Parity Violation in Graviton Non-Gaussianity

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based on JHEP 1108 067 (2011)

Parity violation

Origin of chirality

- CPT invariance is fundamental
- CP & T violations may be transmitted to gravity sector via field eqs.
- In GR, this is generally suppressed

String theory

- candidate for unified theory & quantum gravity
- imples P-violating gravitational interactions *Green & Schwarz*

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right] + S_{\rm PV}$$

$$S_{\rm PV} = \frac{\alpha'}{8\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left(F^a_{\mu\nu}\star F^{a\mu\nu} - \mathcal{R}_{\mu\nu\rho\sigma}\star\mathcal{R}^{\mu\nu\rho\sigma}\right)$$

- Chern-Simons interactions are ubiquitous in string theory

  $\Rightarrow$ detection of P-violation of gravity will shed light on ultimate theory

Gravitational waves

Inflationary universe as HEP laboratory

- best testbed to explore parity violation
- primordial gravitational waves during inflation

  ⇒ P-violation encoded in power spectrum

  - have different GW amplitudes b/w +ve & -ve helicity (circular polarizations)
  - detectable through correlations of CMB

Seto 05, Seto-Taruya 06, Saito-Ichiki-Taruya 07, Sato & Soda 08, Takahashi-Soda 09

Our work

Focus on *non-Gaussianity* of gravitons

- gives another useful measure for the Planck scale physics

Correlators via dual CFT

Maldacena-Pimentel 2011

- discussed graviton non-Gaussianity in dS by studying 3D CFT
- shapes of bispectrum are constrained by *dS isometry* SO(*4,1*)

$$S = \int \mathrm{d}^4x\sqrt{-g}\,\left(\tfrac{1}{2}M_{\rm pl}^2 R + aW^{\mu\nu}{}_{\rho\sigma}W^{\rho\sigma}{}_{\tau\lambda}W^{\tau\lambda}{}_{\mu\nu} - b\star W^{\mu\nu}{}_{\rho\sigma}W^{\rho\sigma}{}_{\tau\lambda}W^{\tau\lambda}{}_{\mu\nu}\right)$$

$W_{\mu\nu\rho\sigma}$:Weyl tensor $\qquad \star W_{\mu\nu\rho\sigma} = \frac{1}{2}\epsilon_{\mu\nu\lambda\tau}W^{\lambda\tau}{}_{\rho\sigma}$

- valid in arbitrary order in derivative expansion

this does not imply that parity violation appears in graviton bispectrum

What we would like to do

- reveal conditions under which the parity-violation arises in bispectrum
- develop a new formalism for computing higher-order graviton correlators

Plan

(I) New formalism for graviton correlators

(II) Graviton bispectrum during inflation

(III) Concluding remarks

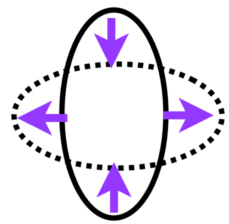
GWs in Minkowski spacetime

Polarization of GWs

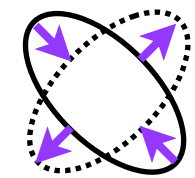
$$\mathrm{d}s^2 = -\mathrm{d}\eta^2 + (\delta_{ij} + \gamma_{ij})\mathrm{d}x^i\mathrm{d}x^j \qquad \gamma_{ii} = \partial_i\gamma_{ij} = 0 \qquad \gamma''_{ij} - \Delta\gamma_{ij} = 0$$

•linear polarization $\gamma_{ij} = \gamma_+ e_{ij}^{(\mathrm{plus})} + \gamma_\times e_{ij}^{(\mathrm{cross})}$

plus mode γ_+



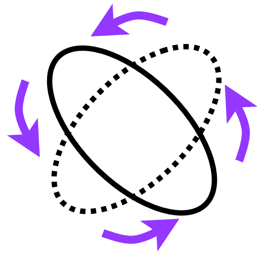
cross mode $\gamma_\times$



•circular polarization $\gamma_{ij} = \gamma^L e_{ij}^{(+)} + \gamma^R e_{ij}^{(-)}$

