What are universal features of gravitating Q-balls?

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I. Introduction

Q-balls:a kind of nontopological solitons consist of scalar fields

Self-gravity may not be important?

Kaup, PR172, 1331 (1968)

⇨ $V(\phi)=\frac{m^2}{2}\phi^2$ flat ×
with self-gravity ○ (mini-boson star)

$$V_{\text{grav.}}(\phi) = \frac{m^2}{2}\phi^2\left[1 + K\ln\left(\frac{\phi}{M}\right)^2\right]$$

gravity-mediation type in the Affleck-Dine (AD) mechanism

flat: $K < 0$ ○ ; $K \geq 0$ ×

gravity: $K \leq 0$ ○ ; $K > 0$ ○ ``Q-balls" surrounded by Q-matter

TT, N. Sakai, PRD**83**, 084046 (2011)

Here, gauge-mediation type
→seek for universal features!!

$$V_{\text{gauge}}(\phi) = m^4 \ln\left(1 + \frac{\phi^2}{m^2}\right)$$

TT, N. Sakai, PRD**84**, 044054 (2011)

II. Model

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{m_{\rm Pl}{}^2}{16\pi} \mathcal{R} + \mathcal{L}_\phi \right)$$

$$\mathcal{L}_\phi \equiv \sqrt{-g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \boldsymbol{\phi} \cdot \partial_\nu \boldsymbol{\phi} - V(\phi) \right\}$$

$\boldsymbol{\phi} = (\phi_1,\ \phi_2)$ is an SO(2)-symmetric scalar field

$$\phi \equiv \sqrt{\boldsymbol{\phi} \cdot \boldsymbol{\phi}} = \sqrt{\phi_1^2 + \phi_2^2}.$$

$$V_{\rm gauge}(\phi) = m^4 \ln\left(1 + \frac{\phi^2}{m^2}\right)$$

$$(\phi_1, \phi_2) = \phi(r)(\cos \omega t, \sin \omega t).$$

normalization

$$\tilde{\phi} = \frac{\phi}{m} \qquad \tilde{V} = \frac{V}{m^4} = \ln(1 + \tilde{\phi}^2)$$

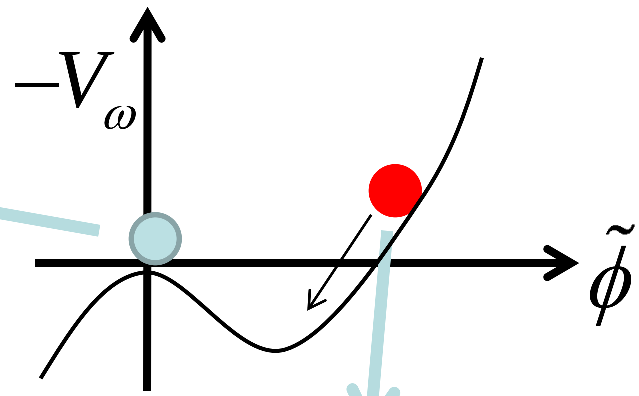
$$\tilde{\omega} = \frac{\omega}{m} \qquad \tilde{t} = mt \qquad \tilde{r} = mr$$

III. flat case

$$\tilde{\phi}'' = -\frac{2}{\tilde{r}}\tilde{\phi}' - \tilde{\omega}^2\tilde{\phi} + \frac{d\tilde{V}_{\text{gauge}}}{d\tilde{\phi}} \Rightarrow V_\omega := \tilde{V}_{\text{gauge}} - \frac{\tilde{\omega}^2\tilde{\phi}^2}{2} = \ln(1+\tilde{\phi}^2) - \frac{\tilde{\omega}^2\tilde{\phi}^2}{2}$$

⇒ Particle motion in $-V_\omega$

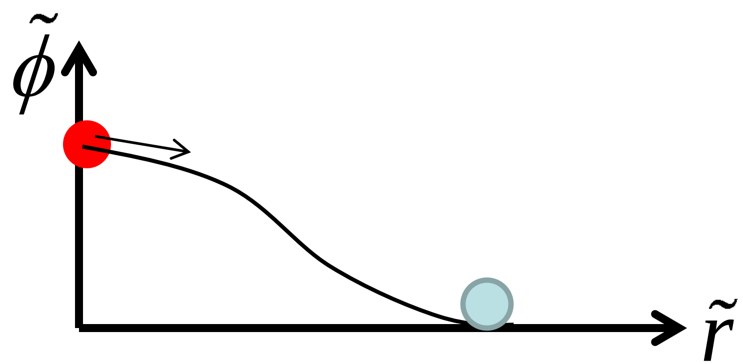
$$\frac{d^2V_\omega}{d\tilde{\phi}^2}(\tilde{\phi}=0) > 0$$



