

# 相対論的ドリフト電流をもつ磁気リコネクション： パルサー風でリコネクションは起きるか

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# current starvation

$$J = \frac{c}{4\pi\lambda} B = en\nu, \quad B \propto \frac{1}{r}, \quad n \propto \frac{1}{r^2}$$

$$\frac{\nu}{c} = \frac{B}{4\pi\lambda en} = \left( \frac{B_{lc}}{4\pi\lambda en_{GJ}\kappa} \right) \left( \frac{r}{r_{lc}} \right) > \frac{1}{2\kappa} \frac{r}{r_{lc}}$$

$$\frac{B_{lc}}{4\pi\lambda en_{GJ}\kappa} \approx \frac{c}{2\lambda\Omega\kappa} > \frac{1}{2\kappa}$$

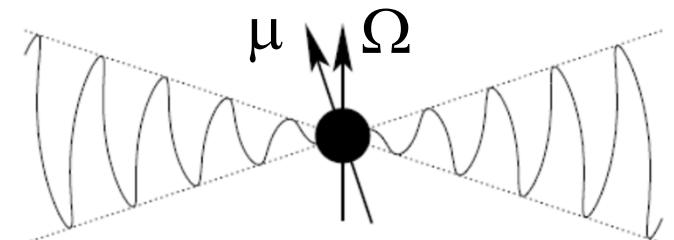
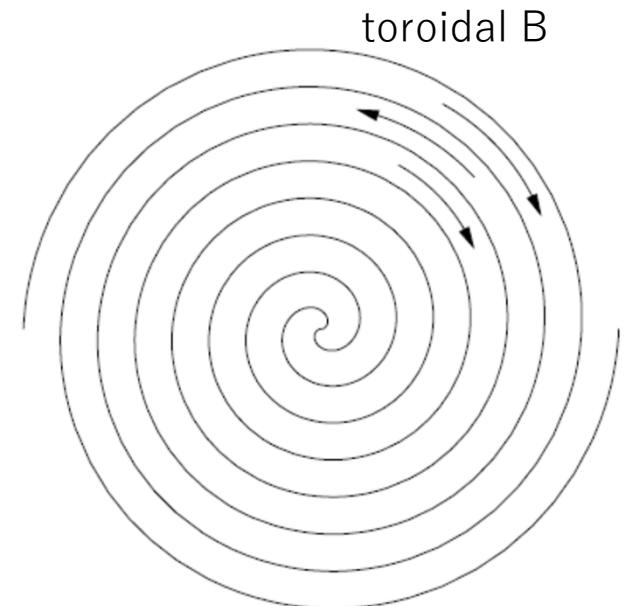
$$n_{GJ} = \frac{\Omega \cdot B}{2\pi e c} \quad \text{Goldreich-Julian density}$$

$\kappa = 10^3 - 10^4$  multiplicity (Arons 1983)

$$\lambda < \frac{c}{\Omega}$$

$$r_{lc} = 10^9 \text{ cm}, \quad r_{shock} = 0.1 \text{ pc} = 3 \times 10^{18} \text{ cm}$$