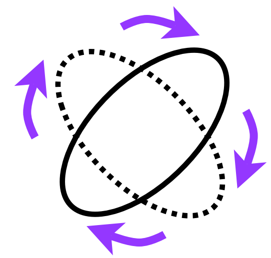
left-handed

$\gamma^L = \frac{1}{\sqrt{2}}(\gamma_+ - \mathrm{i}\gamma_\times)$



right-handed

$\gamma^R = \frac{1}{\sqrt{2}}(\gamma_+ + \mathrm{i}\gamma_\times)$



$$k_j e_{ij}^{(s)}(\boldsymbol{k}) = 0 \qquad e_{ii}^{(s)}(\boldsymbol{k}) = 0 \qquad e_{ij}^{*(s)}(\boldsymbol{k}) = e_{ij}^{(-s)}(\boldsymbol{k}) = e_{ij}^{(s)}(-\boldsymbol{k})$$

$$\epsilon_{ijl}\frac{\partial}{\partial x_l}\left[e_{mj}^{(s)}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\right] = sk e_{im}^{(s)}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \qquad e_{ij}^{(s)}(\boldsymbol{k})e_{ij}^{*(s')}(\boldsymbol{k}) = \delta_{ss'}\ ,$$

s= ±

Helicity variables

Duality operator

define a map $\sim: \gamma_{ij} \mapsto \tilde{\gamma}_{ij} = \epsilon_{ikl}\partial_l \gamma_{kj}$

$$\tilde{\gamma}_{ij} = \tilde{\gamma}_{(ij)} \,, \quad \tilde{\gamma}_{ii} = \partial_i \tilde{\gamma}_{ij} = 0 \,, \quad \tilde{\tilde{\gamma}}_{ij} = -\Delta \gamma_{ij} \,, \qquad \gamma''_{ij} - \Delta \gamma_{ij} = 0$$

$$\gamma^{\pm}_{ij} := \frac{1}{2}(\gamma'_{ij} \mp \mathrm{i}\tilde{\gamma}_{ij})$$

- $\mathbf{C}$-valued, symmetric TT tensor
- form an irrep. of $\mathbf{E}^3$
- correspond to projections onto left & right circular polarizations

c.f. Higaki 1986

imaginary (anti-)self dual Weyl tensor

$$W^{\pm}_{\mu\nu\lambda\sigma} := W_{\mu\nu\lambda\sigma} \pm i * W_{\mu\nu\lambda\sigma}, \qquad *W^{\pm}_{\mu\nu\lambda\rho} = \mp i W^{\pm}_{\mu\nu\lambda\rho}$$

$$(W^{\pm})^3 = 64[(\gamma^{\pm})']^3$$

$$W^{\mu\nu}{}_{\rho\sigma} W^{\rho\sigma}{}_{\tau\lambda} W^{\tau\lambda}{}_{\mu\nu} = \frac{1}{4}\mathrm{Re}(W^+)^3$$

$$*W^{\mu\nu}{}_{\rho\sigma} W^{\rho\sigma}{}_{\tau\lambda} W^{\tau\lambda}{}_{\mu\nu} = \frac{1}{4}\mathrm{Im}(W^+)^3$$

→ W^3 & $*WW^2$ can be treated in a unified manner

W^3 & $*WW^2$ are interchanged into each other under $\partial_\eta \leftrightarrow \mathrm{i}\epsilon_{ijk}\partial_k$

GWs in FLRW

GWs in FLRW

$$ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right] \qquad \partial_i h_{ij} = h_{ii} = 0 \qquad h''_{ij} + 2\frac{a'}{a}h'_{ij} - \Delta h_{ij} = 0$$

$$W^{\pm 0}{}_{i0j} = \frac{1}{2a}[a(h'_{ij} \mp i\epsilon_{ikl}\partial_l h_{jk})]'$$

(radiative parts~Ψ_0, Ψ_4)

$$\Longleftrightarrow$$

$$ds^2 = -d\eta^2 + (\delta_{ij} + \gamma_{(M)ij})dx^i dx^j$$
$$W^{\pm 0}{}_{i0j} = \frac{1}{2}(\gamma'_{(M)ij} \mp i\epsilon_{ikl}\partial_l \gamma_{(M)jk})'$$
$$= (\gamma^{\pm}_{(M)ij})'$$