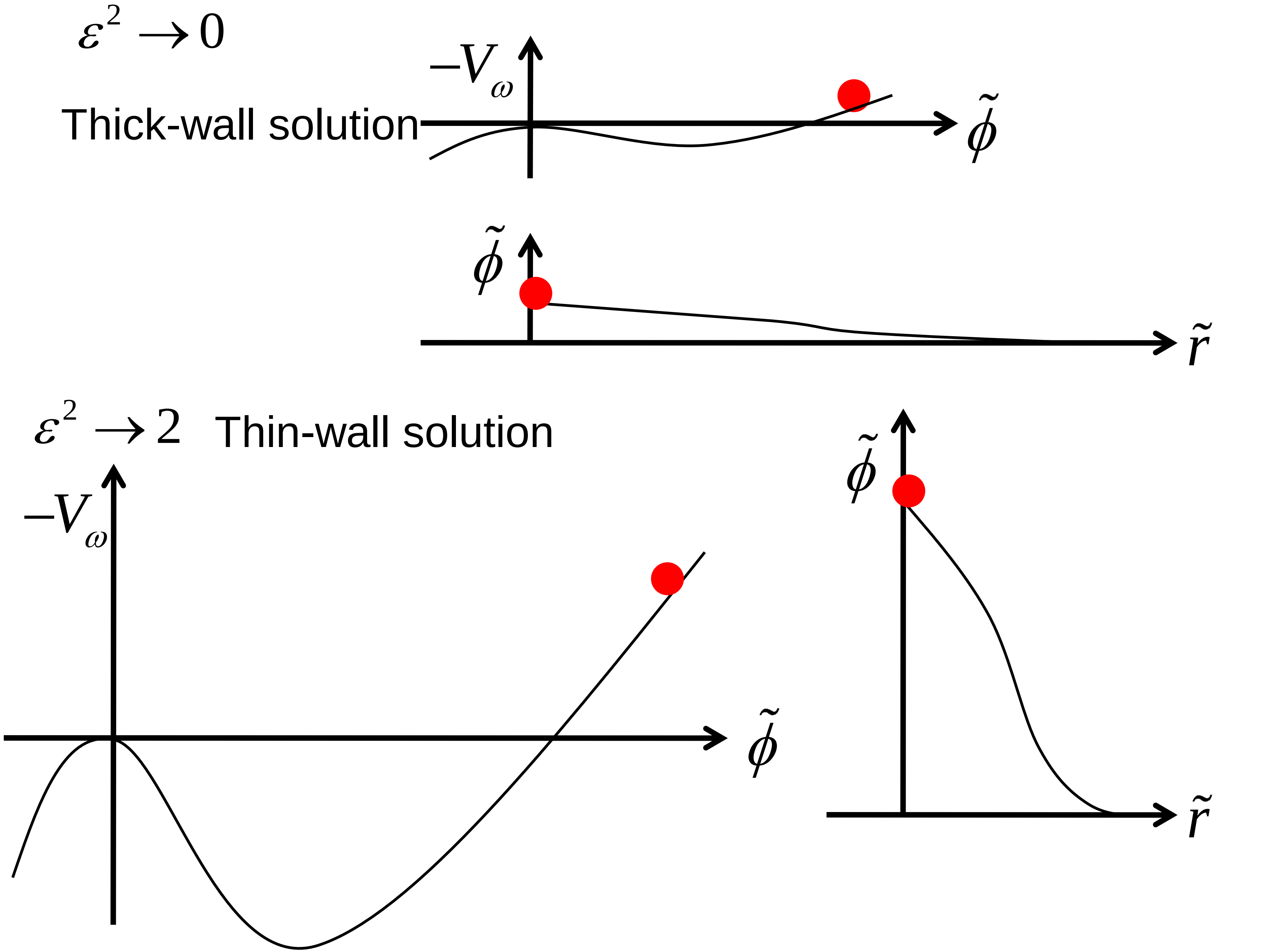
$$\varepsilon^2 := 2 - \tilde{\omega}^2$$

$$0 < \varepsilon^2$$

trivial ⇐ $\max(-V_\omega) > 0$



⇒ $0 < \varepsilon^2 < 2$



$\varepsilon^2 \to 0$
Thick-wall solution
$-V_\omega$
$\tilde{\phi}$
$\tilde{\phi}$
$\tilde{r}$
$\varepsilon^2 \to 2$
Thin-wall solution
$-V_\omega$
$\tilde{\phi}$
$\tilde{\phi}$
$\tilde{r}$

