# 天体計測学特論 I
# Observational Astronomy I

## Lecture 02:
## Optical systems for observations

## Geometrical optics　(I)

# Collimating/Focusing : Optical surface

- Hyperboloidal / Ellipsoidal optical surfaces

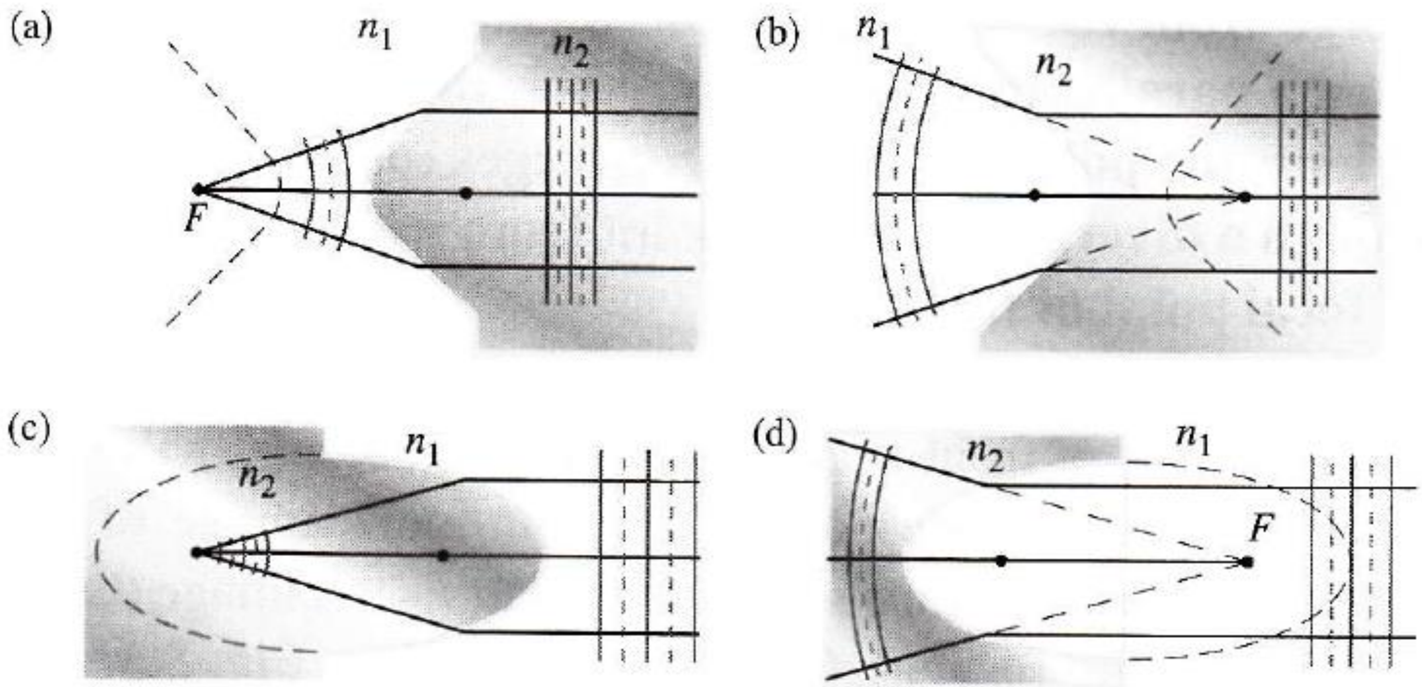
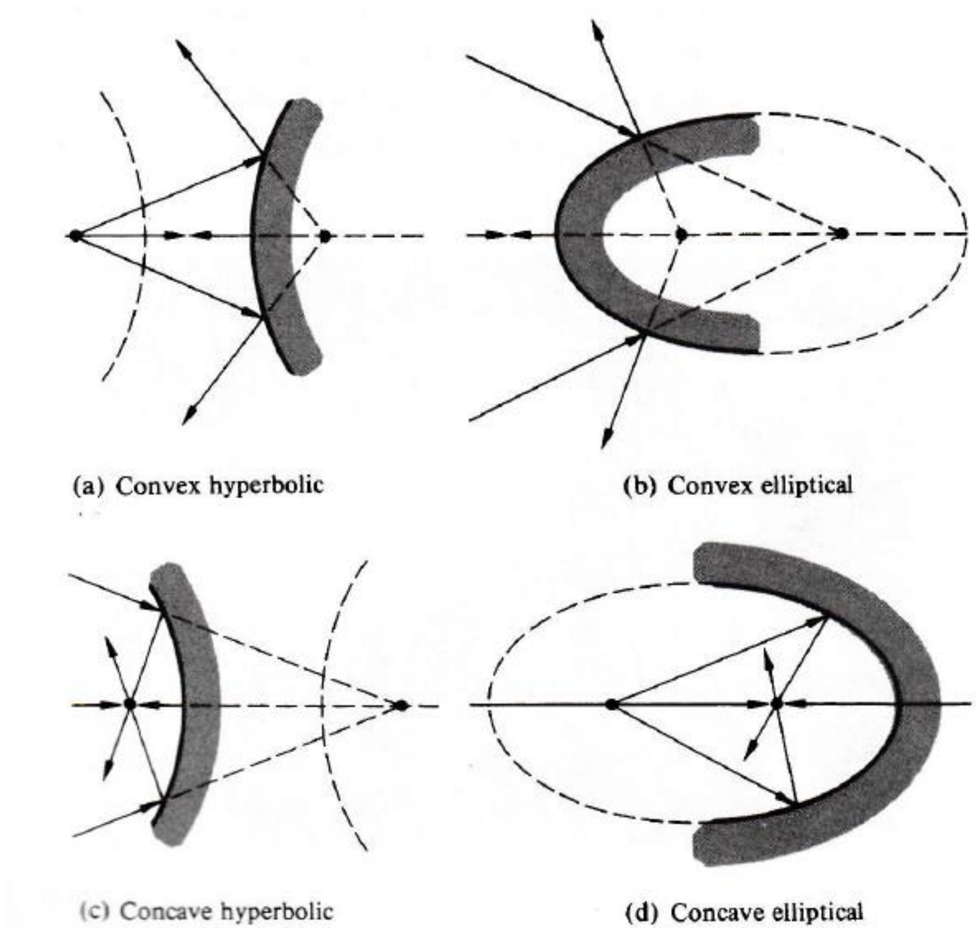



**Figure 5.4** (*a*) and (*b*) Hyperboloidal and (*c*) and (*d*) ellipsoidal refracting surfaces ($n_2 > n_1$) in cross section.

Hecht “Optics”

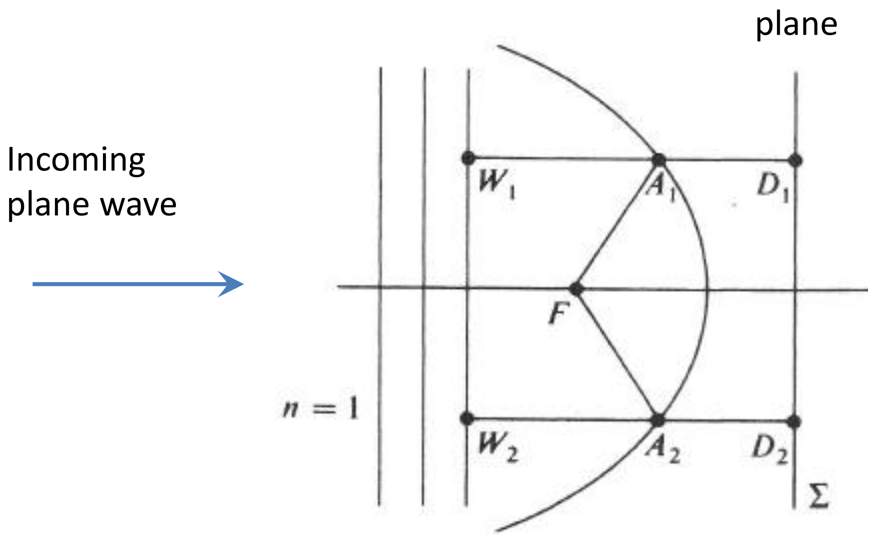
# Focus with Hyperbolic / Elliptical Mirrors




Hecht "Optics"

# Collimating / Focusing : Mirror

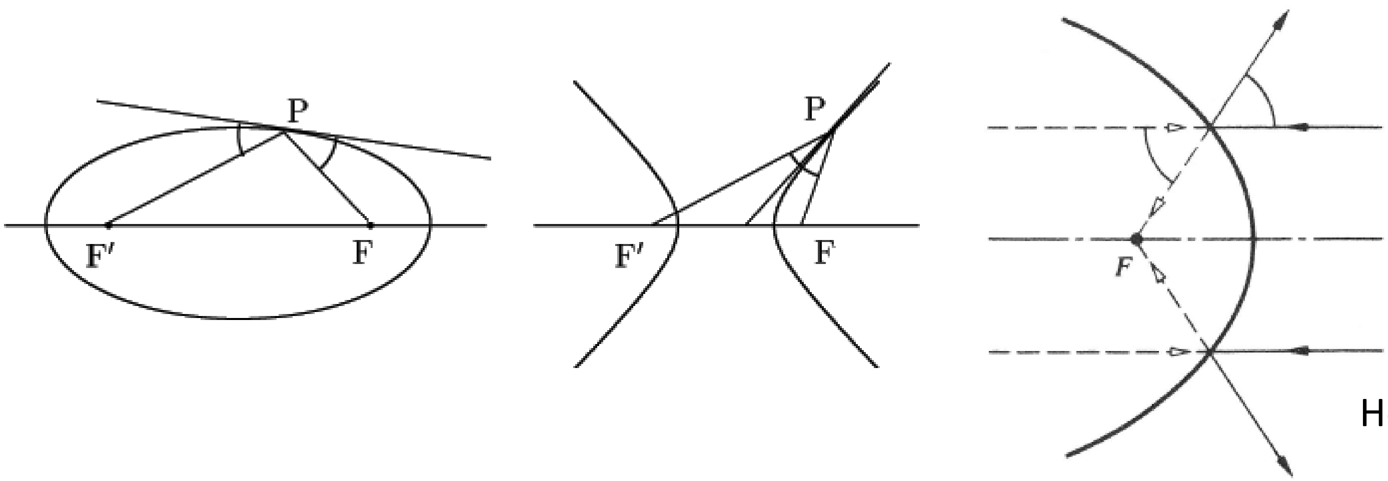
- Ellipse / Parabola / Hyperbola




AF = e AD, e: eccentricity

K = -e^2, K : conic constant

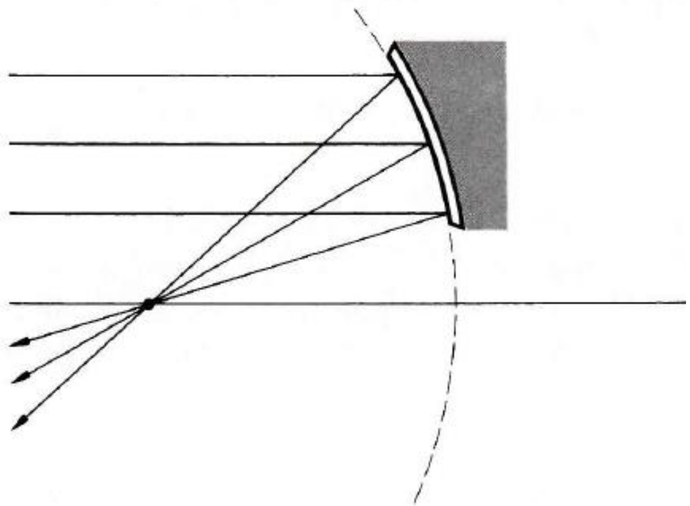
y^2 – 2Rx + (K+1) x^2 = 0




Hecht “Optics”

# Collimating / focusing with parabolic mirrors

- Parabolic and off-axis parabolic mirror



Hecht “Optics”

From Edmund optics

# Parabola (ideal) to the third order approximation

- Expanding the equation for a sphere

$$z = r - r\left(1 - \frac{y^2}{r^2}\right)^{1/2}$$

$$z_{Sph} = \frac{c}{2}y^2 + \frac{c^3}{8}y^4 + \frac{c^5}{16}y^6 + \dots , \qquad c = 1/r$$

Parabola

- Elliptical and hyperbolic surfaces are described with

$$z_{Ell} = \frac{c}{2}y^2 + \frac{c^2}{8a}y^4 + \frac{c^3}{16a^2}y^6 + \dots$$

$$z_{Hyp} = \frac{c}{2}y^2 - \frac{c^2}{8a}y^4 + \frac{c^3}{16a^2}y^6 - \dots \qquad c = \frac{a}{b^2} = \frac{1}{a(1-\varepsilon^2)} ,$$

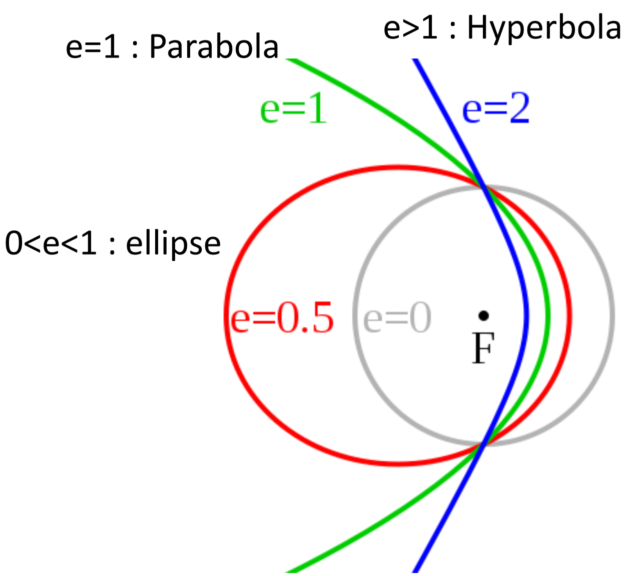
- All of them are the same in the first term (first order), different from the second term (third order).
- All of them can be described by the Schwarzschild (conic) constant bs

$$b_s = -\varepsilon^2$$

$$z = \frac{c}{2}y^2 + \frac{c^3}{8}(1+b_s)y^4 + \frac{c^5}{16}(1+b_s)^2y^6 + \dots$$

# Geometry

- Shape of a surface and corresponding Schwarzschild (conic) constant




| | | |
|---|---|---|
| $b_s = 0,$ | $\varepsilon = 0,$ | circle (sphere) |
| $b_s = -1,$ | $\varepsilon = 1,$ | parabola |
| $-1 < b_s < 0,$ | $0 < \varepsilon < 1,$ | ellipse |
| $b_s < -1,$ | $\varepsilon > 1,$ | hyperbola |

