



Multi-field Open Inflation & Instanton

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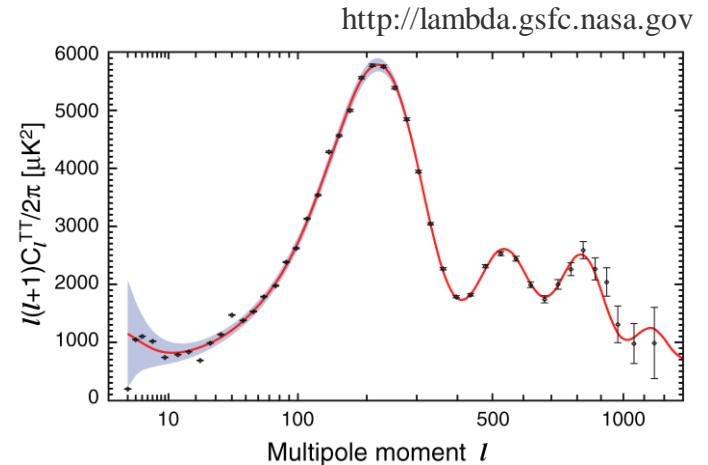
Introductions

Inflation and beyond

□ Inflation solves

- Horizon problem
- Flatness problem
- Monopole problem

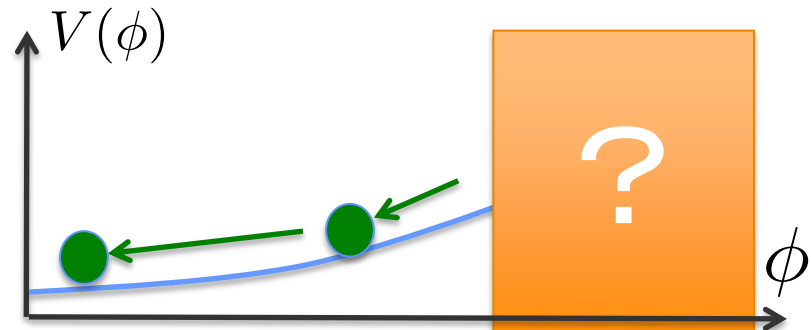
also predicts power spectrum



very good agreement!!

□ But

- How is it implemented in particle physics?
- How does it start?



String landscape

(Susskind, 2003)

- In String theory

- High dimensionality

- Integration out of high energy physics

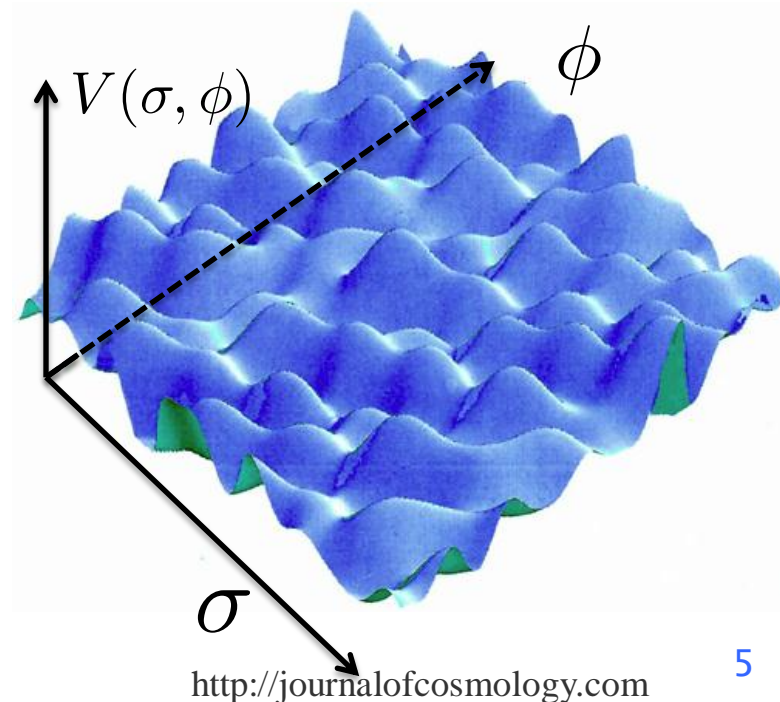
many physical degrees of freedom

Non-trivial structure of vacuum

- implementation of inflation seems possible

- scalar field

- flat potential



String landscape

(Susskind, 2003)

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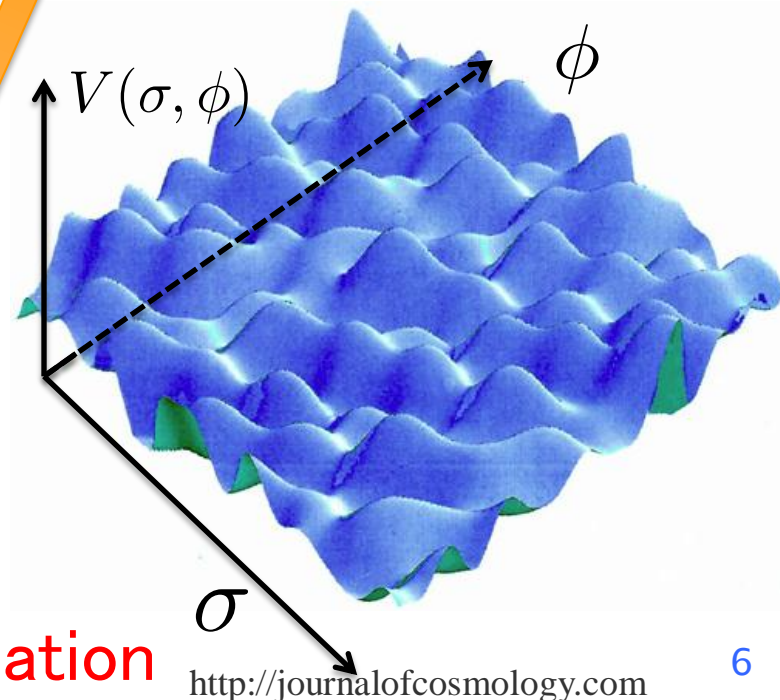
- flat potential

- String theory also implies

- multi-field

- quantum tunneling

Multi-field open inflation



Multi-field open inflation

- Quantum tunneling before slow-roll (in multi-field system)



(Multi-field) Open inflation

- $\Omega_k \sim -0.01$ is most favorable(?) (Freivogel, et al., 2006)



possibility of curvature detection

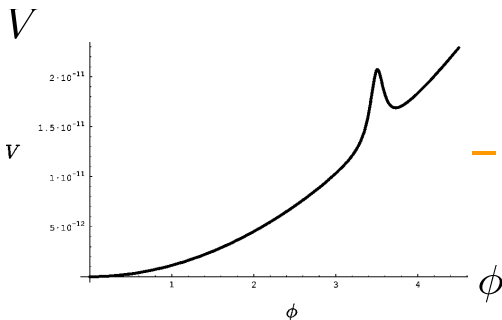
good point of multi-field system

- Open inflation in multi-field system

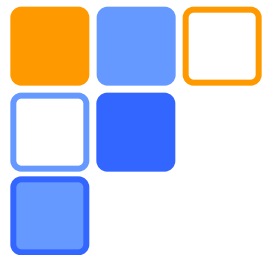
may solve a problem in single-field case



- Only an artificial potential model is known (Linde 1998)



We will study multi-field open inflation!!