IV. gravitating case

$$ds^2 = -\alpha^2(r)dt^2 + A^2(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

$$\Rightarrow \tilde{\phi}'' = -\left(\frac{2}{\tilde{r}} + \frac{\alpha'}{\alpha} - \frac{A'}{A}\right)\tilde{\phi}' - A^2\left(\frac{\tilde{\omega}^2}{\alpha^2}\tilde{\phi} - \frac{d\tilde{V}_{\text{gauge}}}{d\tilde{\phi}}\right)$$

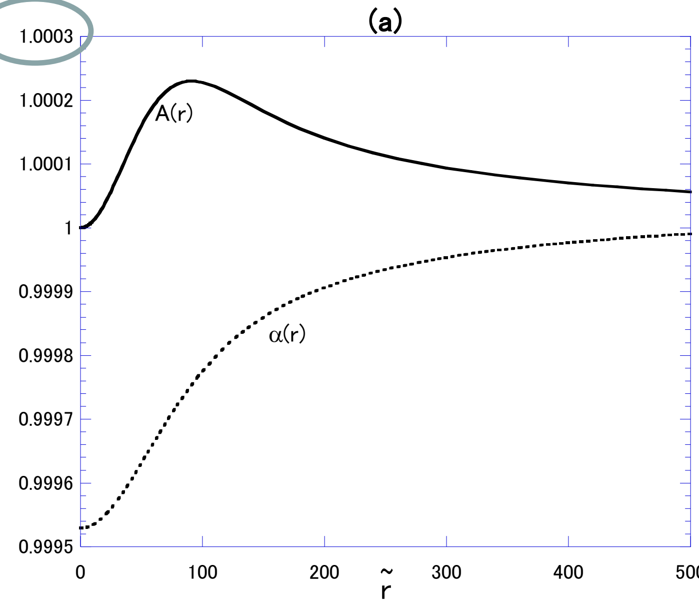
$$\varepsilon^2 := 2 - \frac{\tilde{\omega}^2}{\alpha^2}$$

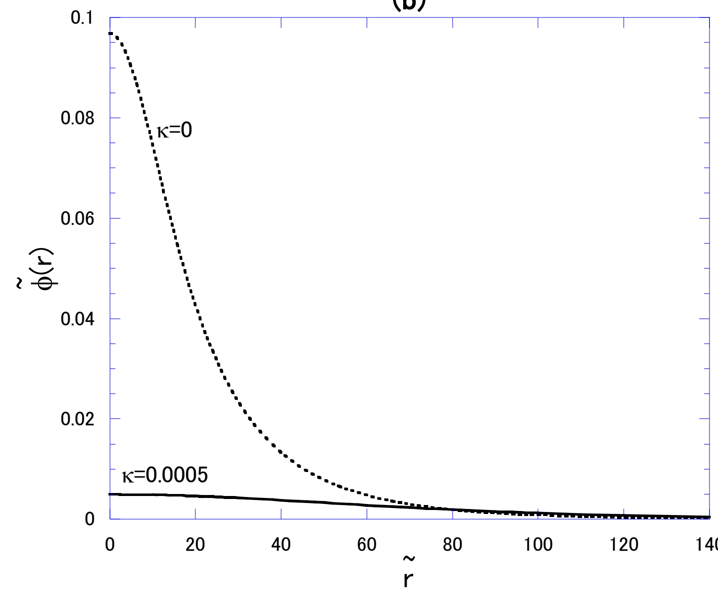
$\Rightarrow$ Particle motion in $-V_\omega$

Signature of ε^2 depends on r. → $-V_\omega$ depends on ``time”.