# Spherical mirrors

- (Spherical) aberration of a spherical mirror

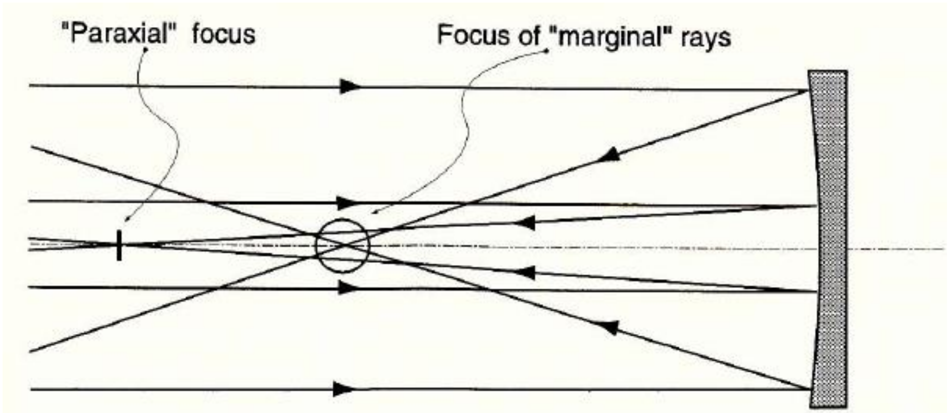
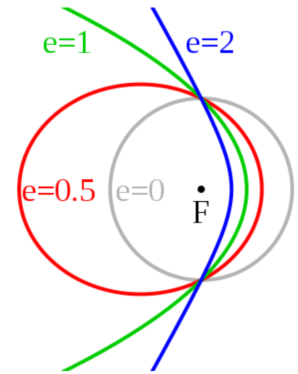
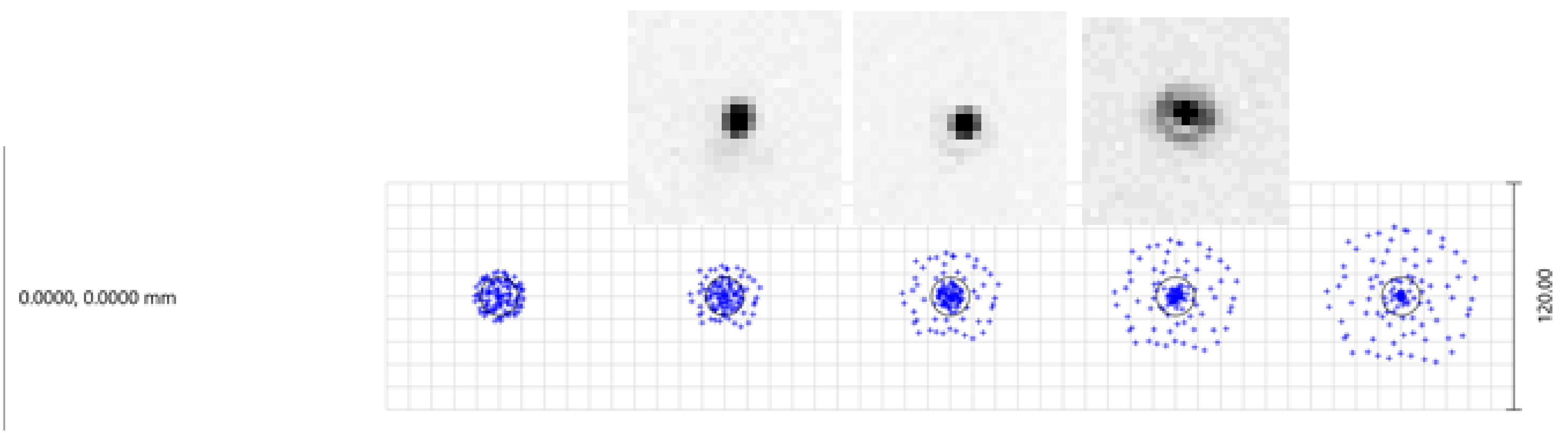



**Fig. 1.2.** Spherical aberration of a spherical concave telescope mirror. "Paraxial" rays are nominally at a negligible height from the axis

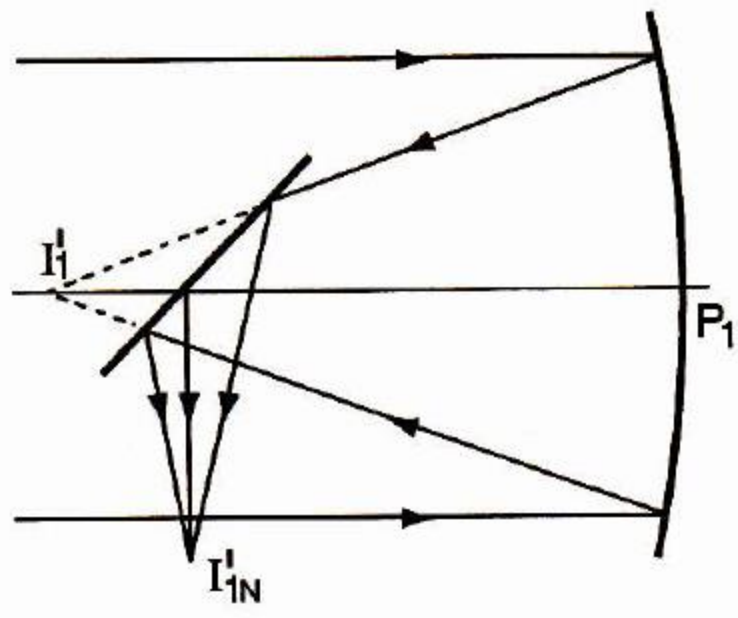



Wilson "Reflecting Telescope Optics I"



# Reflective telescope (0) :

- Newtonian telescope (M1:~Parabolic)

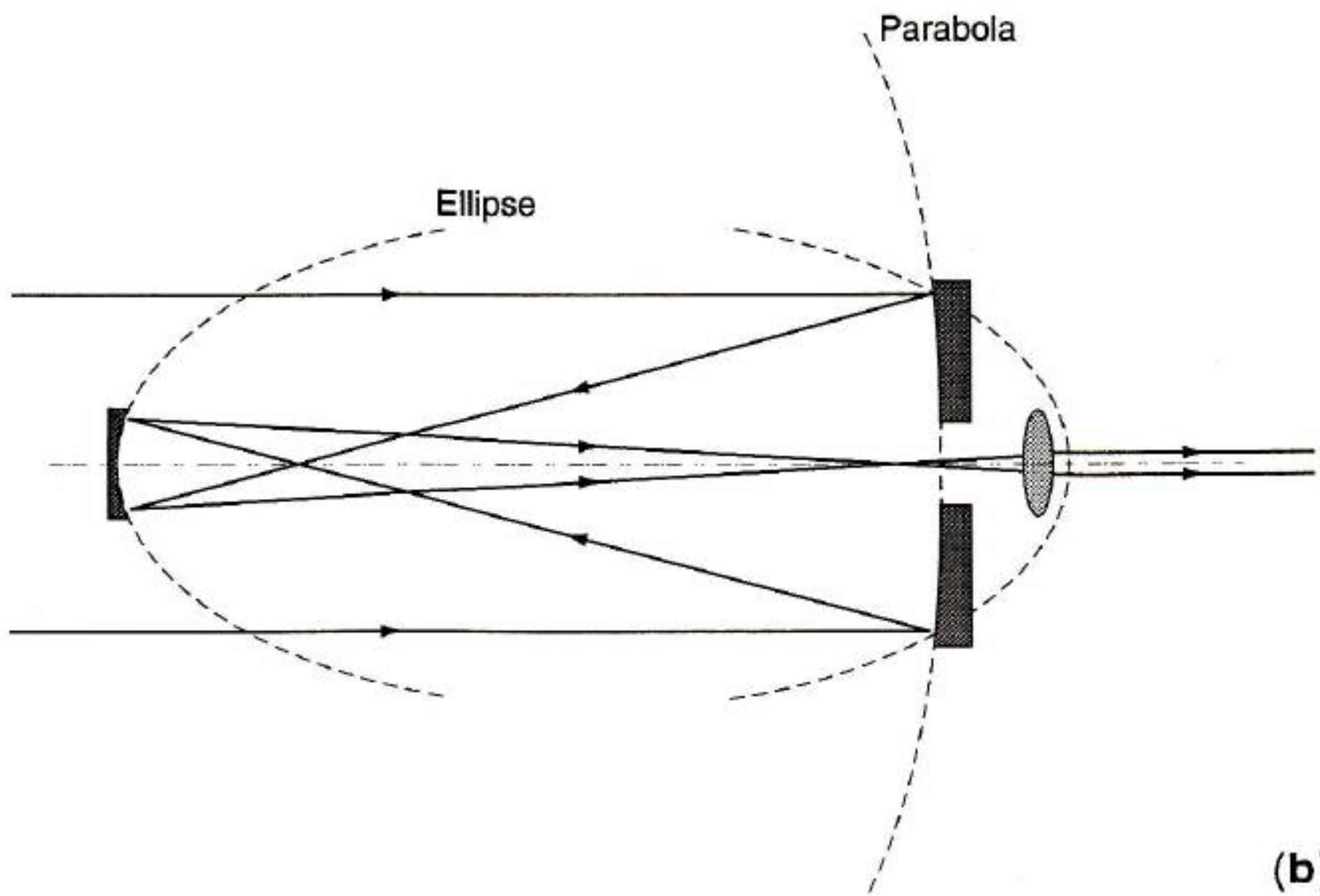



Newton form

Wilson “Reflecting Telescope Optics I”

# Reflective telescope (1) : Gregorian

- The concept




Wilson "Reflecting Telescope Optics I"

# Reflective telescope (1) : Gregorian

- M1:Parabolic +M2:Ellipsoidal

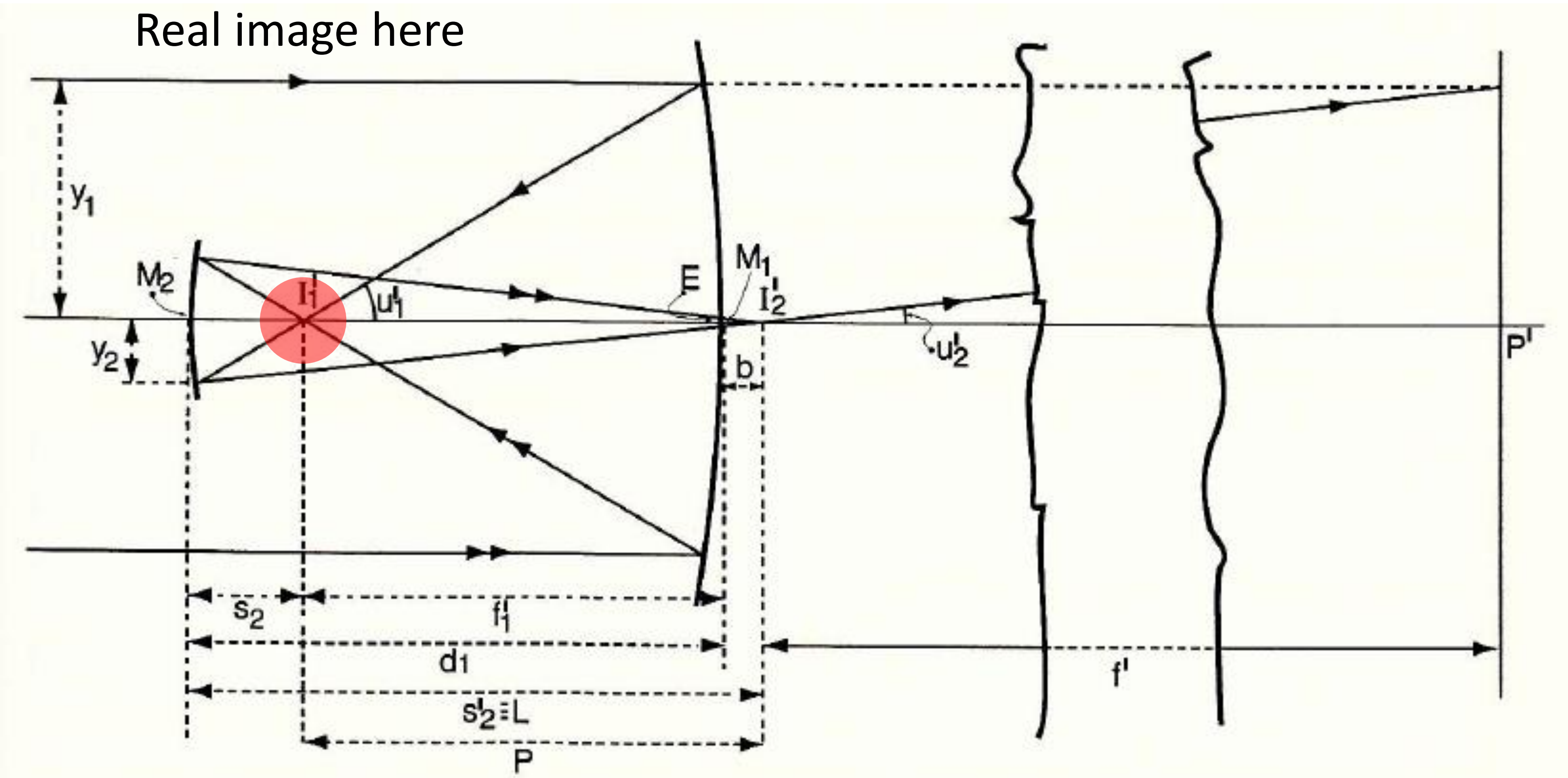
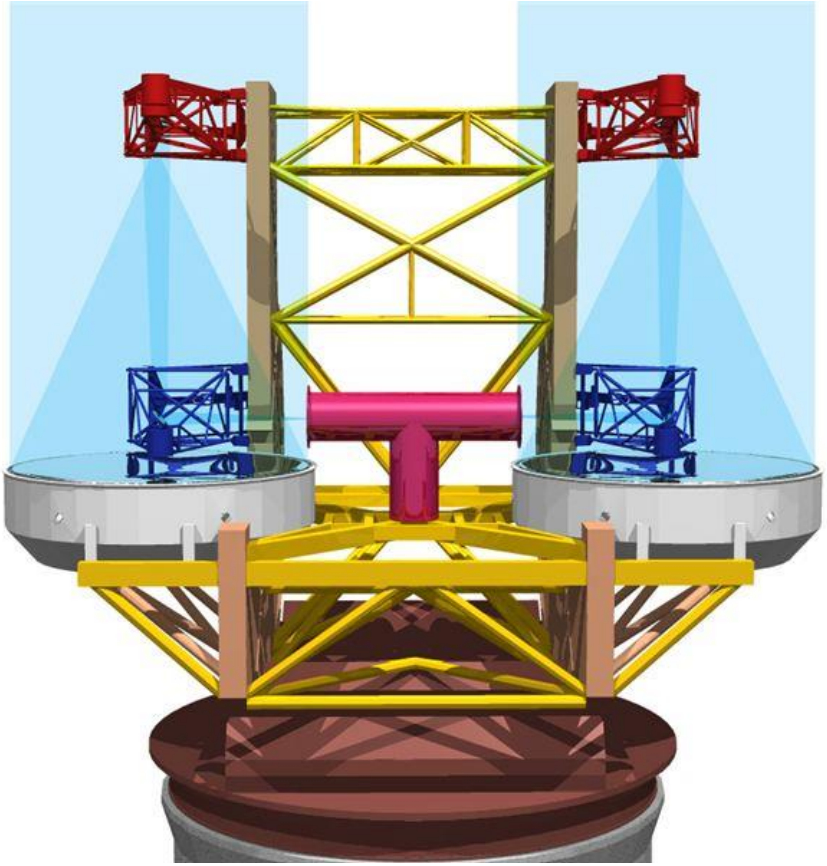


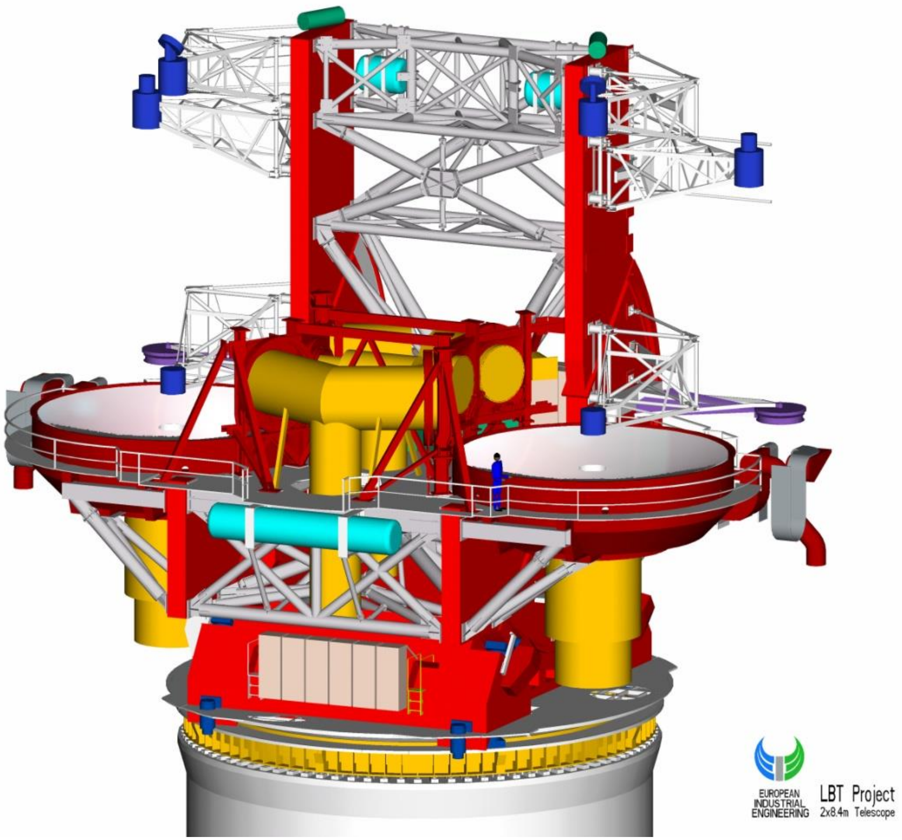

Fig. 2.11. Gaussian optics of a Gregory telescope

Wilson “Reflecting Telescope Optics I”