Multi-field open inflation

Multi-field model with a simple potential

$$m_{pl}^2 = \frac{1}{8\pi G}$$

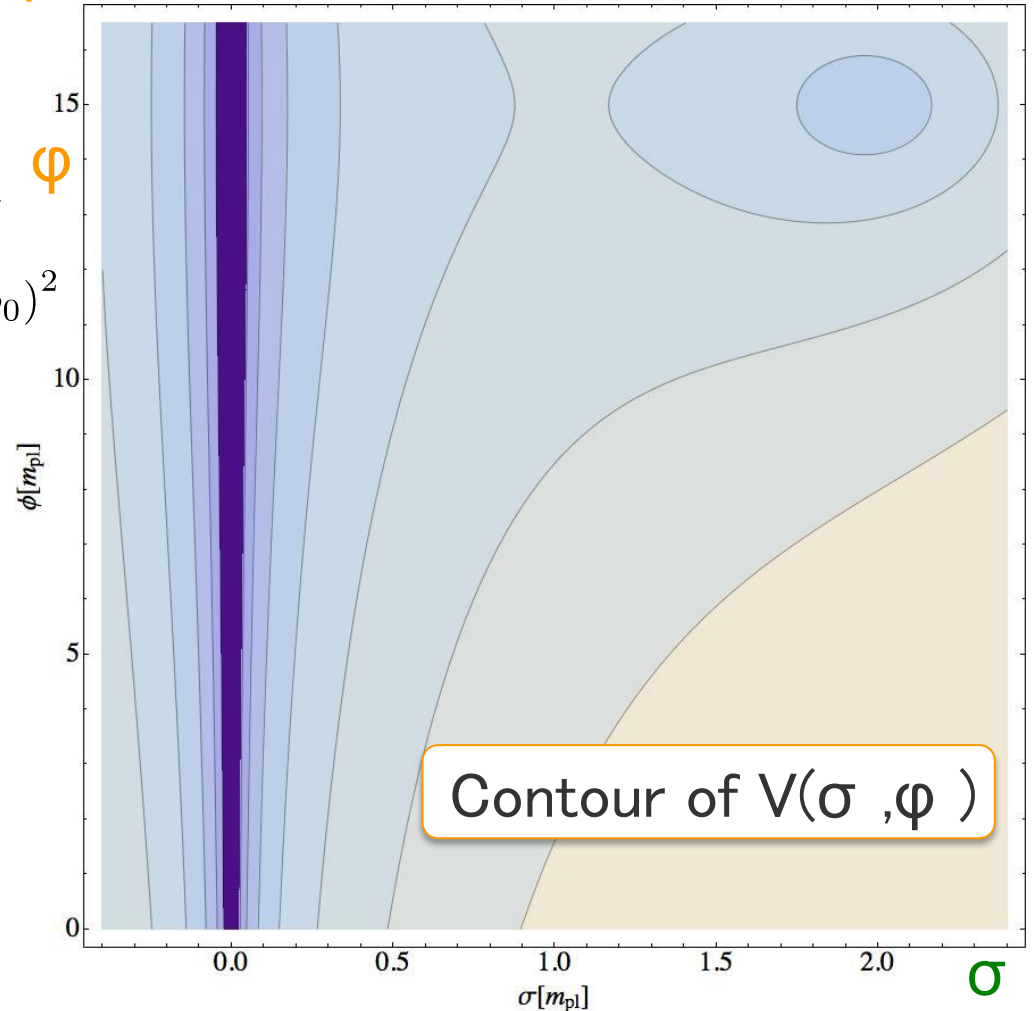
$$\begin{aligned} \alpha &= 0.1 & \beta &= 0.01 \\ \sigma_0 &= 2m_{pl} & m_\phi &= 10^{-6}m_{pl} \\ M_V &= 0.2m_{pl} & \phi_0 &= 15m_{pl} \end{aligned}$$

tunneling field σ inflaton ϕ

$$V(\sigma, \phi) = V_{\text{tun}}(\sigma) + V_{\text{infl}}(\sigma, \phi)$$

$$V_{\text{tun}}(\sigma) = \alpha\sigma^2 \left\{ (\sigma - \sigma_0)^2 + M_V^2 \right\}$$

$$V_{\text{infl}}(\sigma, \phi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{\beta}{2}\sigma^2(\phi - \phi_0)^2$$



Multi-field model with a simple potential

$$m_{pl}^2 = \frac{1}{8\pi G}$$

$$\alpha = 0.1$$

$$\beta = 0.01$$

$$\sigma_0 = 2m_{pl}$$

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tunneling field σ inflaton ϕ

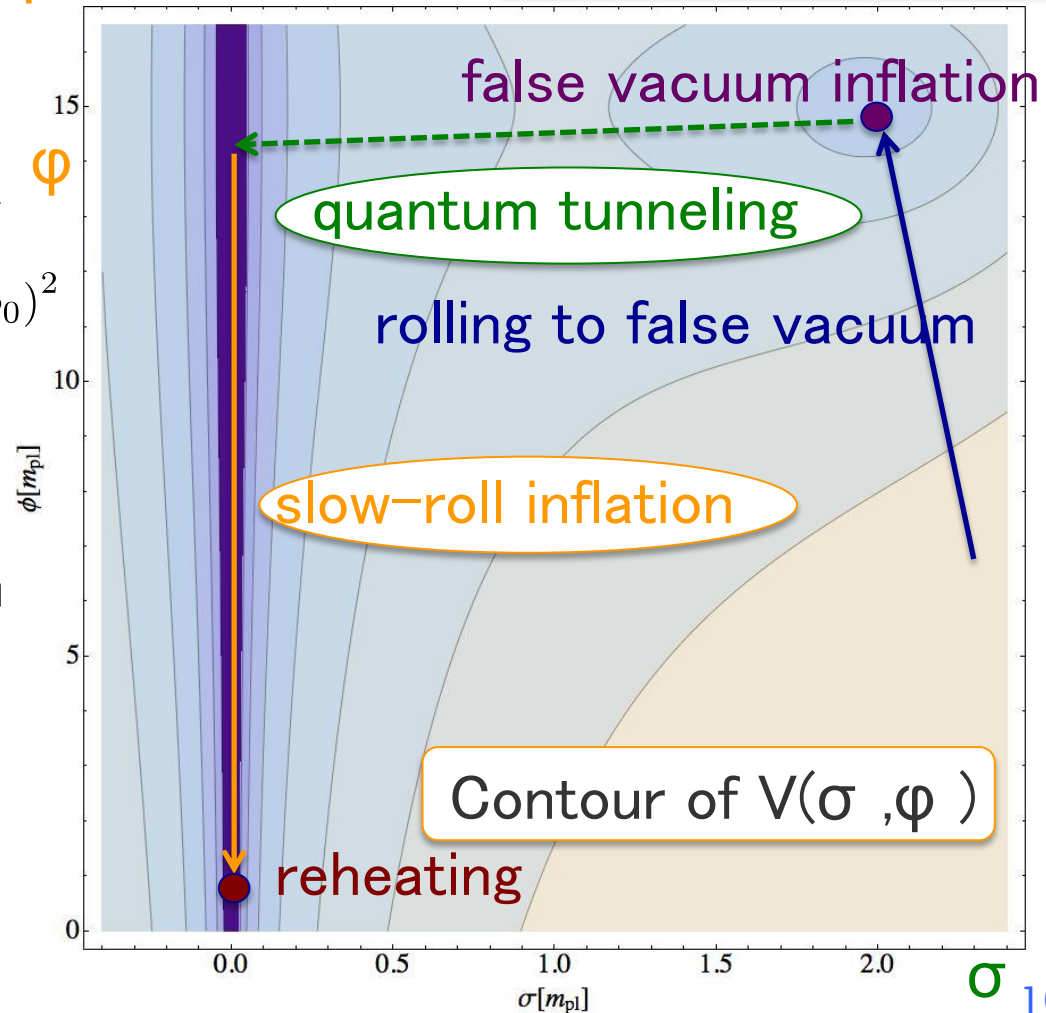
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Scenario

1. rolling to false vacuum
2. false vacuum inflation
3. quantum tunneling
4. slow-roll inflation
5. reheating



Formulation of multi-field tunneling with gravity

- Multi-field extension of Coleman-De Luccia instanton
(Coleman and De Luccia, 1980)