Thick-wall

$\varepsilon^2 = 0.001$

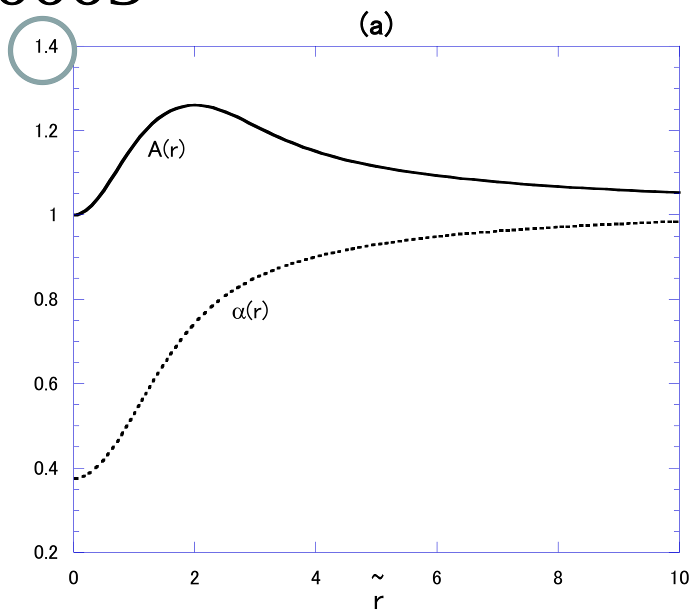


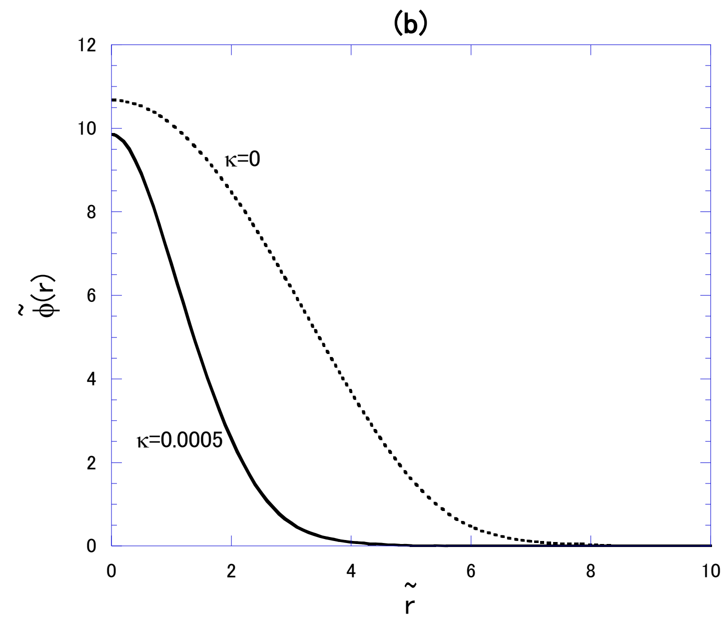


$\kappa := Gm^2 = 0.0005$

Thin-wall

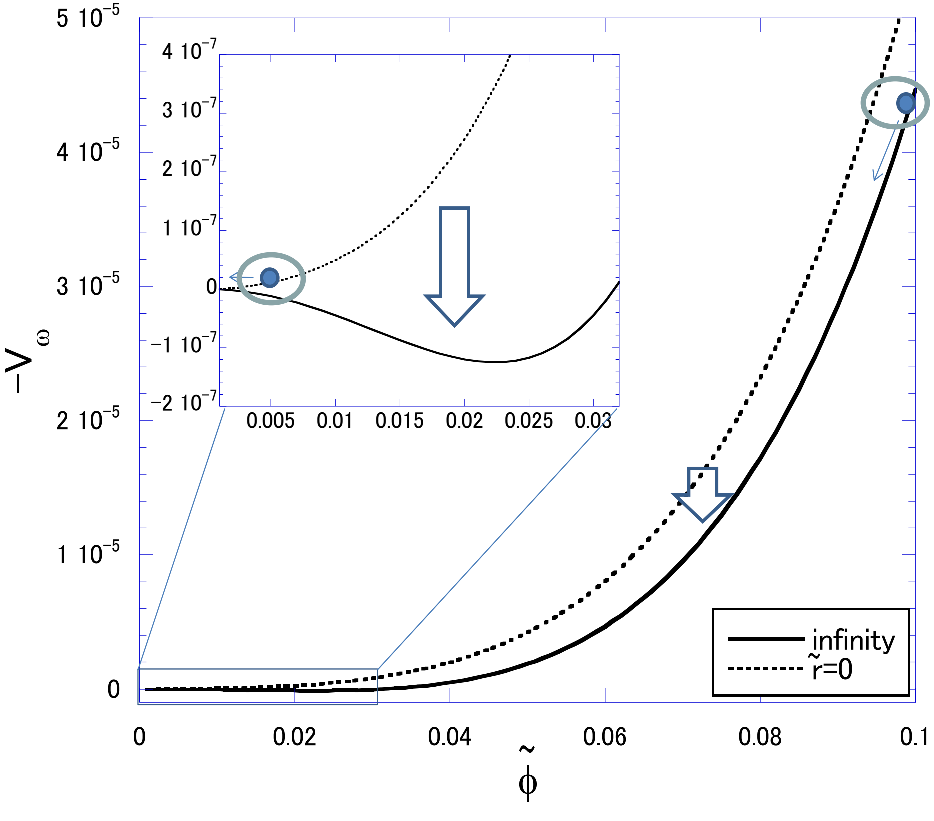