# Reflective telescope (1)： Gregorian

- Large Binocular Telescope

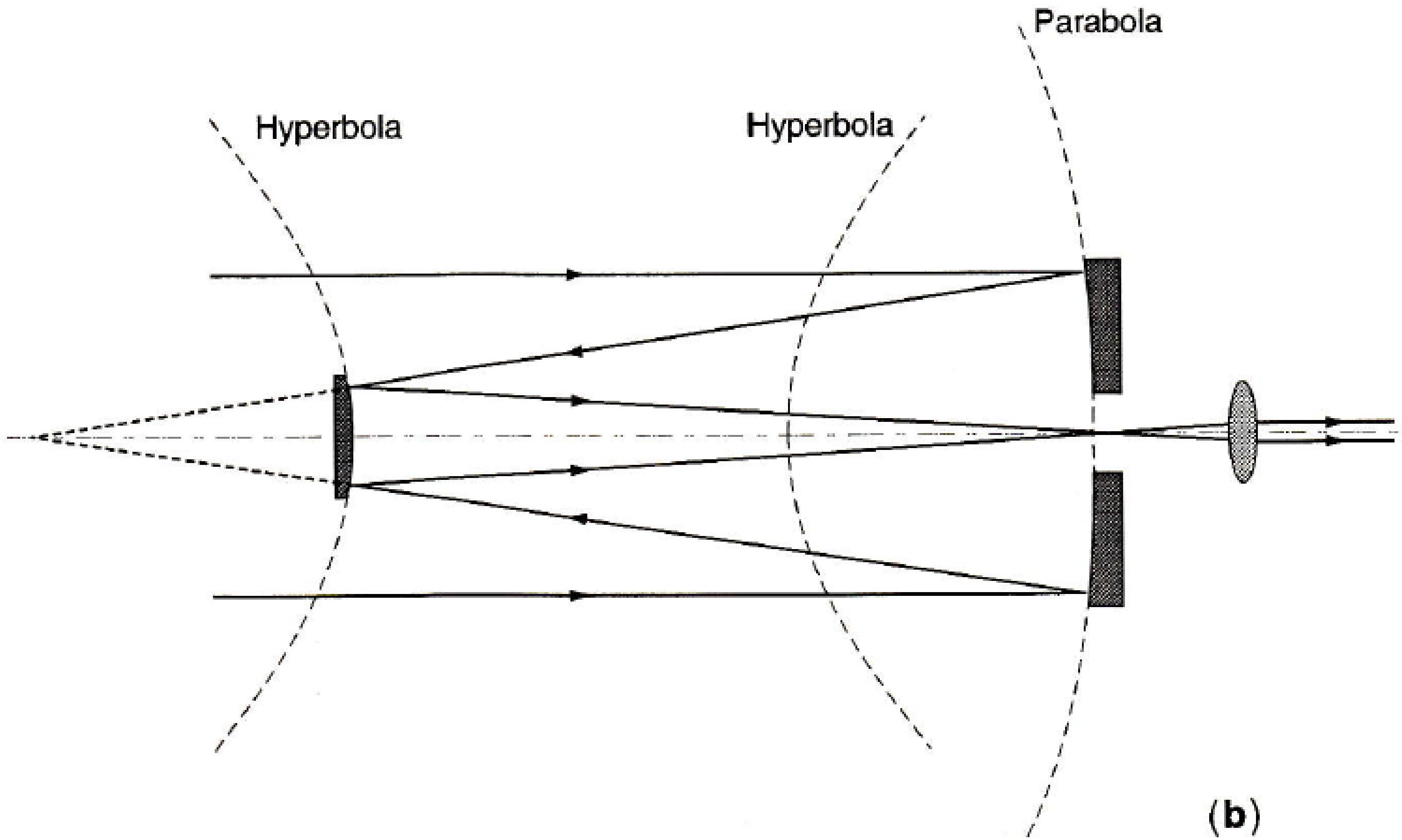





From lbt.org

# Reflective telescope (2) : Cassegrain

- The concept




Wilson “Reflecting Telescope Optics I”

# Reflective telescope (2) : Cassegrain

- M1:Parabolic + M2:Hyperbolic

Imaginary image here

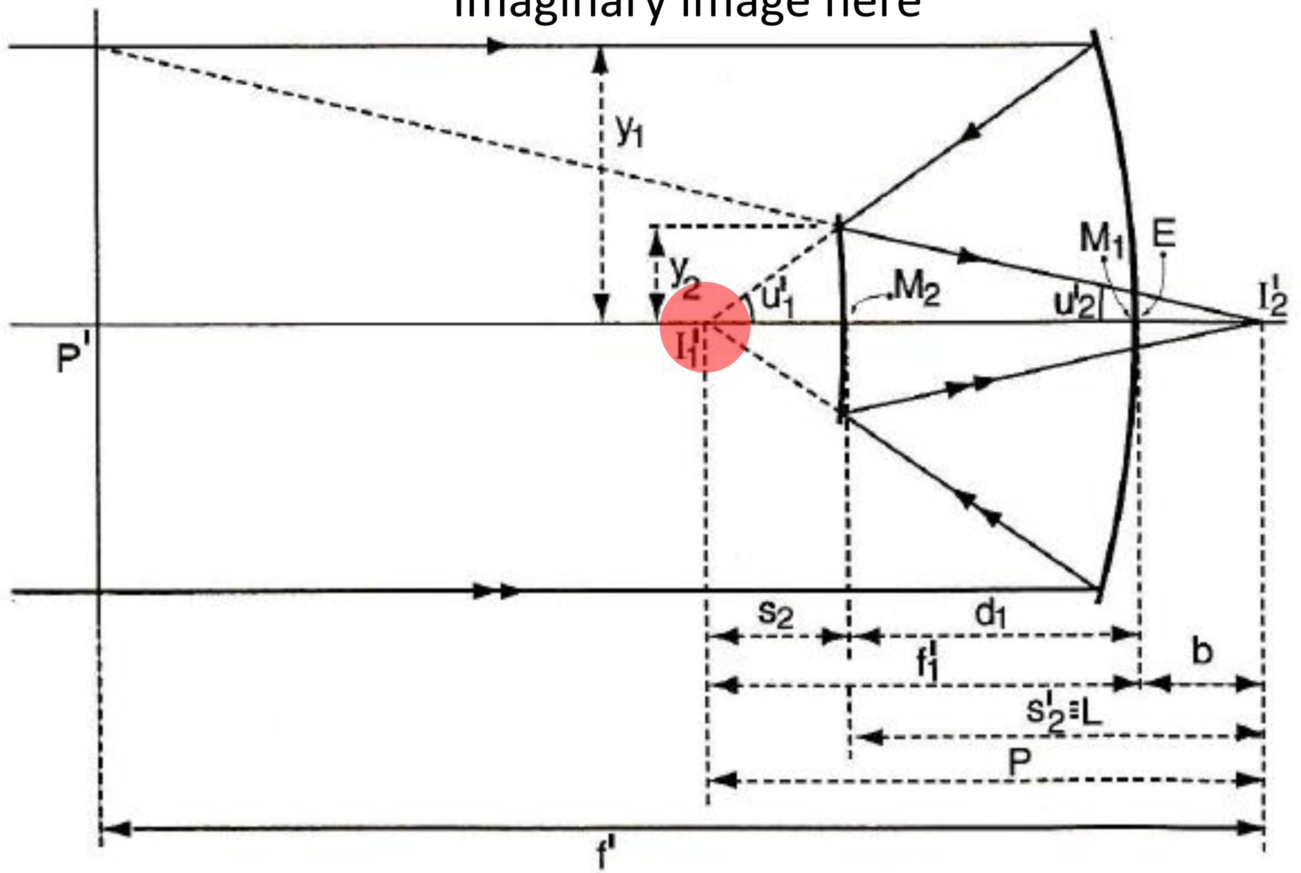



Fig. 2.12. Gaussian optics of a Cassegrain telescope

Wilson "Reflecting Telescope Optics I"

# Reflective telescope (3) : Ritchey-Chretien (Cassegrain)

- M1:Hyperbolic + M2: Hyperbolic

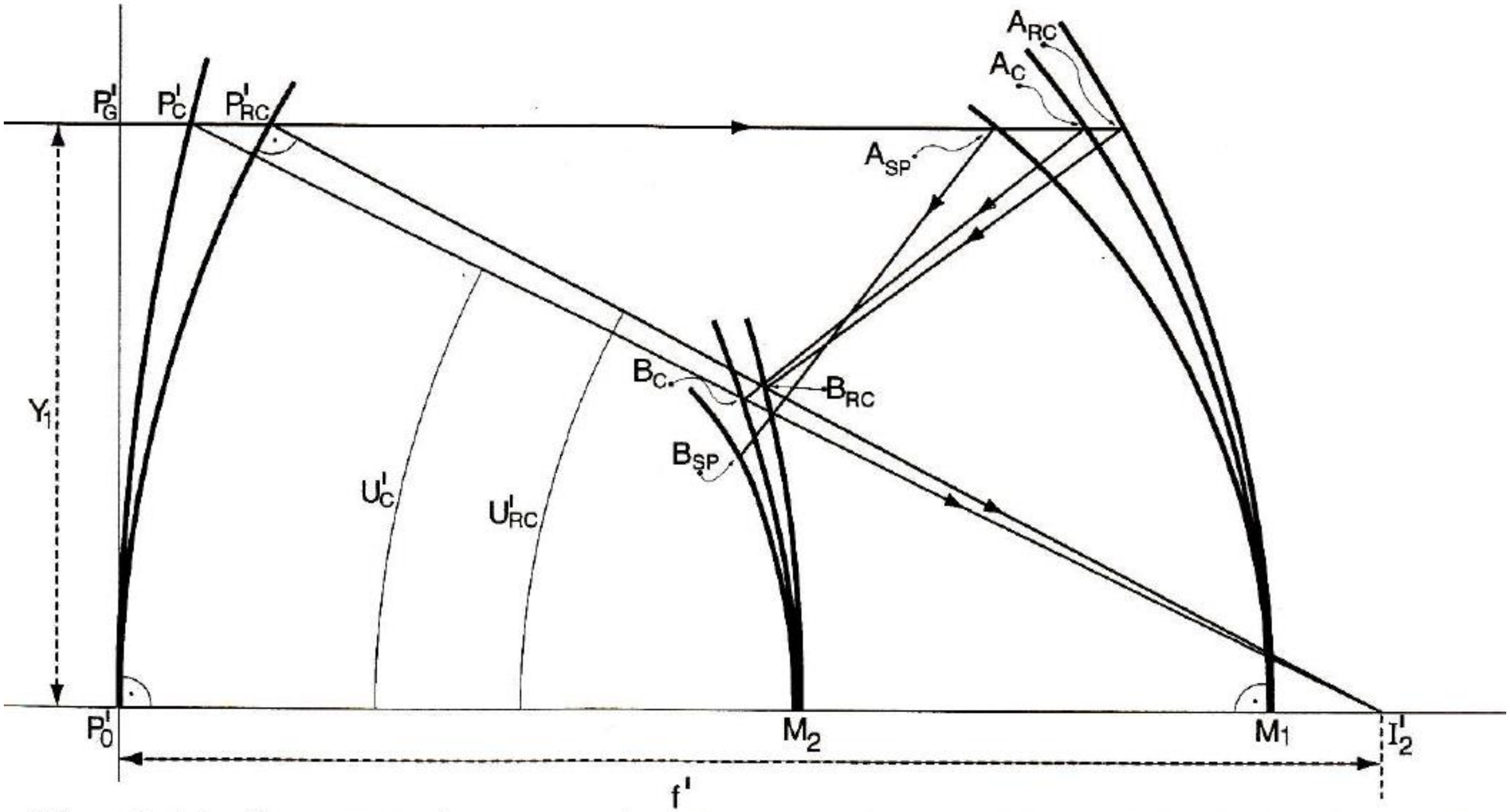
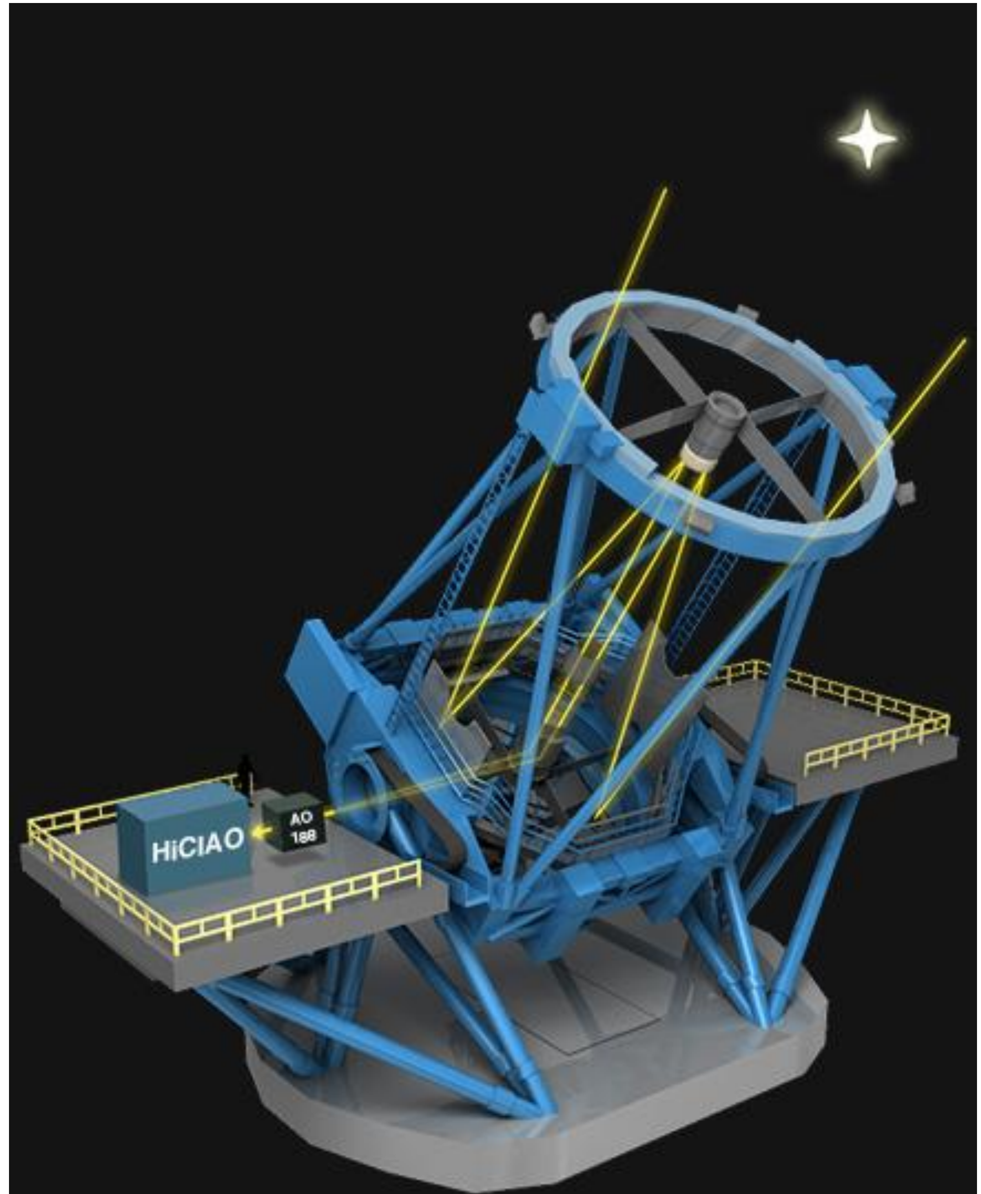



**Fig. 3.11.** Geometrical construction from the sine condition of the form of an RC telescope compared with a classical Cassegrain (from Danjon and Couder [3.24(a)])

Wilson "Reflecting Telescope Optics I"

# Reflective telescope (3) : Ritchey-Chretien (Cassegrain)

- Subaru Telescope




From mtk.nao.ac.jp

# Wavefront and image aberration

- Spherical wavefront forms an ideal image.
- Distorted wavefront (W’) results in the aberration in the image plane.

