



TOHOKU
UNIVERSITY



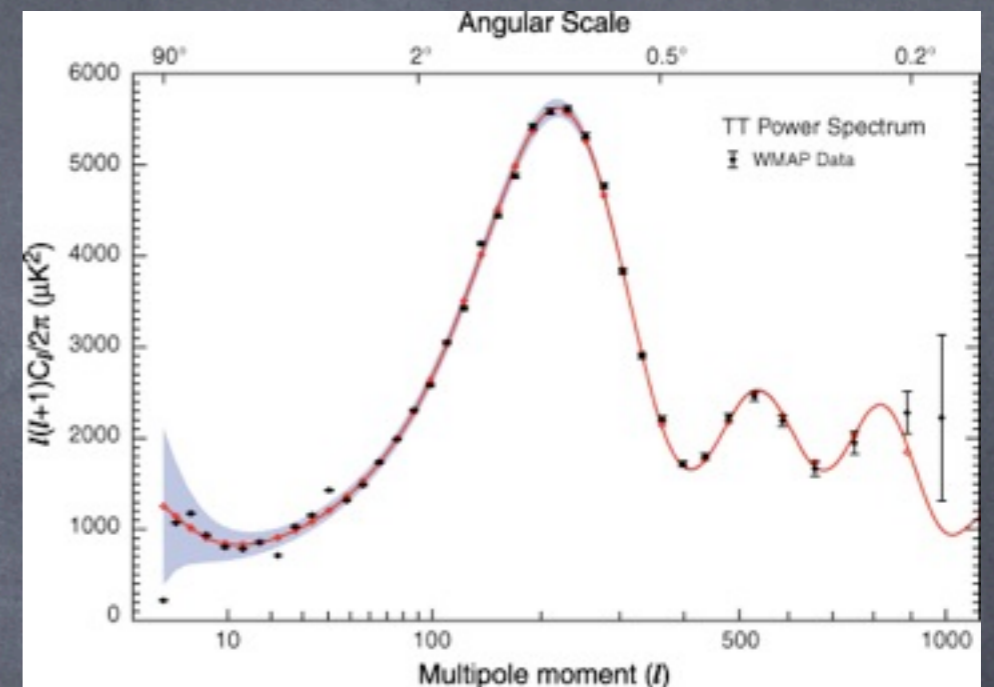
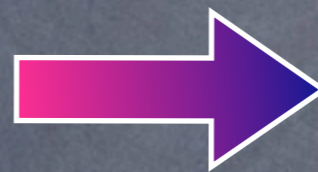
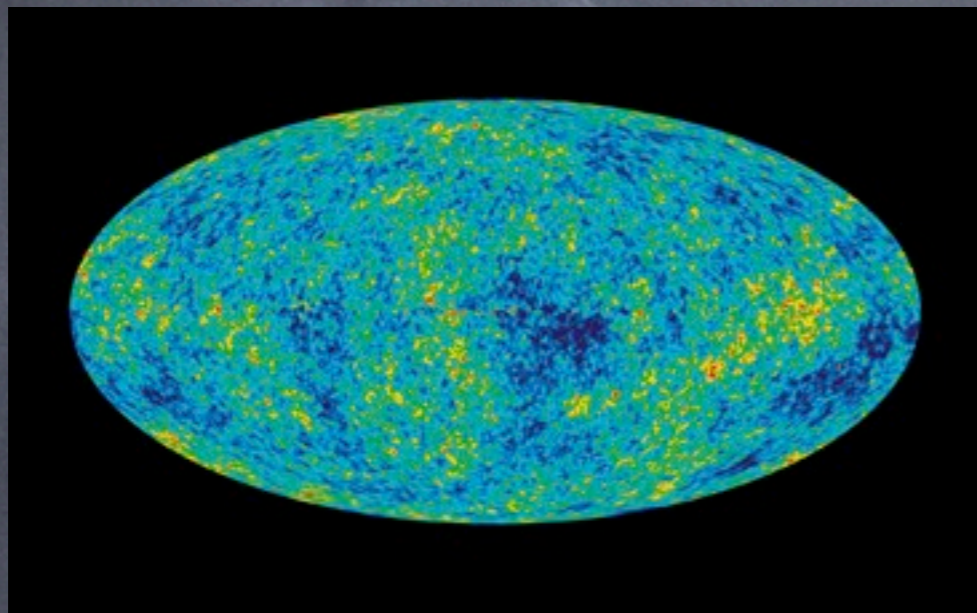
Non-Gaussianity from Hilltop Curvaton

Sep.29@JGGRG21
Tohoku Univ.

Fuminobu TAKAHASHI
(Tohoku U and IPMU)

arXiv:1107.6011 M. Kawasaki, Takeshi Kobayashi, FT

What is the origin of the density perturbation???



CMB temperature anisotropy contains information on the very early universe.

The prime candidate is the inflaton, but..

Life may be more complicated than we expect.



Curvaton

Linde, Mukhanov '97
Enqvist and Sloth '01
Lyth and Wands '01
Moroi and Takahashi '01

1. Light during inflation

$$m \lesssim 0.1 H_{\text{inf}}$$

The light mass can be naturally realized if the curvaton has an approximate shift symmetry.

2. Comes close to dominating the Universe

$$r \equiv \left. \frac{\rho_{\sigma}}{\rho_r} \right|_{\text{decay}} > \mathcal{O}(0.01)$$