Pulsar Wind



e.g. Usov, 1975; Michel 1982; Lyubarsky & Kirk 2001

# Relativistic magnetic reconnection

$T/mc^2 > 1$  relativistic hot plasma

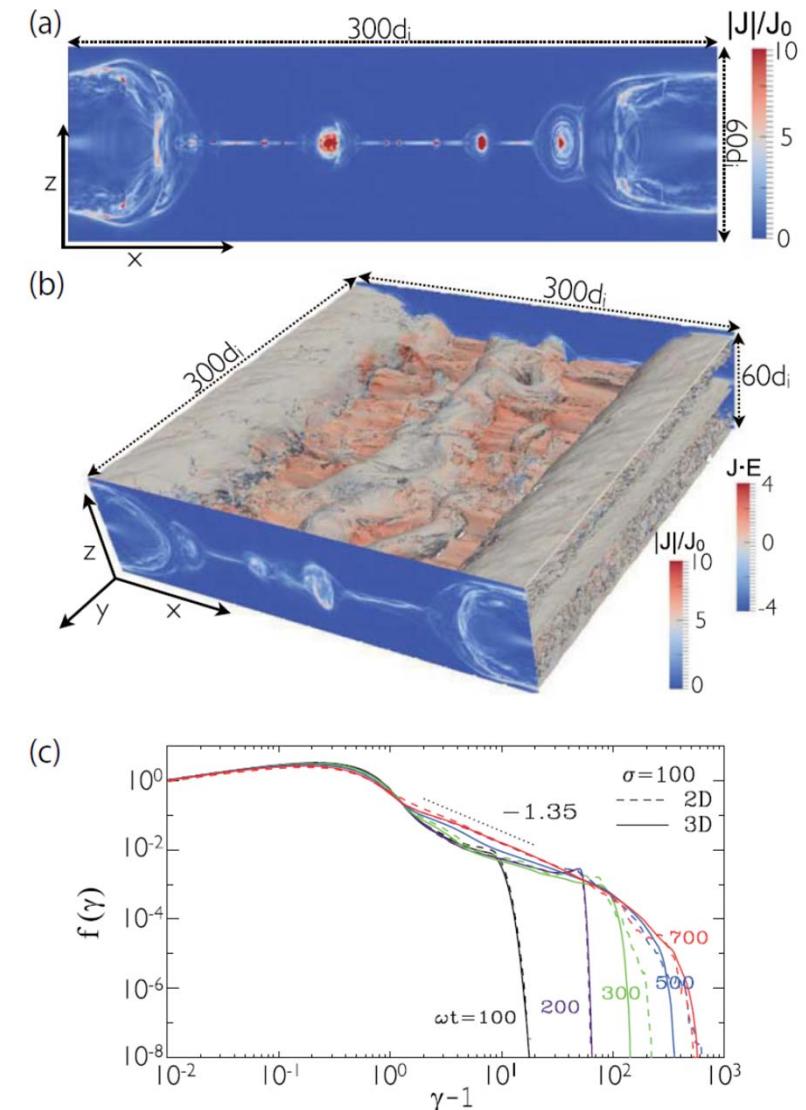
$$P + \frac{1}{8\pi}B^2 = \text{const. } P = 2nT \text{ (pair plasma)}$$

$$\sigma = B^2/8\pi nmc^2 = 2T/mc^2$$

$$v_A = c\sqrt{\sigma/(1+\sigma)} \sim c$$

Non-thermal particle acceleration,  
 Power-law energy spectrum with a hard spectral index  $s < 2$ ,  
 Large reconnection rate (fast reconnection)

e.g. Zenitani & MH, ApJ 2001; Jaroschek+ ApJ 2004;  
 Sironi & Spitkovsky, ApJ 2014; Guto+ PRL 2014