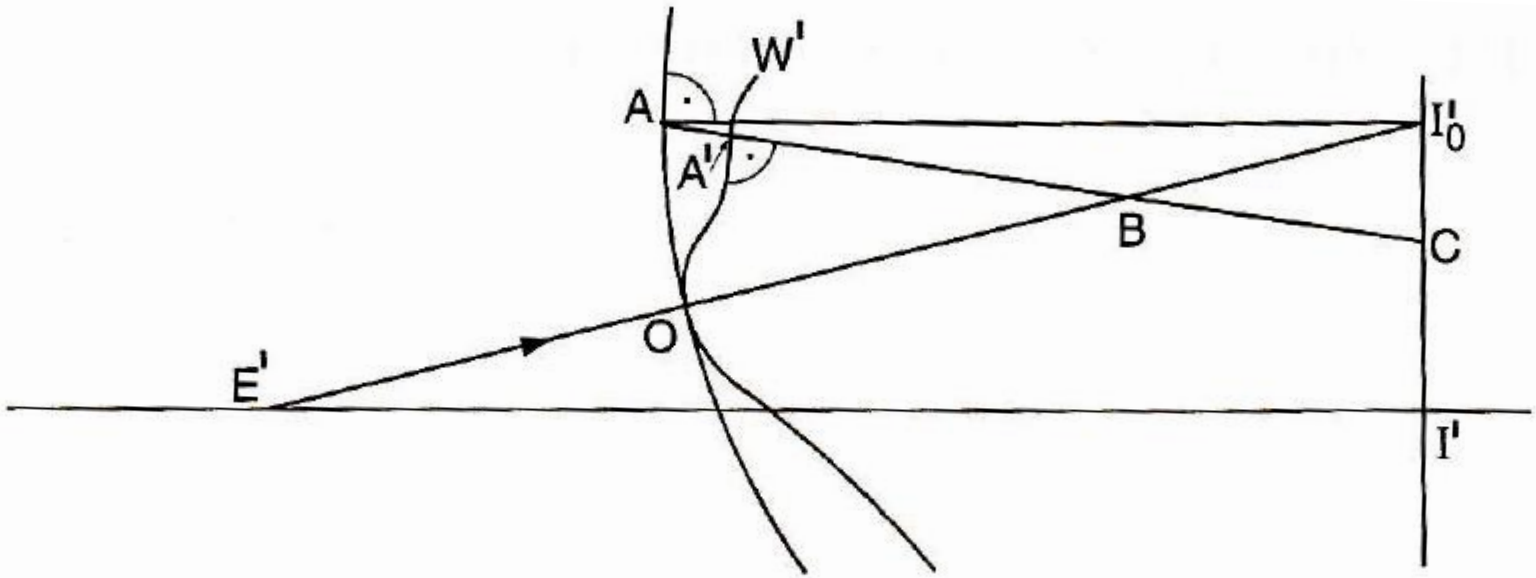
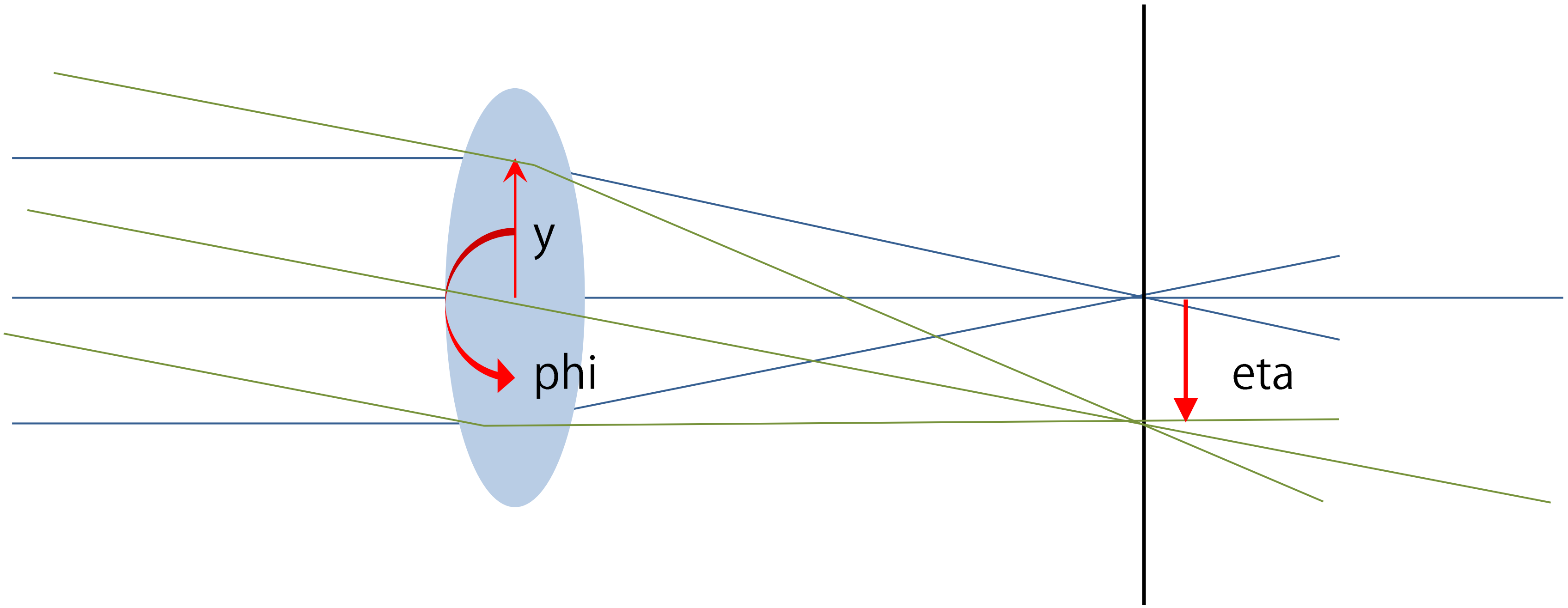



**Fig. 3.1.** Wavefront, longitudinal and lateral aberration

# Describing the wavefront distortion

- Coordinate for an on-axis optics
  - y : the aperture radius
  - eta : the field/image radius
  - phi : azimuth angle



# The Seidel description of the wavefront error

- Third order aberrations :

(spherical) + (coma) + (astigmatism) + (field curvature) + (field distortion)

$$W_3'(y_1,\eta') = \frac{1}{8}\left(\frac{y_1}{y_{m1}}\right)^4 \underline{\sum S_I} + \frac{1}{2}\left(\frac{y_1}{y_{m1}}\right)^3\left(\frac{\eta'}{\eta_m'}\right)\underline{\sum S_{II}}\cos\phi$$
$$+\frac{1}{4}\left(\frac{y_1}{y_{m1}}\right)^2\left(\frac{\eta'}{\eta_m'}\right)^2\left[\underline{(3\sum S_{III}+\sum S_{IV})}\cos^2\phi + \underline{(\sum S_{III}+\sum S_{IV})}\sin^2\phi\right]$$
$$+\frac{1}{2}\left(\frac{y_1}{y_{m1}}\right)\left(\frac{\eta'}{\eta_m'}\right)^3\underline{\sum S_V}\cos\phi$$

- Coefficient for a multi-mirror system can be evaluated by summing a coefficient of each mirror. (nu : id of a surface)

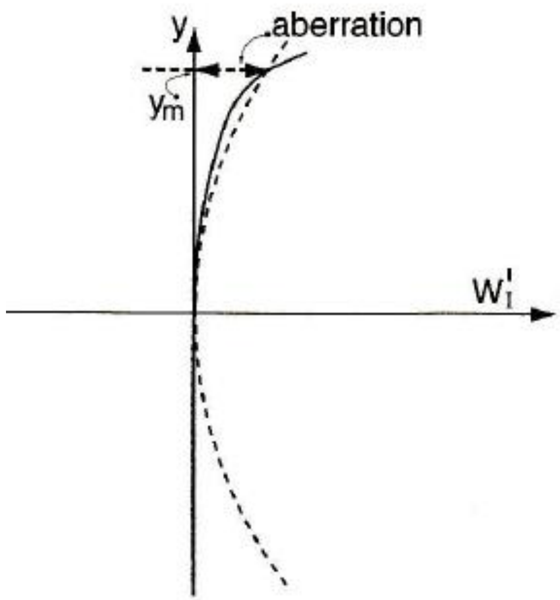
$$\underbrace{\sum S_q}_{\text{total}} = \sum_{\nu}(S_q)_\nu\ ,$$

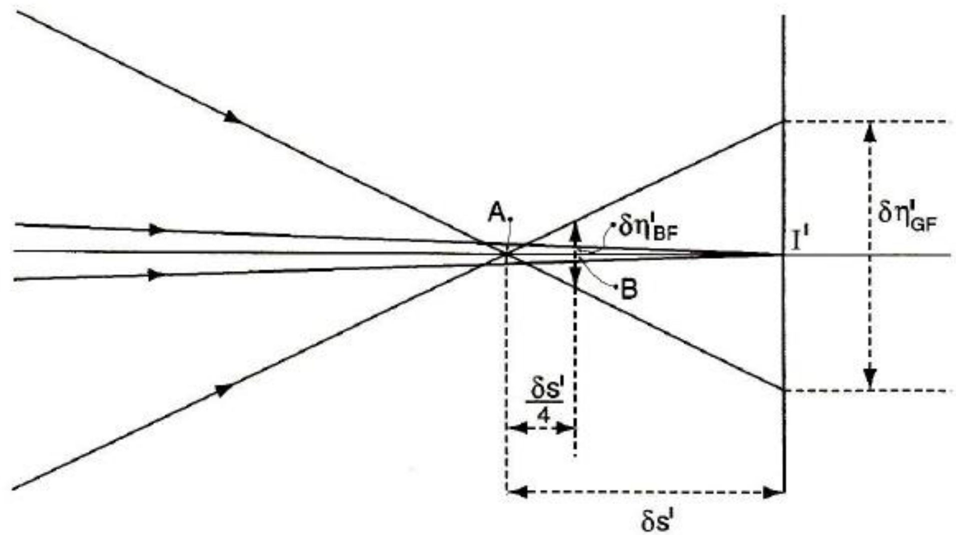
**total**

# Spherical aberration (SI) : 球面収差

$$\frac{1}{8}\left(\frac{y_1}{y_{m1}}\right)^4 \sum S_I$$

- The size of the aberration does not depend on the image height ( $\eta$ )

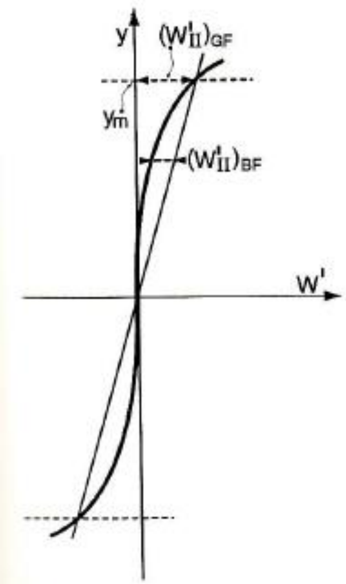


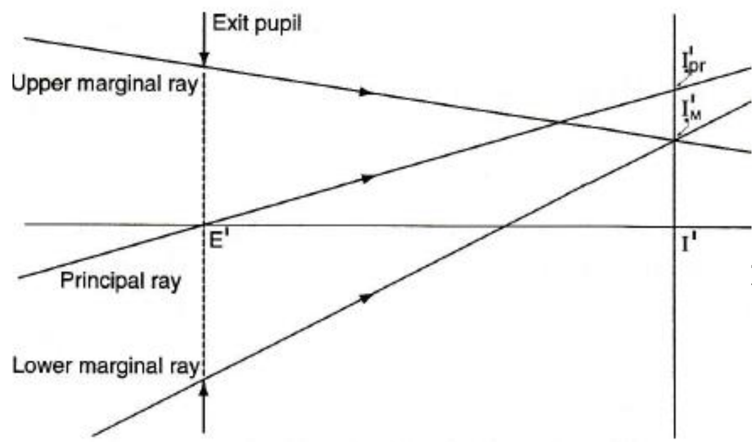



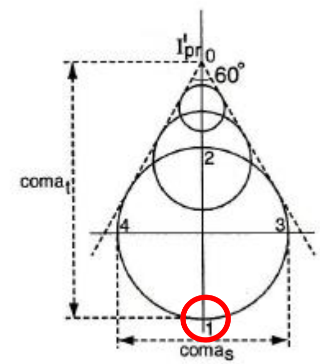
# Coma (SII) : コマ収差

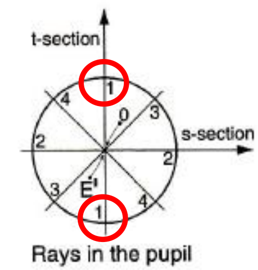
$$\frac{1}{2}\left(\frac{y_1}{y_{m1}}\right)^3\left(\frac{\eta'}{\eta'_m}\right)\sum S_{II}\cos\phi$$

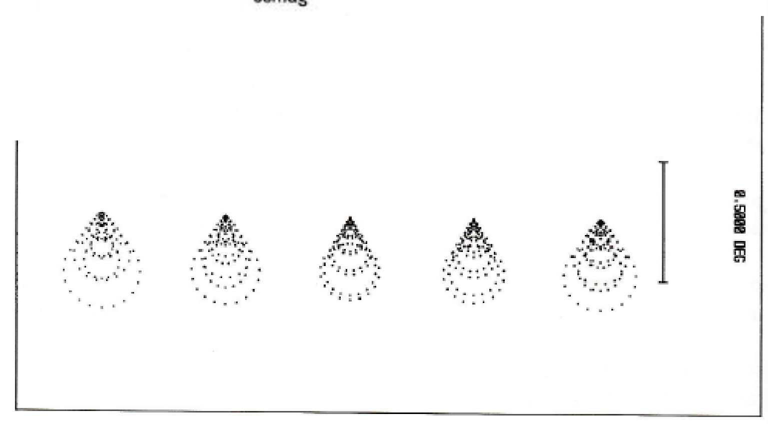
- Dependent on Φ : asymmetric
- The size of the aberration depends on the image height ( $\eta$ ), no-coma aberration at the center of FoV.



# Astigmatism (SIII) : 非点収差

$$(W'_{III}+W'_{IV})_{GF} = \frac{1}{4}\left(\frac{y}{y_m}\right)^2\left(\frac{\eta'}{\eta'_m}\right)^2\Big[(3S_{III}+S_{IV})\cos^2\phi + (S_{III}+S_{IV})\sin^2\phi\Big]$$

$\eta$ term ignored

$$(W'_{III}+W'_{IV})_{GF} = \boxed{\frac{1}{4}\left(\frac{y}{y_m}\right)^2 S_{III}\cos 2\phi} + \frac{1}{4}\left(\frac{y}{y_m}\right)^2 (2S_{III}+S_{IV})$$

- Depends on 2Φ: 180deg symmetry : horizontal vs. vertical
- Depends on image radius ( $\eta$ ).