3. Can generate non-Gaussianity.

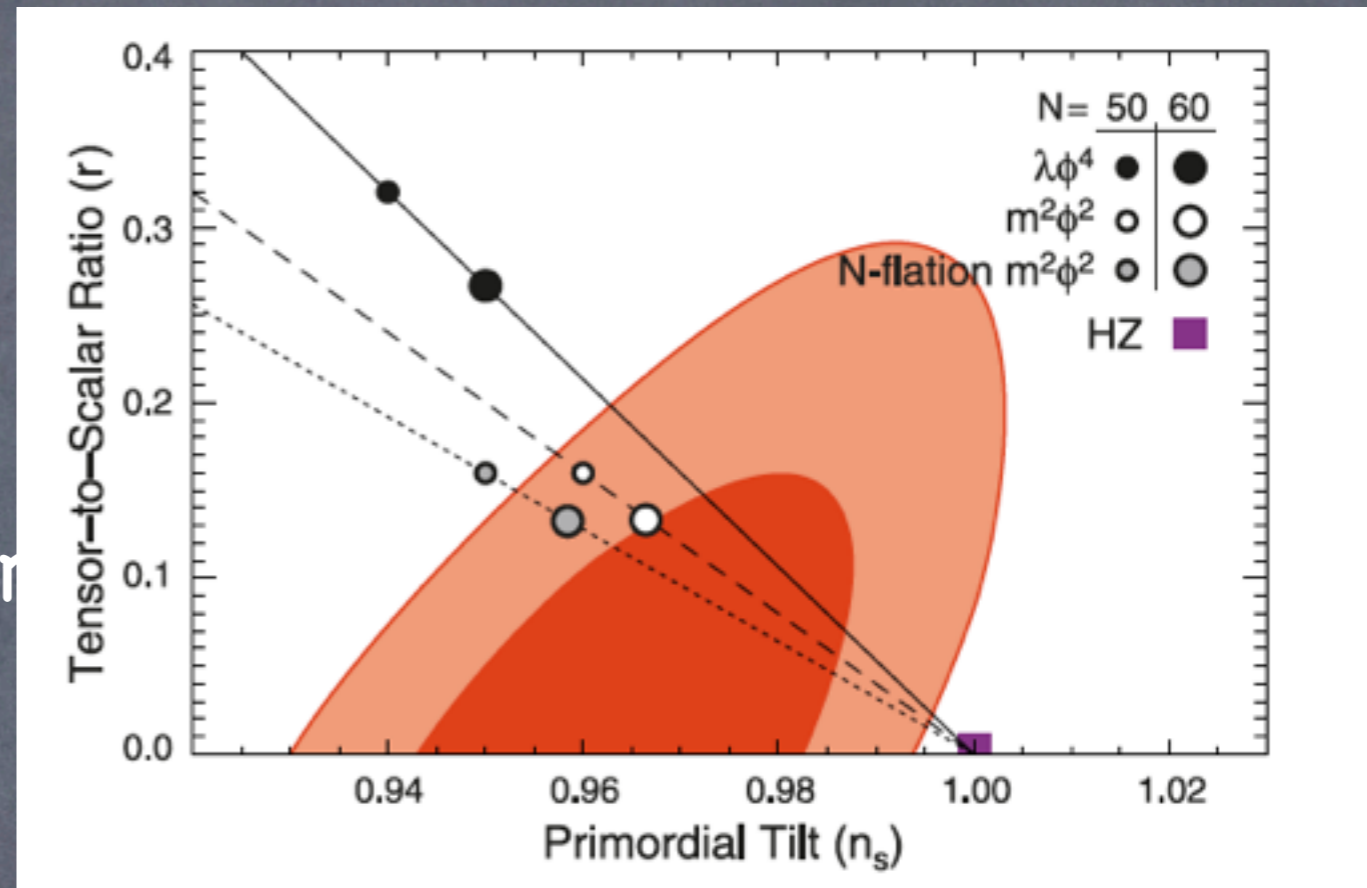
Lore of curvaton

• $f_{NL} \sim 1/r$ (?)

• If we want to have fine-tuning? Another

• $n_s = 1$ (?)

• Two-point functions should be consistent with obs. before talking about three-point function.



What does $n_s < 1$ mean?

Formula: $n_s - 1 \simeq \frac{2 V''(\sigma_*)}{3 H_*^2} + 2 \frac{\dot{H}_*}{H_*^2}$

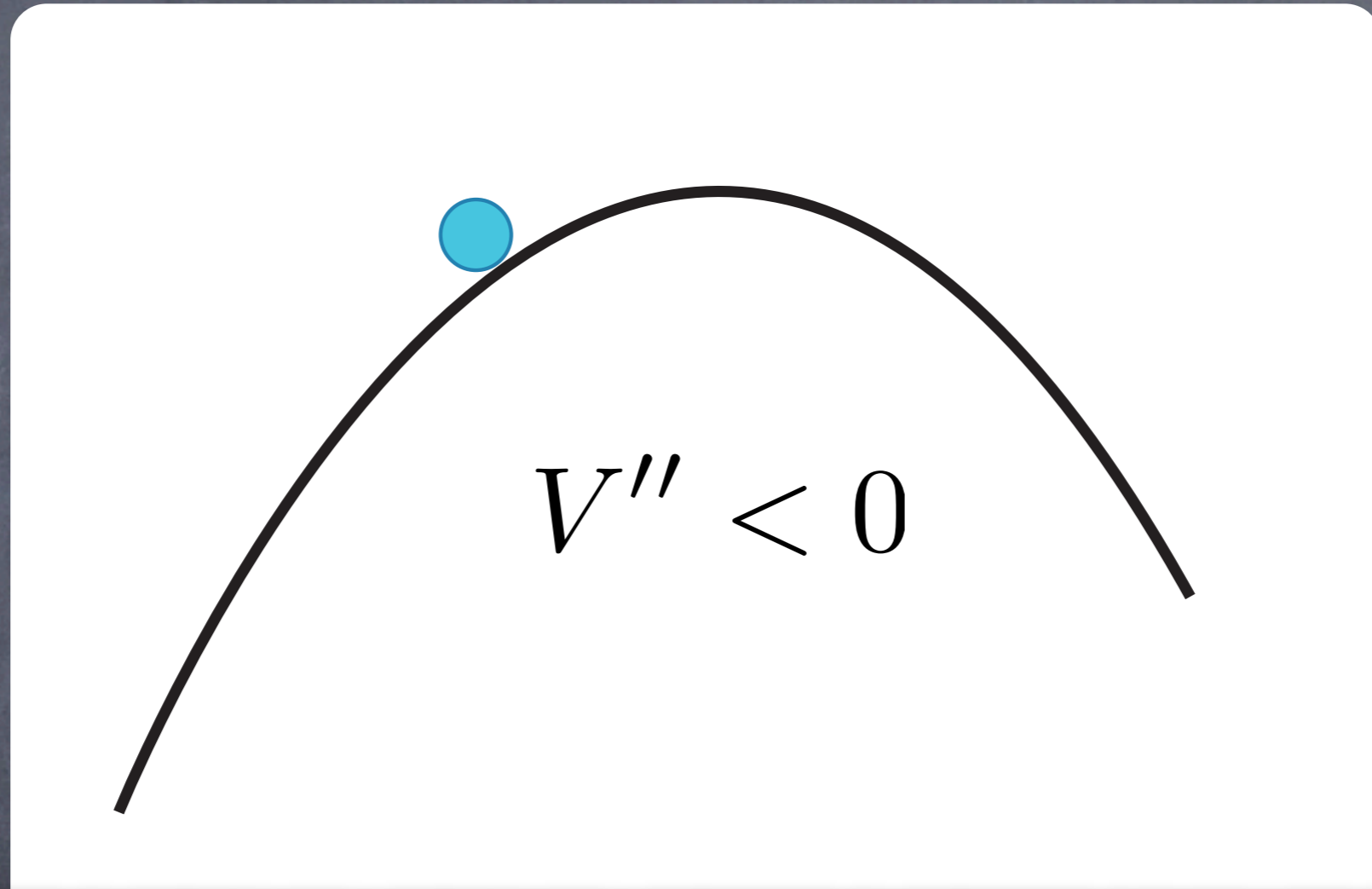
σ : curvaton

In order to obtain the red-tilted spectrum without relying on the inflation models, the curvaton must satisfy

$$V''(\sigma_*) \simeq -\mathcal{O}(0.01) \times H_*^2$$

Negative and (relatively) large mass!

The potential needs to be deviated
from the quadratic potential!



Does the curvaton dominate the Universe?
Does the prediction of the density
perturbation change?