Define $\gamma'_{ij} \equiv a h'_{ij}$ $\longrightarrow$ $W^{\mu}{}_{\nu\lambda\rho}(h) = \frac{1}{a}W^{\mu}{}_{\nu\lambda\rho}(\gamma)\big|_{\text{Minkowski}}$

$$W^{\pm}_{\mu\nu}{}^{\alpha\beta}(h)W^{\pm}_{\alpha\beta}{}^{\gamma\delta}(h)W^{\pm}_{\gamma\delta}{}^{\mu\nu}(h) = \frac{64}{a^9}\gamma'^{\pm}_{ij}\gamma'^{\pm}_{jk}\gamma'^{\pm}_{ki} \qquad W^3 = \tfrac{1}{4}\mathrm{Re}(W^+)^3\,, \quad *WW^2 = \tfrac{1}{4}\mathrm{Im}(W^+)^3$$

$$S_{PV} = -b\int d^4x\sqrt{-g}*W^{\mu\nu}{}_{\rho\sigma}W^{\rho\sigma}{}_{\tau\lambda}W^{\tau\lambda}{}_{\mu\nu} = 8ib\int d\eta d^3x a^{-5}[(\gamma^+)'^3 - (\gamma^-)'^3]$$

$$\longrightarrow \quad H_{\text{int}} = -8ib\int d\eta a^{-5}[(\gamma^+)'^3 - (\gamma^-)'^3]$$

Graviton bispectrum in dS

Pure de Sitter case

$$h_{ij} = \frac{2}{M_{\rm pl}} \int \frac{{\rm d}^3\mathbf{k}}{(2\pi)^{3/2}\sqrt{2k}} \sum_{s=\pm} \left[e_{ij}^{(s)}(\mathbf{k}) u_k(\eta) a_s(\mathbf{k}) e^{{\rm i}\mathbf{k}\cdot\mathbf{x}} + ({\rm h.c.}) \right] \qquad h_{ij}'' + 2\frac{a'}{a}h_{ij}' - \Delta h_{ij} = 0$$

mode function $\quad u_k = \frac{H}{k}(1+ik\eta)\, e^{-ik\eta}\ , \qquad \left[a_s(\boldsymbol{k}), a_s^\dagger(\boldsymbol{k}') \right] = \delta_{ss'}\delta(\boldsymbol{k}-\boldsymbol{k}')$

$$\gamma_{ij}' = a h_{ij}' \qquad \gamma_{ij}^{\pm} := \tfrac{1}{2}(\gamma_{ij}' \mp {\rm i}\tilde{\gamma}_{ij}) \qquad \epsilon_{ijl}\frac{\partial}{\partial x_l}\left[e_{mj}^{(s)}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \right] = s k e_{im}^{(s)}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\,,$$

$$\rightarrow \quad \gamma_{ij}^{+} = -2\int \frac{k\,{\rm d}^3\mathbf{k}}{M_{\rm pl}(2\pi)^{3/2}\sqrt{2k}} \left[e_{ij}^{(+)}(\mathbf{k}) e^{-{\rm i}k\eta} a_+(\mathbf{k}) + e_{ij}^{(-)}(-\mathbf{k}) e^{{\rm i}k\eta} a_-(\mathbf{k}) \right] e^{{\rm i}\mathbf{k}\cdot\mathbf{x}}$$

correlators can be calculable via in-in formalism

$$\langle A(t)\rangle = -i\int_{t_0}^{t} {\rm d}t' \langle [A(t), H_{\rm int}(t')] \rangle \qquad H_{\rm int} = b\int {\rm d}^4x\sqrt{-g} * W W^2$$

$$\langle \gamma_{ij}^{\pm}(\mathbf{p}) (\gamma_{kl}^{\mp}(\eta,\mathbf{k}))' \rangle = 2{\rm i} M_{\rm pl}^{-2} k^2 \Pi_{ij,kl}^{\pm}(\mathbf{p}) \delta^{(3)}(\mathbf{p}+\mathbf{k}) e^{{\rm i}k\eta} \qquad \Pi_{ij,kl}^{\pm}(\mathbf{p}) = e_{ij}^{(\pm)}(\mathbf{p}) e_{kl}^{*(\pm)}(\mathbf{p})$$