- instanton

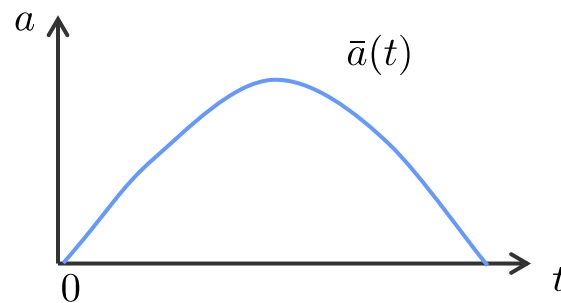
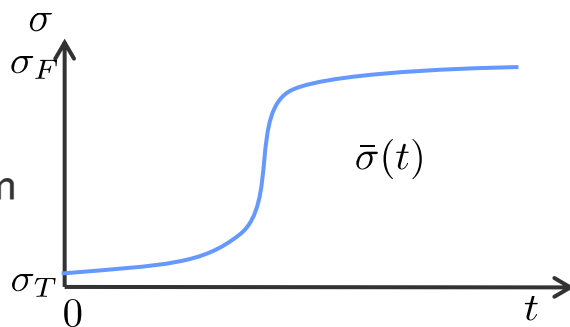
$$\bar{\sigma}(t), \bar{\phi}(t), \bar{a}(t)$$



$O(4)$ -symmetric non-trivial
Euclidean classical path

Euclidean metric $ds_E^2 = dt^2 + \bar{a}^2(t)dx^2$

image of instanton
for single-field system



- inside of nucleated bubble is open Friedmann universe



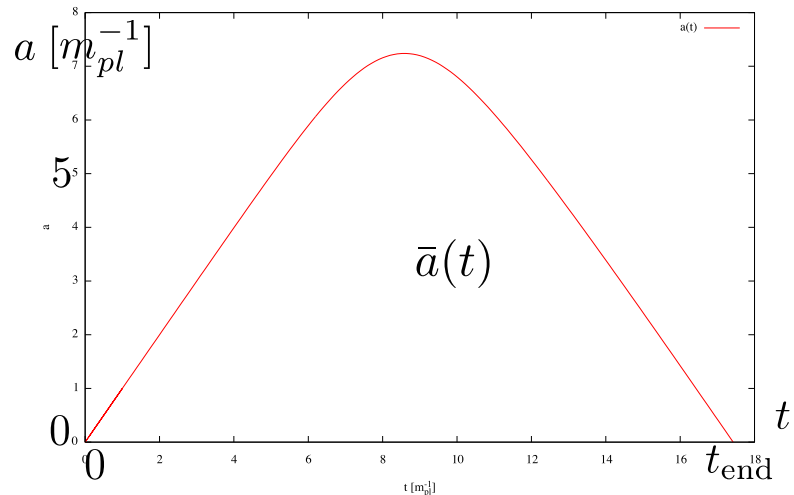
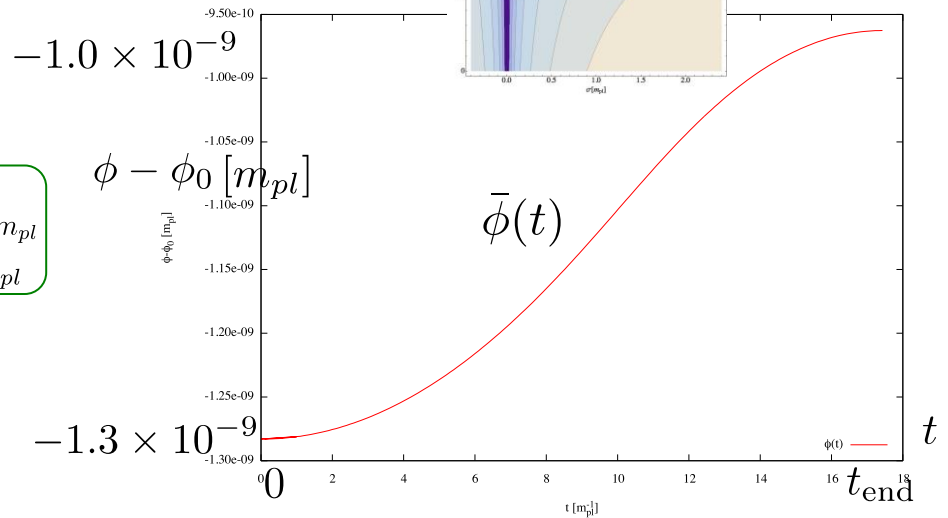
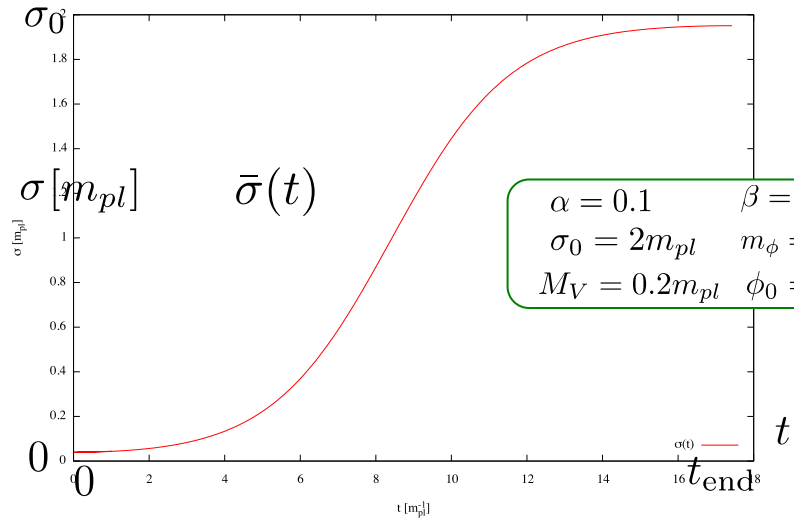
initial state is given by instanton value at $t=0$