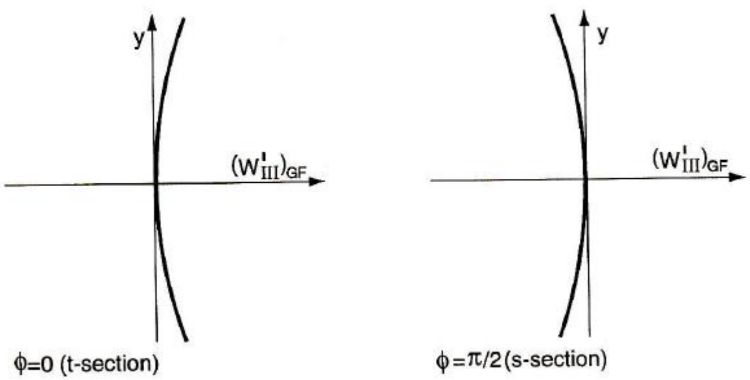



**Fig. 3.19.** Third order astigmatism: wavefront aberration reversal in the $t$- and $s$-sections due to the $\cos 2\phi$ term

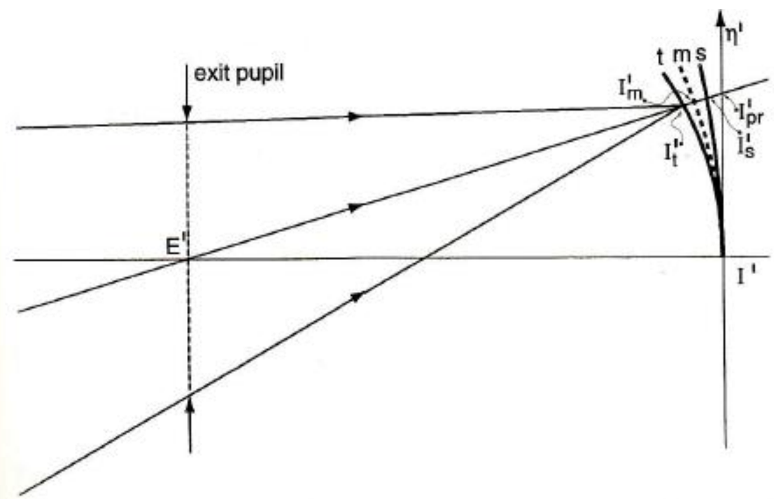
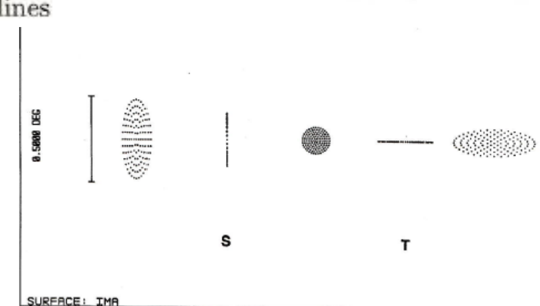



**Fig. 3.20.** Third order astigmatism: astigmatic surfaces and lines



# Field curvature (SIV) : 像面湾曲

$$(W'_{III} + W'_{IV})_{GF} = \frac{1}{4}\left(\frac{y}{y_m}\right)^2 \left(\frac{\eta'}{\eta'_m}\right)^2 \Big[(3S_{III} + S_{IV})\cos^2\phi + (S_{III} + S_{IV})\sin^2\phi\Big]$$

η term ignored

$$(W'_{III} + W'_{IV})_{GF} = \frac{1}{4}\left(\frac{y}{y_m}\right)^2 S_{III}\cos 2\phi + \boxed{\frac{1}{4}\left(\frac{y}{y_m}\right)^2 (2S_{III} + S_{IV})}$$

- Off-focus (y^2) depends on the image radius ( η ).

# Distortion (Sv) : 像面歪曲

- Tilted wavefront (y*cos(phi)) = image shift
- Depends on image radius ( $\eta$ ^3)

$$+\frac{1}{2}\left(\frac{y_1}{y_{m1}}\right)\left(\frac{\eta'}{\eta'_m}\right)^3 \sum S_V \cos\phi$$

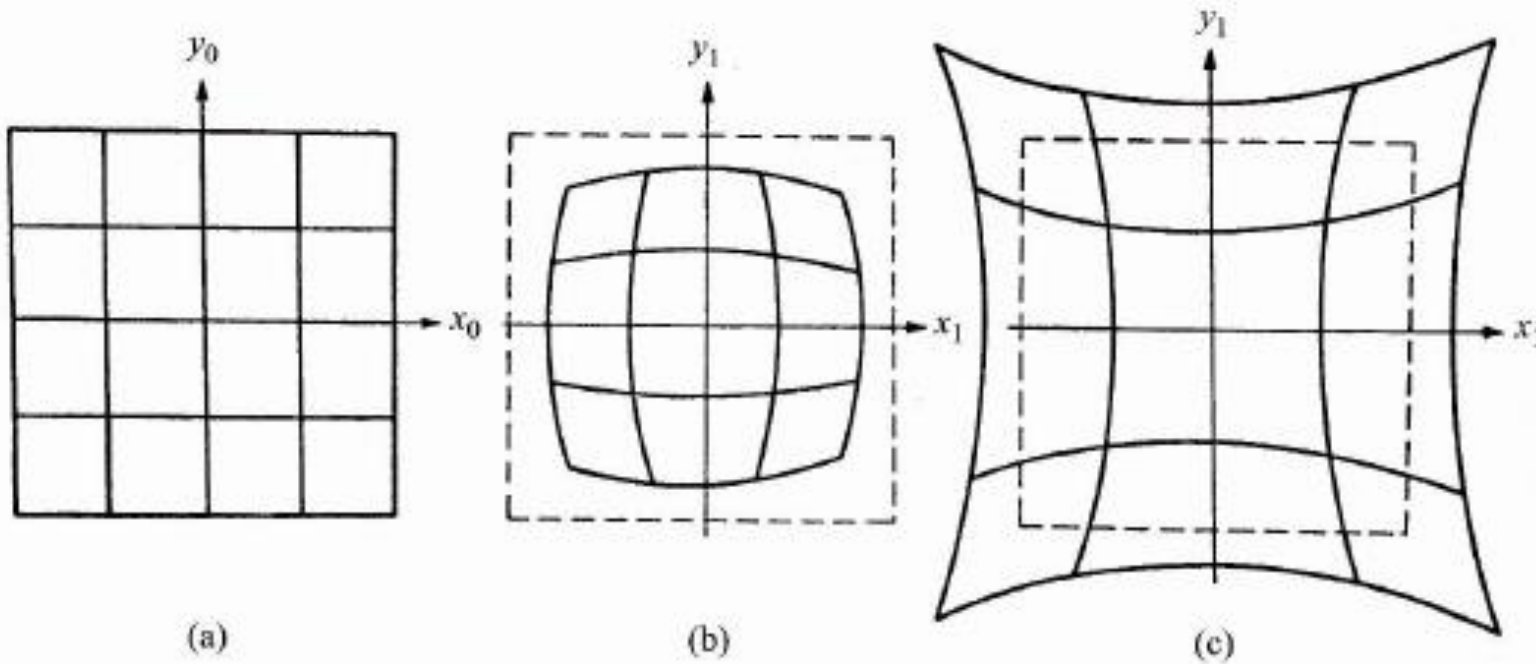



Fig. 5.8 (a) Object. (b) Image in the presence of barrel distortion ($E > 0$). (c) Image in the presence of pincushion distortion ($E < 0$).

# Seidel coefficient of 1-mirror system

- bs: Schwarzschild (conic) constant
- y1: mirror radius
- f1: mirror focal length
- upr1: field radius
- spr1: distance between mirror and aperture stop (=0)

$$
\begin{aligned}
(S_I)_1 &= -\left(\frac{y_1}{f_1'}\right)^4 \frac{f_1'}{4}(1+b_{s1}) \\
(S_{II})_1 &= -\left(\frac{y_1}{f_1'}\right)^3 \frac{1}{4}\left[2f_1' - s_{pr1}(1+b_{s1})\right] u_{pr1} \\
(S_{III})_1 &= -\left(\frac{y_1}{f_1'}\right)^2 \frac{1}{4f_1'}\left[4f_1'(f_1' - s_{pr1}) + s_{pr1}^2(1+b_{s1})\right] u_{pr1}^2 \\
(S_{IV})_1 &= +\frac{H^2}{f_1'}
\end{aligned}
$$

# Seidel coefficient for some basic telescopes

**Table 3.3.** Seidel coefficients for some basic telescope systems. The asterisk denotes the aspheric contribution

| | Case | Surface $\nu$ | $(S_I)_\nu$ | $(S_{II})_\nu$ | $(S_{III})_\nu$ | $(S_{IV})_\nu = -(P_c)_\nu$ | Effective field curvature $2(S_{III})_\nu + (S_{IV})_\nu$ |
|---|---|---|---|---|---|---|---|
| 1. | Spherical mirror (EP at primary) | 1 | +0.25 | −0.5 | +1.0 | −1.0 | +1.0 |
| 2. | Parabolic mirror (EP at primary) | 1 | +0.25 | −0.5 | +1.0 | −1.0 | +1.0 |
| | | 1* | −0.25 | 0 | 0 | 0 | 0 |
| | | Sum | 0 | −0.5 | +1.0 | −1.0 | +1.0 |
| 3. | Classical Cassegrain telescope (EP at primary) $m_2 = -4$ | 1 | +16.0 | −8.0 | +4.0 | −4.0 | +4.0 |
| | | 1* | −16.0 | 0 | 0 | 0 | 0 |
| | | 2 | −4.21875 | +3.86719 | −3.54493 | +13.33333 | +6.24347 |
| | | 2* | +4.21875 | +3.63281 | +3.12825 | 0 | +6.25650 |
| | | Sum | 0 | −0.50000 | +3.58332 | +9.33333 | +16.49997 |
| 4. | Ritchey–Chrétien (RC) Cassegrain telescope (EP at primary) $m_2 = -4$ | 1 | +16.0 | −8.0 | +4.0 | −4.0 | +4.0 |
| | | 1* | −16.58064 | 0 | 0 | 0 | 0 |
| | | 2 | −4.21875 | +3.86719 | −3.54493 | +13.33333 | +6.24347 |
| | | 2* | +4.79940 | +4.13281 | +3.55881 | 0 | +7.11762 |
| | | Sum | 0 | 0 | +4.01388 | +9.33333 | +17.36109 |
| 5. | Dall–Kirkham (DK) Cassegrain telescope (EP at primary) $m_2 = -4$ | 1 | +16.0 | −8.0 | +4.0 | −4.0 | +4.0 |
| | | 1* | −11.78125 | 0 | 0 | 0 | 0 |
| | | 2 | −4.21875 | +3.86719 | −3.54493 | +13.33333 | +6.24347 |
| | | 2* | 0 | 0 | 0 | 0 | 0 |
| | | Sum | 0 | −4.13281 | +0.45507 | +9.33333 | +10.24347 |
| 6. | Spherical primary (SP) Cassegrain telescope (EP at primary) $m_2 = -4$ | 1 | +16.0 | −8.0 | +4.0 | −4.0 | +4.0 |
| | | 1* | 0 | 0 | 0 | 0 | 0 |
| | | 2 | −4.21875 | +3.86719 | −3.54493 | +13.33333 | +6.24347 |
| | | 2* | −11.78125 | −10.14495 | −8.73592 | 0 | −17.47184 |
| | | Sum | 0 | −14.27776 | −8.28085 | +9.33333 | −7.22837 |
| 7. | Classical Gregory telescope (EP at primary) $m_2 = +4$ | 1 | +16.0 | −8.0 | +4.0 | −4.0 | +4.0 |
| | | 1* | −16.0 | 0 | 0 | 0 | 0 |
| | | 2 | +2.53125 | +4.05469 | +6.49502 | −22.22222 | −9.23218 |
| | | 2* | −2.53125 | +3.44531 | −4.68945 | 0 | −9.37890 |
| | | Sum | 0 | −0.50000 | +5.80557 | −26.22222 | −14.61108 |
| 8. | Aplanatic Gregory telescope (EP at primary) $m_2 = +4$ | 1 | +16.0 | −8.0 | +4.0 | −4.0 | +4.0 |
| | | 1* | −15.63265 | 0 | 0 | 0 | 0 |
| | | 2 | +2.53125 | +4.05469 | +6.49502 | −22.22222 | −9.23218 |
| | | 2* | −2.89860 | +3.94531 | −5.37000 | 0 | −10.74000 |
| | | Sum | 0 | 0 | +5.12502 | −26.22222 | −15.97218 |
| 9. | 3-mirror system of Korsch–Design I (EP at primary) $m_2 = -10$ $m_2 = -0.15$ | 1 | +0.843750 | −1.125000 | +1.500000 | −1.500000 | +1.500000 |
| | | 1* | −1.065604 | 0 | 0 | 0 | 0 |
| | | 2 | −0.229711 | +0.654328 | −1.863844 | +5.400000 | +1.672312 |
| | | 2* | +0.437211 | +0.874422 | +1.748844 | 0 | +3.497688 |
| | | 3 | +0.061250 | −0.126052 | +0.259411 | −3.900000 | −3.381178 |
| | | 3* | −0.046896 | −0.277699 | −1.644412 | 0 | −3.288824 |
| | | Sum | 0 | 0 | 0 | 0 | 0 |

# Spot diagram for a Cassegrain telescope

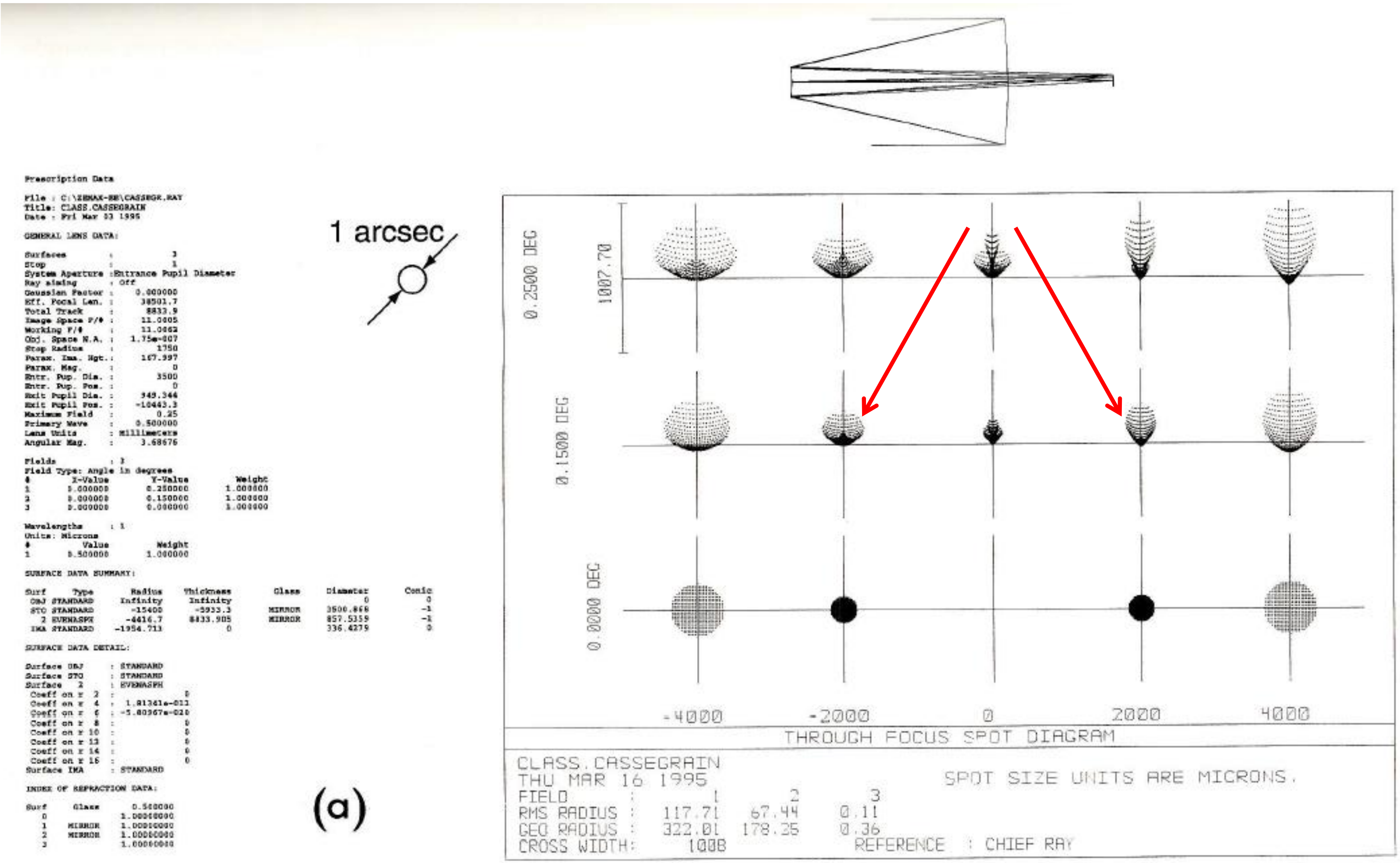



**Fig. 3.4.** (a) Spot-diagrams for a classical Cassegrain telescope with the geometry of the ESO 3.5 m NTT (f/11; $m_2 = -5$) for an optimum field curvature $r_c = -1955$ mm (concave to the incident light)