$\varepsilon^2 = 1.65$

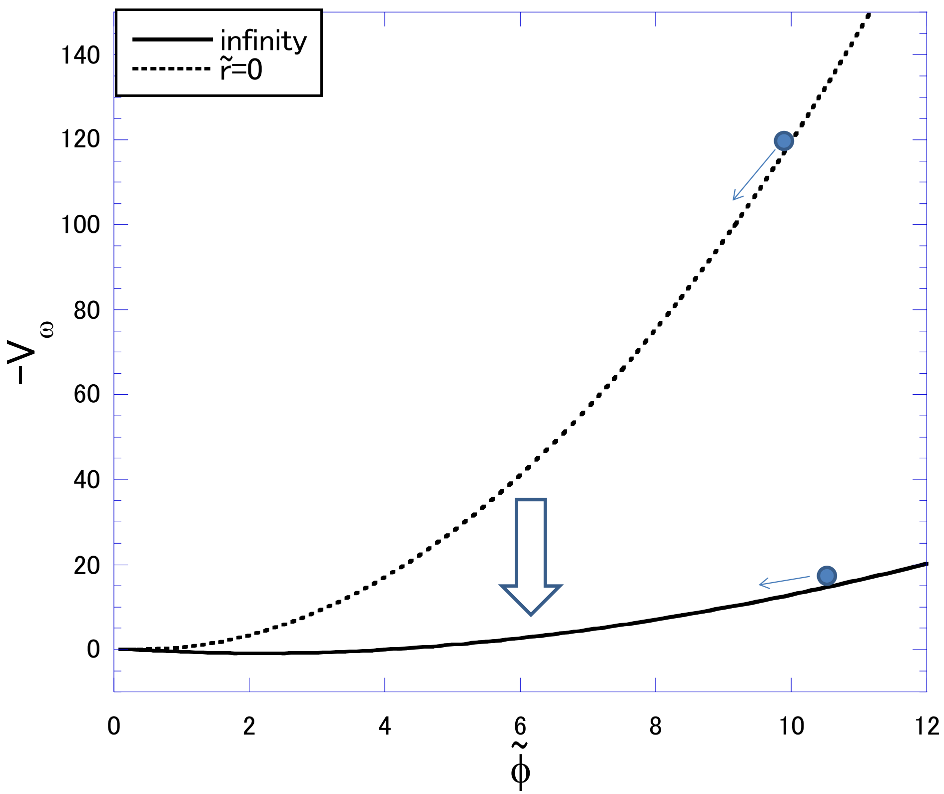




Thick-wall $\varepsilon^2 = 0.001$

Thin-wall $\varepsilon^2 = 1.65$





$-V_\omega$ changes ``quickly”.

For various ε^2 ?

