Gravitational waves from cosmic string - domain wall networks

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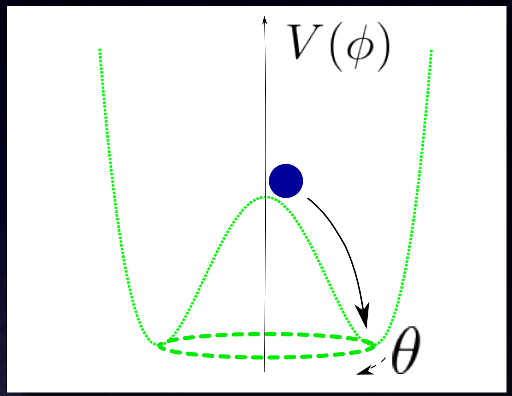
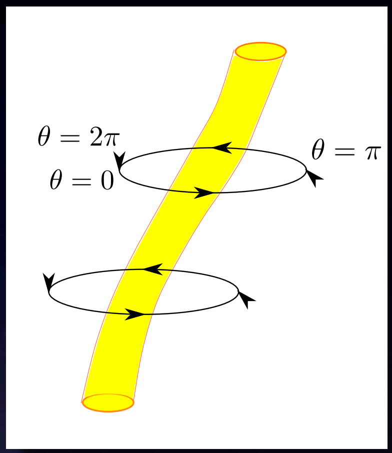
Cosmic strings and domain walls

- Cosmic string ex.) U(1) → 1

complex scalar field ϕ



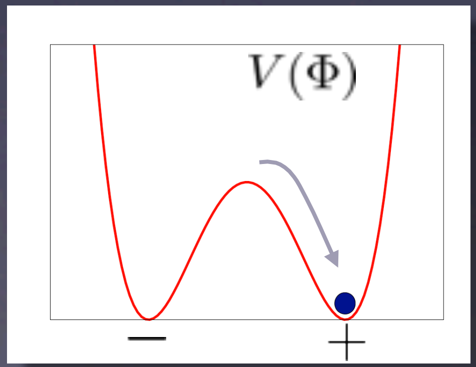
field space



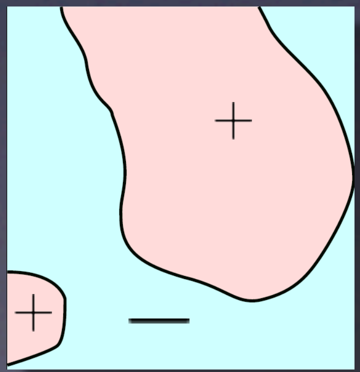
coordinate space

- Domain wall ex.) Z_2 → 1

real scalar field Φ



field space



coordinate space

2011年9月30日金曜日

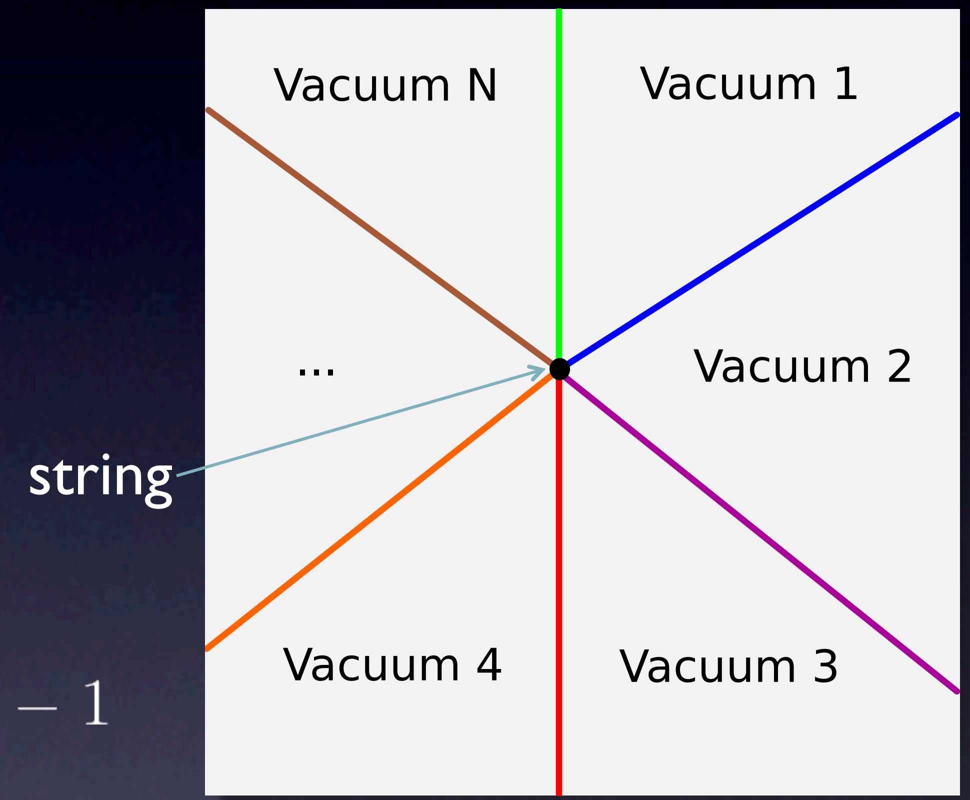
Hybrid defects

- Sequence of symmetry breaking
- ex.) U(1) → Z_N → 1

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{1}{4}(\phi^*\phi - \eta^2)^2 + (m^2\eta^2/N^2)(\cos N\theta - 1)$$

N discrete vacua at

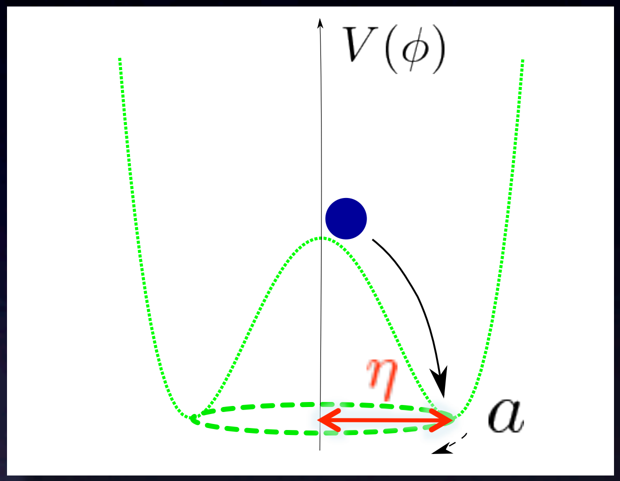
$$\phi = \eta e^{i\theta}, \quad \theta = \frac{2\pi k}{N}, \quad k = 0, 1, \ldots, N-1$$



