Generation of the primordial magnetic fields from the non-adiabatic fluctuations at the pre-recombination era

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Reference: 1109.0691[astro-ph.CO]

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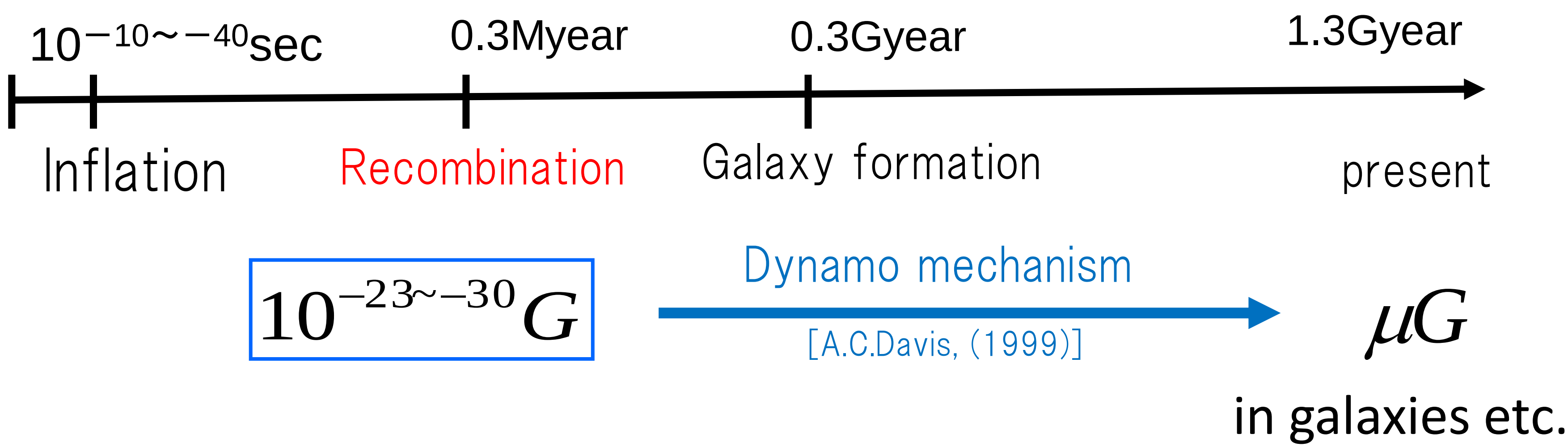
INTRODUCTION

Evolution of magnetic fields



Scenario of the magnetic field

→Weak seed fields are generated in the early universe

→Amplified by the dynamo after galaxy formation

Generation mechanism of seed fields

→ Second-order cosmological perturbations at pre-recombination era

Comparison to previous works

- Kobayashi+(2007), Maeda+(2009)

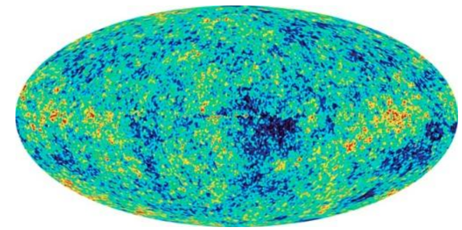
 They assume that the adiabatic perturbations only exist initially.

→They show that the magnetogenesis does not appear on the 1st-order tight coupling approximation.

➡ If the initial non-adiabatic perturbations exist, we show the generation of the magnetic fields on the 1st-order tight coupling approximation and evaluate the power spectrum.

＊The non-adiabatic fluctuations called here is the same as the isocurvature fluctuations called usually.

Constraints of the non-adiabatic fluctuations



Adiabatic fluctuations are main component in CMB anisotropic fluctuations.
However, the non-adiabatic fluctuations can still exist as subdominant component.

