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Hawking temperature for near-equilibrium black holes

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Introduction

- The fact that black holes possess thermodynamic properties has been intriguing in gravity
  - Hawking temperature, Black hole entropy
- The AdS/CFT has opened up new insights about these properties
  - Stationary black holes correspond to thermal equilibrium state of dual field theories at finite-temperature
- It is interesting to explore generalization for non-stationary spacetime

What is problem?

- In stationary cases Hawking temperature is given by surface gravity of (Killing) horizon. How about dynamical cases?
  - No Killing horizon. Which horizon?
  - If we defined temperatures by using some quantities associated with event horizon, how can we observe it at asymptotic region? It is puzzling from causality.

- Our approach: we try to define temperature from spectrum of Hawking radiation
  - For gravitational collapse, Barcelo et al. (2011) have considered.
  - We will focus our attention on eternal BHs because we are interested in near-equilibrium systems

Conventional derivation of the Hawking radiation

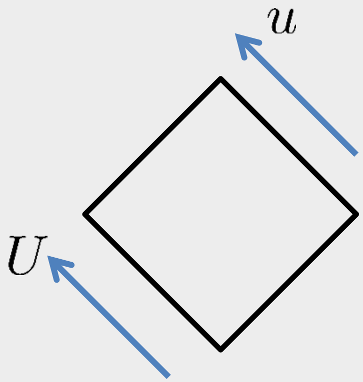
- We introduce two null coordinates
  - u: asymptotic time (natural time for asymptotic observers)
  - U: Kruskal time (an affine parameter on the past horizon)
- Bogoliubov transfomation between two sets of modes $u_\omega \propto e^{-i\omega u}, \quad \bar{u}_{\omega'} \propto e^{-i\omega' U}$

$$\left.\begin{matrix}\alpha_{\omega\omega'} \\ \beta_{\omega\omega'}\end{matrix}\right\} = \pm \frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{\infty} \mathrm{d}u \frac{\mathrm{d}U}{\mathrm{d}u} e^{\pm i\omega u - i\omega' U(u)}.$$

$$U(u) = -\exp(-\kappa u)$$

$$\frac{|\beta_{\omega\omega'}|^2}{|\alpha_{\omega\omega'}|^2} = e^{-2\pi\omega/\kappa} \qquad |\beta_{\omega\omega'}|^2 \propto \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

Thermal spectrum



Extension to time-dependent spacetime

- We use wave packets which are localized in both of time and frequency domains

$$\psi_\omega(u) = \frac{1}{\sqrt{4\pi\omega}} w_{\Delta u}(u - u_0) e^{-i\omega u}$$

$w_{\Delta u}(x)$: a window function which goes rapidly to zero outside Δu

Δu : a duration of observation

u_0 : a time of observation

We expect to be able to define temperature, if the background spacetime would evolve sufficiently slowly within the above time interval

Saddle-point approximation

- Bogoliubov coefficients for wave packets

$$\left.\begin{matrix} A_{\omega\omega'} \\ B_{\omega\omega'} \end{matrix}\right\} = \pm \frac{i}{2\pi}\sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{\infty} w_{\Delta u}(u-u_0)\frac{\mathrm{d}U}{\mathrm{d}u} e^{\pm i\omega u - i\omega' U(u)}\mathrm{d}u$$

This integral can be evaluated as

Geometric optics $(\omega \gg \kappa)$

$$\int_{-\infty}^{\infty} w_{\Delta u}(u-u_0)\exp\phi(u)\mathrm{d}u \simeq w_{\Delta}(u_* - u_0)e^{\phi(u_*)}\sqrt{\frac{2\pi}{|\phi''(u_*)|}}$$

$$\begin{cases} \phi(u) = \log U'(u) \pm i\omega u - i\omega' U(u) \\ \phi'(u) = -\kappa(u) \pm i\omega - i\omega' U'(u) \\ \phi''(u) = -\kappa'(u) + i\omega' U'(u)\kappa(u) \end{cases}$$

Saddle point:

$$\phi'(u_*) = 0$$

We have defined $\kappa(u) \equiv -\frac{\mathrm{d}}{\mathrm{d}u}\log\frac{\mathrm{d}U}{\mathrm{d}u}$

Adiabatic expansion

- To clarify what slowly evolving is, we will define adiabatic expansion

$$\kappa(u) = \kappa_0[1 + \epsilon f(u)] \qquad |f(u)| < 1 \quad \text{for} \quad |u - u_0| < \Delta u$$

$$\Rightarrow \quad \frac{\delta\kappa}{\kappa_0} < \epsilon \quad (\delta\kappa \equiv |\kappa(u) - \kappa_0|)$$