$$\sigma(0) = \bar{\sigma}(0) \quad \phi(0) = \bar{\phi}(0) \quad a(0) = 0$$

We will

1. construct a multi-field instanton
2. evolve the universe inside bubble

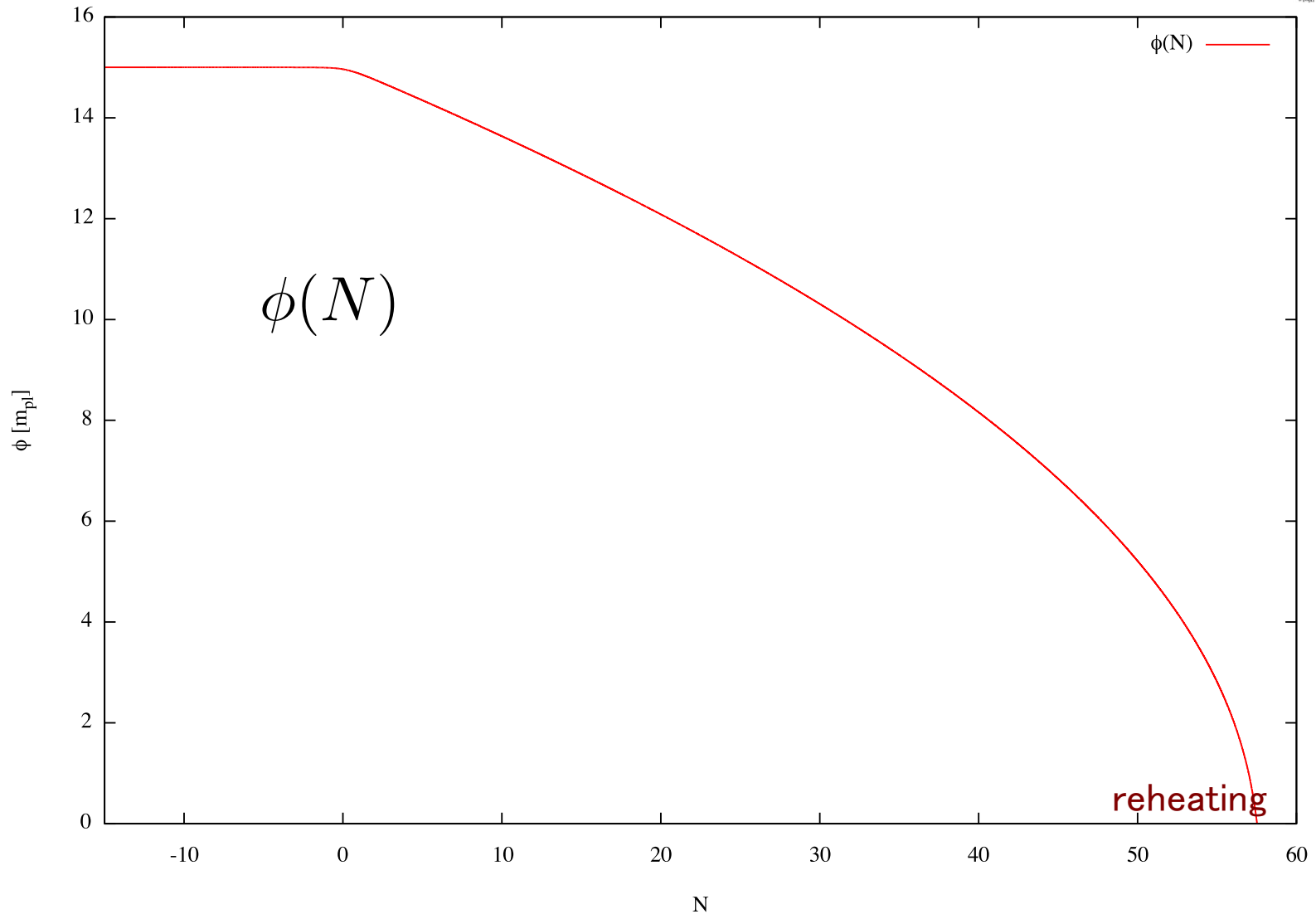
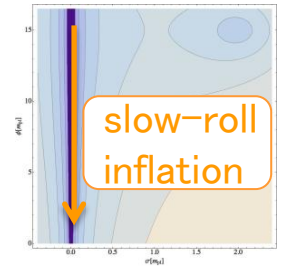
Multi-field instanton



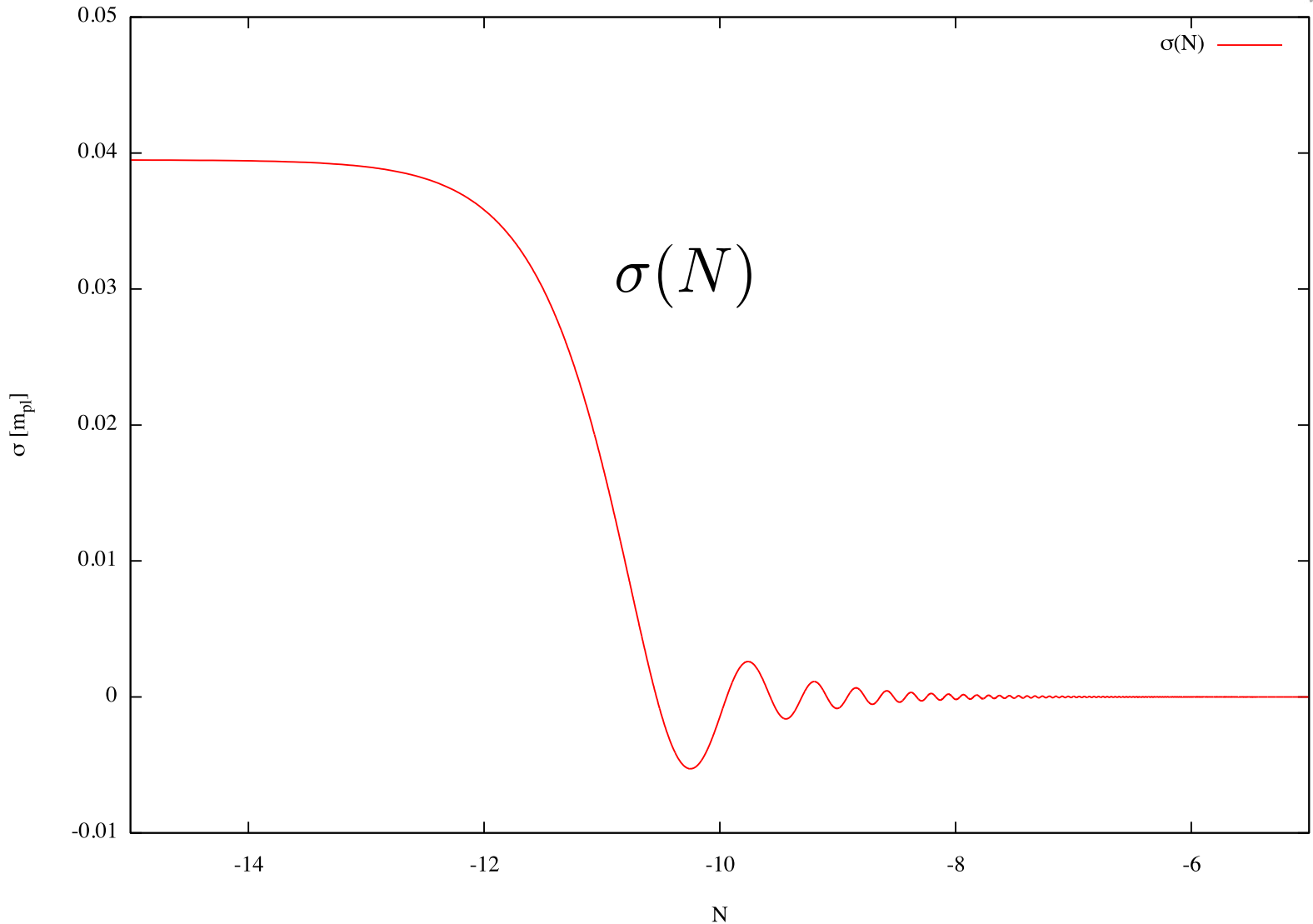
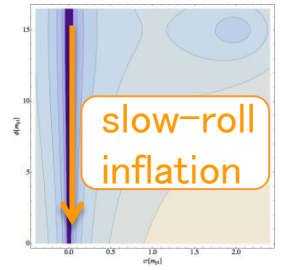
- multi-field instanton with gravity is explicitly constructed for the first time
- inflaton ϕ moves during tunneling but a little for this parameter choice
- instanton value at $t=0$ gives the initial state of bubble