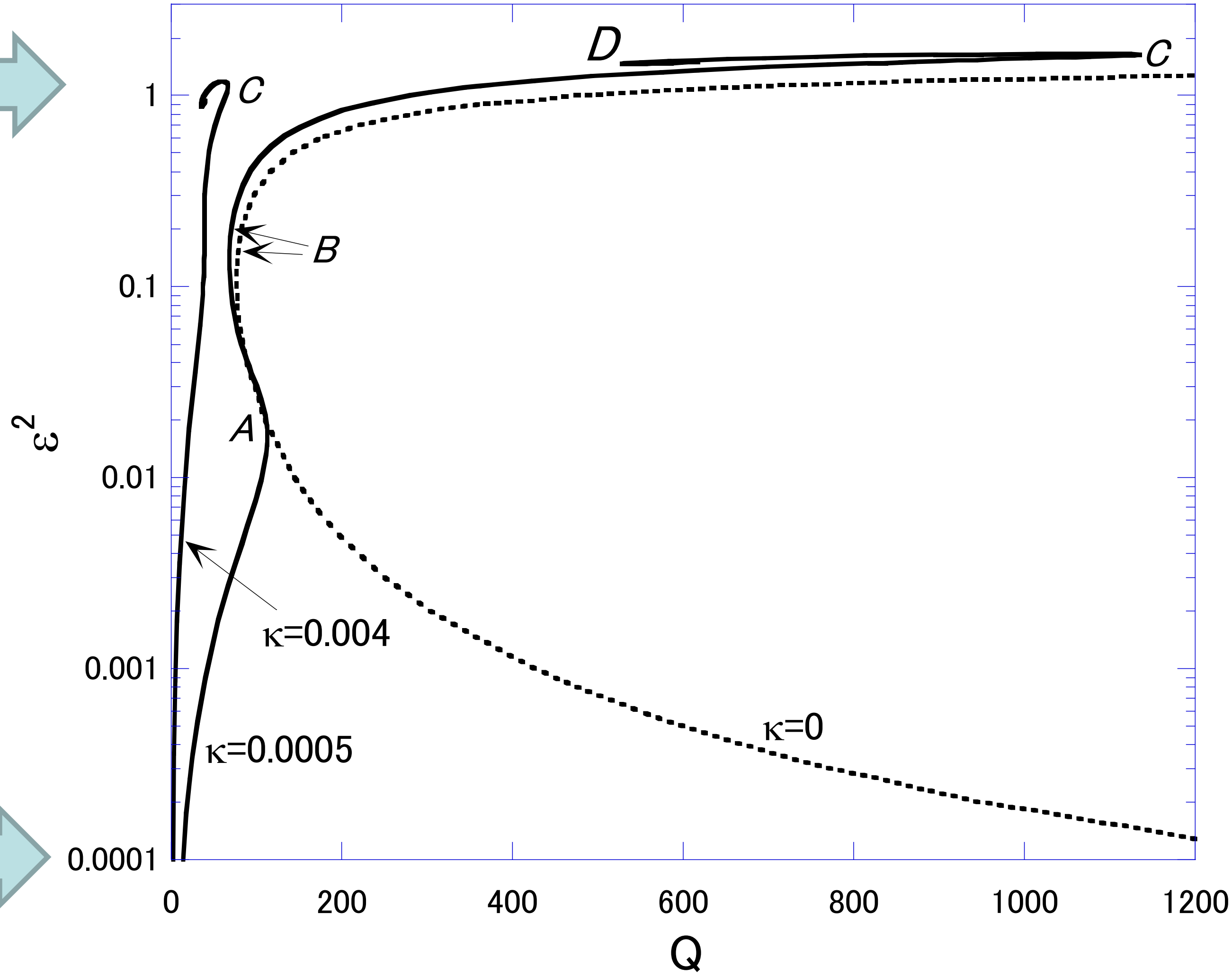
strong gravity

WHY ???

$\kappa = 0 \quad Q \to \infty$

$\kappa \neq 0 \quad Q \to 0$

weak gravity

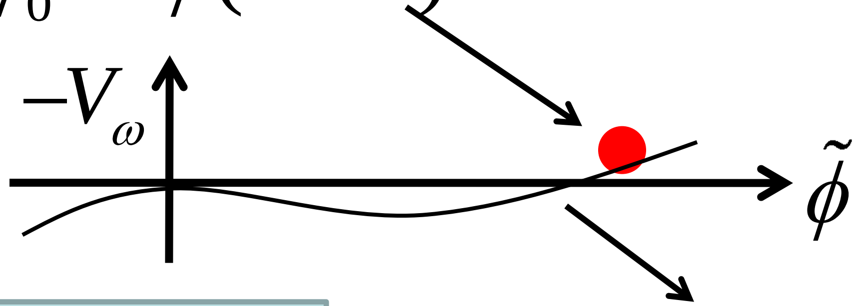


V. thick-wall case

take up to first order in h,f.

weak gravity $\alpha^2 = 1 + h(r)$ $A^2 = 1 + f(r)$

Let us evaluate $\phi_0 := \tilde{\phi}(\tilde{r} = 0)$



$$\varepsilon^2 + \tilde{\omega}^2 h(0) - {\phi_0}^2 \cong 0$$

Evaluate from $V_\omega = 0$

We used Maclaurin expansion and neglected $O({\phi_0}^5)$

For $\kappa = 0 \longrightarrow \phi_0 \cong \varepsilon$

For $\kappa \neq 0 \longrightarrow$ We should compare $h(0)$ with ε^2

$$-G_t^t + G_i^i = \left(\frac{\tilde{r}^2 \alpha'}{A}\right)' = 8\pi\kappa \tilde{r}^2 A\alpha \left(\frac{\tilde{\omega}^2 \tilde{\phi}^2}{\alpha^2} - V\right)$$

