Introduction

- We are conducting a concept study on a wide field of regard (FoR) Multi-Object Adaptive Optics (MOAO) system for Thirty Meter Telescope (TMT-AGE: TMT - Analyzer for Galaxies in the Early universe [1]).
- Because the number density of high-redshift galaxies is small, which are key science targets of TMT-AGE, enlarging the FoR of an MOAO system up to around 10° is critical.
- We propose a new tomographic reconstruction method which increases a FoR of MOAO System. The new method use atmospheric wind profiles, wind speeds and directions at each altitude, and WFS measurements at previous time steps to increase the number of virtual measurement points of atmospheric turbulence layers for tomographic reconstruction.
- The wind profiles are derived using the atmospheric turbulence structure estimated with tomographic reconstruction.

Methods

- New Tomographic Reconstruction
  - Under the assumption of Taylor’s frozen-flow hypothesis, evolution of atmospheric turbulences can be described as the shift of atmospheric layers due to the wind. If the wind profiles are known, we can build the relation between measurements at previous time steps and current atmospheric turbulence.
  - The linear equation between current atmospheric turbulence $\phi$ and Shack Hartmann wavefront sensor (WFS) measurements $s$ at both of current and previous time steps is given as follows.
    \[
    \begin{bmatrix}
    s(t) \\
    s(t - \Delta t)
    \end{bmatrix} = \begin{bmatrix}
    \Gamma \\
    \Gamma'
    \end{bmatrix} \phi(t) + 
    \begin{bmatrix}
    \eta(t) \\
    \eta(t - \Delta t)
    \end{bmatrix},
    \]
    Here, $\Gamma$ and $\Gamma'$ are influence matrices between $\phi(t)$ and $s(t)$ and $s(t - \Delta t)$, respectively, and $\eta$ is a noise vector.
  - The solution for Eq.(1) is constructed with Minimum Variance Reconstruction.
    \[
    \left((\Gamma')^T \Gamma + \sigma^2 L^T L\right) \phi(t) = (\Gamma')^T \begin{bmatrix}
    s(t) \\
    s(t - \Delta t)
    \end{bmatrix}
    \] (2)
    The Eq.(2) is solved with Conjugate Gradient Method.

- Estimation of Wind Speeds and Directions
  - Tomographic reconstruction provides the phase map of atmospheric turbulence at each altitude. Wind profiles can be estimated from the time variation of the phase on the maps.
  - We use a temporal auto correlation of estimated slope for deriving wind profiles, since a slope correlation has a sharper peak than a wavefront correlation.
  - Wind speed and direction at each altitude are calculated with tracking a peak of temporal auto correlation of estimated slope which moves depending on wind speed and direction at this altitude and a time delay for the temporal correlation.

Results

- Wind profiles for the new tomography method are derived using the temporal auto correlation of estimated slope.
- Assuming 100% frozen-flow and perfect information of atmospheric layer altitudes.
- The measurements at 0.1s previous time step are used, $\Delta t = 0.1s$.

Table 1: Atmospheric Model and Wind Estimation Error

<table>
<thead>
<tr>
<th>Altitude [km]</th>
<th>0</th>
<th>0.1</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind speed [m/s]</td>
<td>7.617</td>
<td>8.600</td>
<td>9.729</td>
<td>12.307</td>
<td>18.952</td>
<td>24.213</td>
<td>57.753</td>
</tr>
<tr>
<td>The error of wind speeds [m/s]</td>
<td>0.080</td>
<td>0.044</td>
<td>0.068</td>
<td>0.381</td>
<td>0.389</td>
<td>2.370</td>
<td>1.142</td>
</tr>
<tr>
<td>The error of wind direction [deg]</td>
<td>0.00</td>
<td>0.02</td>
<td>0.04</td>
<td>0.06</td>
<td>0.01</td>
<td>0.06</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Discussion

- Since the displacement error within 0.1s due to wind estimation error is less than 25cm, which is half of DM segment size, a tomographic error due to the error of wind estimation is smaller than the fitting error.
- In reality, the time scale of frozen flow is limited and the correlation peak becomes fainter with time [3]. This affects the accuracy of the detection of the correlation peaks.
- We will evaluate the accuracy of the wind estimation method with the uncertainties of the altitudes of atmospheric layers, for example uncertainty with estimation by SLODAR [4], in a future work.

Conclusion

We propose a new tomographic reconstruction method using measurements at previous time steps and wind information to increase the FoR of MOAO systems and evaluate the method with numerical simulations. We also introduce a method to estimate wind speeds and directions at each altitude. The wind profiles can be estimated with sufficient accuracy under assumption of 100% frozen flow and perfect knowledge of altitudes of atmospheric turbulences. With the estimate of wind speeds and directions, the new tomographic reconstruction method reduces WFE to less than 250nm for the wider LGS configuration in 8° FoR (Fig.2).

Reference