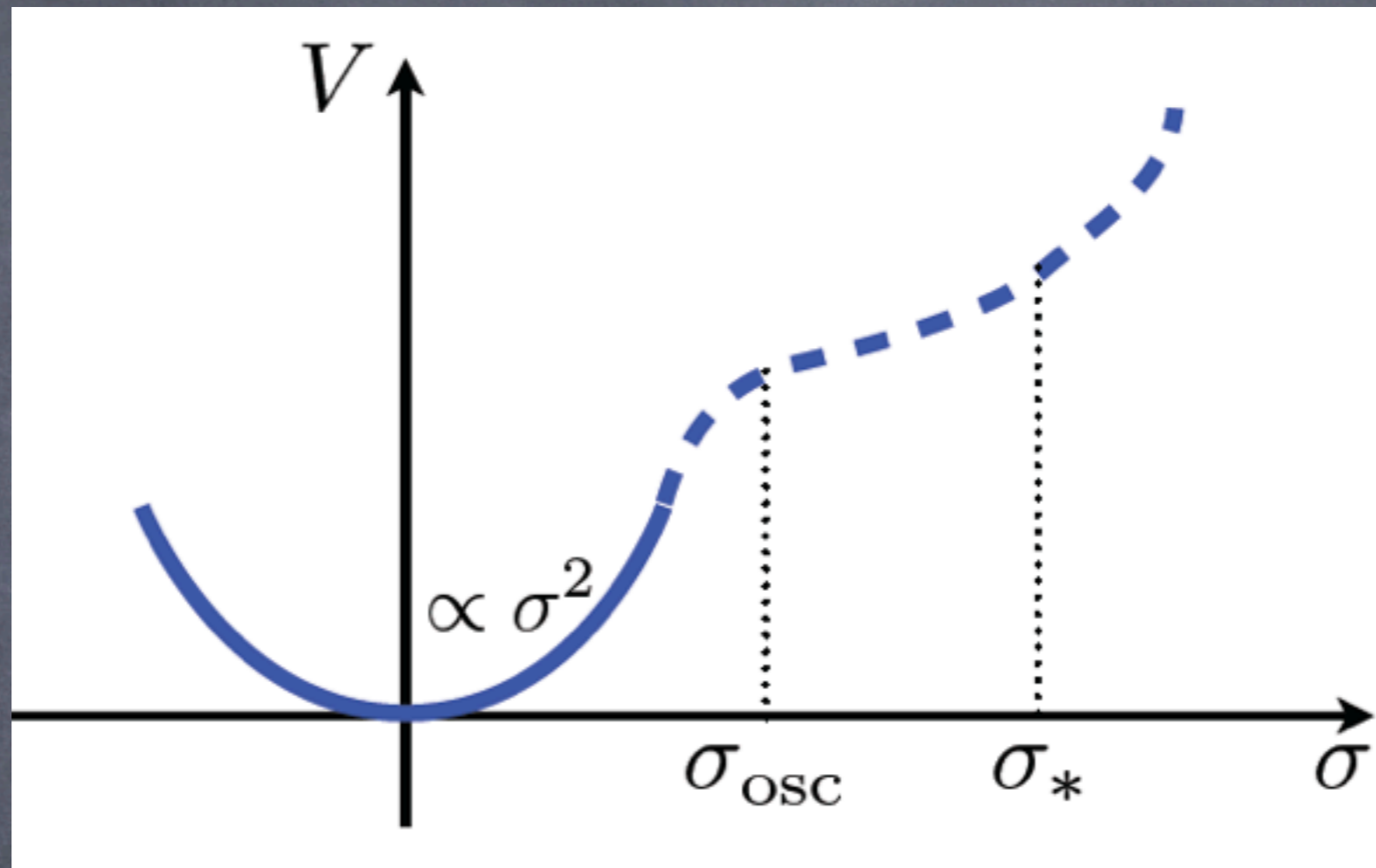
For the curvaton mechanism to work, the energy fraction should be sizable at the decay.

There are two possibilities.

1. Very long lifetime

2. Initial position close to the hilltop.

Very rough sketch



$$\zeta \sim c_1 \frac{\delta \rho_\sigma}{\rho_\sigma} - c_2 \frac{\delta H_{\text{osc}}}{H_{\text{osc}}}$$

non-uniform onset of oscillations

A bit more detail

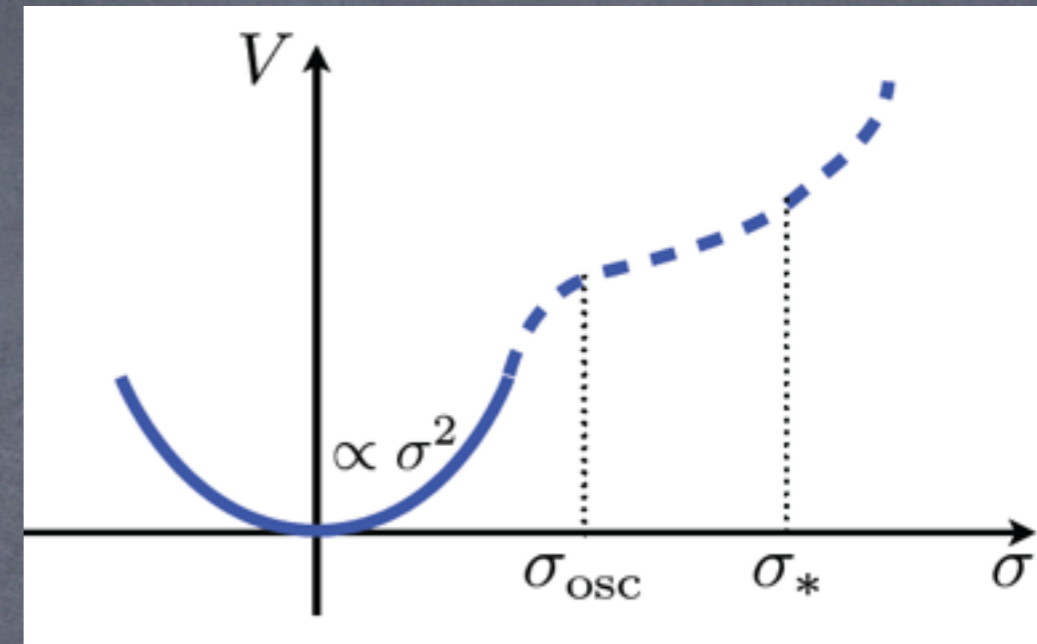
We evaluate σ_{osc} as a function of σ_* using an attractor solution.

$$cH(t)\dot{\sigma} \simeq -\frac{\partial V(\sigma)}{\partial \sigma}$$

(This is valid as long as $\left| \frac{V''}{cH^2} \right| \ll 1$)

We define H_{osc} as $\left. \frac{\dot{\sigma}}{H\sigma} \right|_{\text{osc}} = 1$