# Spot diagram for a Ritchey-Chretian telescope

```
Prescription Data

File : C:\ZEMAX-EE\RC.RAY
Title: RITCHEY-CHRETIEN
Date : Fri Mar 03 1995

GENERAL LENS DATA:

Surfaces            :            3
Stop                :            1
System Aperture     :Entrance Pupil Diameter
Ray aiming          : Off
Gaussian Factor     :     0.000000
Eff. Focal Len.     :      38501.7
Total Track         :      8833.89
Image Space F/#     :      11.0005
Working F/#         :      11.0005
Obj. Space N.A.     :    1.75e-007
Stop Radiur         :         1750
Parax. Ima. Hgt.    :      167.997
Parax. Mag.         :            0
Entr. Pup. Dia.     :         3500
Entr. Pup. Pos.     :            0
Exit Pupil Dia.     :      949.344
Exit Pupil Pos.     :     -10443.2
Maximum Field       :         0.25
Primary Wave        :     0.500000
Lens Units          : Millimeters
Angular Mag.        :      3.68676

Fields              : 3
Field Type: Angle in degrees
#       X-Value        Y-Value        Weight
1       0.000000       0.250000       1.000000
2       0.000000       0.150000       1.000000
3       0.000000       0.000000       1.000000

Wavelengths         : 1
Units: Microns
#         Value        Weight
1       0.500000       1.000000

SURFACE DATA SUMMARY:

Surf     Type        Radius    Thickness      Glass    Diameter    Conic
 OBJ STANDARD      Infinity     Infinity                      0        0
 STO EVENASPH        -15400      -5933.3     MIRROR    3500.868       -1
   2 EVENASPH       -4416.7     8833.895     MIRROR    857.9522       -1
 IMA STANDARD      -1880.66            0               335.9057        0

SURFACE DATA DETAIL:

Surface OBJ         : STANDARD
Surface STO         : EVENASPH
 Coeff on r  2      :             0
 Coeff on r  4      :  8.15183e-016
 Coeff on r  6      :             0
 Coeff on r  8      :             0
 Coeff on r 10      :             0
 Coeff on r 12      :             0