➡ The non-adiabatic fluctuations are constrained by CMB observations etc. (Komatsu+(2011), Sollom+(2009) and so on)

$$\frac{P_{na}}{P_a} \equiv \frac{A}{1-A}\left(\frac{k}{k_c}\right)^{n_2-1}$$

➡

$$A \lesssim 0.1 - 0.01$$
$$n_2 \simeq 2 - 4$$

A: the ratio of CDM non-adiabatic fluctuations to the sum of adiabatic and non-adiabatic fluctuations.

MAGNETIC GENERATION FROM NON-ADIABATIC PERTURBATIONS

Basic equations

- The EoMs of the photon and baryon

Photon: $\nabla_\nu T_{(\gamma)}{}_i{}^\nu = \kappa_i^{\gamma p} + \kappa_i^{\gamma e}$

Proton: $\nabla_\nu T_{(p)}{}_i{}^\nu = e n_p E_i + \kappa_i^{pe} + \kappa_i^{p\gamma}$

Electron: $\nabla_\nu T_{(e)}{}_i{}^\nu = -e n_e E_i - \kappa_i^{pe} + \kappa_i^{e\gamma}$

Scattering term

Thomson scattering

$$\begin{cases} \kappa_i^{\gamma e} = -a\sigma_T n_e \rho_\gamma (v_{(\gamma)i} - v_{(e)i}) \\ \kappa_i^{\gamma p} = -a\dfrac{m_e^2}{m_p^2}\sigma_T n_p \rho_\gamma (v_{(\gamma)i} - v_{(p)i}) \end{cases}$$

Coulomb scattering

$$\kappa_i^{pe} = -ae^2 n_p n_e \eta_C (v_{(p)i} - v_{(e)i})$$

(σ_T : Cross section of the Thomson scattering η_C : electric resistivity)

・Maxwell equation (induction eq.)

$$(a^3 B^i)' = -\varepsilon^{ijk}\partial_j\left[a(1+\phi)E_k\right] - \varepsilon^{ijk}(a v_j E_k)'$$

The prime is the derivative in respect to the conformal time η.

$$F_{\mu\nu} = u_{(\gamma)\mu}E_\nu - u_{(\gamma)\nu}E_\mu + \varepsilon_{\mu\nu\rho}B^\rho, u_{(\gamma)\mu}E^\mu = u_{(\gamma)\mu}B^\mu = 0$$

$u_{(\gamma)\mu}$:Four velocities of the photon $\varepsilon_{\mu\nu\rho}$:anti symmetric tensor

ϕ :metric perturbations

→ Show that the generation of the magnetic fields from fluctuations

"Ohm's law" and magnetic fields

Subtracting the equations of proton and electron leads to the equation like "Ohm's law". ($\delta v_{(pe)i} << \delta v_{(\gamma b)i}$)

$$E_i = \frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e} a\rho_\gamma \delta v_{(\gamma b)i}$$

$$\left(\beta = \frac{m_e}{m_p}\right)$$

Substituting this equation into the induction equation.

$$(a^3 B^i)' = -\frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e} a^2 \varepsilon^{ijk}\left[\partial_j(\rho_\gamma \delta v_{(\gamma b)k}) + \rho_\gamma \partial_j \phi \delta v_{(\gamma b)k} + \frac{1}{a^2}(\rho_\gamma a^2 v_j \delta v_{(\gamma b)k})'\right]$$

Next solve $\delta v_{(\gamma b)i}$ using tight coupling approximation.

Tight coupling approximation

→ Interaction is strong （ $T^{-1}\tau << 1$ ）

dynamical timescale　scattering timescale

Expand physical quantities with respect to $T^{-1}\tau$.