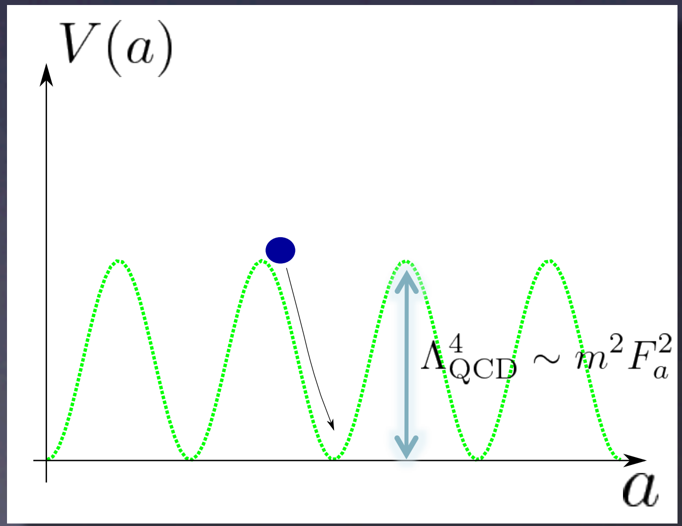
- Domain walls bounded by strings
- Naturally arise in some particle physics models (such as axions)

Axion cosmology and topological defects

- Axion : Pseudo-Nambu-Goldstone boson from spontaneous breaking of U(1)PQ
- Peccei-Quinn (PQ) field ϕ



- $T \lesssim F_a$ $F_a = \eta/N_{\rm DW} \sim 10^{10-11}{\rm GeV}$
  - Spontaneous breaking of U(1)PQ
  - Formation of cosmic strings
- $T \sim \Lambda_{\rm QCD}$ $U(1)_{\rm PQ} \rightarrow Z_{N_{\rm DW}}$
  - Axion acquires a mass (QCD effect)



- $H \lesssim m_a$ $(T \lesssim 1{\rm GeV})$
  - Spontaneous breaking of ZNDW
  - Formation of domain walls

Axionic domain wall problem

- Domain wall number N_{DW}

  $$N_{\rm DW} = {\rm Tr}[Q_{\rm PQ}(q) I(q)]$$: depend on models

- If $N_{DW} > 1$, string-wall networks are stable
  - come to overclose the universe (domain wall problem)
  - However, it can be avoided if we introduce a bias (unstable domain walls)

- Existence of string-wall networks in the early universe
  - Source of the stochastic gravitational wave backgrounds observed today

Numerical simulations

- Solve the classical field equations for ϕ on 3D lattice

$$\bar{\phi}_1'' - \nabla^2\bar{\phi}_1 = -\lambda\bar{\phi}_1(|\bar{\phi}|^2 - a^2) + \frac{a^4 m^2}{N_{\rm DW}|\bar{\phi}|}\sin\theta\sin N_{\rm DW}\theta$$

$$\bar{\phi}_2'' - \nabla^2\bar{\phi}_2 = -\lambda\bar{\phi}_2(|\bar{\phi}|^2 - a^2) + \frac{a^4 m^2}{N_{\rm DW}|\bar{\phi}|}\cos\theta\sin N_{\rm DW}\theta$$

$$\phi = \phi_1 + i\phi_2$$

$$\phi \equiv \bar{\phi}/a \qquad \phi' = \frac{d\phi}{d\tau}$$

$$d\tau = dt/a$$

(τ : conformal time)

- Input parameters

Scheme	Leap frog
Number of grids	256×256×256
Era	Radiation dom.
Initial time	2 (in unit of η^{-1})
Final time	25
Time resolution	0.01
Box size	60
λ	0.1
m	0.1

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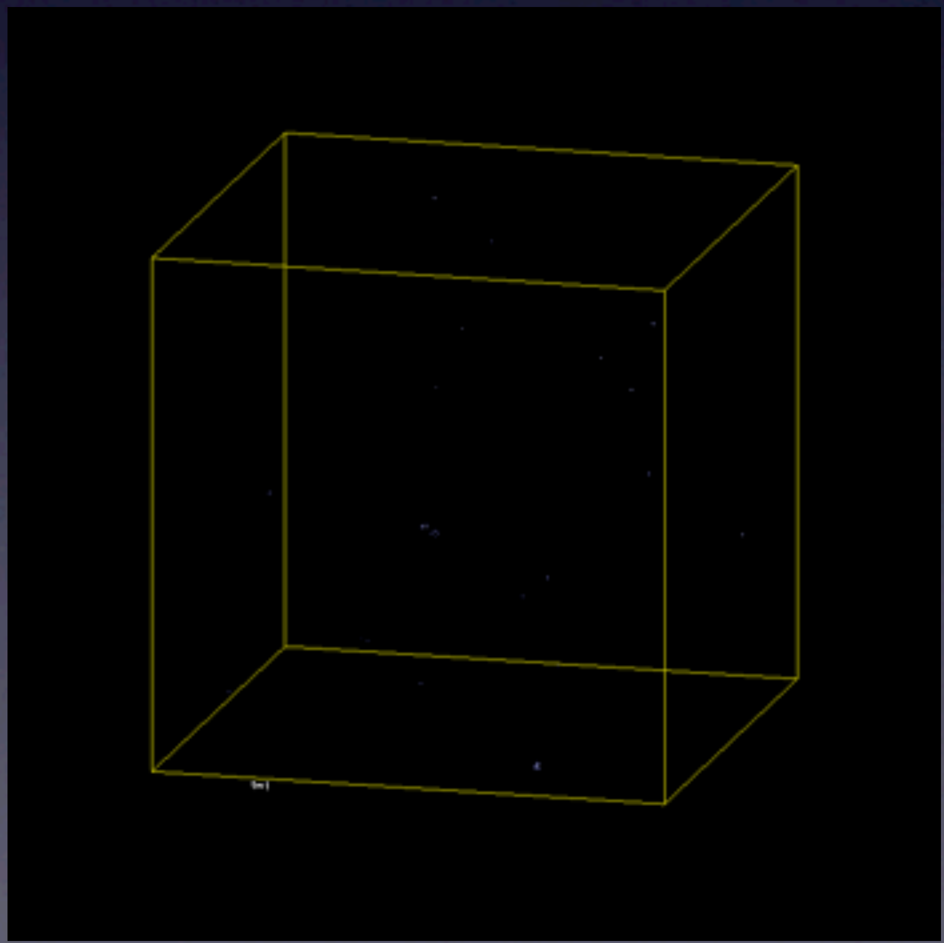

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Calculation of GW spectrum

- Linearized theory

$$ds^2 = -dt^2 + a^2(\delta_{ij} + h_{ij})dx^i dx^j$$

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2}{a^2}h_{ij} = \frac{16\pi G}{a^2}T_{ij}^{TT} \qquad \rho_{\rm gw}(t) = \frac{1}{32\pi G}\langle \dot{h}_{ij}(t,\mathbf{x})\dot{h}_{ij}(t,\mathbf{x})\rangle_V$$