 Coeff on r 14      :             0
 Coeff on r 16      :             0
Surface   2         : EVENASPH
 Coeff on r  2      :             0
 Coeff on r  4      :  2.10756e-012
 Coeff on r  6      :    -7.76e-020
 Coeff on r  8      :             0
 Coeff on r 10      :             0
 Coeff on r 12      :             0
 Coeff on r 14      :             0
 Coeff on r 16      :             0
Surface IMA         : STANDARD

INDEX OF REFRACTION DATA:

Surf     Glass       0.500000
   0                 1.00000000
   1    MIRROR       1.00000000
   2    MIRROR       1.00000000
   3                 1.00000000
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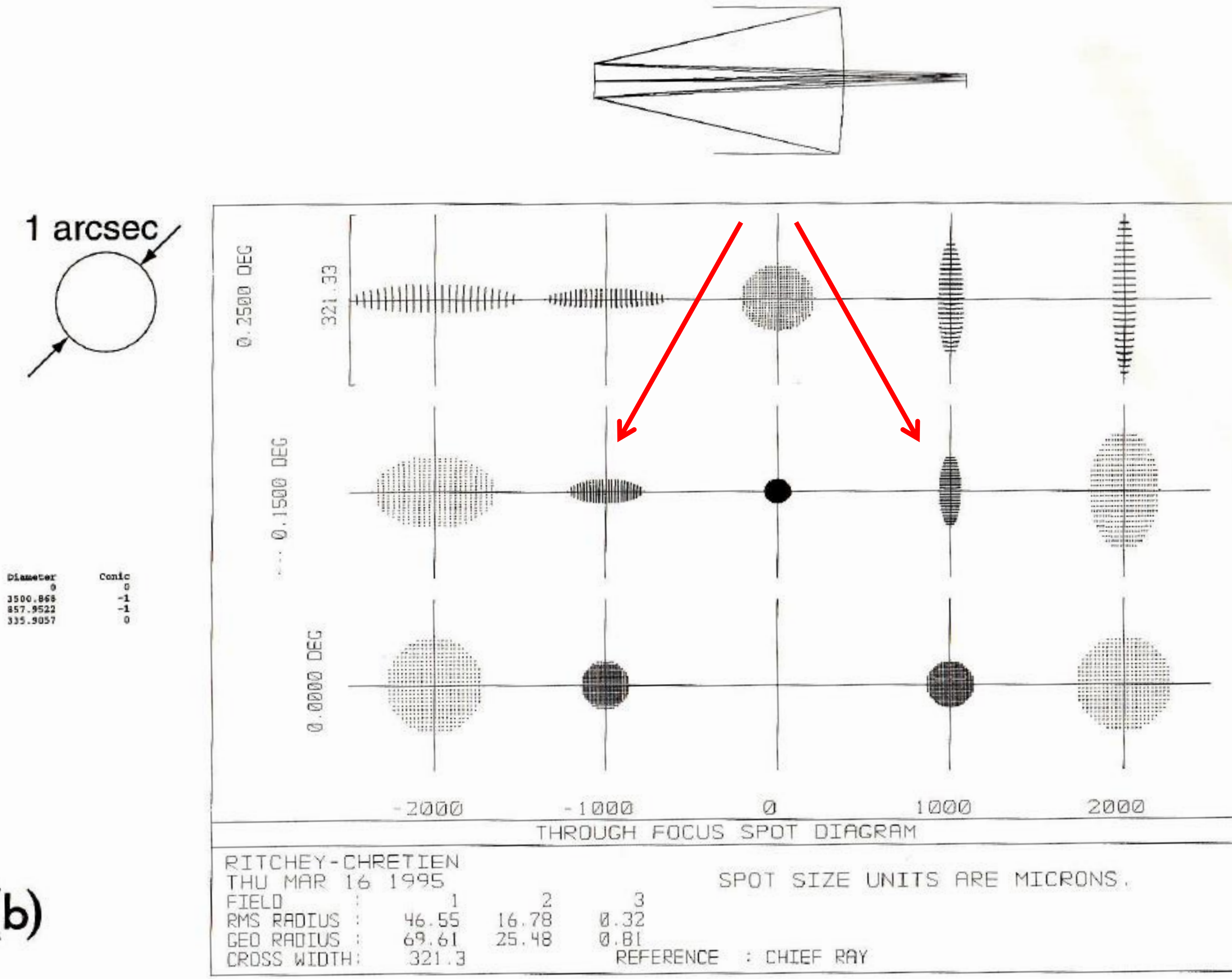
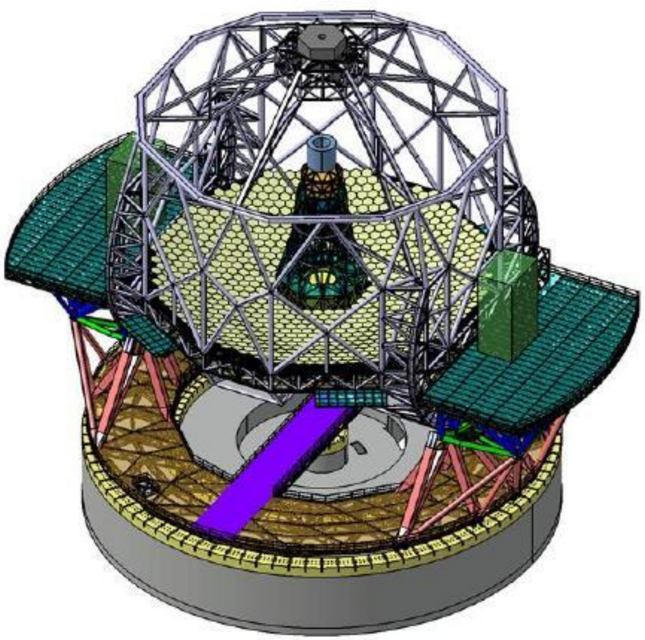


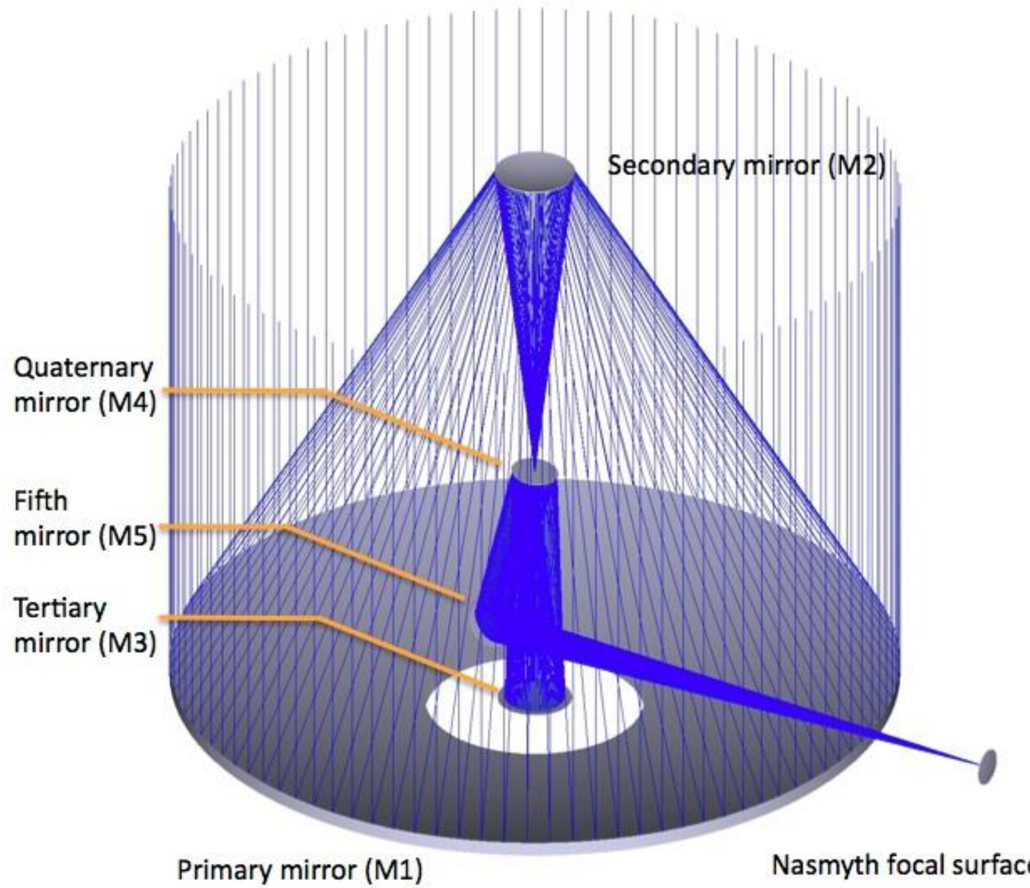

(b)

**Fig. 3.4.** (b) Spot-diagrams for an RC aplanatic telescope with the geometry of the ESO 3.5 m NTT (f/11; $m_2 = -5$) for an optimum field curvature $r_c = -1881$ mm

# Reflective telescope (4) : Three-mirrror anastigmat

- European-Extremely Large Telescope

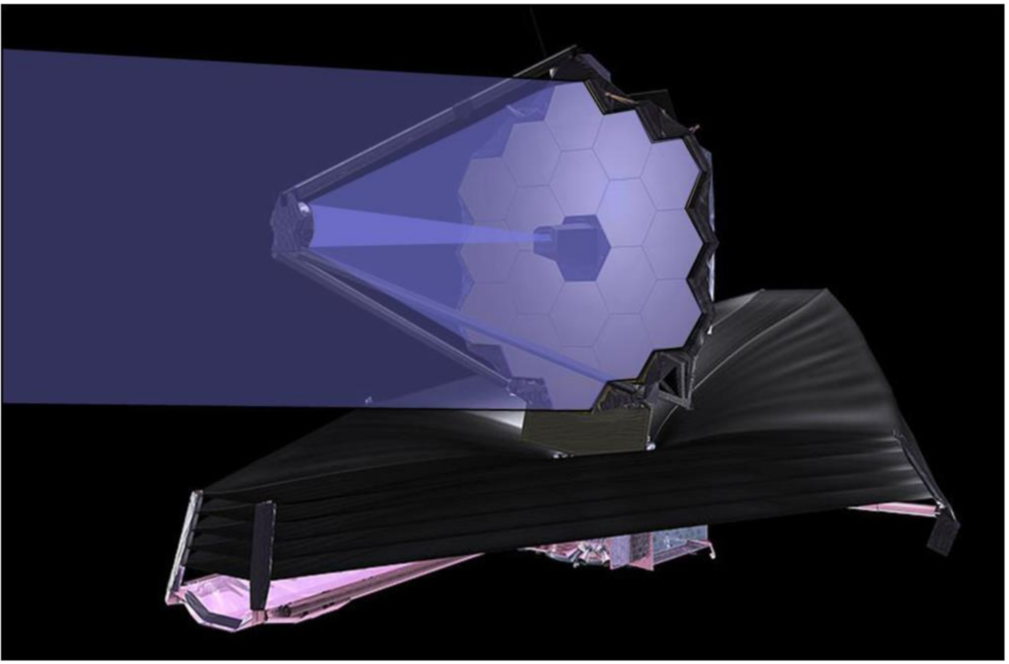


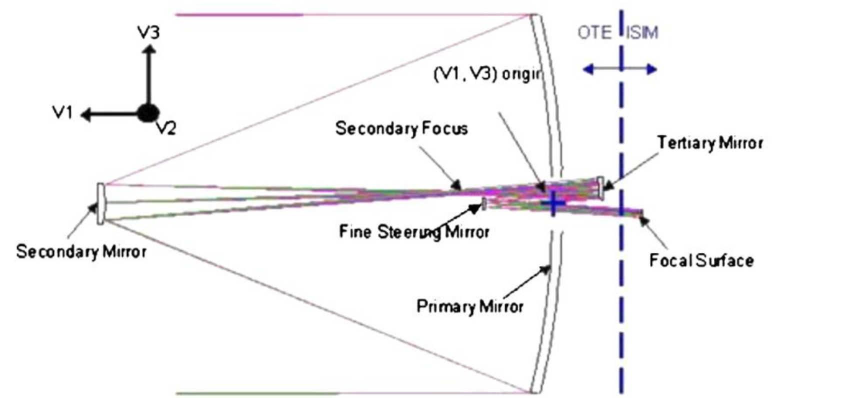



From eso.org

# Reflective telescope (4) : Three-mirrror anastigmat

- James-Webb Space telescope





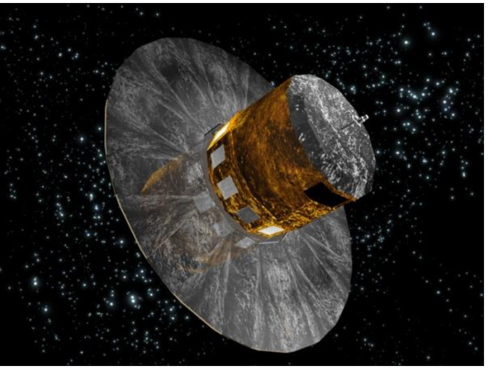

| Component | RoC (mm) | Surface | Conic | V1 (mm) | V2 (mm) | V3 (mm) | Phys. Size (mm) |
|---|---|---|---|---|---|---|---|
| Primary | 15879.7 | concave | -0.9967 | 0 | 0 | 0 | 6605.2 |
| Secondary | 1778.9 | convex | -1.6598 | 7169.0 | 0 | 0 | 738 |
| Tertiary | 3016.2 | concave | -0.6595 | -796.3 | 0 | -0.19 | 728 x 517 |
| Fine Steering Mirror | | flat | | 1047.8 | 0 | -2.36 | 172.5 |

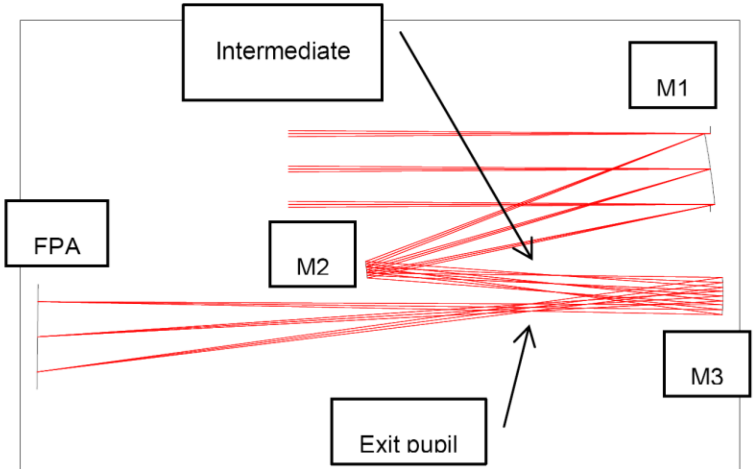
PM/SM EFL ≈ 59400 mm
OTE EFL ≈ 131400 mm

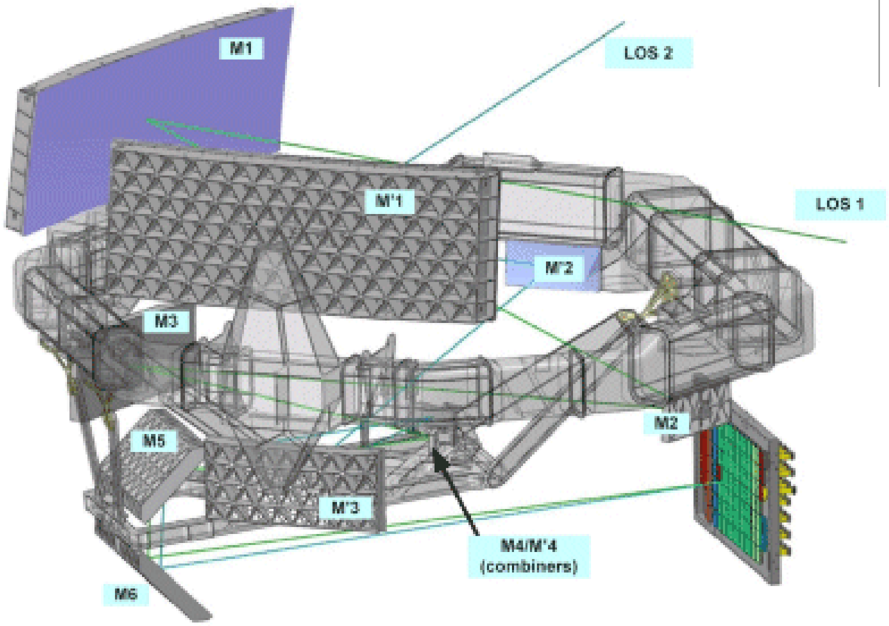
From SPIE digital library

# Reflective telescope (5) : Three-mirrror anastigmat

- GAIA : combine two FoVs with 106.5deg separation. 1.45x0.5m aperture with 35m focal length. 60mas x 180 mas sampling with 0.7deg x 0.7deg FoV.