→ Particles are almost the same velocity.

$$\vec{v}_\alpha = \vec{v}_\alpha^{(0)} + \vec{v}_\alpha^{(I)} + \vec{v}_\alpha^{(II)} + \cdots$$

（ $T^{-1}\tau$: expansion parameter）

common velocity　the differences of the common velocity

$$\Rightarrow \delta\vec{v}_{\alpha\beta} = \delta\vec{v}_{\alpha\beta}^{(I)} + \delta\vec{v}_{\alpha\beta}^{(II)} + \cdots$$

・Expand the electromagnetic fields also

$$\vec{E} = \vec{E}^{(I)} + \vec{E}^{(II)} + \cdots \qquad \vec{B} = \vec{B}^{(I)} + \vec{B}^{(II)} + \cdots$$

Using this method, we calculate $\delta v_{(\gamma b)i}^{(I,1)}$ and $\delta v_{(\gamma b)i}^{(I,2)}$.

Evaluation of $\delta v_{(\gamma b)i}$

Subtracting the equations of photon and baryon leads to

$$(\delta v_{(\gamma b)i})' + 4H\delta v_{(\gamma b)i} + \frac{\rho_\gamma'}{\rho_\gamma}(v_{(\gamma)i} + \chi_i) - \frac{n'}{n}(v_{(b)i} + \chi_i) - (\phi + 2\psi)\left[\frac{\rho_\gamma'}{\rho_\gamma}v_{(\gamma)i} - \frac{n'}{n}v_{(b)i} + (\delta v_{(\gamma b)i})' + 4H\delta v_{(\gamma b)i}\right] - 5\psi'\delta v_{(\gamma b)i}$$
$$+\frac{1}{4}\frac{\partial_i \rho_\gamma}{\rho_\gamma} + \partial_j(v_{(\gamma)i}v^j_{(\gamma)}) - \frac{1}{1+\beta}\partial_j(v_{(p)i}v^j_{(p)} + \beta v_{(e)i}v^j_{(e)}) = -\alpha\delta v_{(\gamma b)i}$$

$$\left(\alpha \equiv \frac{1+\beta^2}{1+\beta}(1+R)\frac{a\sigma_T\rho_\gamma}{m_p} \qquad R \equiv \frac{3m_p(1+\beta)n}{4\rho_\gamma}\right)$$

Flow of the derivation of magnetic fields

(1) Evaluate $\delta v_{(\gamma b)i}$ in the above equation at order by order in the TCA and cosmological perturbation

(2) Substitute the obtained results in the equation of the magnetic fields

We obtain

$$\delta v^{(I,1)}_{(\gamma b)i} = \frac{1}{\alpha^{(0)}}\left[H v_i^{(1)} - \frac{1}{4}\frac{\partial_i \delta\rho_\gamma^{(1)}}{\rho_\gamma^{(0)}} \right]$$

$$\delta v^{(I,2)}_{(\gamma b)i} = \frac{1}{\alpha^{(0)}}\left[H(v_i^{(2)} + \chi_i^{(2)} - \frac{\alpha^{(1)}}{\alpha^{(0)}} v_i^{(1)}) - (\psi^{(1)}{}' - \frac{1}{3}\partial_\ell v^{(1)\ell}) v_i^{(1)} - H(\phi^{(1)} + 2\psi^{(1)}) v_i^{(1)} - \frac{1}{4}\left\{ \frac{\partial_i \delta\rho_\gamma^{(2)}}{\rho_\gamma^{(0)}} - \left(\frac{\delta\rho_\gamma^{(1)}}{\rho_\gamma^{(0)}} + \frac{\alpha^{(1)}}{\alpha^{(0)}} \right) \frac{\partial_i \delta\rho_\gamma^{(1)}}{\rho_\gamma^{(0)}} \right\} \right]$$

$$- \frac{1}{\alpha^{(0)}} \frac{R^{(0)}}{1+R^{(0)}} C \delta v^{(I,1)}_{(\gamma b)i}$$

↑

New contribution from the non-adiabatic fluctuation

$$C(\vec{x}) \equiv \frac{\delta n^{(1)}}{n^{(0)}} - \frac{3\delta\rho_\gamma^{(1)}}{4\rho_\gamma^{(0)}}$$

($H = \frac{a'}{a}$ 、I: order of the TCA, 0,1,2;order of the cosmological perturbation)