- "Green function" method J. Dufaux, et al., PRD76, 123517 (2007)

$$\Omega_{\rm gw}(k,\tau) \equiv \frac{d\rho_{\rm gw}/d\ln k}{\rho_c(\tau)} = \frac{4}{3\pi V}\frac{G^2}{a^4H^2}S_k(\tau)$$

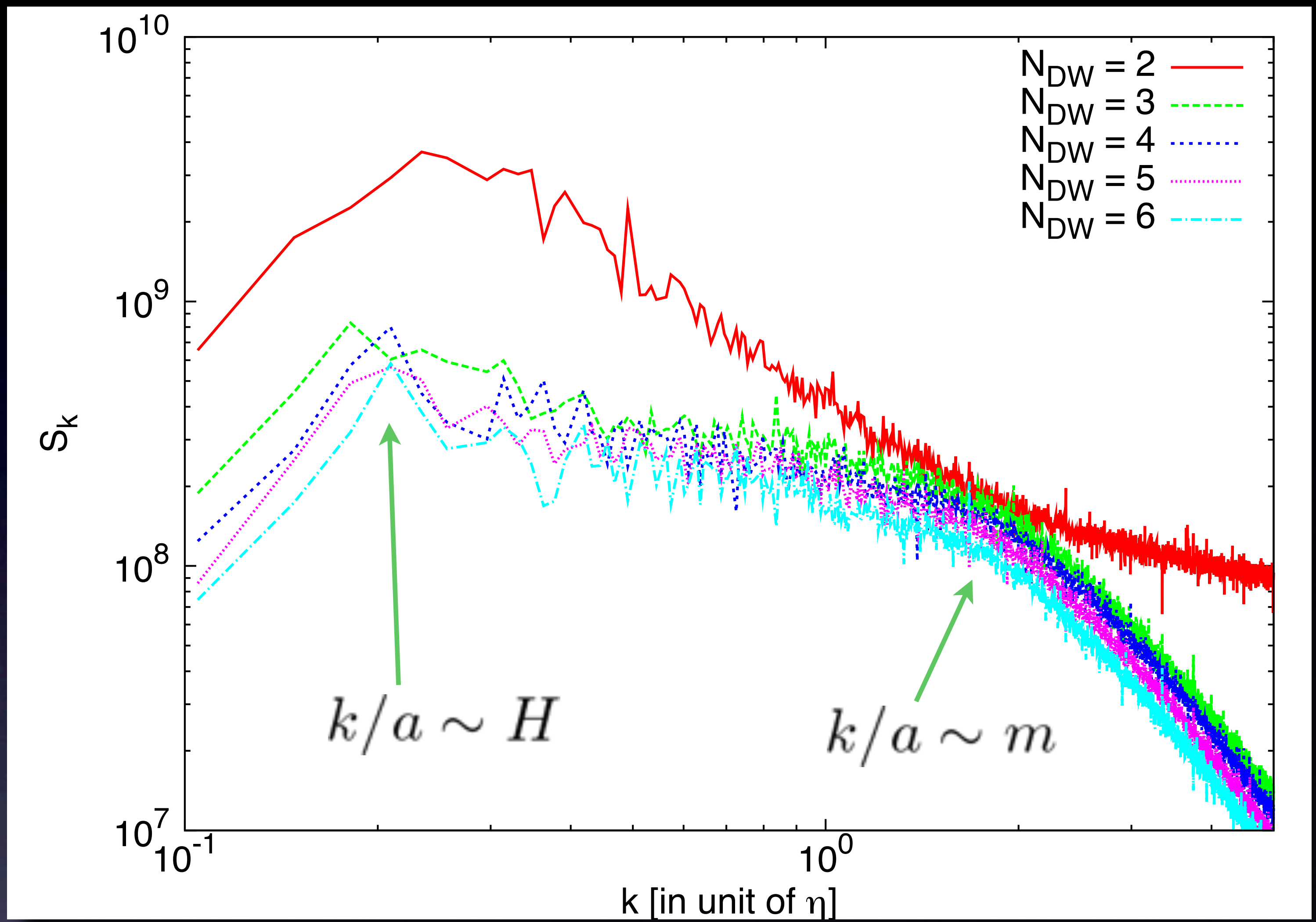
$$S_k(\tau) \equiv \int d\Omega_k \sum_{ij}\left(\left|C_{ij}^{(1)}\right|^2 + \left|C_{ij}^{(2)}\right|^2\right) \qquad \rho_c(\tau) = \frac{3H^2}{8\pi G}$$

$$C_{ij}^{(1)} = -\int_{\tau_i}^{\tau} k d\tau' \sin(k\tau')a(\tau')T_{ij}^{TT}(\mathbf{k},\tau')$$

$$C_{ij}^{(2)} = \int_{\tau_i}^{\tau} k d\tau' \cos(k\tau')a(\tau')T_{ij}^{TT}(\mathbf{k},\tau')$$

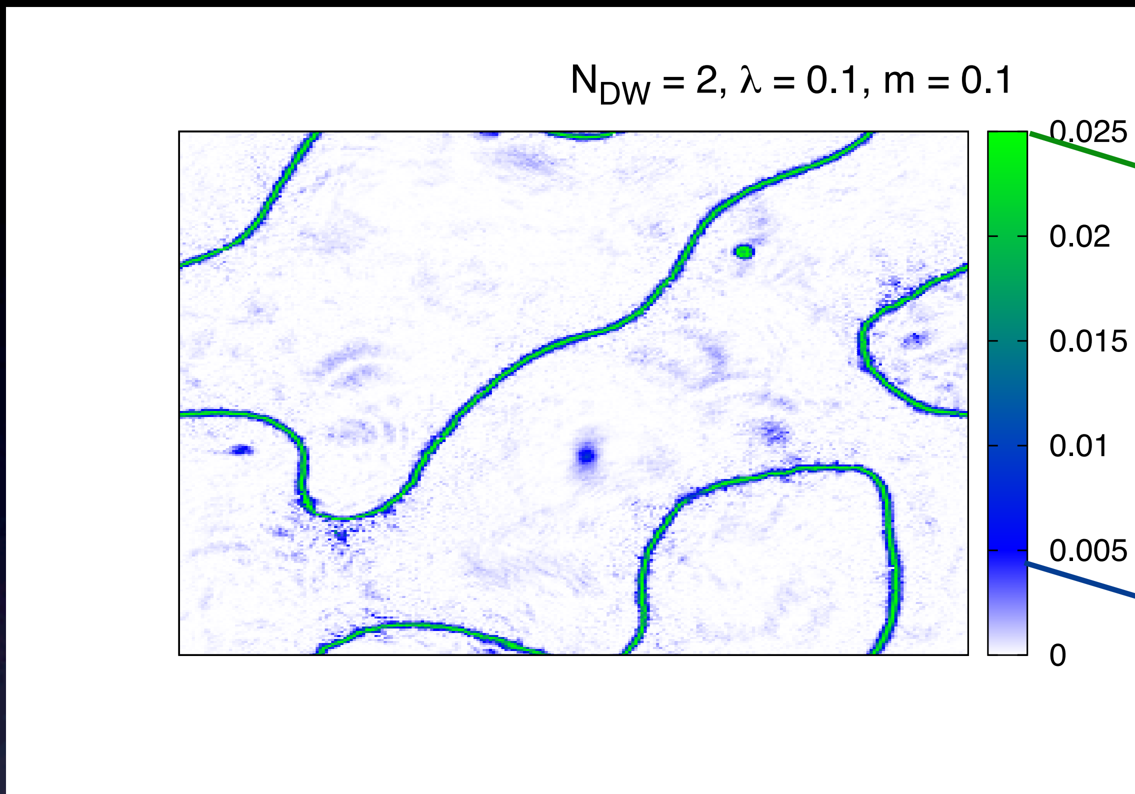
$$T_{ij}^{TT} \sim \nabla_i\phi\nabla_j\phi$$