# Relativistic magnetic reconnection

relativistic hot plasma

$$T/mc^2 > 1$$

$$P + \frac{1}{8\pi}B^2 = \text{const.}$$

- fast reconnection,
- rapid magnetic energy release

(Theory) Zelenyi & Krasnosel'skihk, Sov. Astron. 1977  
(Simulation) Zenitani & MH, ApJ 2001; Jaroschek+ ApJ 2004;  
Sironi & Spitkovsky, ApJ 2014; Guto+ PRL 2014

relativistic drift velocity

$$v_d \approx c$$

$$-\nabla P + \frac{en}{c} v_d \times B = 0$$



So far,

- probably fast reconnection,
- Probably rapid magnetic energy release,

But, not yet investigated..

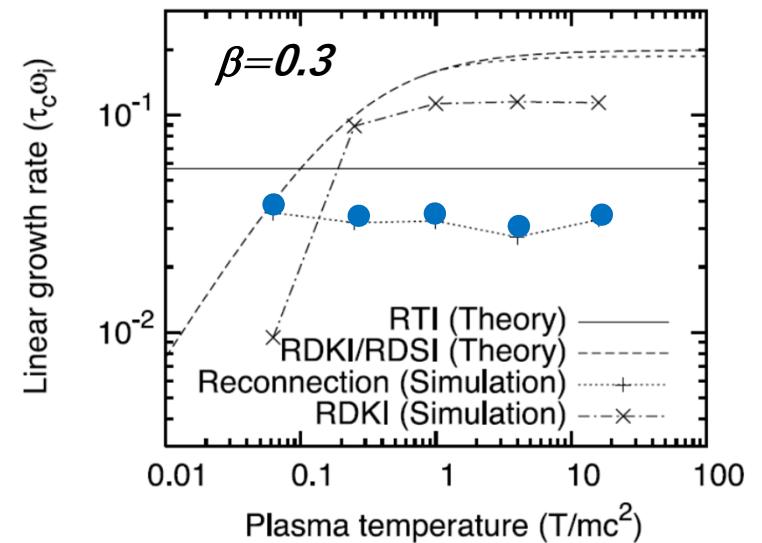
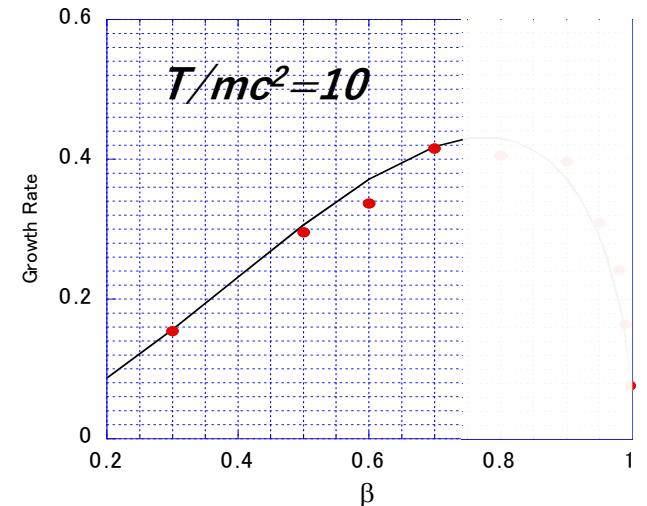
# tearing instability