Magnetic fields

- The equation of the magnetic fields

After calculations, we obtain the evolution equation of the magnetic fields.

$$(a^3 B^i)' = \frac{1-\beta^3}{1+\beta}\frac{\sigma_T}{e} a^2 \rho_\gamma^{(0)} \left[\frac{2H}{\alpha^{(0)}} a^2 \omega^{(2)i} + \boxed{\frac{\varepsilon^{ijk}}{\alpha^{(0)}} \frac{R^{(0)}}{1+R^{(0)}} \partial_j C \delta v^{(I,1)}_{(\gamma b)k}} \right]$$

New contribution from the non-adiabatic fluctuations and 1st order TCA!

$\omega^{(2)i}$:photon's vorticity $\qquad \alpha^{(0)} \equiv \frac{4\beta(1+\beta^2)}{3(1+\beta)}(1+R^{(0)})\frac{1}{\tau_T}$

$\beta = m_e / m_p \qquad R^{(0)} = \rho_b^{(0)} / \rho_\gamma^{(0)}$

POWER SPECTRUM OF THE GENERATED FIELDS

Process of the evaluation

- Flow of the calculation

→Solve the evolution of the perturbations

→Substitute the above results into
the equation of the fields

→Evaluate <$B_i B^i$>

$$\left\langle \vec{B}_i(\vec{k})\vec{B}^{*i}(\vec{K})\right\rangle \equiv \frac{2\pi^2}{k^3} P_B(k)\delta(\vec{k}-\vec{K})$$

Power spectrum of the magnetic fields

$$\frac{2\pi^2}{k^3}P_B(k) = \left(\frac{1-\beta^3}{1+\beta}\frac{\sigma_T\rho_{\gamma 0}}{ea^3}\right)^2 (2\pi^2)^2 \int d^3p |\vec{k}\times\vec{p}|^2 P_{na}(p) P_a(|\vec{k}-\vec{p}|) \times$$
$$\times \int_0^\eta d\eta_1 \int_0^\eta d\eta_2 a_1^{-2} a_2^{-2} \{g(\vec{k},\vec{p},\eta_1) g(\vec{k},\vec{p},\eta_2) + f(\vec{k},\vec{p},\eta_1) f(\vec{k},\vec{p},\eta_2)\}$$

$$f(\vec{k},\vec{p},\eta) \equiv \frac{1}{(2\pi)^{3/2}} \frac{\eta^2 \bar{R}^{(0)}}{2(1+\bar{R}^{(0)})} \frac{1}{\bar{\alpha}^{(0)}} |\vec{k}-\vec{p}|^2 \frac{j_1(y)}{y},$$

$$g(\vec{k},\vec{p},\eta) \equiv \frac{1}{(2\pi)^{3/2}} \frac{1}{\bar{\alpha}^{(0)}} \frac{|\vec{k}-\vec{p}|^2}{2\eta(1+\bar{R}^{(0)})} \int d\eta' \frac{(\eta')^2 \bar{R}^{(0)}}{1+\bar{R}^{(0)}} \frac{j_1(y')}{y'}$$