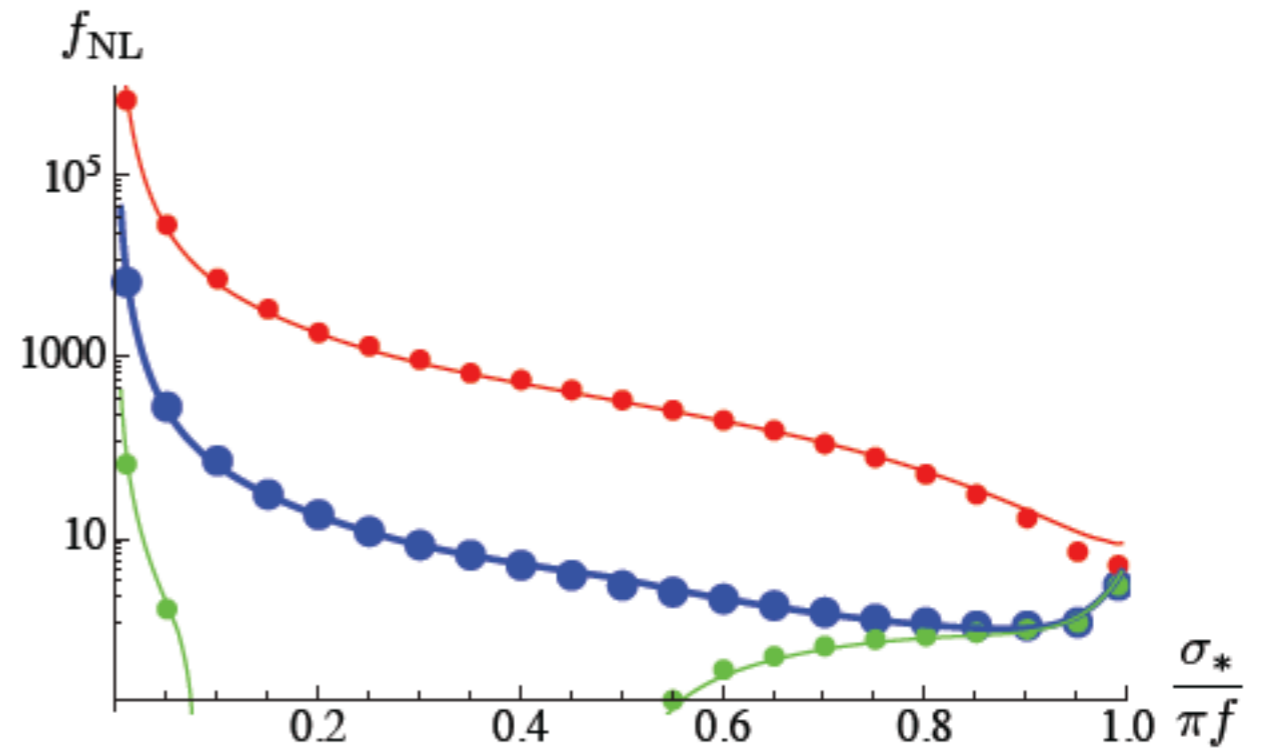
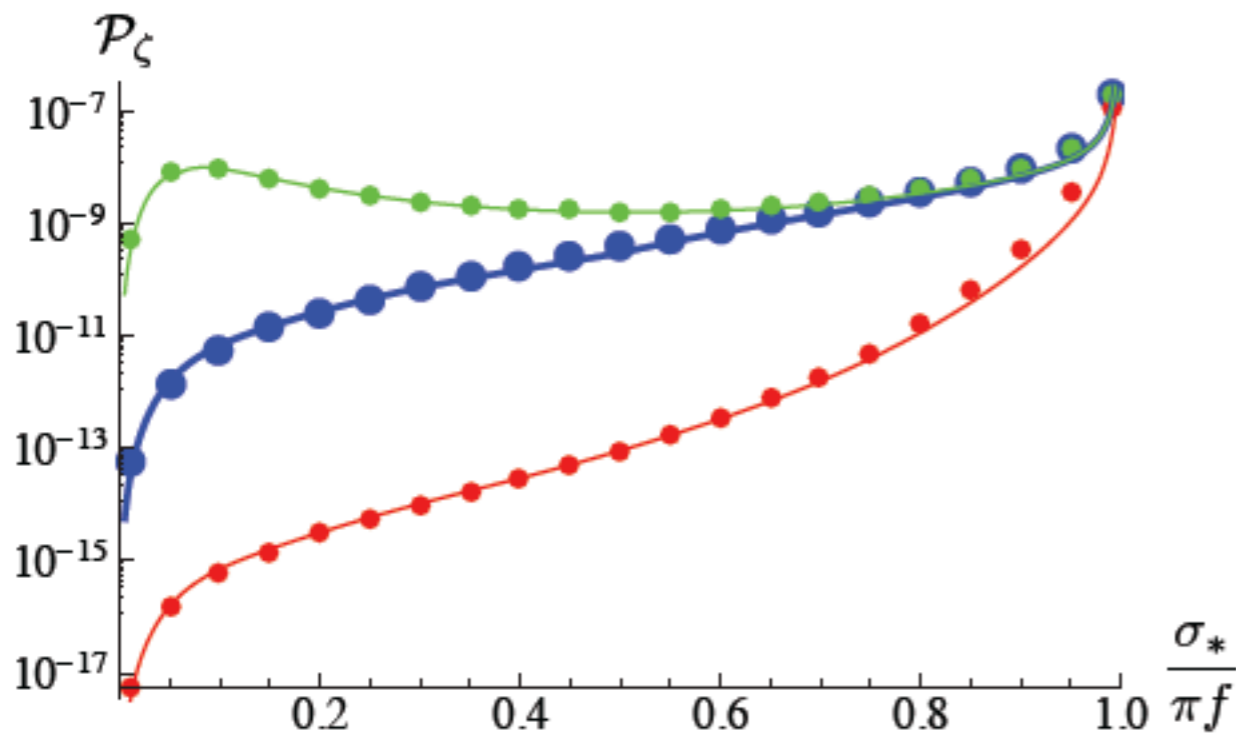
Using the δN -formalism, we estimated the density perturbation analytically.



Results

$$\frac{\partial \mathcal{N}}{\partial \sigma_*} = \frac{r}{4 + 3r} (1 - X(\sigma_{\text{osc}}))^{-1} \left\{ \frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} \right\} \frac{V'(\sigma_{\text{osc}})}{V'(\sigma_*)}.$$

$$5 \partial^2 \mathcal{N} (\partial \mathcal{N})^{-2}$$



$$2(c-3) \left(\frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} \right)$$

$$\rho_r |_{\text{dec}}$$

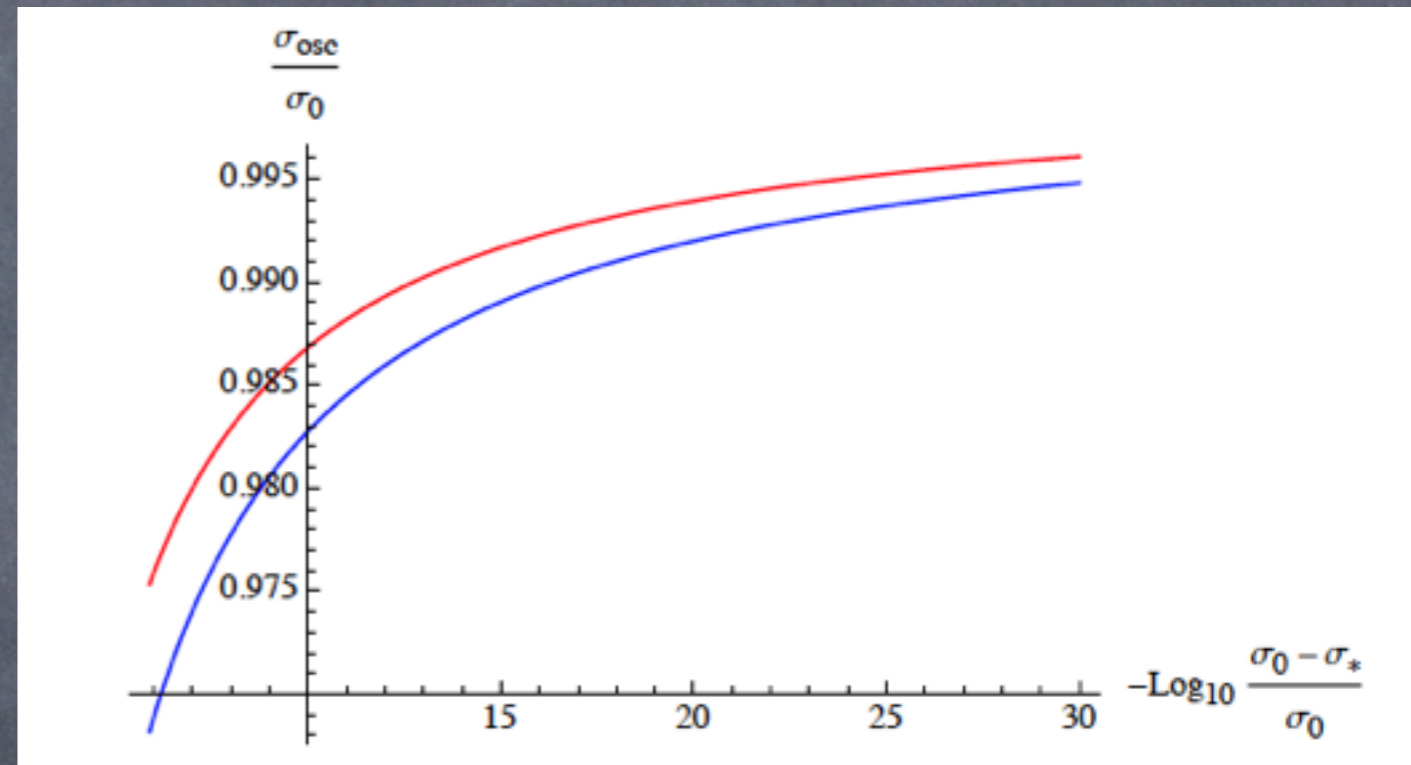
N.B. X represents the fluctuation of H_{osc} , and it vanishes in the case of the quadratic potential.

Hilltop curvaton

$$V(\sigma) = V_0 - \frac{1}{2}m^2(\sigma - \sigma_0)^2$$

$$\mathcal{P}_\zeta^{1/2} \simeq \frac{3r}{4+3r} \frac{\sigma_0 - \sigma_{\text{osc}}}{\sigma_0 - \sigma_*} \frac{H_{\text{inf}}}{2\pi\sigma_{\text{osc}}},$$

$$f_{\text{NL}} \simeq \frac{5(4+3r)}{18r} \frac{\sigma_{\text{osc}}}{\sigma_0 - \sigma_{\text{osc}}},$$



The curvature perturbation rapidly increases while f_{NL} only mildly logarithmically increases in the hilltop limit.