$$\Gamma_\beta = 1/\sqrt{1 - \beta^2} \approx O(1)$$

growth rate increases with increasing  $\beta = v_d/c$

$$\gamma_{MR}\tau_c = \begin{cases} (1 - k^2\lambda^2)\beta^{\frac{3}{2}} \left(\frac{2T}{mc^2}\right)^{-1/2}, & T \ll mc^2 \\ (1 - k^2\lambda^2)\beta^{\frac{3}{2}}, & T \gg mc^2 \end{cases}$$

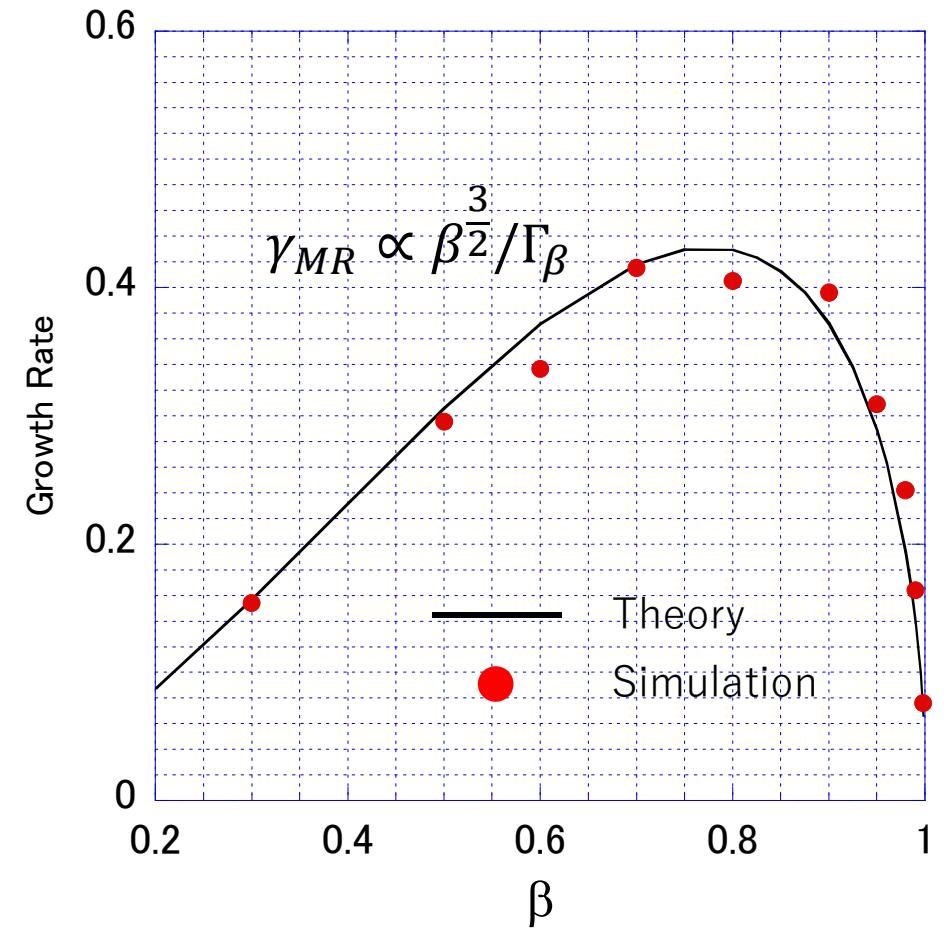
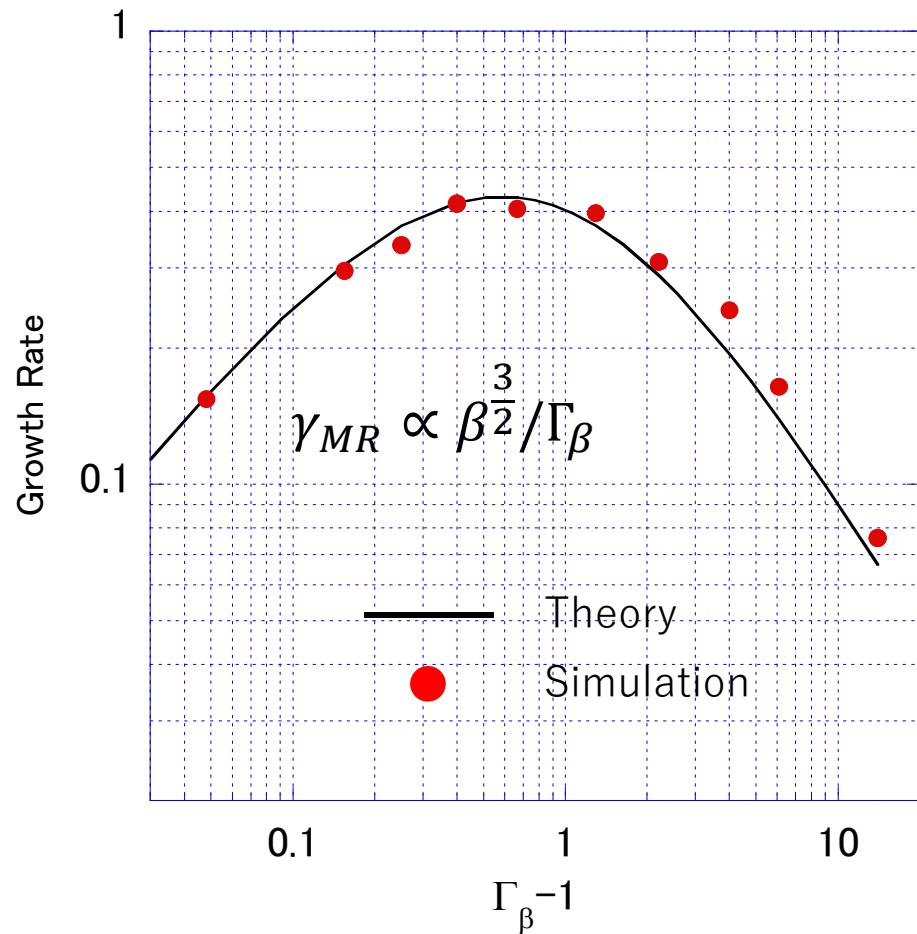
(Zelenyi & Krasnosel'skikh, Sov. Astron. 1977)



(Zenitani & MH, ApJ, 2007)