GW spectrum



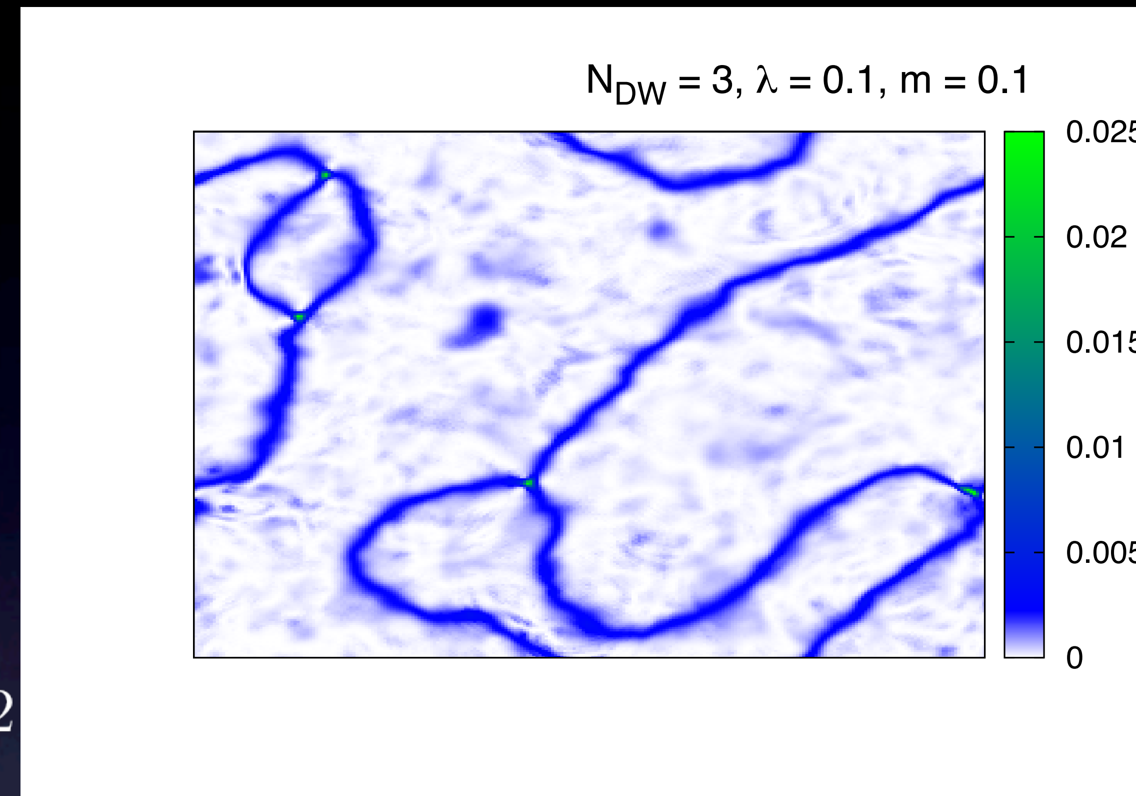
- $N_{DW}>2$: spectrum similar to that of simple Z_2 model with real scalar field see M. Kawasaki and KS, JCAP09(2011)008
  - Nearly flat spectrum extends between two relevant scales (Hubble radius and wall width)
- $N_{DW} = 2$: different from others

Field configuration in $N_{DW} = 2$



$V_{\rm string} \simeq \lambda\eta^4/4$

$V_{\rm wall} \simeq m^2\eta^2/2$



- $V_{\rm string}$ distributs like a wall rather than string
- Width is determined by the size of strings $\delta_{\rm string} \sim (\sqrt{\lambda}\eta)^{-1} \ll \delta_{\rm wall} \sim m^{-1}$

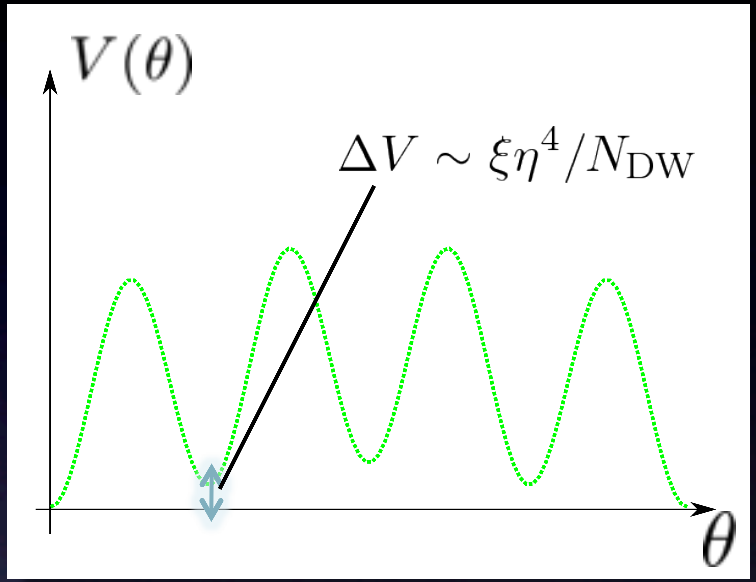
  → spectrum extends to higher frequency

- Produced only if $N_{\rm DW} = 2$ and $\dfrac{V_{\rm string}}{V_{\rm wall}} \simeq \dfrac{\lambda\eta^4/4}{m^2\eta^2/2} \lesssim 10$

  (axion model : $V_{\rm string}/V_{\rm wall} \sim (F_a/\Lambda_{\rm QCD})^4 \sim 10^{48}$)

Termination of GW production

- GW production is terminated when string-wall networks collapse



- The collapse is caused by a bias
  - Lifts degenerate vacua
  - Phenomenologically parameterized as

P. Sikivie, PRL48, 1156 (1982)

$$V(\theta) \to V(\theta) + \delta V, \quad \delta V = -\xi\eta^3(\phi + {\rm h.c.})$$