No parity violation

$$\langle\gamma^{+}_{i_1 j_1}(0,\mathbf{p}_1)\gamma^{+}_{i_2 j_2}(0,\mathbf{p}_2)\gamma^{+}_{i_3 j_3}(0,\mathbf{p}_3)\rangle$$

$$= 384\mathrm{i}bH^5 M_{\mathrm{pl}}^{-6}(2\pi)^3(p_1p_2p_3)^2\delta^{(3)}(\mathbf{p})\frac{5!}{p^6}\left[\Pi^{+}_{i_1j_1,kl}(\mathbf{p}_1)\Pi^{+}_{i_2j_2,lm}(\mathbf{p}_2)\Pi^{+}_{i_3j_3,mk}(\mathbf{p}_3)+\Pi^{-}_{i_1j_1,kl}(\mathbf{p}_1)\Pi^{-}_{i_2j_2,lm}(\mathbf{p}_2)\Pi^{-}_{i_3j_3,mk}(\mathbf{p}_3)\right]$$

$$\mathbf{p}=\sum_i \mathbf{p}_i$$

- project onto left & right handed circular polarizations

$$h^R := h_{ij}e^{*(+)}_{ij}\,,\quad h^L := h_{ij}e^{*(-)}_{ij} \qquad h_{ij}(\eta=0,\mathbf{p}) = -\frac{H}{p^2}[\gamma^{+}_{ij}(\mathbf{p})+\gamma^{-}_{ij}(\mathbf{p})]$$

$$\langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle$$

$$= -384i(2\pi)^3(bH)^2(H/M_{\mathrm{pl}})^6F(p_2,p_2,p_3)\left[\frac{5!}{(p_1+p_2+p_3)^2}-\frac{5!}{(p_1+p_2+p_3)^2}\right]\delta^{(3)}(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)=0$$

$$F(k_1,k_2,k_3) = -\frac{(k_1+k_2+k_3)^3\,(k_1+k_2-k_3)\,(k_2+k_3-k_1)\,(k_3+k_1-k_2)}{64k_1^2k_2^2k_3^2}$$

$$\langle h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle = 0 \qquad \langle h^R(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle = 0$$

$$\langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle - \langle h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle = 0$$
$$\langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle - \langle h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle = 0$$

In dS universe, parity violation does not appear in graviton non-Gaussianity.

Slow roll corrections

Consider deviations away from de Sitter

- expand terms by slow roll parameter $\epsilon := -\dot{H}/H^2 \,,\qquad a \sim (-H_*\eta)^{-1-\epsilon}$

I. mode function

$$u_k(\eta) = \sqrt{\frac{\pi k}{2H_*}} e^{i\pi\nu/2 - i\pi/4} (-H_*\eta)^{\nu} H_\nu^{(1)}(-k\eta) \;,$$

$$h_{ij}(0,\mathbf{p}) = -\frac{H_*}{p^2} C\left[\gamma_{ij}^{+}(0,\mathbf{p}) + \gamma_{ij}^{-}(0,\mathbf{p})\right]$$

$$C = 1 + \epsilon(2 - \gamma - \log 2 + \log H_*/k)$$

II. interaction Hamiltonian

$$H_{\rm int} = -8{\rm i}b \int {\rm d}\eta {\rm d}^3 x a^{-5}[(\gamma^+)'^3 - (\gamma^-)'^3]$$

$$(\gamma_{ij}^{\pm})' \;\to\; (\gamma_{({\rm dS})ij}^{\pm})' + \epsilon\chi_{ij}^{\pm}$$

III. cosmic expansion

$$\int_{-\infty}^{0} d\eta(-\eta)^{5+5\epsilon} e^{-i(p_1+p_2+p_3)\eta} \quad \sim -\frac{5!}{(p_1+p_2+p_3)^6}\left(1 + \frac{5\pi}{2} i\epsilon\right)$$

Parity violation

$$\langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle = -\langle h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle$$

$$= -64(2\pi)^4\epsilon(bH_*^2)(H_*/M_{\rm pl})^6\delta^{(3)}(\mathbf{p})F(p_1,p_2,p_3)\frac{6!}{p^6}$$

$$\left(F(k_1,k_2,k_3) = -\frac{(k_1+k_2+k_3)^3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{64k_1^2k_2^2k_3^2} \qquad \mathbf{p} = \sum_i \mathbf{p}_i \right)$$