# reconnection with relativistic drift velocity ( $\beta=v_d/c$ ): PIC simulation

$$T/mc^2 = 10$$
$$\Gamma_\beta = 1/\sqrt{1 - \beta^2}$$



# collisionless conductivity in Ohm's law (heuristic)

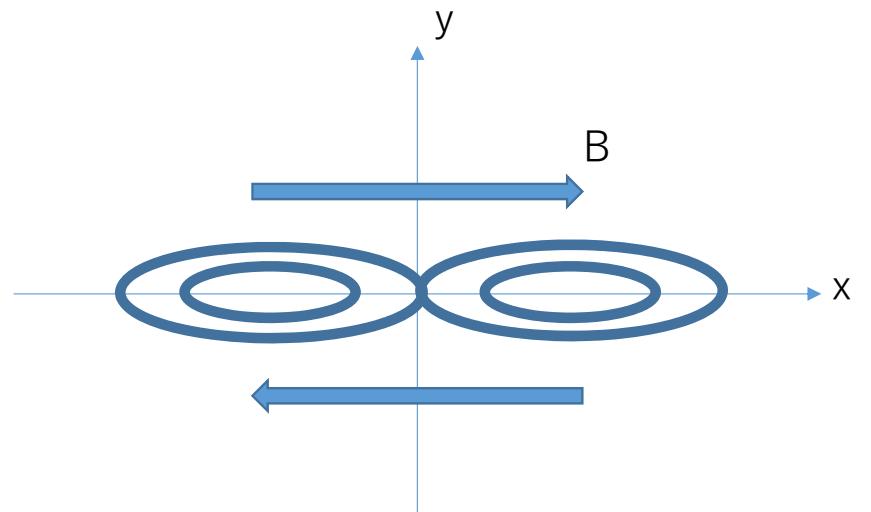
near the neutral sheet,  $B \sim 0$

$$m \frac{dv}{dt} = eE - v_c mv \approx 0$$

$$v_c = kv_{th}$$
 Landau resonance around X-point

$$J_z = env_z = \sigma E_z$$

$$\sigma = \frac{ne^2}{m} \frac{1}{v_c} = \frac{ne^2}{m} \frac{1}{kv_{th}}$$



# collisionless conductivity (Landau resonance)

$$f_0 = \frac{\bar{N}}{4\pi m^2 c \Theta K_2(\frac{mc^2}{\Theta})} \exp \left[ -\frac{\Gamma_\beta}{\Theta} (E + c\beta p_z) \right]$$

$$\beta = v_d/c \quad T = \Gamma_\beta \Theta$$

$$\Gamma_\beta = 1/\sqrt{1 - \beta^2}$$

$$f_0 \propto \exp \left[ -\frac{m}{2\Theta} (v_x^2 + v_y^2 + (v_z - v_d)^2) \right]$$

$$\left\{ \frac{\partial}{\partial t} + \nu \cdot \frac{\partial}{\partial x} + e \left( \frac{\nu}{c} \times B_0 \right) \cdot \frac{\partial}{\partial p} \right\} f_1 = \left\{ e \left( E_1 + \frac{\nu}{c} \times B_1 \right) \cdot \frac{\partial}{\partial p} \right\} f_0$$

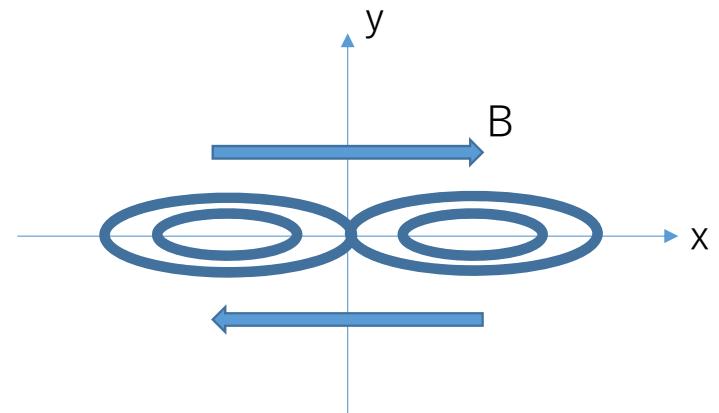
$$E_1 = -\frac{1}{c} \frac{\partial}{\partial t} A_1$$

$$B_1 = \nabla \times A_1$$

$$\left\{ \frac{\partial}{\partial t} + \nu \cdot \frac{\partial}{\partial x} \right\} f_1 \approx \frac{ef_0}{cT} \left\{ \left( \frac{\partial}{\partial t} + \nu \cdot \frac{\partial}{\partial x} \right) (v_d A_{1z}) - \frac{\partial}{\partial t} (v_z A_{1z}) \right\}$$

$$\frac{\partial}{\partial t} + \nu \cdot \frac{\partial}{\partial x} = -i(\omega - kv_x)$$

$$f_1 \approx \frac{ef_0}{cT} \left( v_d A_{1z} - \frac{\omega}{\omega - kv_x} v_z A_{1z} \right)$$



# collisionless conductivity (non-relativistic)

$$f_0 = n \left( \frac{m}{2\pi T} \right)^{3/2} \exp \left[ -\frac{m}{2\Theta} (v_x^2 + v_y^2 + (v_z - v_d)^2) \right]$$

$$f_1 \approx \frac{ef_0}{cT} \left( v_d A_{1z} - \frac{\omega}{\omega - kv_x} v_z A_{1z} \right)$$