  - Affects as a pressure on walls → annihilate them
  - Life time of the networks

$$t_{\rm dec} \sim \sigma/\Delta V \sim m/N_{\rm DW}\xi\eta^2$$

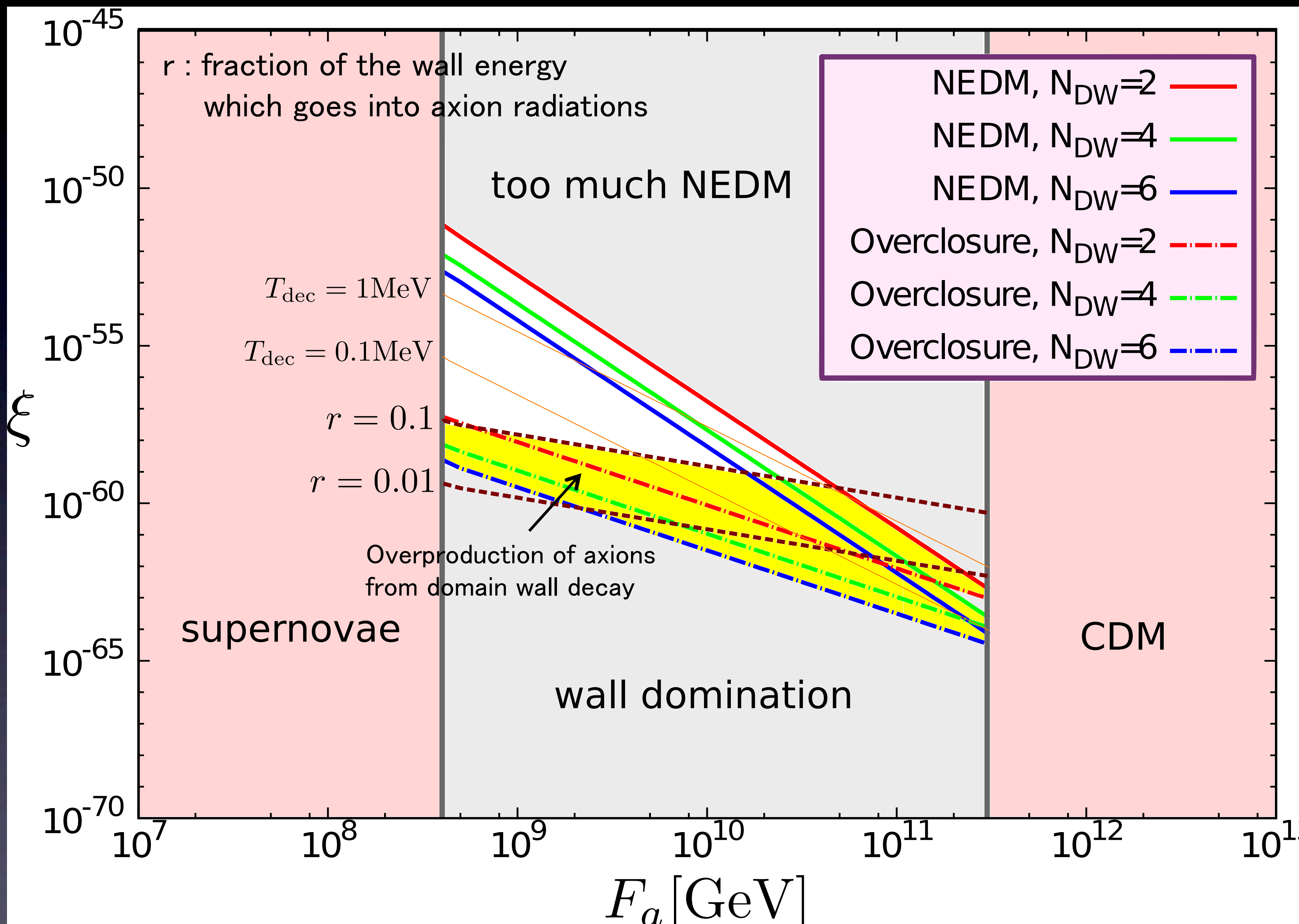
$\sigma \sim m\eta^2/N_{\rm DW}^2$: the surface energy density of domain walls

- The present GW energy density is obtained as

$$\rho_{\rm gw}(t_0) = (a(t_{\rm dec})/a(t_0))^4 \rho_{\rm gw}(t_{\rm dec}) \sim 10^{-5}(t_{\rm dec}/t_0)^2 G\sigma^2$$

Constraints for bias parameter

T. Hiramatsu, M. Kawasaki and KS, JCAP08(2011)030

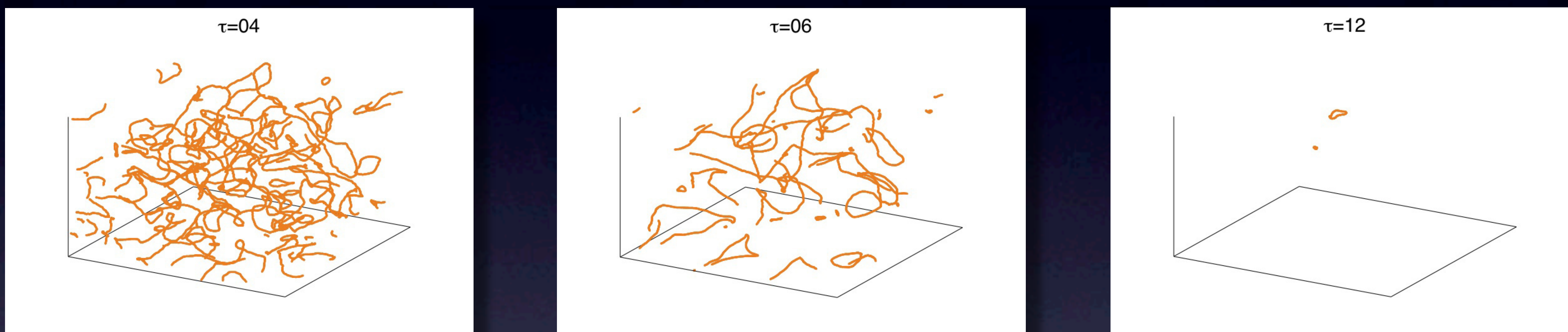


The white region is still allowed

$$\xi \sim 10^{-58} \quad \rightarrow \quad (\Omega_{gw})_{today} \sim 10^{-12}$$

$N_{DW} = 1$

- $N_{DW} = 1$; walls quickly disappear (no domain wall problem)



- Strings decay due to the domain wall tension
- Axions produced by the decay contribute to the cold dark matter (CDM) component of the universe
- Estimation of CDM abundance [work in progress]

Summary

- Calculate GW spectrum produced by string-domain wall networks
  - Based on 3D lattice simulations
  - Spectrum is similar to that calculated in simple model in which discrete Z_2 symmetry is spontaneously broken (except the model with $N_{DW}=2$)
- For axion models, the signal can be observed in future GW interferometers
- Estimation of axion CDM abundance produced by the decay of networks is left as future works