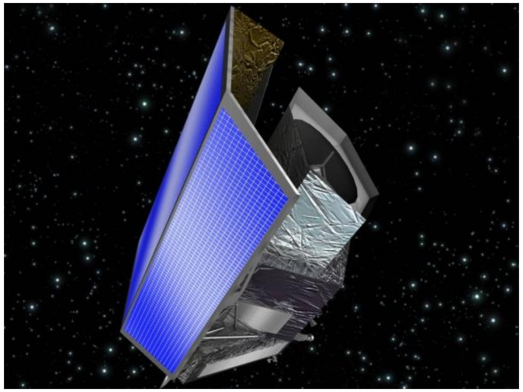


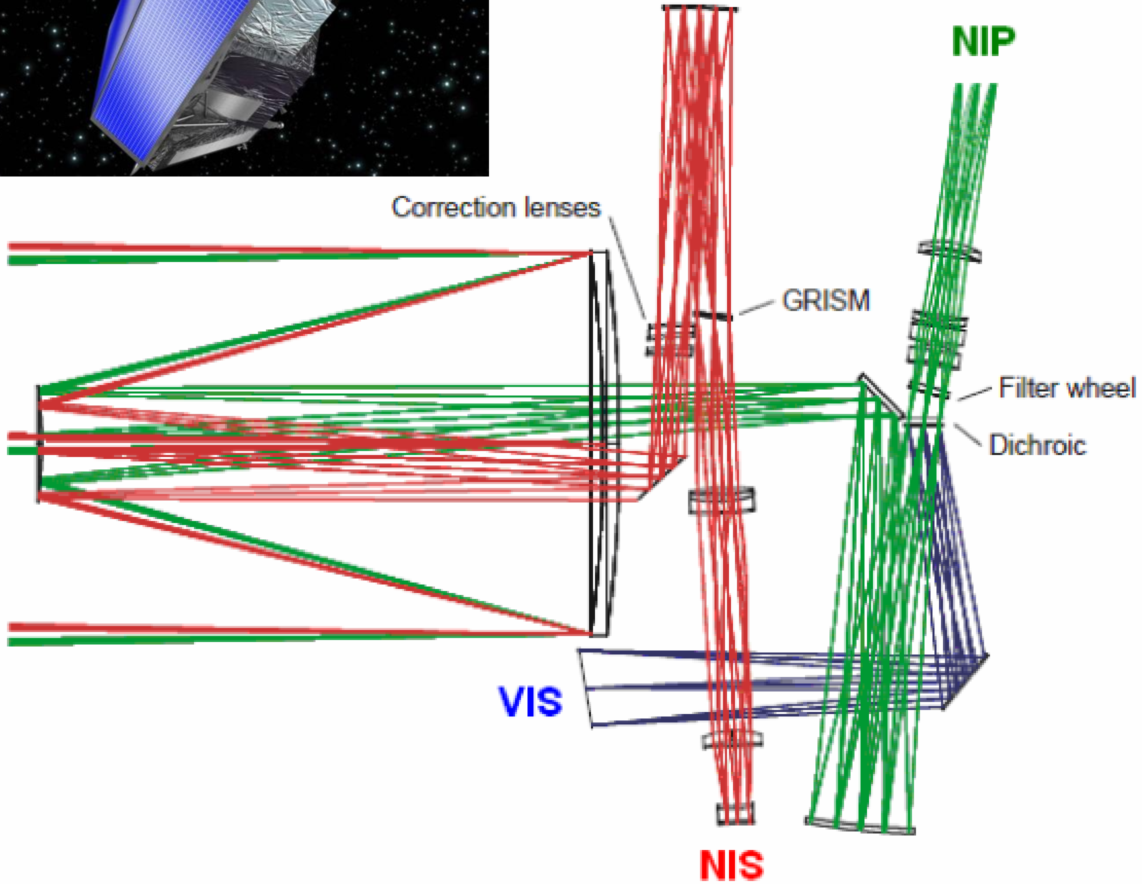



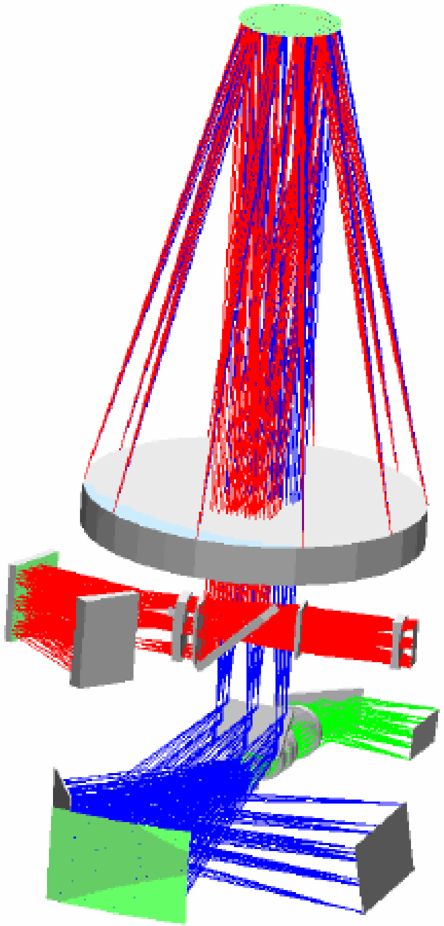

From sci.esa.int

# Reflective telescope (4) : Three mirrror Korsch

- Euclid mission



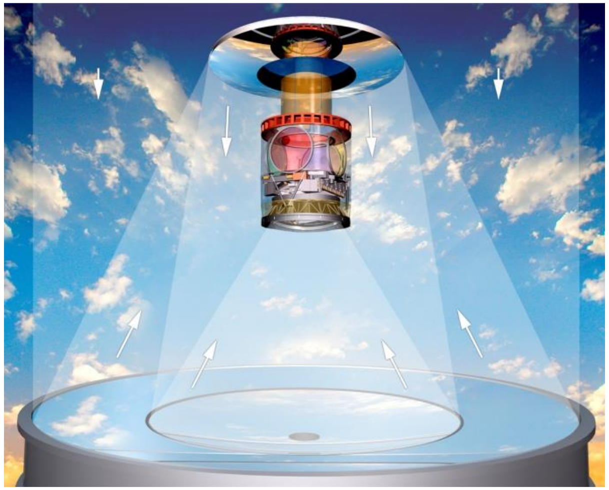


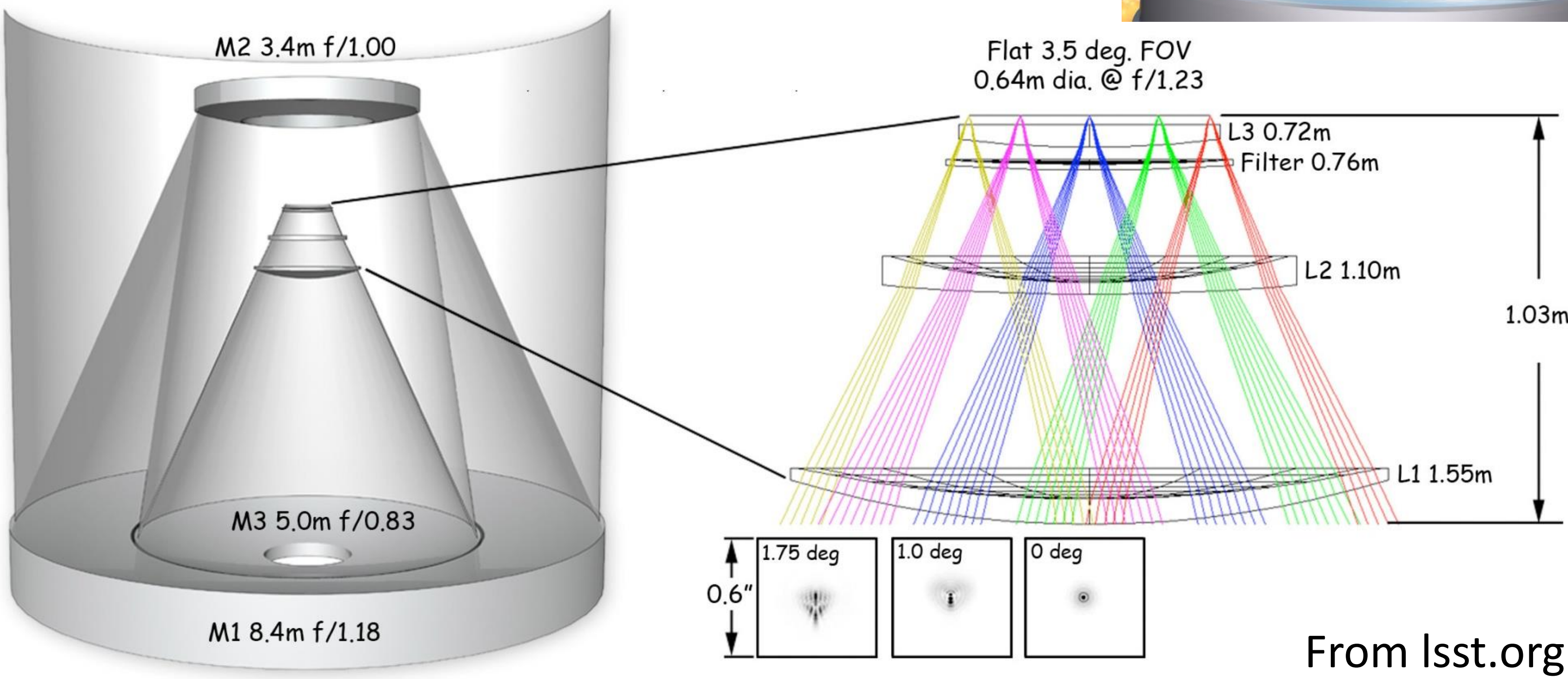



From euclid.cnes.fr

# Reflective telescope (4) : Three mirrrors

- Large Synoptic Survey telescope

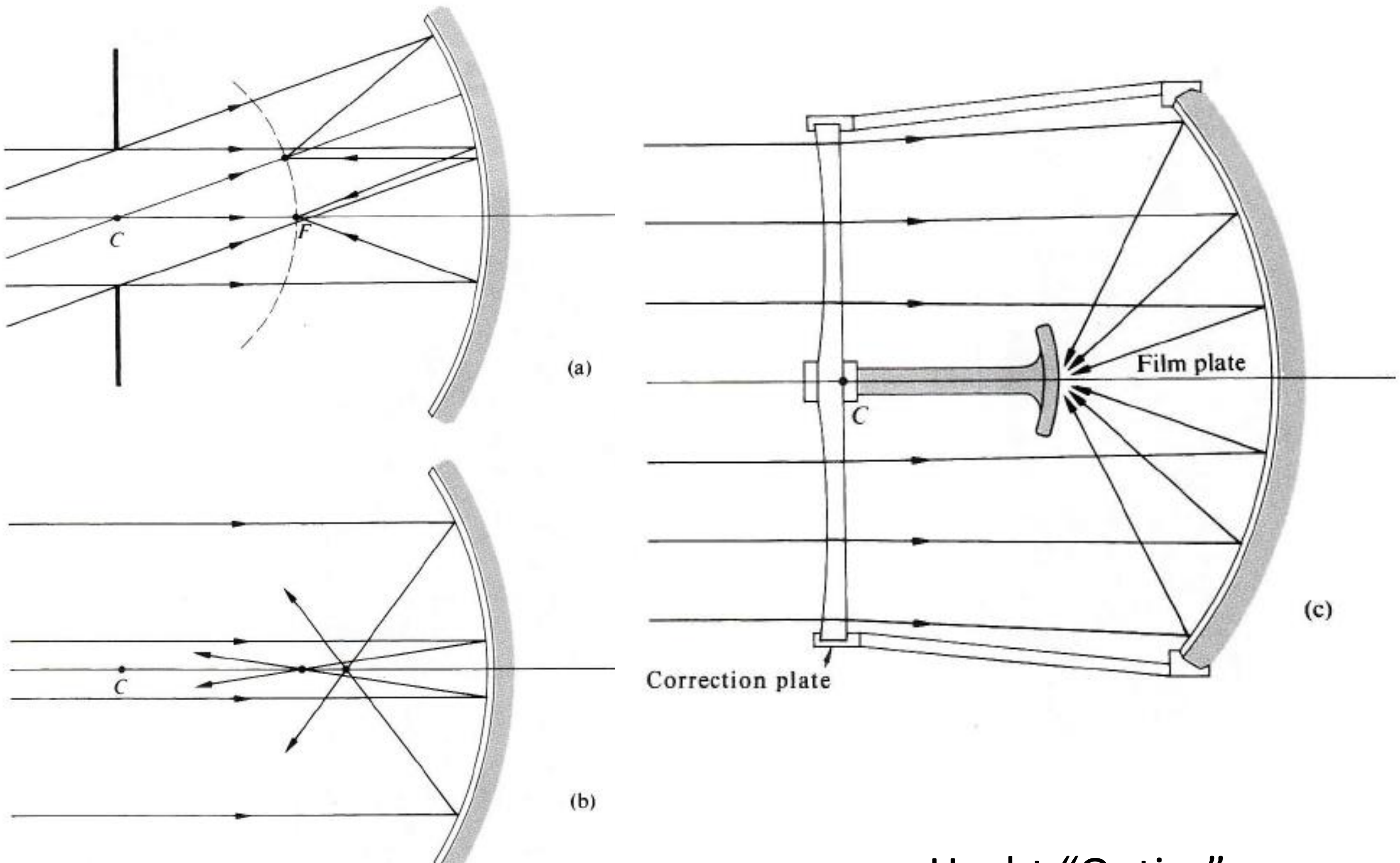





From lsst.org

# Schmidt telescope

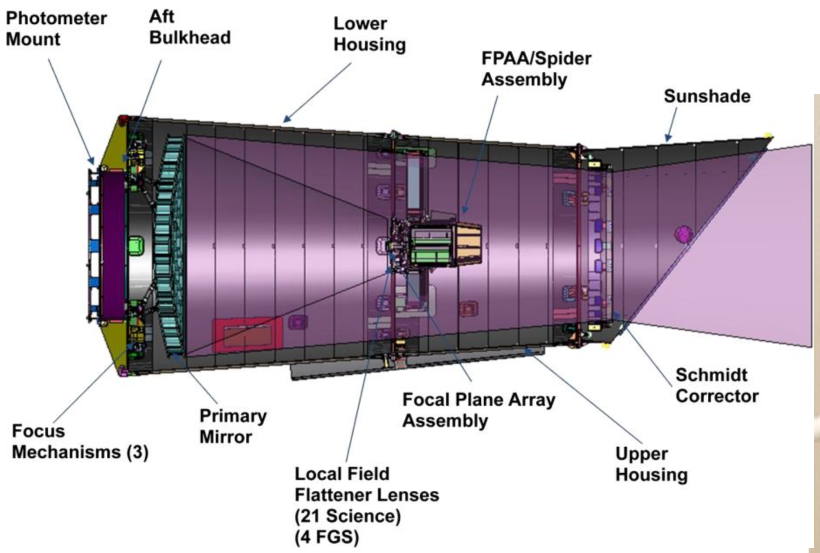
- Catoptric (reflecting) + Dioptric (refracting) = Catadioptric
- Schmidt design for a wide-field telescope
- Correction plate correcting the spherical aberration (b)




Hecht “Optics”

# Curved focal plane of a Schmidt telescope

- Kepler mission telescope is a Schmidt telescope to cover wide field of view.




# Important concept (1) : pupil

- Pupil = image of the aperture stop
- Entrance pupil : Aperture Stop seen from object side
- Exit pupil : Aperture Stop seen from image side
- Chief ray : any ray from an off-axis object point pass through the center of the aperture stop

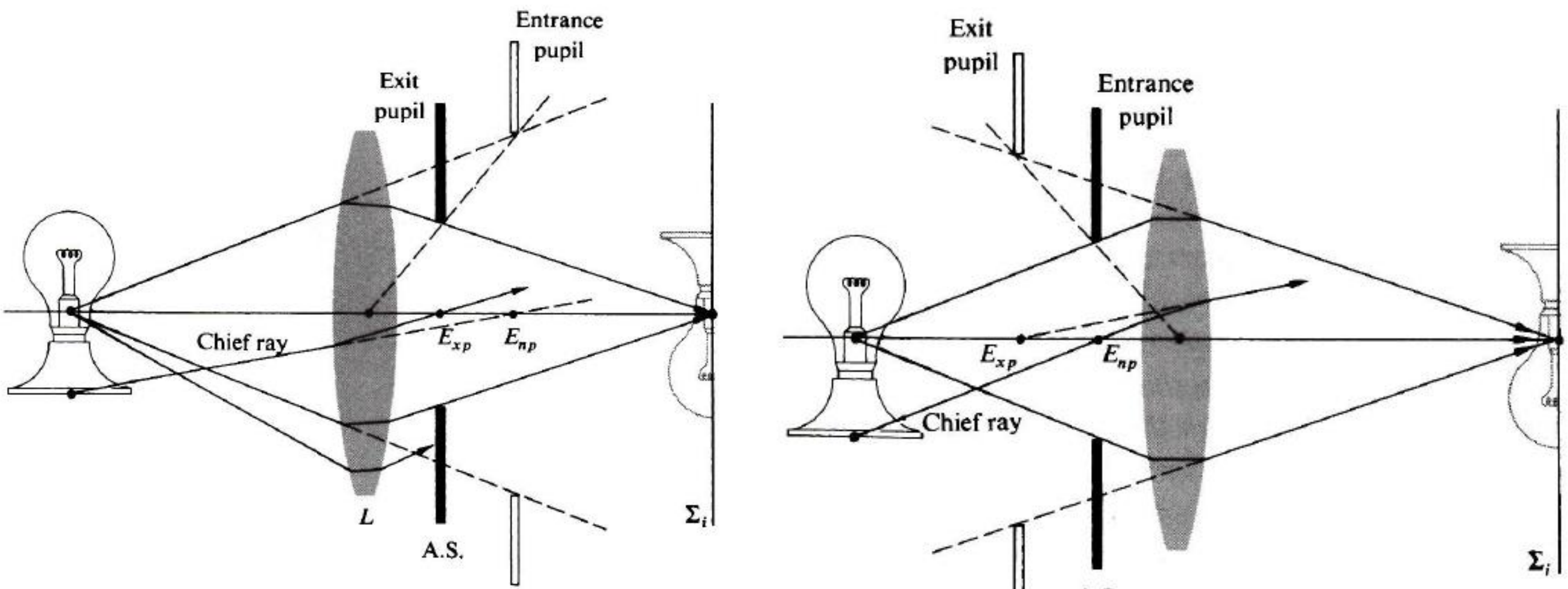
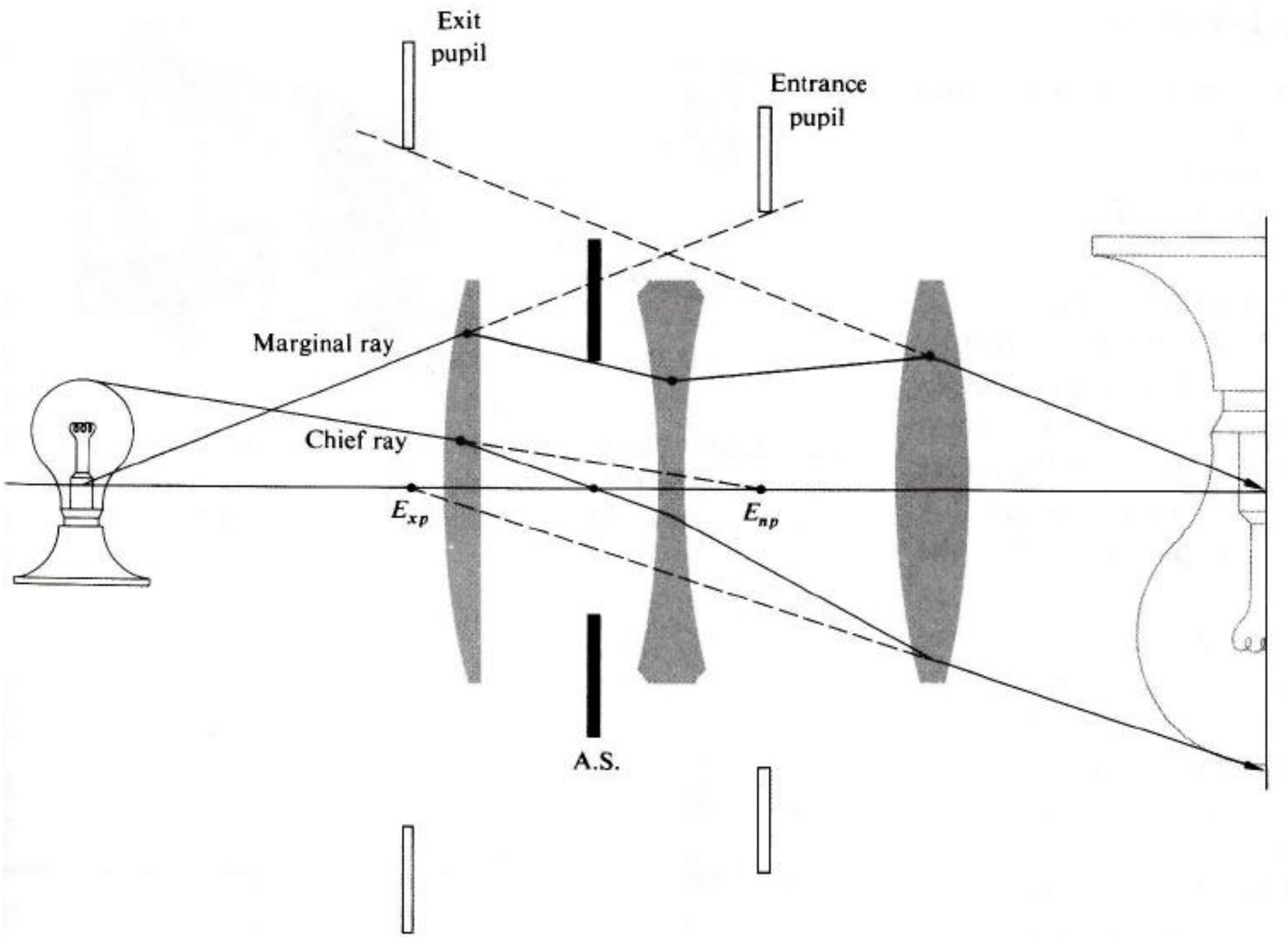



**Figure 5.34** Entrance pupil and exit pupil.

Hecht "Optics"

# Important concept (1) : pupil

- Pupil in a three lens system

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# Important concept (2) : Vignetting

- Vignetting in a two lens system

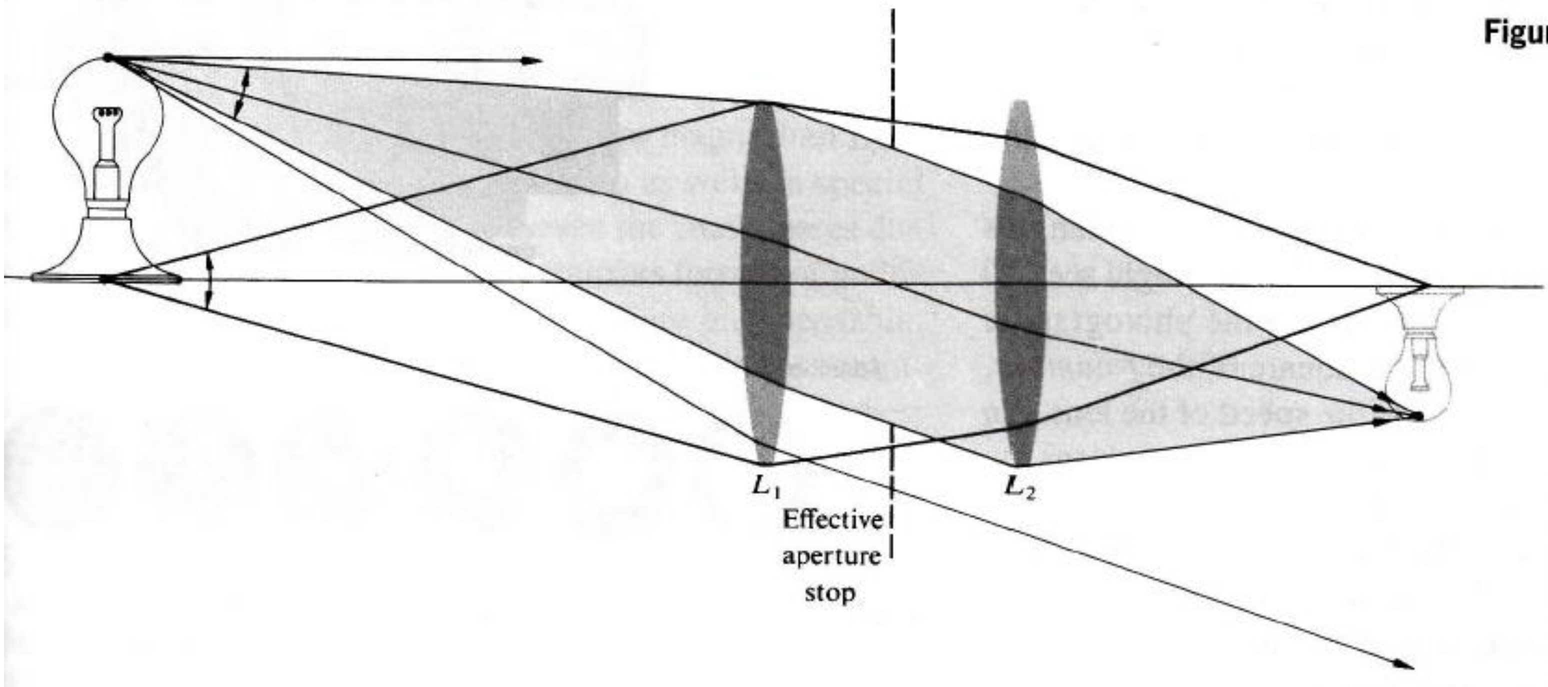


**Figure 5.37** Vignetting.

Hecht "Optics"

# Important concept (3) : Telecentricity

- Telecentric imaging system
  - Telecentricity is important for Astrometry, Multi-object spectroscopy etc.
- Exit pupil is at infinity

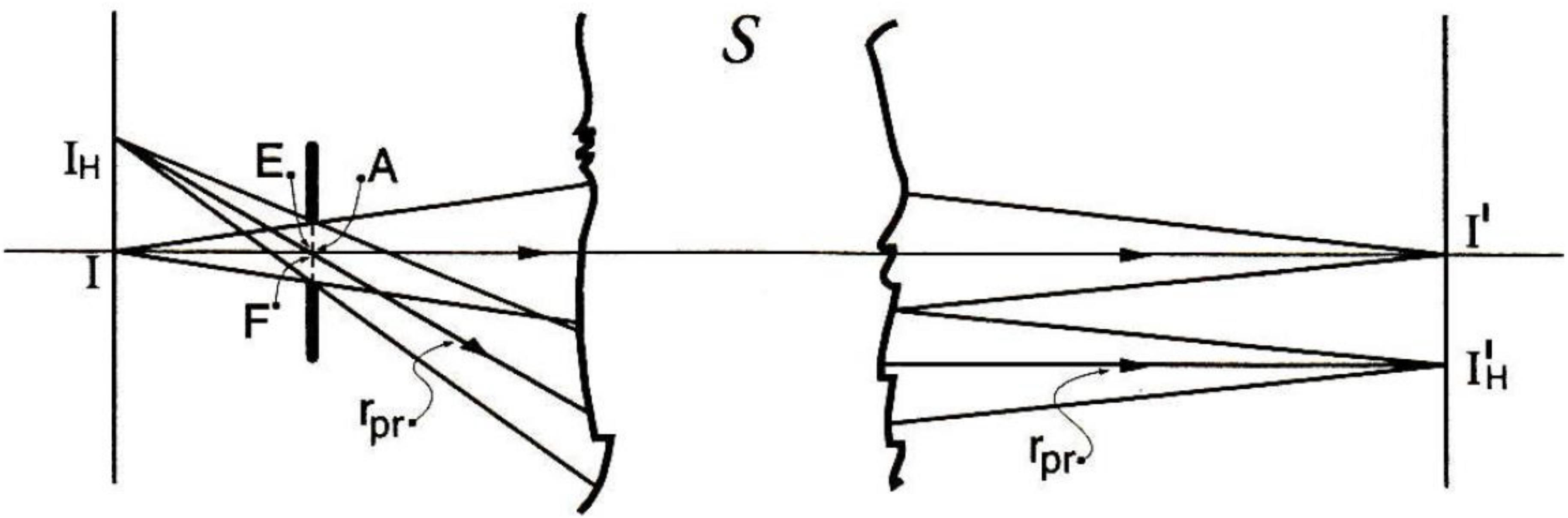


**Fig. 2.7.** Telecentric aperture stop

Wilson "Reflecting Telescope Optics I"