$\varepsilon^2 \to 0, i.e., \tilde{\omega}^2 \to 2$ and weak gravity

$$\left(\tilde{r}^2 h'\right)' \cong 16\pi\kappa \tilde{r}^2 \tilde{\phi}^2$$

We assume $\tilde{\phi}(\tilde{r}) \cong \phi_0 < 1$ for $\tilde{r} < \frac{C}{\varepsilon}$ C=const.

and approximating $h(\frac{C}{\varepsilon}) \cong h(\infty) = 0$

$$h(0) \cong -\frac{8}{3}\pi\kappa \phi_0^{\ 2} \frac{C^2}{\varepsilon^2}$$

substitute into $\varepsilon^2 + \tilde{\omega}^2 h(0) - \phi_0^{\ 2} \cong 0$

$$\phi_0{}^2 = \frac{3\varepsilon^4}{8\pi\kappa C^2 + 3\varepsilon^2}$$

as in the flat case

$$\varepsilon^2 > \kappa C^2 \longrightarrow \phi_0 \cong \varepsilon \quad \Rightarrow \quad Q \propto \tilde{\phi}^2 \tilde{r}^3 \cong \phi_0{}^2 \frac{1}{\varepsilon^3} \cong \frac{1}{\varepsilon}$$

$$\varepsilon^2 < \kappa C^2 \longrightarrow \phi_0 \cong \frac{\varepsilon^2}{2C}\sqrt{\frac{3}{2\pi\kappa}}$$

$$\Downarrow$$

$$Q \propto \tilde{\phi}^2 \tilde{r}^3 \cong \phi_0{}^2 \frac{1}{\varepsilon^3} \cong \varepsilon \to 0$$

V. conclusion

Gravitating Q-balls $V_{\text{gauge}}(\phi) = m^4 \ln\left(1 + \frac{\phi^2}{m^2}\right)$

➡ The thick-wall limit is completely different from the flat case.

$Q \to 0$ exists !!

➡ From the analytic estimation, our results hold **for general potentials if a positive mass term is a leading order.**