Appendix

Initial Conditions

- Treat ϕ_1 and ϕ_2 as two independent real scalar fields with correlation function

$$\langle \phi_i(\mathbf{k})\phi_i(\mathbf{k}')\rangle = \frac{1}{2k}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \qquad \phi = \phi_1 + i\phi_2$$

$$\langle \dot{\phi}_i(\mathbf{k})\dot{\phi}_i(\mathbf{k}')\rangle = \frac{k}{2}(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}') \qquad (i = 1,2)$$

- No correlation in the k space
  - Generate $\phi_i(\mathbf{k})$ as Gaussian with

$$\langle |\dot{\phi}(\mathbf{k})|^2\rangle = \frac{k}{2}V_b \qquad \langle |\phi(\mathbf{k})|^2\rangle = \frac{1}{2k}V_b$$

$$\langle \phi(\mathbf{k})\rangle = \langle \dot{\phi}(\mathbf{k})\rangle = 0$$

$V_b \simeq (2\pi)^3\delta^{(3)}(0)$

: volume of the simulation box

  - Fourier transform to obtain $\phi_i(\mathbf{x})$ and $\dot{\phi}_i(\mathbf{x})$

Comments on the numerical study

- One must consider three extremely different length scales
  - Core of the string

    $$\delta_s \sim 1/\sqrt{\lambda}\eta \sim \text{const.} > \text{lattice spacing} \sim a(t)$$

  - Width of the wall

    $$\delta_w \sim m^{-1} \sim \text{const.} > \text{lattice spacing} \sim a(t)$$

  - Hubble radius

    $$H^{-1} \sim t < \text{simulation box}$$

- At the final time of the simulation, the core of the string is marginally resolvable

$$\frac{H^{-1}}{a(\tau)\delta x} = \frac{N\tau}{L} \simeq 107, \quad \frac{\delta_w}{a(\tau)\delta x} = \frac{N}{Lm}\left(\frac{\tau_i}{\tau}\right) \simeq 3.42, \quad \frac{\delta_s}{a(\tau)\delta x} = \frac{N}{L\lambda^{1/2}}\left(\frac{\tau_i}{\tau}\right) \simeq 1.08$$

at $\tau = \tau_f = 25\eta^{-1}$

$\delta x = L/N$: lattice spacing

Effect on Big Bang Nucleosynthesis (BBN)

- Domain walls dominates the energy density of the universe at the temperature

$$T \simeq 8 \times 10^{-2} \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)^{1/2} \mathrm{MeV}$$

➡ The wall domination occurs after the BBN epoch

- During the BBN epoch, domain walls contributes as an extra particle d.o.f.

$$\rho_{\mathrm{extra}}(t_{\mathrm{BBN}}) = \frac{\pi^2}{30}\frac{7}{8}(N_\nu - 3)T_{\mathrm{BBN}}^4 = \rho_{\mathrm{wall}}(t_{\mathrm{BBN}}) = \sigma H_{\mathrm{BBN}}$$

- Observations indicate $N_\nu \lesssim 4$

- However, the contribution from domain walls is negligible

$$N_\nu - 3 = 8.4 \times 10^{-2} \times \left(\frac{F_a}{10^{12}\mathrm{GeV}}\right)$$

Magnitude of GW

- Suppose that gravitational waves are generated at some length scale R_*
- The energy of gravitational waves