Application to NG curvaton

$$V(\sigma) = \Lambda^4 \left[1 - \cos\left(\frac{\sigma}{f}\right) \right]$$

(1) non-hilltop case

We assume

$$n_s = 0.96$$

$$\Gamma_\sigma \sim \frac{1}{16\pi} \frac{m^3}{f^2} = \frac{1}{16\pi} \frac{\Lambda^6}{f^5},$$

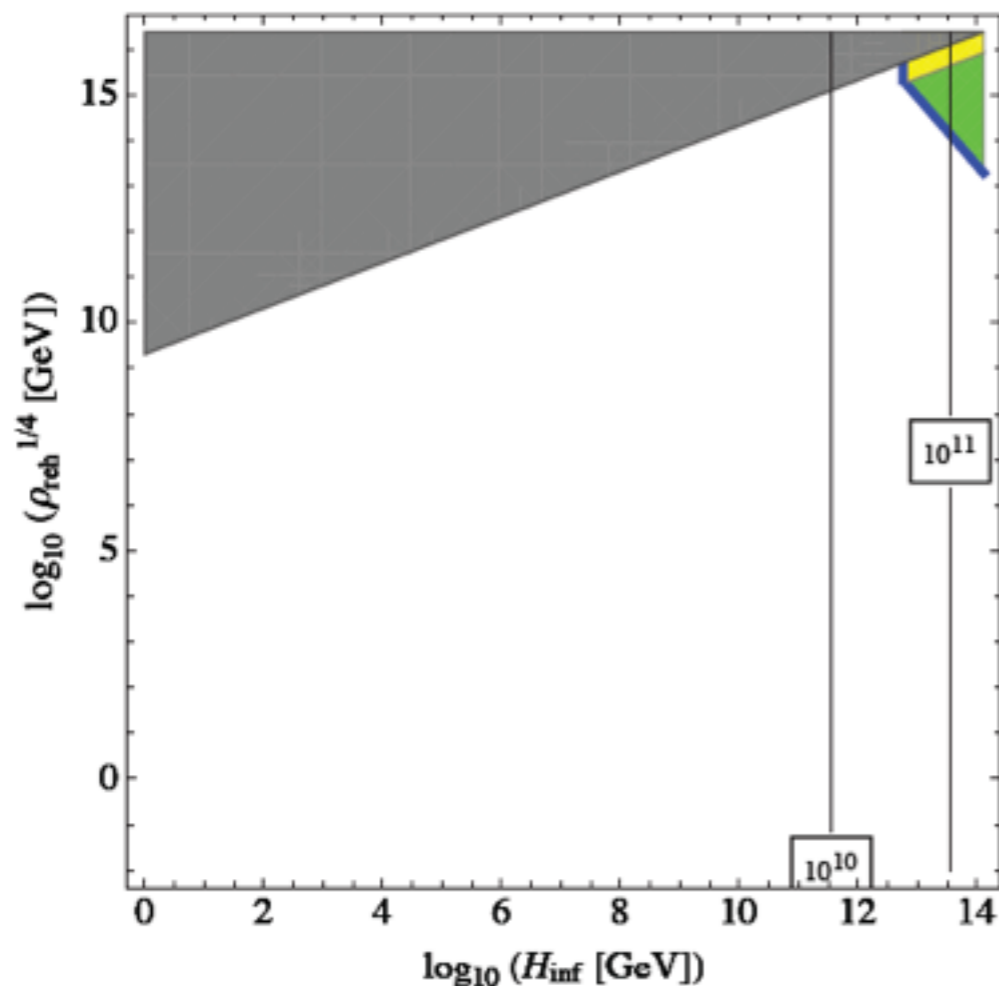


Figure 11: $\sigma_*/\pi f = 3/4$, $r > 1$.

Very high T_R and H_{inf} are required;

because the curvaton soon starts to oscillate after inflation in the non-hilltop case.

Application to NG curvaton

(2) hilltop case

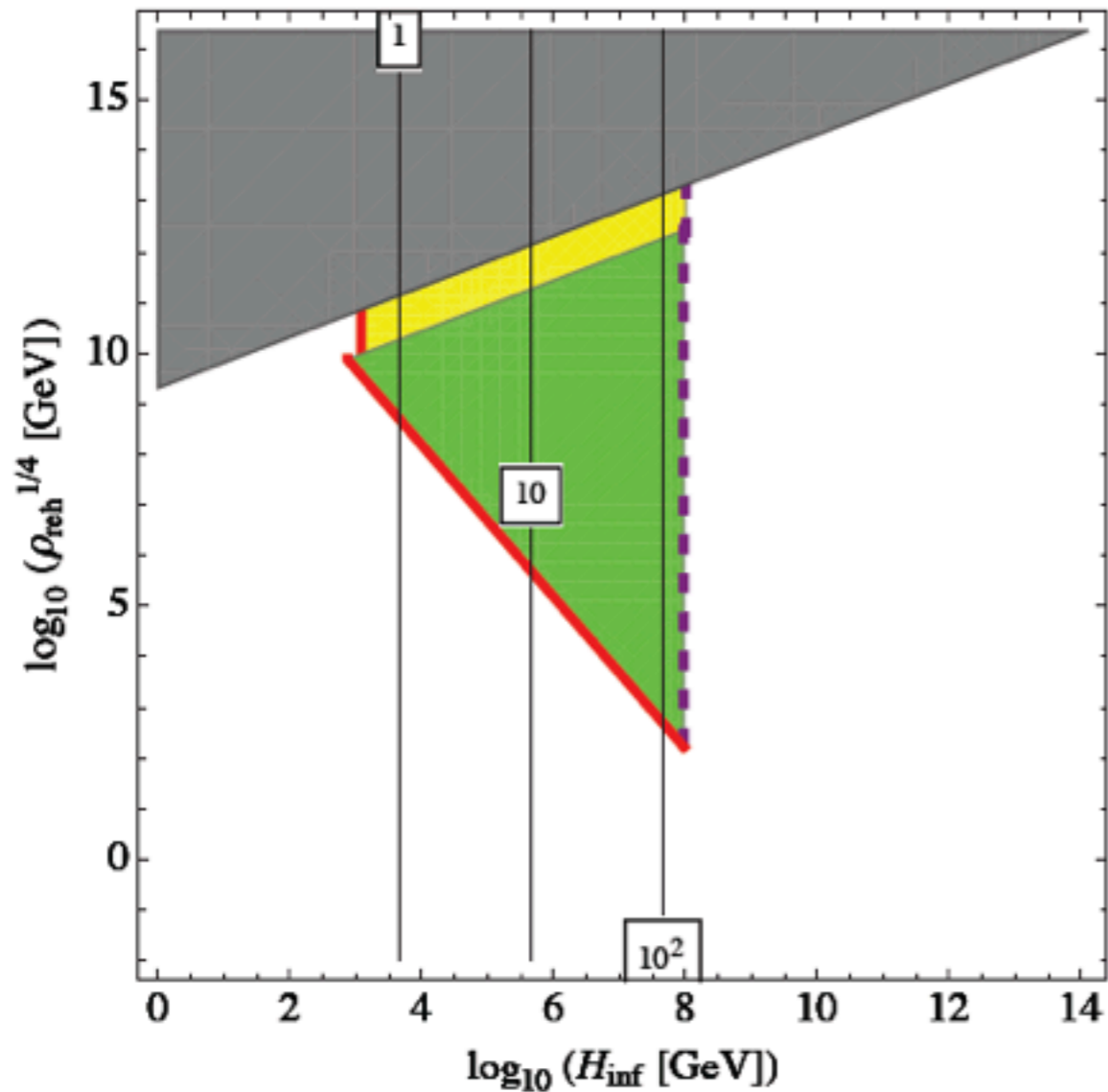


Figure 15: $\sigma_*/\pi f = 1 - 10^{-8}$, $r > 1$. $f_{\text{NL}} \sim 20$ in most of the allowed region.