- Expanding $U(u)$ with respect to ε

$$U'(u) = \exp\left(-\int_{u_0}^{u} \kappa(u')\mathrm{d}u'\right) = U_0' e^{-\kappa_0 \delta u}(1 - \epsilon\kappa_0 g(u) + \mathcal{O}(\epsilon^2)),$$

$$U(u) = \int_{u_0}^{u} U'(u')\mathrm{d}u' = U_0 + U_0'\left(\frac{1 - e^{-\kappa_0\delta u}}{\kappa_0} - \epsilon\kappa_0 \int_{u_0}^{u} e^{-\kappa_0\delta u'} g(u')\mathrm{d}u' + \mathcal{O}(\epsilon^2)\right),$$

$$U''(u) = -\kappa(u)U'(u) = -\kappa_0(1 + \epsilon f(u))U_0' e^{-\kappa_0\delta u}(1 - \epsilon\kappa_0 g(u) + \mathcal{O}(\epsilon^2))$$

Adiabatic expansion

- Also, we will expand the saddle point and solve the saddle point equation order by order

$$u_* = u_*^{(0)} + \epsilon u_*^{(1)} + \mathcal{O}(\epsilon^2)$$

$$\begin{aligned} 0 = \phi'(u_*) = &- \kappa_0 r e^{\mp i\theta} - i\omega' U_0' e^{-\kappa_0 \delta u_*^{(0)}} \\ &+ \epsilon \kappa_0 \left[-f(u_*^{(0)}) + i\omega' U_0' e^{-\kappa_0 \delta u_*^{(0)}} \left(u_*^{(1)} + g(u_*^{(0)}) \right) \right] + \mathcal{O}(\epsilon^2) \end{aligned}$$

$$\phi(u_*) - \phi(u_0) = \phi_0 + \epsilon \phi_1 + \mathcal{O}(\epsilon^2), \quad \phi''(u_*) = \phi_0'' + \epsilon \phi_1'' + \mathcal{O}(\epsilon^2)$$

To guarantee that the purturbative expansion is valid, we should require each of correction term to be sufficiently smaller than its leading term

Sufficient condition

$\epsilon \ll 1$ ⇒ $$\frac{|B_{\omega\omega'}|^2}{|A_{\omega\omega'}|^2} = \exp\left(-\frac{2\pi\omega}{\kappa(u_0)} \right)$$

Surface gravity for past horizon

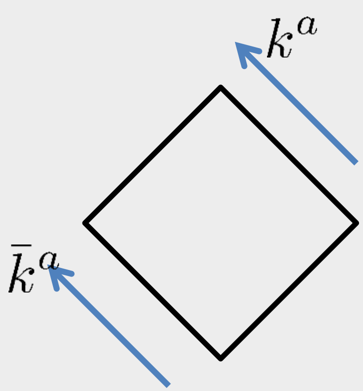
- Geometrical meaning of $\kappa(u)$
  - In-going null vectors with respect to u and U

$$k^a = \left(\frac{\partial}{\partial u}\right)^a, \quad \bar{k}^a = \left(\frac{\partial}{\partial U}\right)^a \quad (k^a = U'(u)\bar{k}^a)$$

  - Geodesic equation

$$\begin{aligned} k^a \nabla_a k^b =& k^a \nabla_a (U'(u)\bar{k}^b) \\ =& (k^a \nabla_a U'(u))\bar{k}^b + (U')^2 \bar{k}^a \nabla_a \bar{k}^b \\ =& -\kappa(u) k^b \end{aligned}$$

$$\kappa(u) = -\frac{\mathrm{d}}{\mathrm{d}u} \log \frac{\mathrm{d}U}{\mathrm{d}u}$$