$$E_{\rm gw} \sim GM_{\rm DW}^2/R_* \sim G\sigma^2 R_*^3$$

where $M_{\rm DW} \sim \sigma R_*^2$: the mass energy of domain walls

$\sigma \sim m\eta^2/N_{\rm DW}^2$: the surface energy density of domain walls

- The energy density of gravitational wave

$$\rho_{\rm gw} \sim E_{\rm gw}/R_*^3 \sim G\sigma^2$$

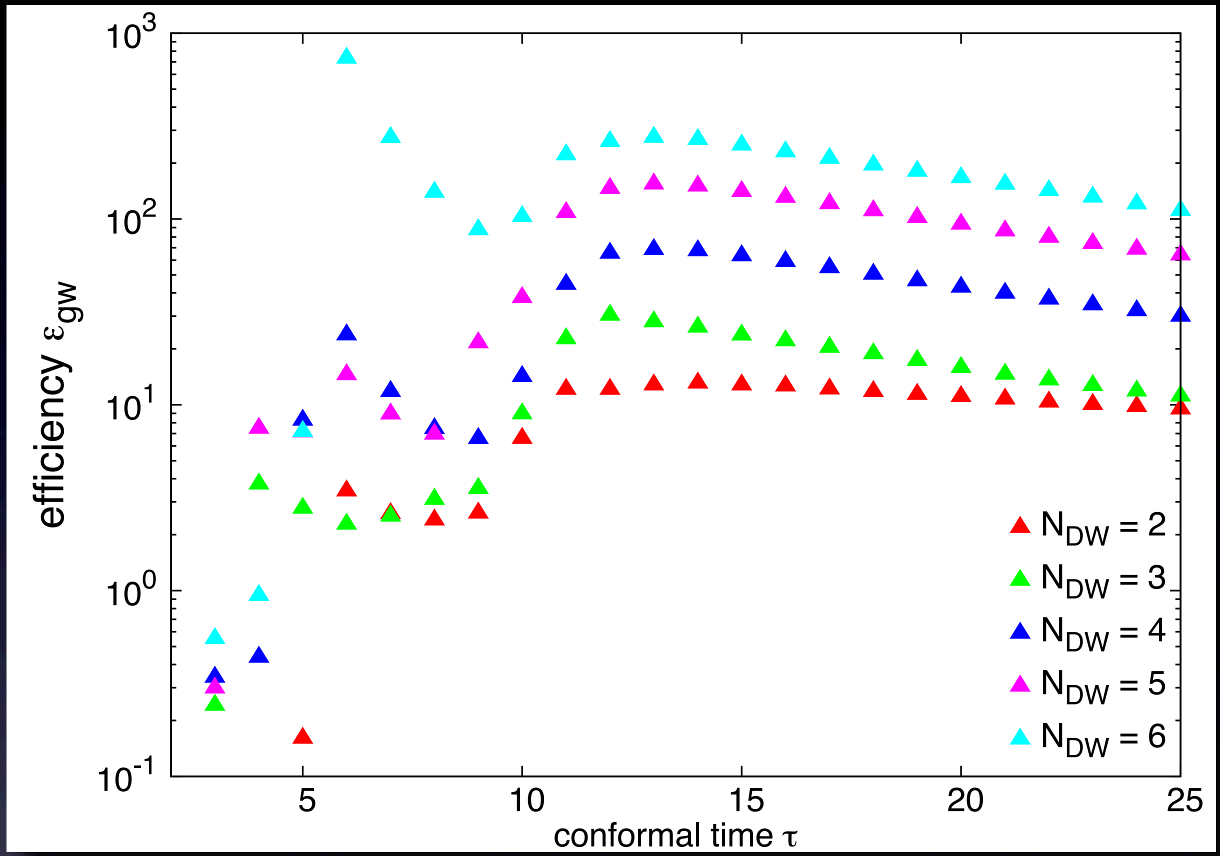
does not depend on the scale R_*

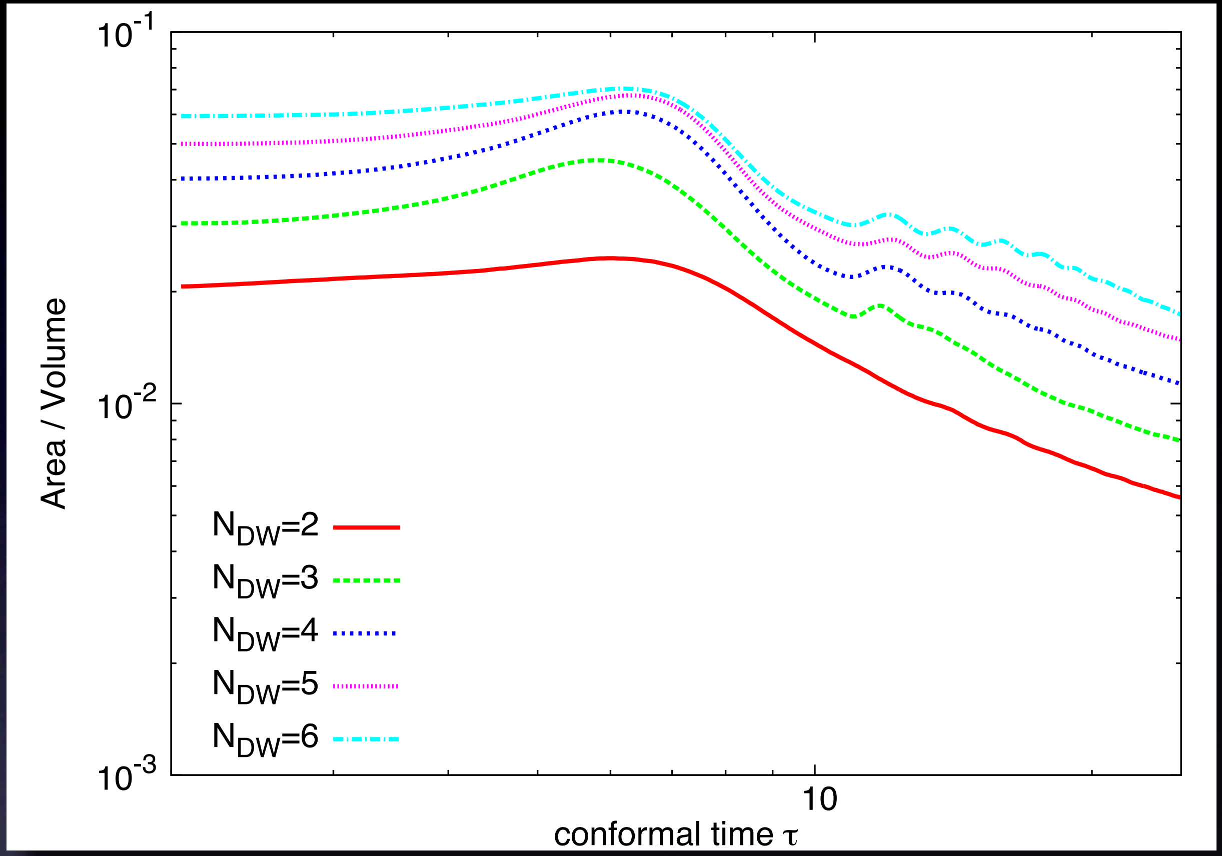
- define the efficiency parameter

$$\rho_{\rm gw}^{\rm (sim)} \equiv \epsilon_{\rm gw} G\sigma^2$$

$\rho_{\rm gw}^{\rm (sim)}$: energy density of gravitational waves calculated in numerical simulations

Efficiency and area density





- $\epsilon_{\rm gw} \sim \mathcal{O}(10-100)$
- Scaling solution : $(\text{comoving})\ \text{Area}/\text{Volume} \propto \tau^{-1}$
- The amplitude of GWs is enhanced if N_{DW} is large

Observable?

- Emission of GWs is terminated at

Intensity

$$\Omega_{\rm gw}h^2 \equiv \frac{1}{\rho_c(t_0)}\frac{d\rho_{\rm gw}(t_0)}{d\log f} \sim 10^{-5}\frac{\rho_{\rm gw}(t_*)}{\rho_c(t_*)}$$

$$\sim 5\times 10^{-12}\times\left(\frac{4}{N_{\rm DW}}\right)^6\left(\frac{10^{-58}}{\xi}\right)^2\left(\frac{10^{10}{\rm GeV}}{F_a}\right)^4$$

Spectrum extends from

$$f = \frac{a(t_*)}{a(t_0)}H_* \sim 2\times 10^{-11}\times\left(\frac{N_{\rm DW}}{4}\right)^{3/2}\left(\frac{\xi}{10^{-58}}\right)^{1/2}\left(\frac{F_a}{10^{10}{\rm GeV}}\right)^{3/2}{\rm Hz}$$