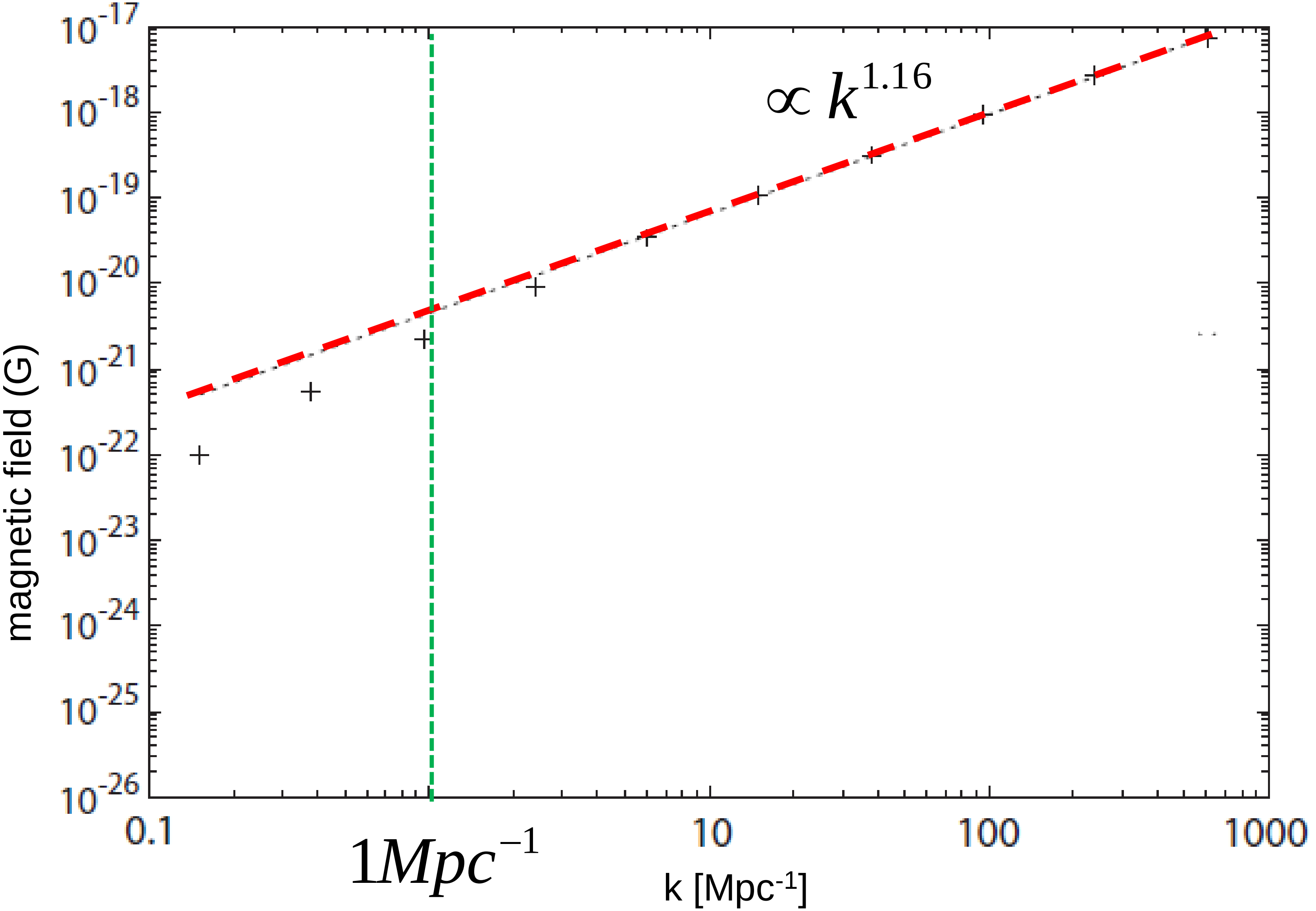
$$\left(y = |\vec{k}-\vec{p}|\eta/\sqrt{3} \right)$$

- Power spectrum of the non-adiabatic fluctuations of baryon

$$\frac{P_{na}}{P_a} \equiv \left(\frac{\Omega_{CDM}}{\Omega_b}\right) \frac{A}{1-A} \left(\frac{k}{k_c}\right)^{n_2-1}$$

This factor expresses conversion of non-adiabatic fluctuation of CDM to one of baryon

Results($n_I = 4, A \approx 0.01$)



SUMMARY

Summary

- We show that the non-adiabatic fluctuations generate the magnetic fields at pre-recombination era in the 1st-order tight coupling.

- The amplitude of the generated fields is $B_{eq} \sim 10^{-21}\mathrm{G}$ at $1Mpc^{-1}$ which is enough to be amplified to the magnetic fields in galaxies and the power spectrum is proportional to $k^{1.16}$.