→ $\langle (h^R)^3\rangle - \langle (h^L)^3\rangle \neq 0$

Parity violation shows up & is proportional to slow-roll parameter

$$\langle h^L(0,\mathbf{p}_1)h^L(0,\mathbf{p}_2)h^R(0,\mathbf{p}_3)\rangle = \langle h^R(0,\mathbf{p}_1)h^R(0,\mathbf{p}_2)h^L(0,\mathbf{p}_3)\rangle$$

$$= 192i(2\pi)^3\epsilon H_*^2(H_*/M_{\rm pl})^6 J(p_1,p_2,p_3)(A_- - A_-^*)\delta^{(3)}(\mathbf{p})$$

→ No parity violation

This sort of parity violation can be observed in CMB

Shiraishi-Nitta-Yokoyama 2011

Concluding remarks

Summary

- a new formalism for computing graviton correlators

→ provides us w/ unified treatment of W^3 & $*WW^2$

- for Weyl cubic interactions parity violation does not show up in exact de Sitter, but it does in the slow roll case

Outlooks

- a single field effective field theory admits parity-odd interaction *S. Weinberg 2008*

$$L_0 = R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \qquad L_1 = f_1(\phi)(\nabla\phi)^4 + f_2(\phi)W_{abcd}W^{abcd} + f_3(\phi)W_{abcd} \star W^{abcd}$$

$$\star W_{abcd} = \frac{1}{2}\epsilon_{abef}W^{ef}{}_{cd}$$

W_{abcd}: Weyl tensor

(conformal) isometries in dS

- Conformal Killing vectors $\mathcal{L}_{\zeta_{(i)}} g_{ab} = 2\phi_{(i)} g_{ab}\,, \quad \phi_{(i)} = \frac{1}{4}\nabla_a \zeta^a_{(i)}$

RW metric $\mathrm{d}s^2 = a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)$. admits maximal set of 15 CKVs

$$P_\mu = \partial_\mu \quad D = x^\mu \partial_\mu\,, \quad M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu\,, \quad K_\mu = 2x_\mu D - (x_\nu x^\nu) P_\mu$$

$$[D, K_\mu] = K_\mu\,, \quad [D, P_\mu] = P_\mu\,, \quad [K_\mu, P_\mu] = 2(\eta_{\mu\nu} D + 2M_{\mu\nu}) \qquad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$$

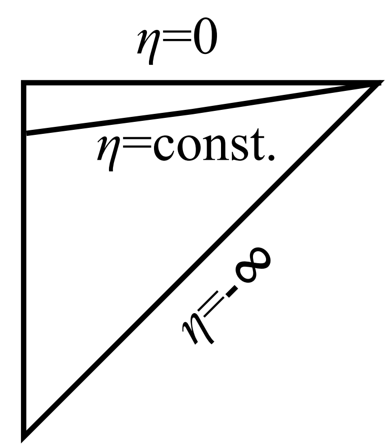
$$[K_\mu, M_{\nu\rho}] = \eta_{\mu\nu} K_\rho - \eta_{\mu\rho} K_\nu\,, \quad [P_\mu, M_{\nu\rho}] = \eta_{\mu\nu} P_\rho - \eta_{\mu\rho} P_\nu\,, \quad [M_{\mu\nu}, M_{\rho\sigma}] = -\eta_{\mu\rho} M_{\sigma\nu} + \cdots .$$

$\phi_{(\mathrm{i})}$ are given by

$$\phi_{P_0} = \mathcal{H}\,, \qquad \phi_{K_0} = -2\eta - (\eta^2 + x^i x_i)\mathcal{H}\,, \qquad \phi_{M_{0i}} = -\mathcal{H} x_i$$

$$\left.\begin{aligned} \phi_D &= 1 + \eta\mathcal{H}\,, \\ \phi_{K_i} &= 2x_i(1 + \eta\mathcal{H}) \end{aligned}\right\} \text{ isometry for dS } (\mathcal{H} = -1/\eta)$$

$$\mathcal{H} = \mathrm{d}(\ln a)/\mathrm{d}\eta$$



K_{i} generates special CT of $\mathbf{E}^3$ for late time (η~0)

⇒ restricts allowed correlators for tensors w/ conformal dimension 2 (=stress tensor)