$$\phi_0{}^2 = \frac{3\varepsilon^4}{8\pi\kappa C^2 + 3\varepsilon^2}$$

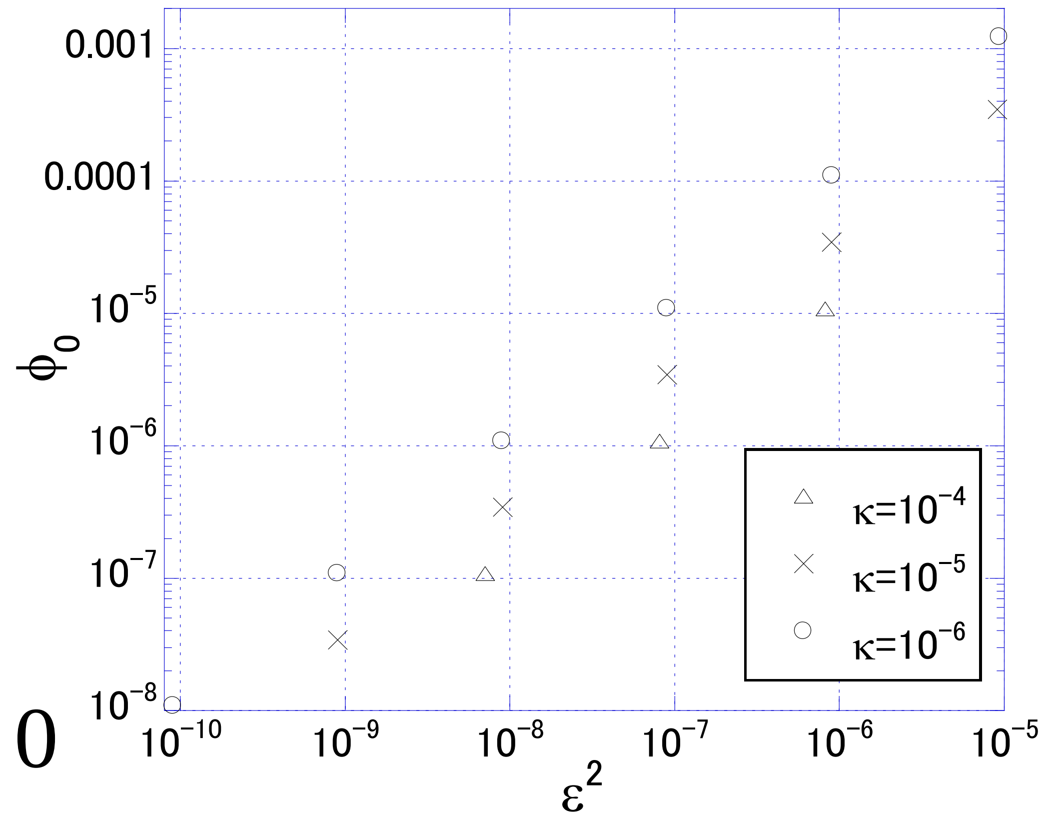
as in the flat case

$$\varepsilon^2 > \kappa C^2 \longrightarrow \phi_0 \cong \varepsilon \quad \Rightarrow \quad Q \propto \tilde{\phi}^2 \tilde{r}^3 \cong \phi_0{}^2 \frac{1}{\varepsilon^3} \cong \frac{1}{\varepsilon}$$

$$\varepsilon^2 < \kappa C^2$$

$$\phi_0 \cong \frac{\varepsilon^2}{2C}\sqrt{\frac{3}{2\pi\kappa}}$$

$$Q \propto \tilde{\phi}^2 \tilde{r}^3 \cong \phi_0{}^2 \frac{1}{\varepsilon^3} \cong \varepsilon \to 0$$



important quantities

$$Q \equiv \int d^3x \sqrt{-g} g^{\mu\nu} (\phi_1 \partial_\nu \phi_2 - \phi_2 \partial_\nu \phi_1)$$

Total energy (Hamiltonian)

$$E = \lim_{r \to \infty} \frac{r^2 \alpha'}{2GA}.$$

$$\tilde{E} := \frac{mE}{M^2}, \tilde{Q} := \frac{m^2 Q}{M^2}$$

Basic equations

$$A' + \frac{A}{2r}(A^2 - 1) = \frac{4\pi}{m_{\mathrm{Pl}}{}^2} r A^3 \left(\frac{\phi'^2}{2A^2} + \frac{\omega^2 \phi^2}{2\alpha^2} + V \right)$$

$$\alpha' + \frac{\alpha}{2r}(1 - A^2) = \frac{4\pi}{m_{\mathrm{Pl}}{}^2} r \alpha A^2 \left(\frac{\phi'^2}{2A^2} + \frac{\omega^2 \phi^2}{2\alpha^2} - V \right)$$

$$' \equiv d/dr$$

$$E \equiv \int d^3x(\mathcal{H}_{\rm G} + \mathcal{H}_\phi) = \lim_{r\to\infty} \frac{r^2\alpha'}{2GA}.$$

$$\pi^{ij} = \frac{\partial \mathcal{L}_{\rm G}}{\partial \dot{h}_{ij}}, \qquad \mathcal{H}_{\rm G} \; = \; \pi^{ij}\dot{h}_{ij} - \mathcal{L}_{\rm G}$$

$$\mathcal{P}_a = \frac{\partial \mathcal{L}_\phi}{\partial \dot{\phi}_a} = \frac{\sqrt{-g}}{\alpha^2}\dot{\phi}_a, \qquad \mathcal{H}_\phi = \mathcal{P}_a\dot{\phi}_a - \mathcal{L}_\phi$$

サイズの議論

mini-BS　1/mの半径→GM~1/m →　$M \sim \frac{M_p^2}{m}$

BS　Φ^4　の相互作用項の存在が本質的 → e.g.,

$$U(\Phi) = m^2\Phi^2(1+\frac{2\pi\Lambda}{M_p^2}\Phi^2) \xrightarrow{\Lambda大} \Phi \simeq M_{\rm Pl}/\sqrt{\Lambda}$$

$$\longrightarrow U \sim \frac{m^2 M_p^2}{\Lambda} \longrightarrow \frac{1}{m_{re}} \sim \frac{\sqrt{\Lambda}}{m}$$

GM~1/m_re ⇨ $M \sim \frac{M_p^2\sqrt{\Lambda}}{m} \sim \frac{M_p^3}{m^2}$

m(neutron) 程度→ M=M(太陽)