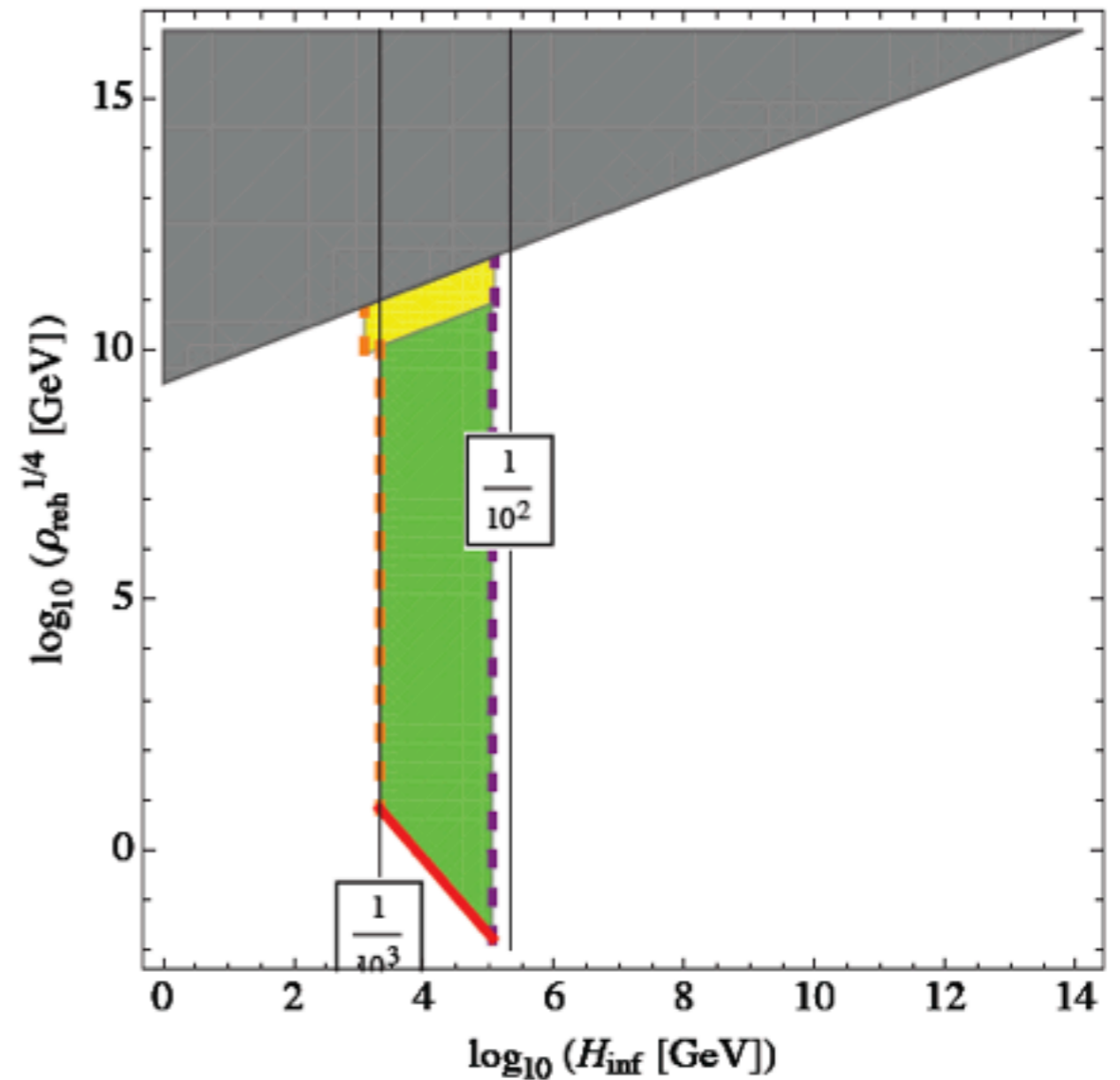


Figure 16: $\sigma_*/\pi f = 1 - 10^{-11}$, $r > 1$. $f_{\text{NL}} \sim 30$ in most of the allowed region.

Summary

- The red-tilted power spectrum suggests a **negative** curvature of the curvaton potential:

$$V''(\sigma_*) \simeq -\mathcal{O}(0.01) \times H_*^2$$

- In the hilltop limit, **f_{NL} can be as large as $O(10)$ for dominant curvaton**, and it is not sensitive to the initial deviation from the maximum.
- **In the case of the NG curvaton, $f_{\text{NL}} = 10-30$.**

back-up slides

Spectral Index

Using $\frac{\partial \mathcal{N}}{\partial \sigma_*} = (\text{function of } r, \sigma_{\text{osc}}) \times \frac{1}{V'(\sigma_*)}$,

we obtain

$$\begin{aligned} n_s - 1 &= \frac{d}{d \ln k} \ln \left(\frac{H_*}{V'(\sigma_*)} \right)^2 \\ &\approx \frac{2}{3} \frac{V''(\sigma_*)}{H_*^2} + 2 \frac{\dot{H}_*}{H_*^2}, \end{aligned}$$

• For non-tachyonic curvaton,

$$n_s - 1 \geq \frac{2\dot{H}}{H^2}$$

On the other hand, assuming single-field canonical slow-roll inflaton,

$$\frac{1}{M_p} \left| \frac{d\phi}{d\mathcal{N}} \right| \geq \sqrt{1 - n_s}$$

Therefore, $n_s = 0.96$ requires a super-Planckian field range for the inflaton.

Attractor Solution

see arXiv:1107.6011

EOM:
$$\ddot{\sigma} + 3H(t)\dot{\sigma} + \frac{\partial V(\sigma)}{\partial \sigma} = 0$$

can be approximated by

$$cH(t)\dot{\sigma} \simeq -\frac{\partial V(\sigma)}{\partial \sigma},$$

if the following is satisfied.