"surface gravity" for past horizon

Application to AdS-Vaidya

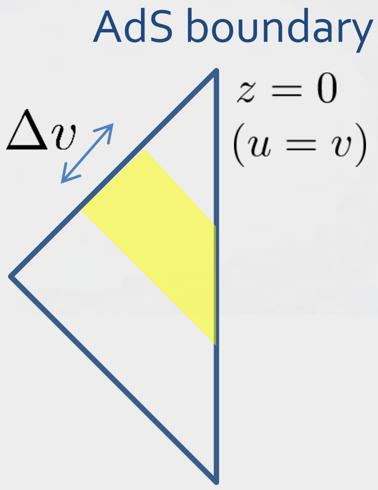
- A toy model of thermalization for CFT fluid
  - Mass injection into BH

$$ds^2 = \frac{1}{z^2}[-(1-2m(v)z^4)dv^2 - 2dvdz + d\vec{x}_3^2]$$

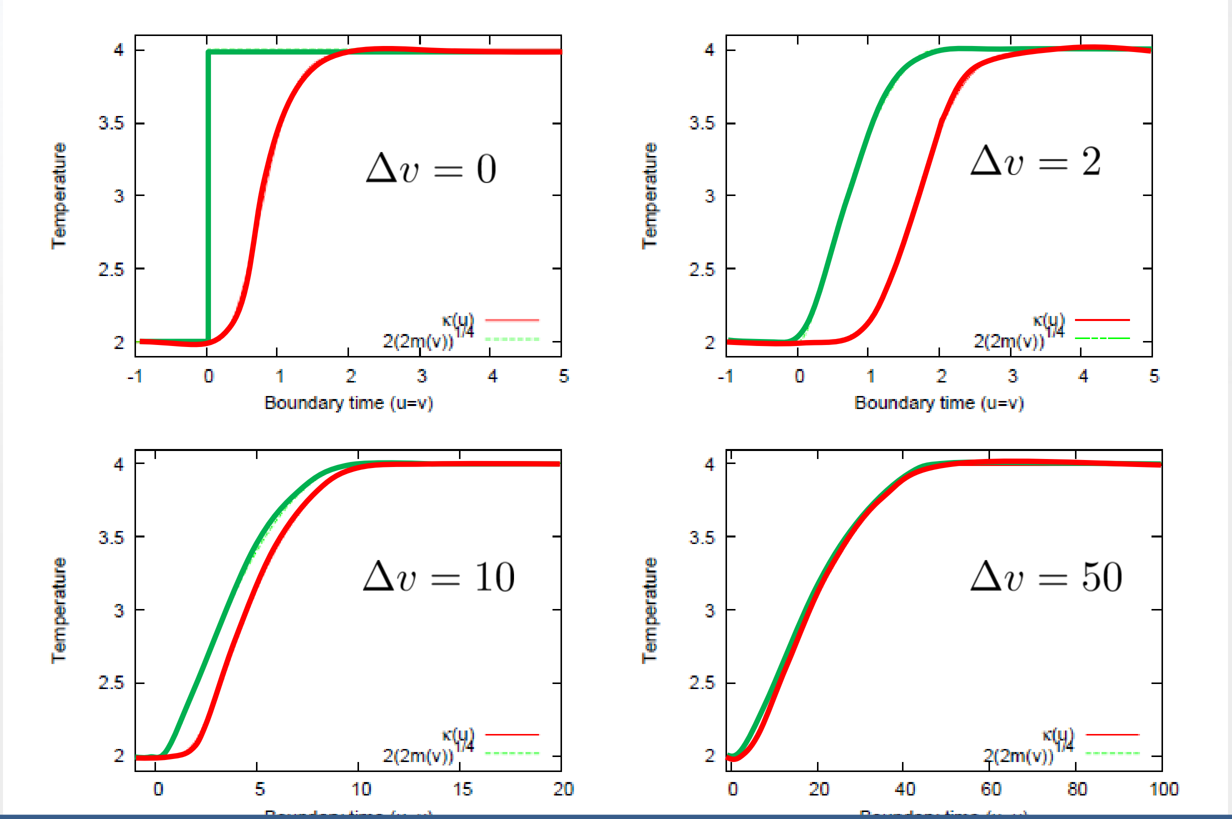
Mass function:

$$m(v) = \begin{cases} m_0 + \Delta m & (v > \Delta v) \\ m_0 + \Delta m \sin^2 \dfrac{\pi v}{2\Delta v} & (0 \leq v \leq \Delta v) \\ m_0 & (v < 0) \end{cases}$$

Δv : time-scale of injection



Results



- Even if the mass injection is so rapid, the temperature will gradually change.
- The delay time is determined by typical thermal scale 1/T.

Summary

- We have discussed Hawking temperature for non-stationary spactimes
  - Hawking temperature is determined by surface gravity for the past horizon
- We have applied it to AdS-Vaidya as a toy model of thermalization process in AdS/CFT
  - Even if the spacetime rapidly changes, the temperature will gradually evolve and its time-scale is bounded by typical thermal time-scale