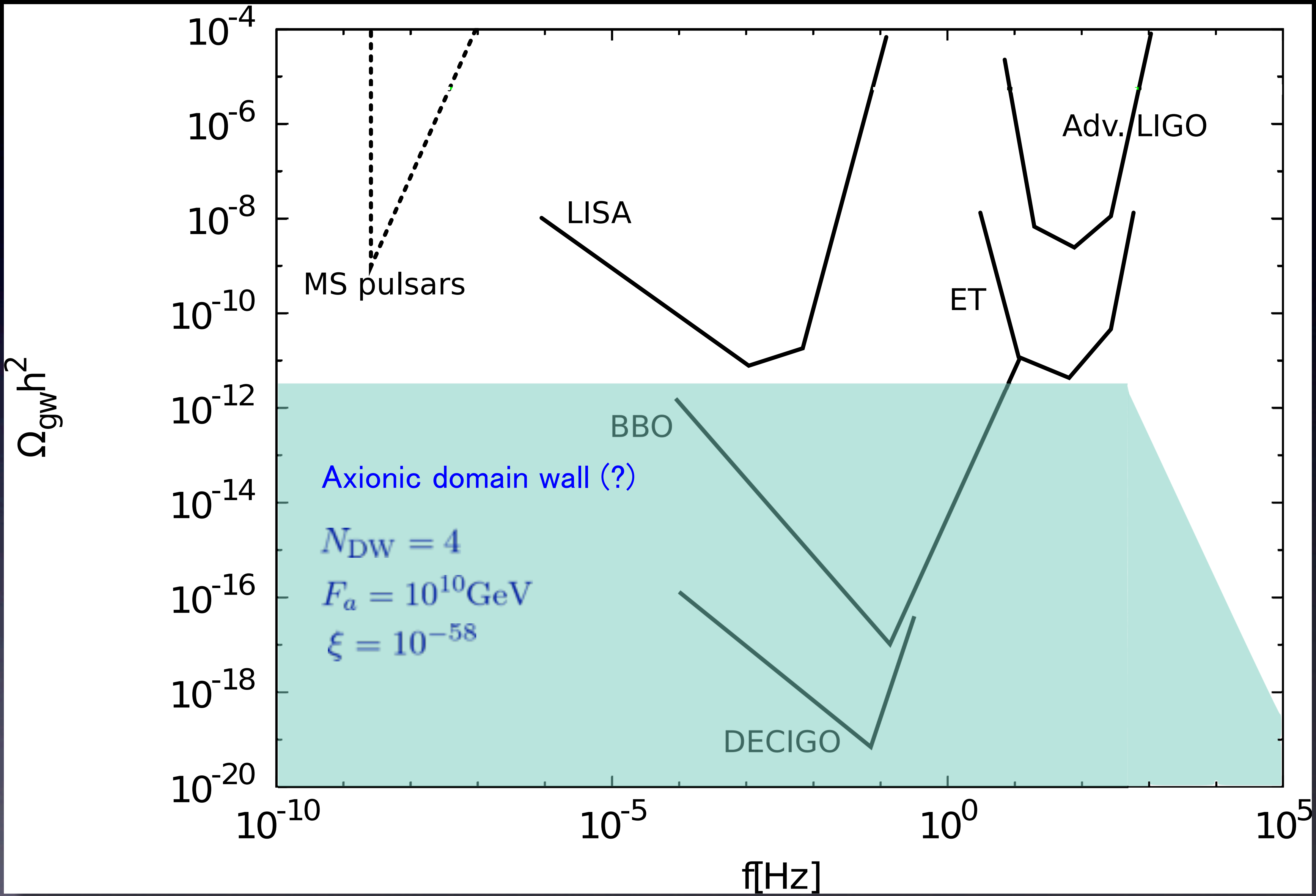
to

$$f = \frac{a(t_*)}{a(t_0)}m \sim 6\times 10^{2}\times\left(\frac{4}{N_{\rm DW}}\right)^{3/2}\left(\frac{10^{-58}}{\xi}\right)^{1/2}\left(\frac{10^{10}{\rm GeV}}{F_a}\right)^{5/2}{\rm Hz}$$

cf. DECIGO $\Omega_{\rm gw}h^2 \sim 10^{-20}$ at $f\sim 10^{-1}{\rm Hz}$

- Future experiments can detect signals → probe axion models

Schematics



$\Omega_{gw}h^2$
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
10^{-10}
10^{-5}
10^{0}
10^{5}
f[Hz]
MS pulsars
LISA
Adv. LIGO
ET
BBO
DECIGO
Axionic domain wall (?)
$N_{DW} = 4$
$F_a = 10^{10}$GeV
$\xi = 10^{-58}$

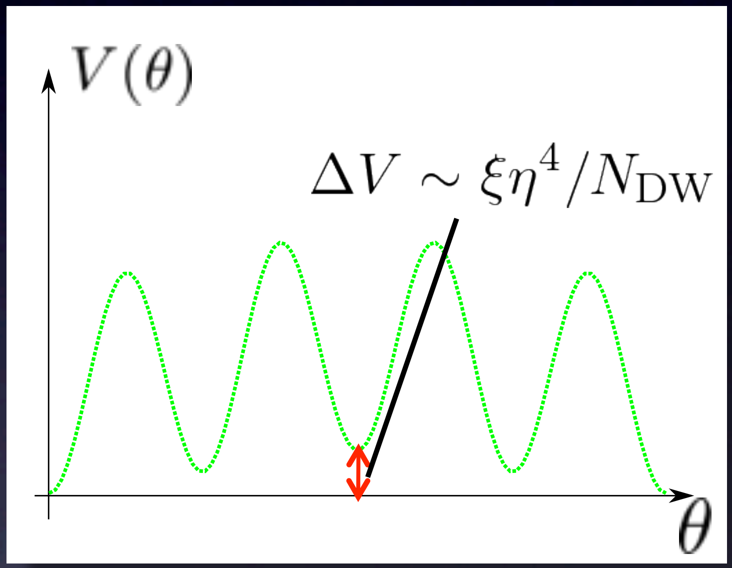
Bias

- $\xi \neq 0$
- Two forces acting on domain walls
  - Tension (straightens the wall)

$$p_T \sim \sigma/R \sim m\eta^2/N_{\rm DW}^2 R$$

  - Pressure (collapses the wall)

$$p_V \sim \Delta V \sim \xi\eta^4/N_{\rm DW}$$



- Pressure dominates when

$$p_T \sim p_V \quad \rightarrow \quad R \sim m/N_{\rm DW}\xi\eta^2$$

  - Decay time of walls

$$t_{\rm dec} \sim R \sim m/N_{\rm DW}\xi\eta^2$$

Bounds for ξ

- Neutron Electric Dipole Moment (NEDM)
  - The bias term δV spoils the PQ solution to the strong CP problem
  - Shifts the θ value from zero $\quad \theta \sim \frac{\xi\eta^2}{m^2} < 10^{-10}$
  - Observation of NEDM ➡ Upper bound for ξ
- Overclosure bound
  - Wall dominates when

$$\rho_{\rm wall} \simeq \sigma/t \sim \rho_c \simeq 1/Gt^2$$

$$\Rightarrow \quad t_{\rm WD} \sim 1/G\sigma$$

  - Require $t_{\rm dec} < t_{\rm WD}$ ➡ Lower bound for ξ

Cold axions from domain walls

- Decay of domain walls ➡ production of axions
  - The fraction r of the wall energy goes into axion radiations $\rho_a(t_{\rm dec}) = r\rho_{\rm wall}(t_{\rm dec})$
  - Radiated axions are barely relativistic with Lorents factor $\gamma \simeq 60$ S. Chang, C. Hagmann and P. Sikivie, PRD59, 023505 (1999)

    ➡ become CDM component of the universe
- Abundance of cold axions from domain walls at the time of equality between matter and radiation

$$\Omega_a(t_{\rm eq}) \equiv \frac{\rho_a(t_{\rm dec})}{\rho_c(t_{\rm eq})} \approx 3\times10^{-29}\times r\xi^{-1/2}N_{\rm DW}^{-3/2}\left(\frac{60}{\gamma}\right)\left(\frac{0.15}{\Omega_M h^2}\right)\left(\frac{10^{12}{\rm GeV}}{F_a}\right)^{1/2} < \frac{1}{2}$$

  - Another lower bound on ξ
  - r must be small