➡ evolution after tunneling

Evolution of inflaton ϕ after tunneling



Evolution of tunneling field σ after tunneling

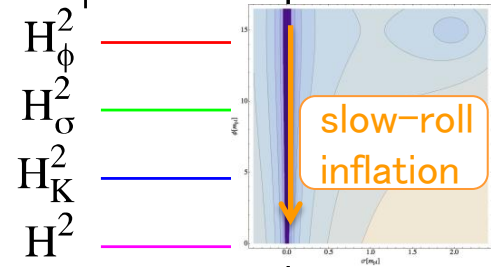


Evolution after tunneling

$$\alpha = 0.1 \quad \beta = 0.01$$

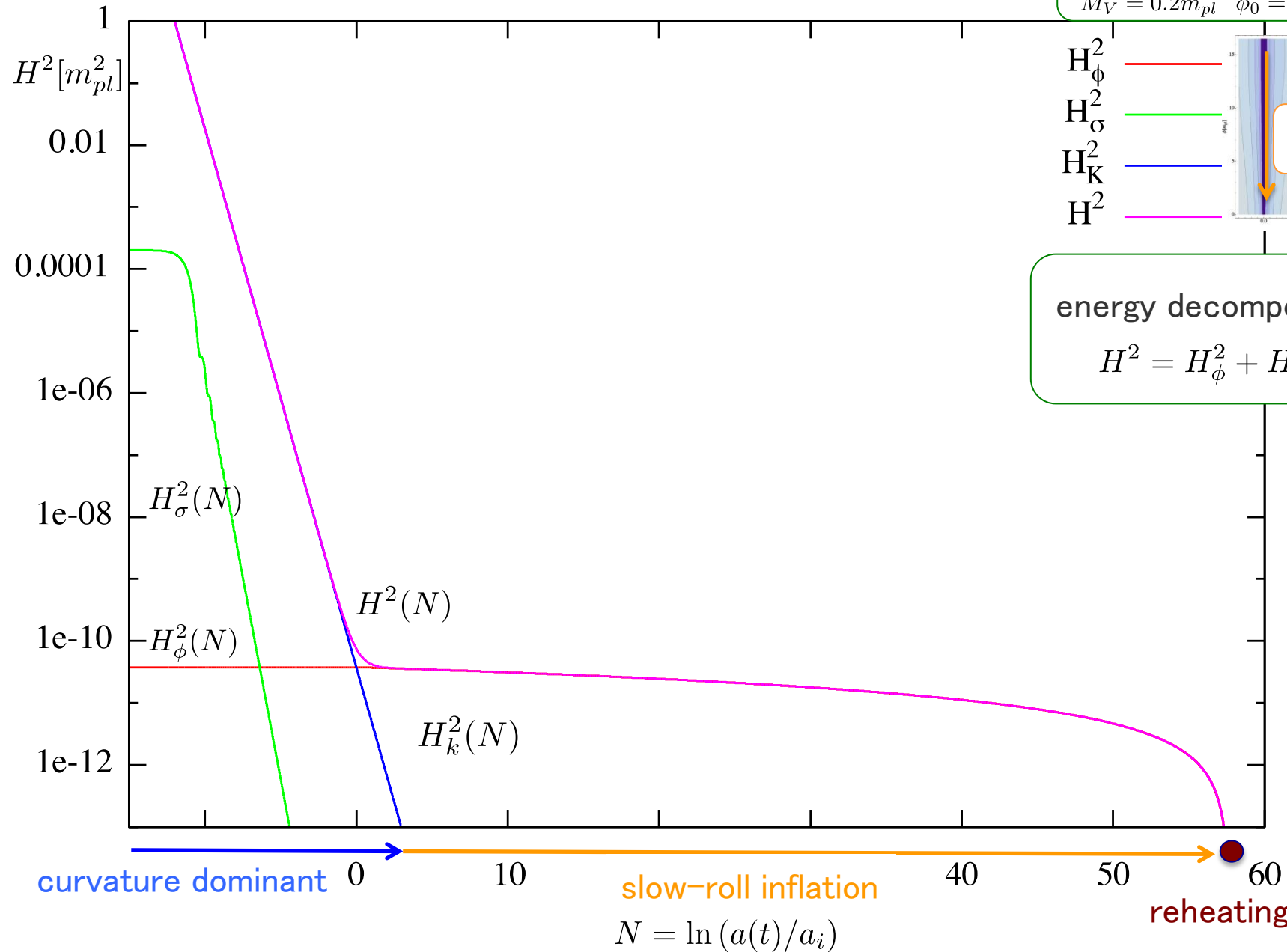
$$\sigma_0 = 2m_{pl} \quad m_\phi = 10^{-6}m_{pl}$$

$$M_V = 0.2m_{pl} \quad \phi_0 = 15m_{pl}$$



energy decomposition

$$H^2 = H_\phi^2 + H_\sigma^2 + H_k^2$$



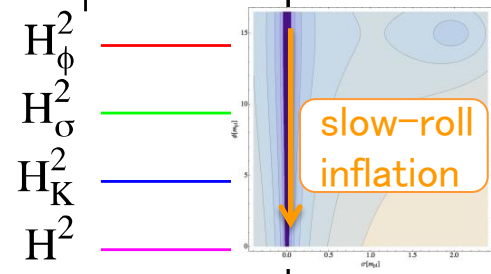
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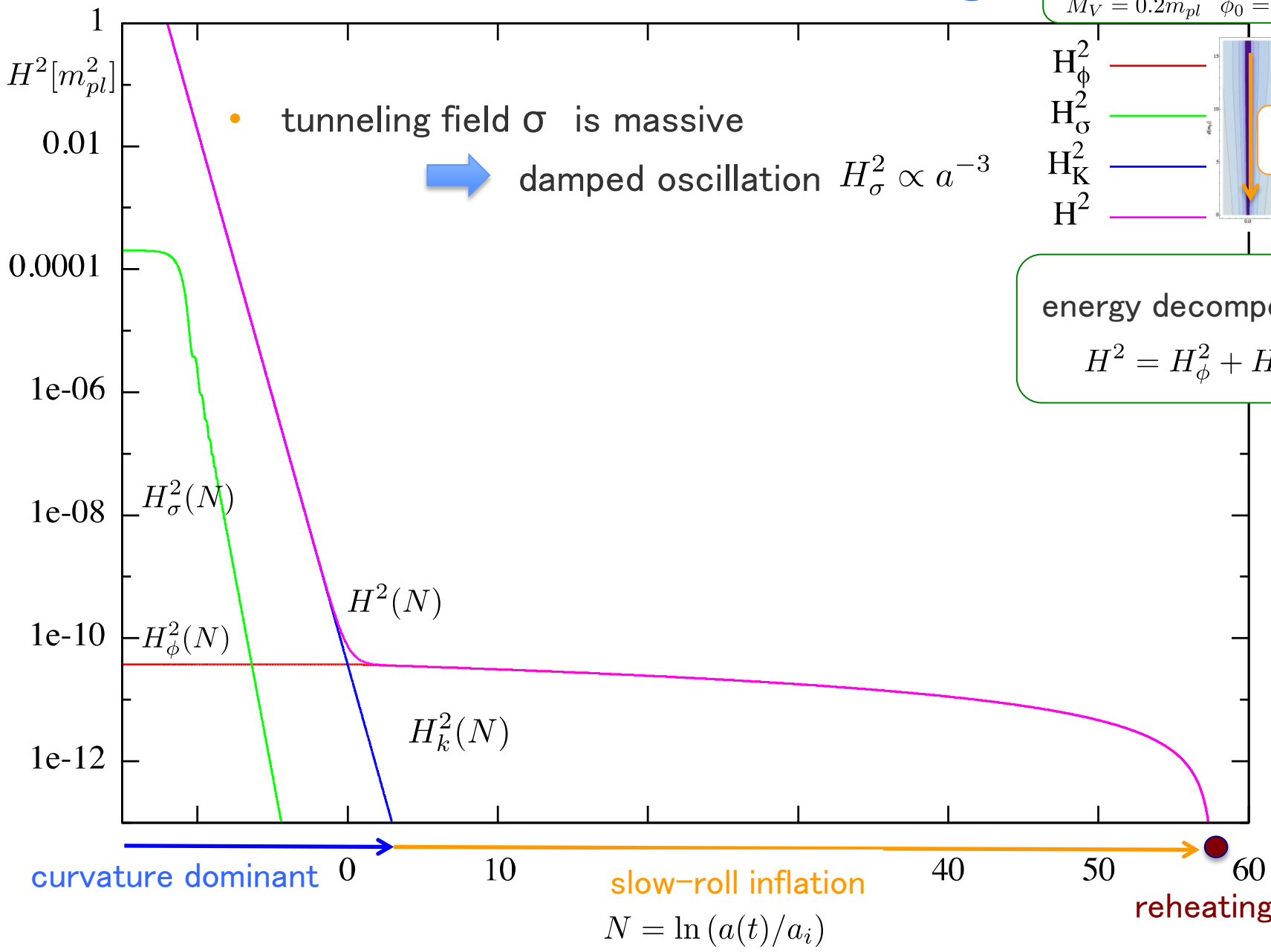
$$M_V = 0.2m_{pl} \quad \phi_0 = 15m_{pl}$$

