestep

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)\rho + \nabla_j(\rho\Delta v^j) = 0$$

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)(\rho\Delta v^i) + \nabla_j(\rho\Delta v^j\Delta v^i) = S^i_{\rho v}$$

$$\left(\frac{\partial}{\partial t} + v^j_{\rm eq}\nabla_j\right)h = -\Delta v^j\nabla_j h - (\Gamma - 1)h\nabla_j\Delta v^j$$ =0

➡ $\nabla_j(\rho\Delta v^j) = 0$

➡ $\triangle P = S_p$ Boundary Condition: P=0 at the stellar surface

Procedure

Same procedure as MAC method in NS incompressible fluid

(McKee et al. 08)

1. Time update $\rho \quad \rho\Delta v^i$
2. Poisson equation $\nabla_i\nabla^i\psi = \nabla_i(\rho\Delta v^i)^*$

   Boundary Condition: $\psi = 0$ at the stellar surface
3. Anelastic condition $\rho\Delta v^i = (\rho\Delta v^i)^* - \nabla^i\psi$
4. Pressure poisson equation $\triangle P = S_p$

6. Summary

We propose a natural extension to the nonlinear regime of the anelastic approximation to focus on the dynamics of secular timescale

- Constraint scheme seems to be stable through evolution (Difficult treatment goes to pressure poisson equation)
- Anelastic approximation is a relaxation from the incompressible fluid approximation
  (the next relaxation stage is low Mach number approximation)

Future Study

- Code development and tests
- Nonlinear dynamics of r-mode instability