$$\left| \frac{V''}{cH^2} \right| \ll 1,$$

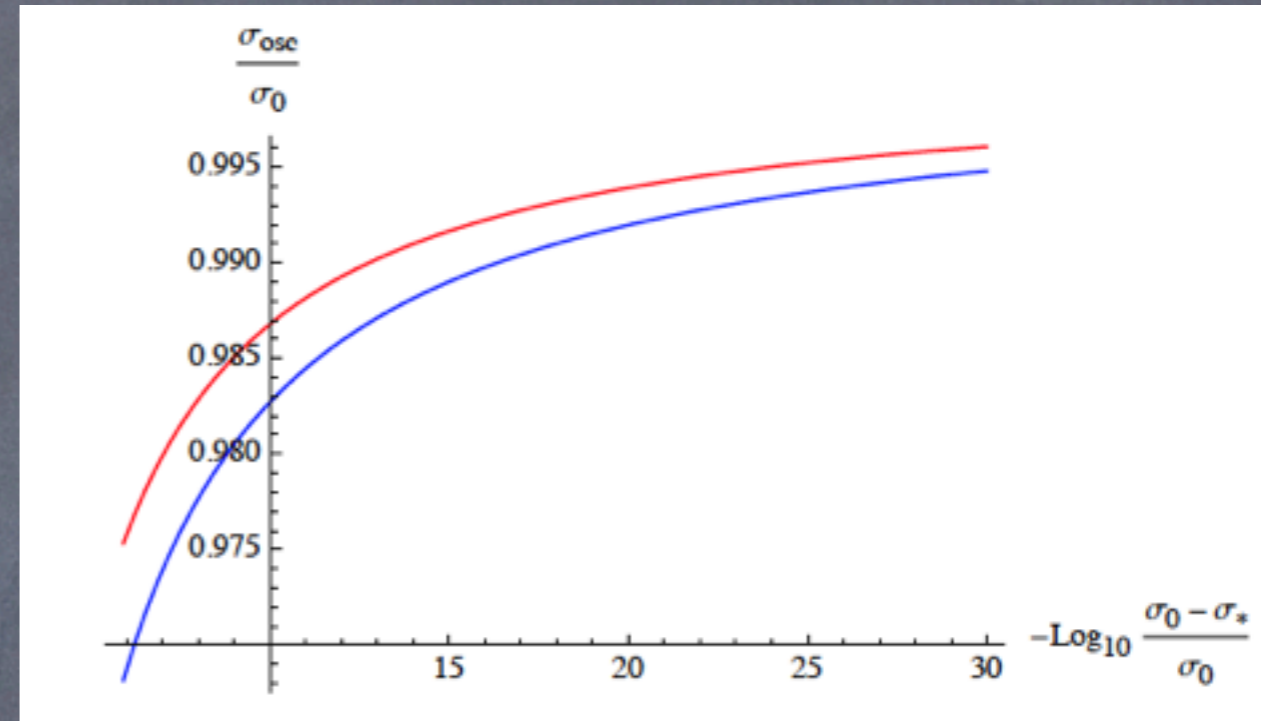
Hilltop limit

$$V(\sigma) = V_0 - \frac{1}{2}m^2(\sigma - \sigma_0)^2$$

$$\mathcal{P}_\zeta^{1/2} \simeq \frac{3r}{4 + 3r} \frac{\sigma_0 - \sigma_{\text{osc}}}{\sigma_0 - \sigma_*} \frac{H_{\text{inf}}}{2\pi\sigma_{\text{osc}}},$$

$$f_{\text{NL}} \simeq \frac{5(4 + 3r)}{18r} \frac{\sigma_{\text{osc}}}{\sigma_0 - \sigma_{\text{osc}}},$$

$$n_s - 1 = -\frac{2}{3} \frac{m^2}{H_{\text{inf}}^2} < 0,$$



f_{NL} increases VERY SLOWLY in the hilltop limit.

pseudo-Nambu-Goldstone Curvaton

black : $t_{\text{osc}} \gtrsim t_{\text{reh}}$

blue : $r \gtrsim 1$

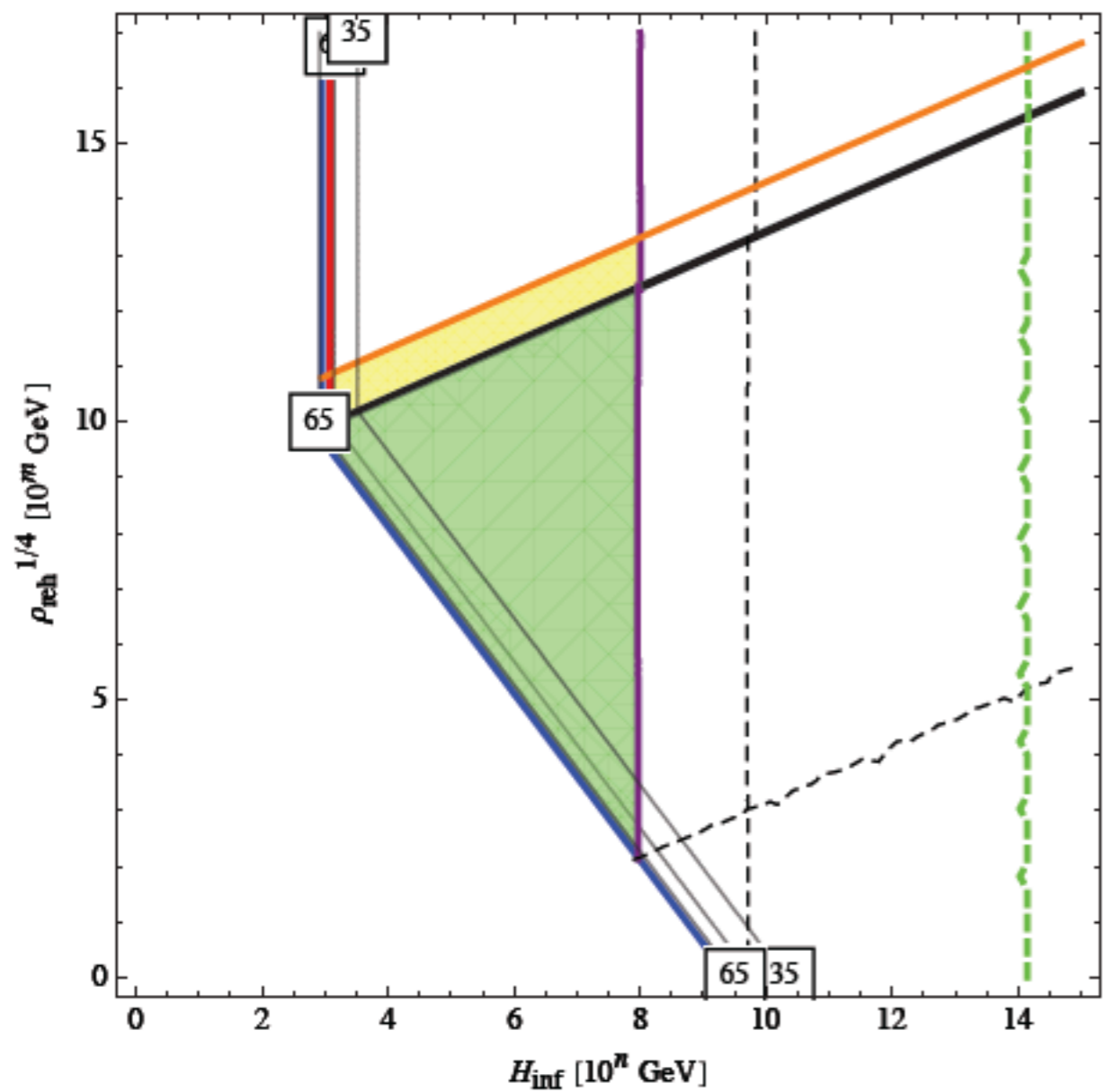
pink dashed : BBN

purple : curvaton subdominant at onset of oscillation
(energy density smaller than 100% (1%) for
dominant (subdominant) curvaton at t_{osc})