• tunneling field σ is massive
 → damped oscillation $H_\sigma^2 \propto a^{-3}$



energy decomposition

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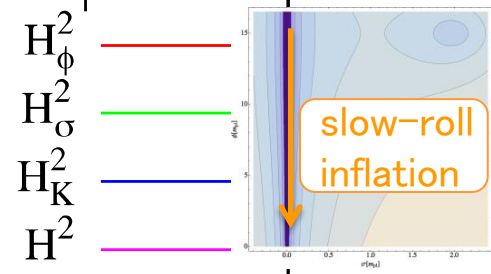
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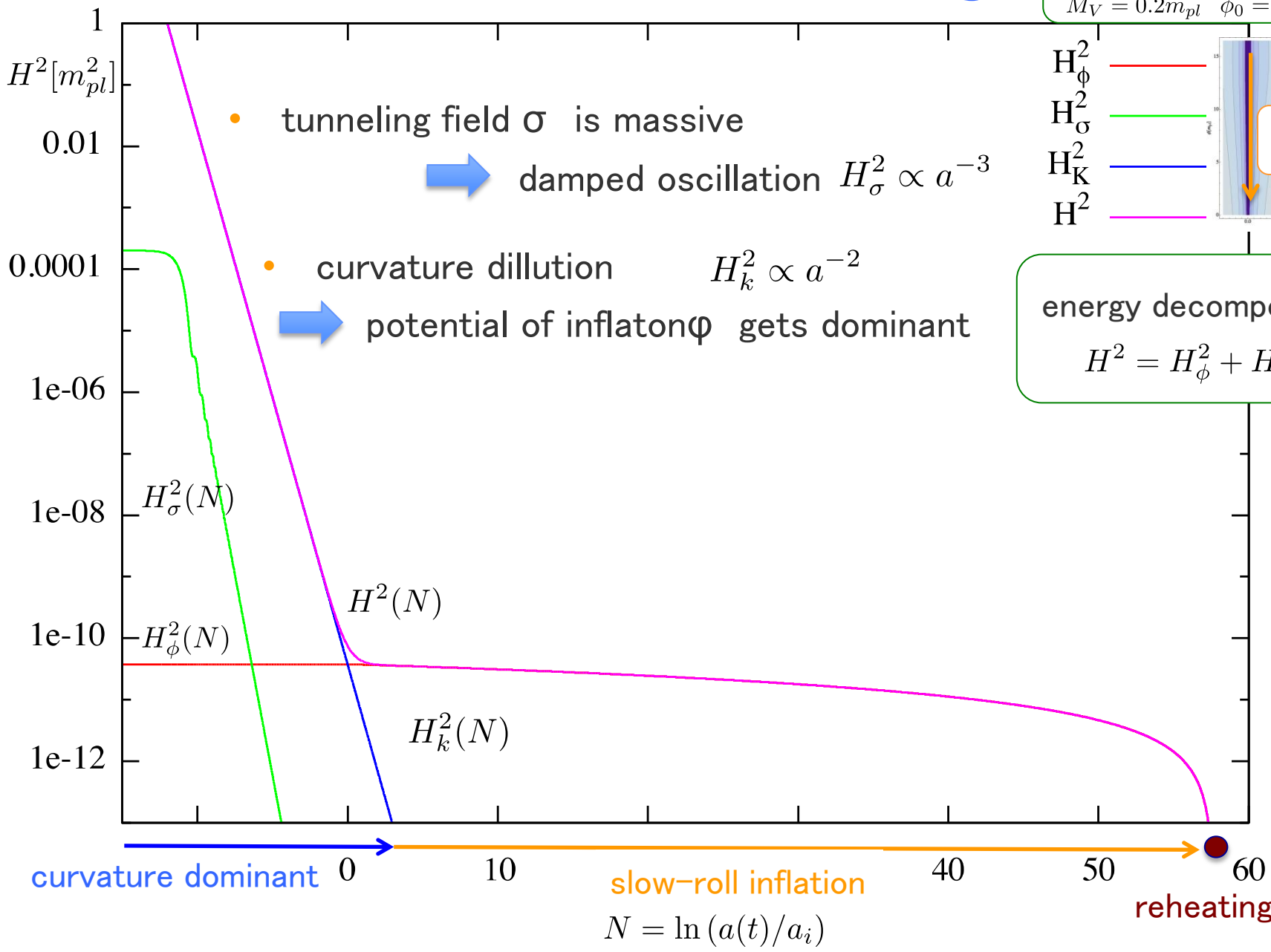
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- tunneling field σ is massive
 - damped oscillation $H_\sigma^2 \propto a^{-3}$
- curvature dilution $H_k^2 \propto a^{-2}$
 - potential of inflaton ϕ gets dominant



energy decomposition

$$H^2 = H_\phi^2 + H_\sigma^2 + H_k^2$$

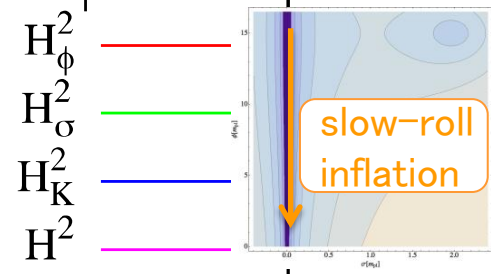


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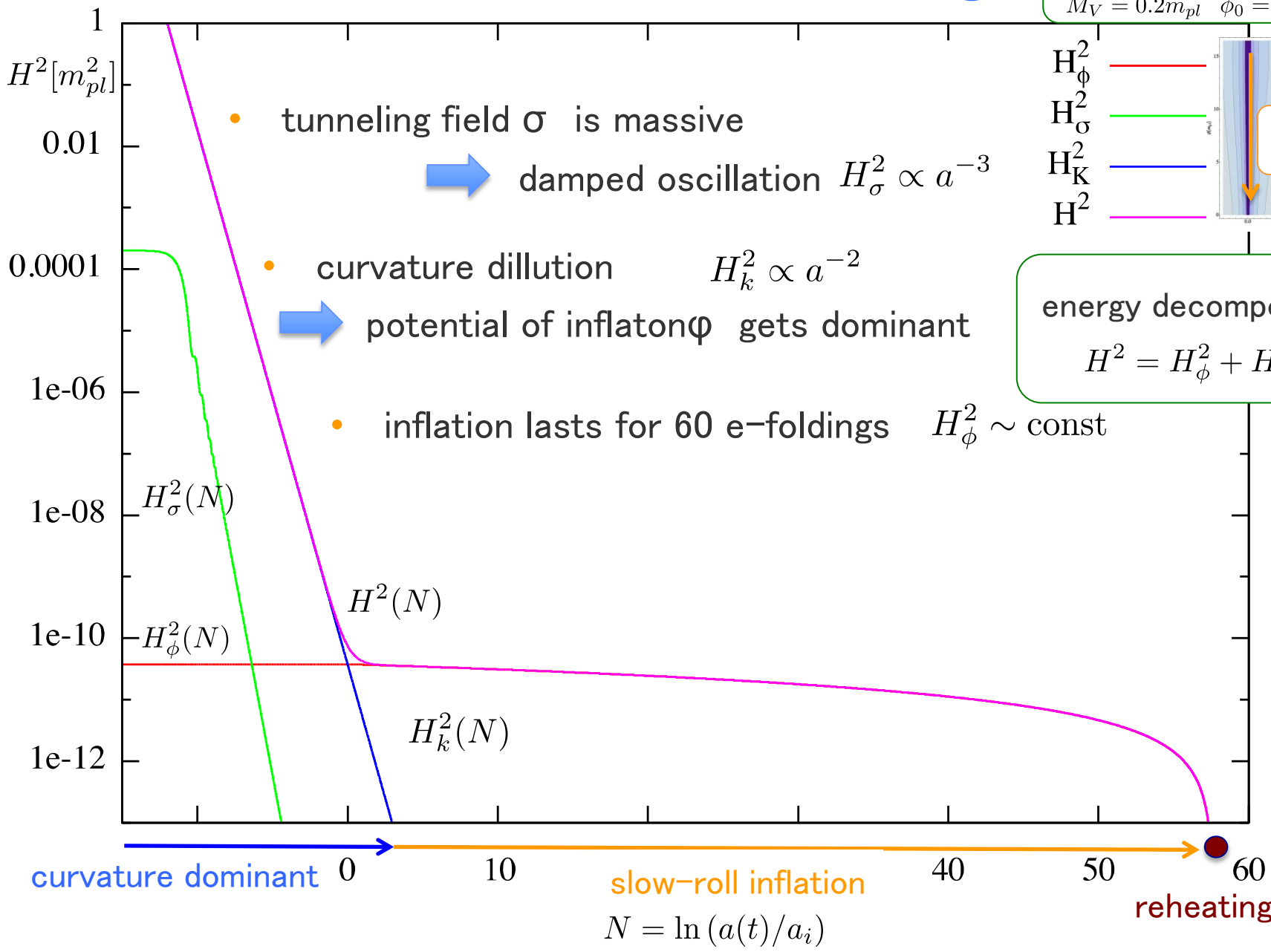
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energy decomposition

$$H^2 = H_\phi^2 + H_\sigma^2 + H_k^2$$

- tunneling field σ is massive
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 - potential of inflaton ϕ gets dominant
- inflation lasts for 60 e-foldings $H_\phi^2 \sim \text{const}$

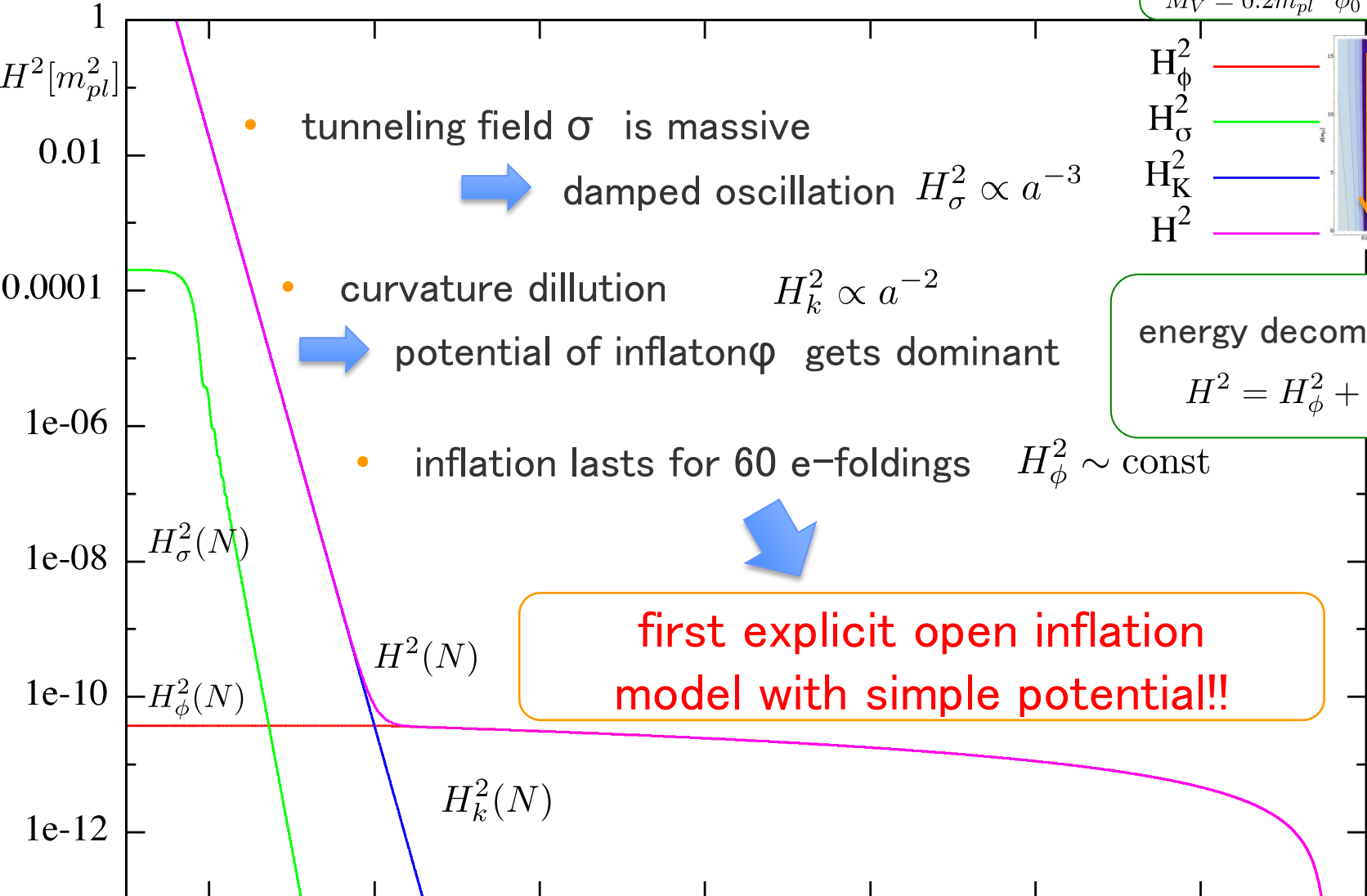


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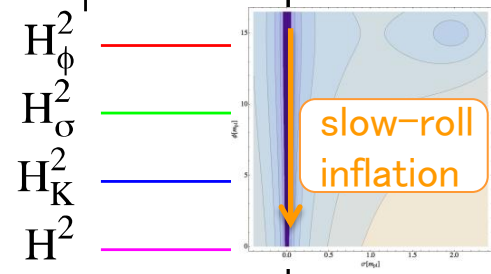


- tunneling field σ is massive
 \rightarrow damped oscillation $H_\sigma^2 \propto a^{-3}$

- curvature dilution $H_K^2 \propto a^{-2}$
 \rightarrow potential of inflaton ϕ gets dominant

- inflation lasts for 60 e-foldings $H_\phi^2 \sim \text{const}$

first explicit open inflation model with simple potential!!



energy decomposition

$$H^2 = H_\phi^2 + H_\sigma^2 + H_K^2$$

curvature dominant slow-roll inflation reheating

$N = \ln(a(t)/a_i)$

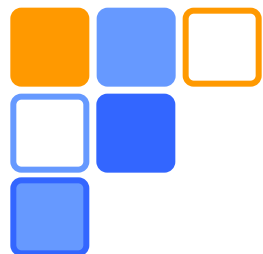


Conclusions and Discussions



Conclusions

- We studied about Multi-field open inflation, which is motivated by string landscape
- The Coleman De Luccia instanton method was extended to the multi-field case
- Multi-field instanton with gravity and the evolution inside the bubble were explicitly calculated
- Our multi-field open inflation model is the first explicit open inflation model with a simple potential



Discussions and Future works

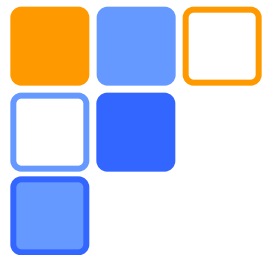
- Method to calculate a quantum fluctuation in open inflation seems possible to be applied to our model

(Garriga, Montes, Sasaki, Tanaka (1998))

- Interaction between heavy oscillating tunneling field and inflaton may produce some interesting features in power spectrum
- Quantum tunneling changes the state of the universe from Bunch–Davies vacuum, and this may produce a characteristic non–gaussianity
(now we are working!)
- If we are very lucky, we may find an evidence of our model, or string landscape, from observations



Appendix



Interaction effect on decay rate



naive question about multi-field dynamics

Q. Do multi-field dynamics make decay rate larger, or smaller?

- decay rate $\propto e^{-B[\bar{\sigma}, \bar{\phi}, \bar{a}]}$ $B[\bar{\sigma}, \bar{\phi}, \bar{a}] = S_E[\bar{\sigma}, \bar{\phi}, \bar{a}] - S_E[\sigma_F, \phi_F, a_F]$

(Coleman and De Luccia, 1980)