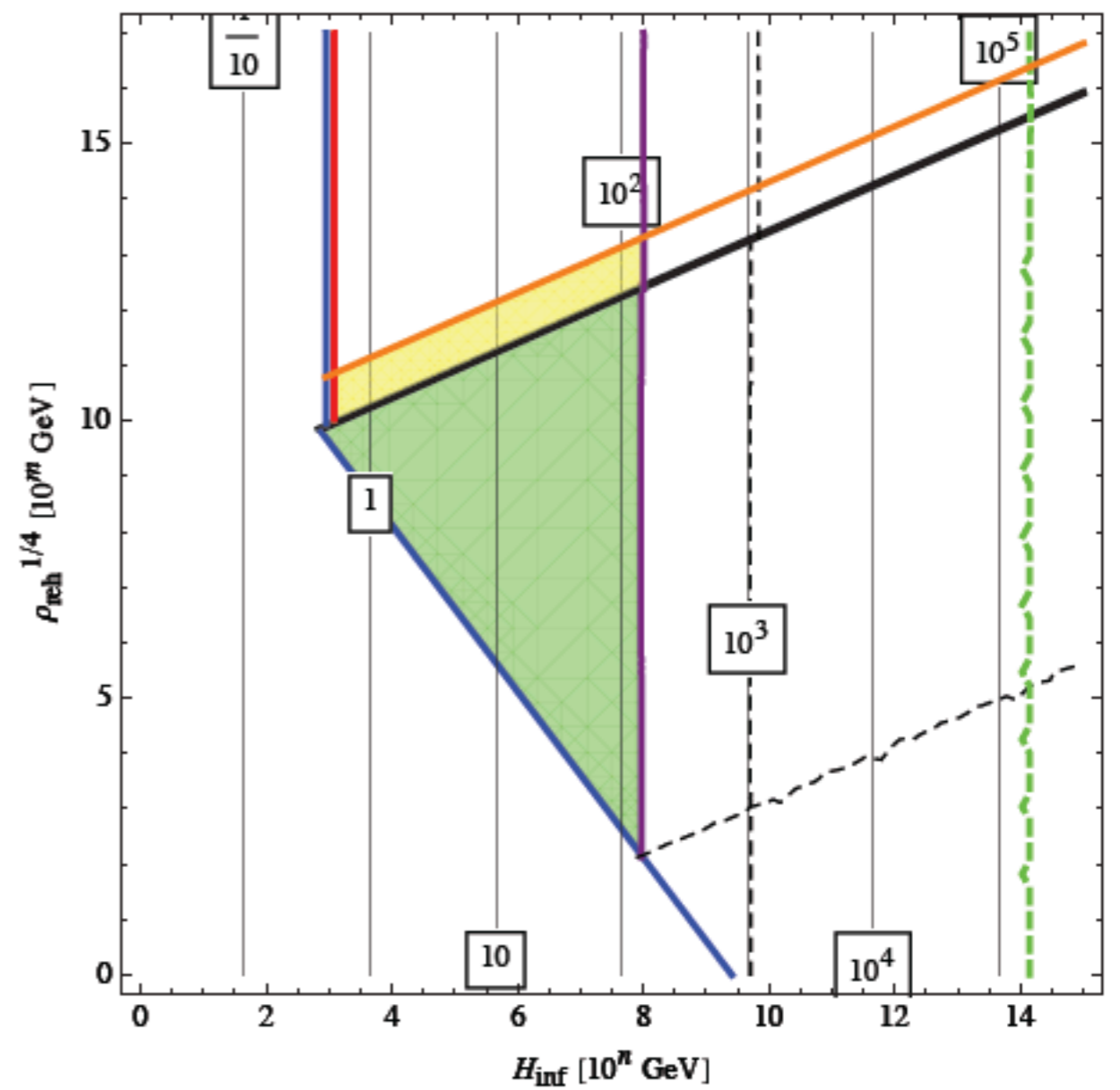
red : $f_{\text{NL}} < 74$

green dashed : GW bound

orange : $t_{\text{end}} < t_{\text{reh}}$

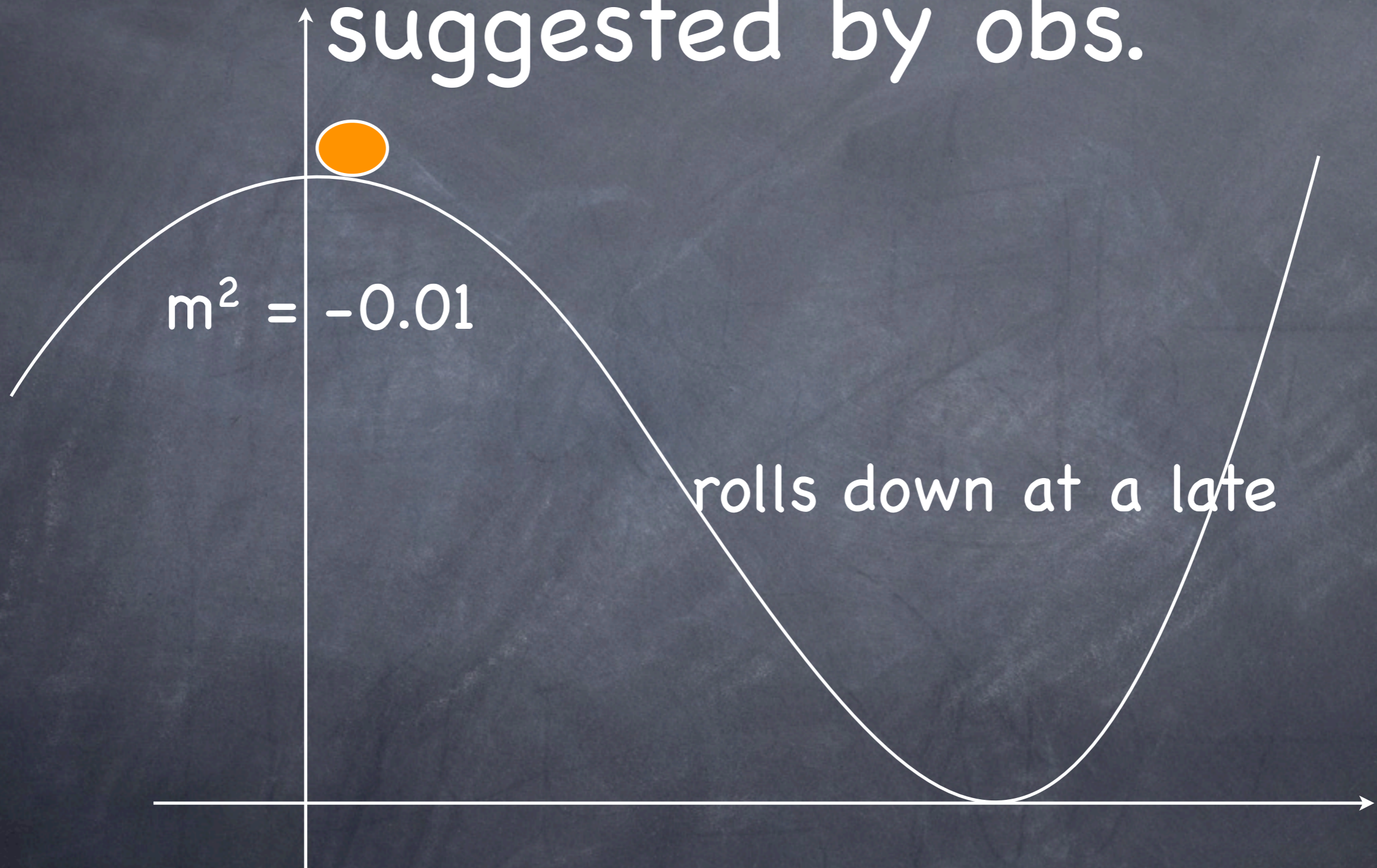


fNL ~ 35 in most of the allowed region
(numbers in boxes : fNL)



curvaton decay energy density^{1/4} ~
temperature in terms of GeV

Hilltop Curvaton is suggested by obs.



We consider a Nambu-Goldstone boson type model.

Any prediction?

- Yes!

- Predicts non-gaussianity f_{NL} as

$$f_{\text{NL}} = 20-40$$

for most parameter space, and

- non-Gaussianity is generated for the dominant curvaton.

Analysis

$$\begin{aligned}
 f_{\text{NL}} &= \frac{5}{6} \frac{\partial^2 \mathcal{N}}{\partial \sigma_*^2} \left(\frac{\partial \mathcal{N}}{\partial \sigma_*} \right)^{-2} \\
 &= \frac{40(1+r)}{3r(4+3r)} + \frac{5(4+3r)}{6r} \left\{ \frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} \right\}^{-1} \left[(1-X(\sigma_{\text{osc}}))^{-1} X'(\sigma_{\text{osc}}) \right. \\
 &\quad + \left. \left\{ \frac{V'(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} \right\}^{-1} \left\{ \frac{V'''(\sigma_{\text{osc}})}{V(\sigma_{\text{osc}})} - \frac{V'(\sigma_{\text{osc}})^2}{V(\sigma_{\text{osc}})^2} - \frac{3X'(\sigma_{\text{osc}})}{\sigma_{\text{osc}}} + \frac{3X(\sigma_{\text{osc}})}{\sigma_{\text{osc}}^2} \right\} \right. \\
 &\quad \left. \left. + \frac{V'''(\sigma_{\text{osc}})}{V'(\sigma_{\text{osc}})} - (1-X(\sigma_{\text{osc}})) \frac{V'''(\sigma_*)}{V'(\sigma_{\text{osc}})} \right] .
 \end{aligned}$$