- multi-field instanton

$$\bar{\sigma}(t), \bar{\phi}(t), \bar{a}(t) \quad \rightarrow \quad B = B[\bar{\sigma}, \bar{\phi}, \bar{a}]$$

- instanton when neglecting dynamical freedom of φ

$$\bar{\sigma}_0(t), \bar{\phi}_0(t), \bar{a}_0(t)$$

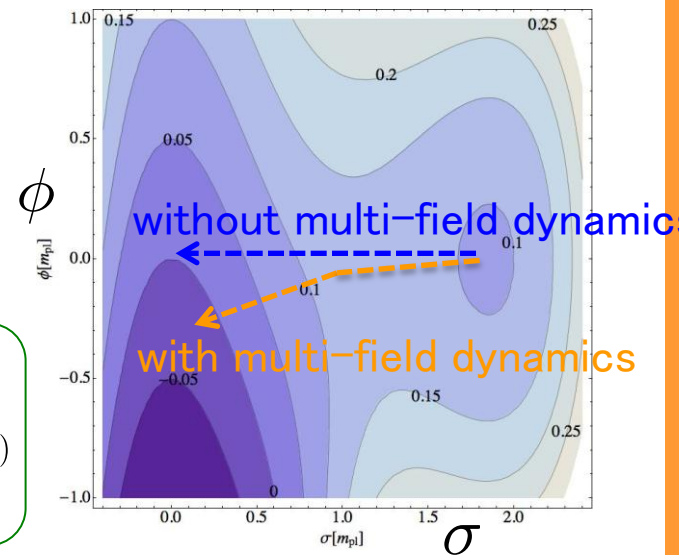
- staying at false vacuum

$$\bar{\phi}_0(t) = \phi_F$$

- single field instanton for $V_0(\sigma) = V(\sigma, \phi_F)$

$$\bar{\sigma}_0(t), \bar{a}_0(t)$$

$$\rightarrow \quad B_0 = B[\bar{\sigma}_0, \bar{\phi}_0, \bar{a}_0]$$



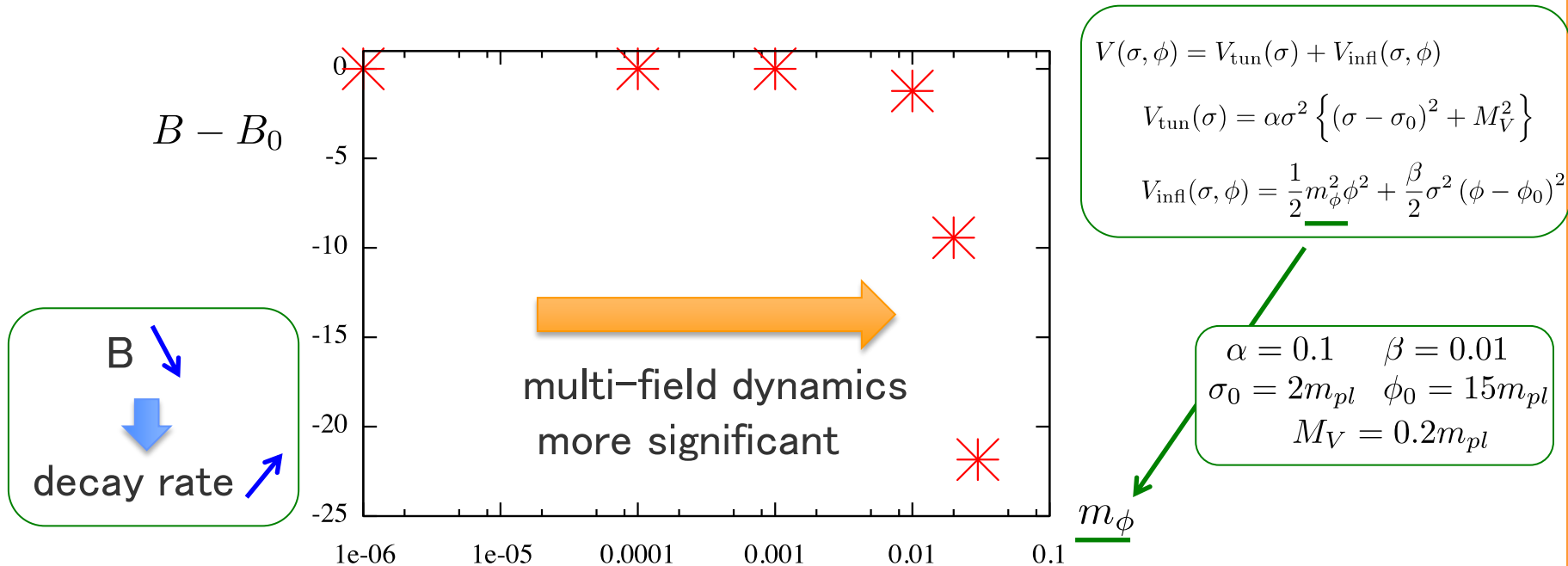
Contour of $V(\sigma, \varphi)$

- $B - B_0$ comes from the multi-field dynamics effect

Multi-field dynamics effect on decay rate

$$B[\bar{\sigma}, \bar{\phi}, \bar{a}] = S_E[\bar{\sigma}, \bar{\phi}, \bar{a}] - S_E[\sigma_F, \phi_F, a_F]$$

- decay rate $\propto e^{-B[\bar{\sigma}, \bar{\phi}, \bar{a}]}$
- difference of B due to multi-field dynamics $B - B_0$



A. The multi-field dynamics make decay rate larger!!



Decomposition of energy density in the universe after tunneling

- Hubble parameter $H^2 = \frac{1}{3m_{pl}^2} \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\sigma}^2 + V(\sigma, \phi) \right) + \frac{1}{a^2}$
- inflaton ϕ $H_\phi^2 = \frac{1}{3m_{pl}^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\sigma_T, \phi) \right)$
- tunneling field σ $H_\sigma^2 = \frac{1}{3m_{pl}^2} \left(\frac{1}{2}\dot{\sigma}^2 + V(\sigma, \phi) - V(\sigma_T, \phi) \right)$
- curvature $H_k^2 = \frac{1}{a^2}$

m_ϕ dependence of the system

m_ϕ changes not only the strength of interaction but also the non-interaction part

$$V(\sigma, \phi) = V_{\text{tun}}(\sigma) + V_{\text{infl}}(\sigma, \phi)$$

$$V_{\text{tun}}(\sigma) = \alpha \sigma^2 \left\{ (\sigma - \sigma_0)^2 + M_V^2 \right\}$$

$$V_{\text{infl}}(\sigma, \phi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{\beta}{2} \sigma^2 (\phi - \phi_0)^2$$

different m_ϕ

different

$$V_0(\sigma) = V(\sigma, \phi_F)$$

$$\bar{\sigma}_0(t), \bar{\phi}_0(t), \bar{a}_0(t)$$

$$B_0 = B[\bar{\sigma}_0, \bar{\phi}_0, \bar{a}_0]$$

$$\alpha = 0.1 \quad \beta = 0.01$$

$$\sigma_0 = 2m_{pl} \quad \phi_0 = 15m_{pl}$$

$$M_V = 0.2m_{pl}$$

large m_ϕ \Rightarrow large $V_0(\sigma_F)$
 \Rightarrow small V''/H^2

- in the case of the very large m_ϕ , there may exist only a Hawking-Moss instanton (barrier is effectively small)

	B	B_0	ΔB
$m_\phi = 10^{-6} m_{pl}$	12109.11	12109.11	0.00
$m_\phi = 10^{-4} m_{pl}$	12108.10	12108.10	0.00
$m_\phi = 10^{-3} m_{pl}$	12008.71	12008.71	0.00
$m_\phi = 10^{-2} m_{pl}$	6328.61	6329.83	-1.23
$m_\phi = 2 \times 10^{-2} m_{pl}$	2196.44	2205.89	-9.44
$m_\phi = 3 \times 10^{-2} m_{pl}$	844.13	865.98	-21.84