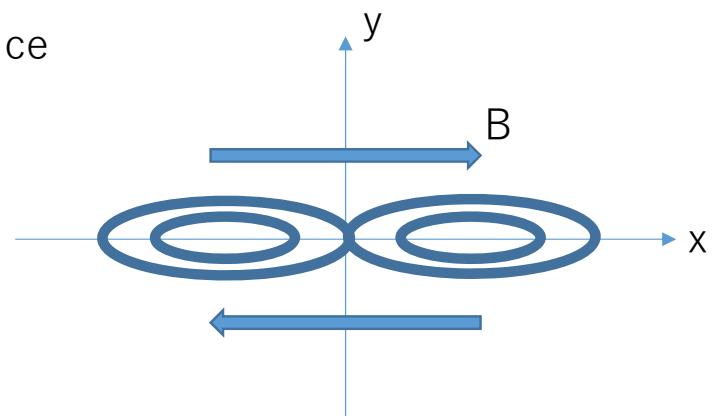
$$f_1^{ad} \approx \frac{ef_0}{cT} v_d A_{1z} \quad f_1^{res} \approx -\frac{ef_0}{cT} \frac{\omega}{\omega - kv_x} v_z A_{1z}$$

$$J_{1z} \approx \iiint e v_z f_1^{res} d^3 v \approx \frac{ne^2}{m} \frac{1}{kv_{th}} E_{1z}$$

Landau resonance

$$\sigma = \frac{ne^2}{m} \frac{1}{kv_{th}}$$

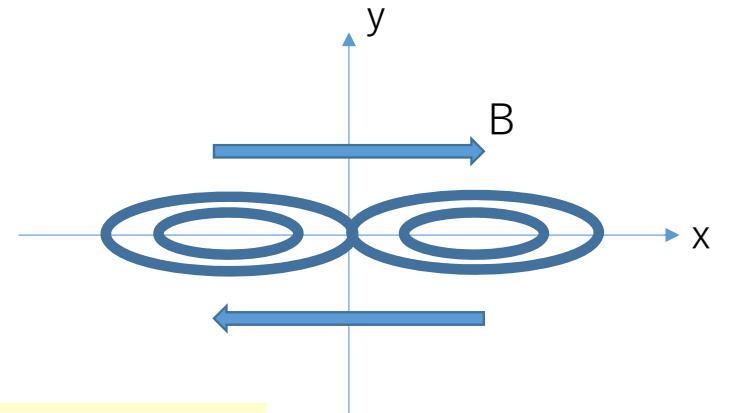
$$E_1 = -\frac{1}{c} \frac{\partial}{\partial t} A_1$$



# collisionless conductivity (relativistic)

$$f_0 = \frac{\bar{N}}{4\pi m^2 c \Theta K_2(\frac{mc^2}{\Theta})} \exp\left[-\frac{\Gamma_\beta}{\Theta}(E + c\beta p_z)\right]$$

$$f_1 \approx -\frac{ef_0}{cT} \frac{\omega}{\omega - kv_x} v_z A_{1z}$$



$$\begin{aligned} J_{1z} &\approx \iiint e v_z \frac{ef_0}{cT} \frac{\omega}{\omega - kv_x} v_z A_{1z} d^3 p \\ &\approx \left(\frac{\bar{N}\Gamma_\beta e^2}{m}\right) \left(\frac{\Gamma_\beta}{kc}\right) \left(\frac{\Gamma_\beta mc^2}{\Theta}\right) E_{1z} = \left(\frac{ne^2}{m}\right) \left(\frac{\Gamma_\beta}{kc}\right) \left(\frac{mc^2}{T}\right) E_{1z} \end{aligned}$$

$$\sigma^{non-rel} = \frac{ne^2}{m} \frac{1}{kv_{th}}$$

$$\sigma^{rel} = \left(\frac{ne^2}{m}\right) \left(\frac{\Gamma_\beta}{kc}\right) \left(\frac{mc^2}{T}\right)$$

# tearing instability ( $\Gamma_\beta \gg 1$ )

$$\Gamma_\beta \equiv 1/\sqrt{1 - \beta^2}$$

$$\gamma_{MR}\tau_c = \begin{cases} (1 - k^2\lambda^2)\beta^{\frac{3}{2}}\left(\frac{2\Gamma_\beta T}{mc^2}\right)^{-1/2}, & \Gamma_\beta T \ll mc^2 \\ (1 - k^2\lambda^2)\frac{\beta^{\frac{3}{2}}}{\Gamma_\beta}, & \Gamma_\beta T \gg mc^2 \end{cases}$$

growth rate decreases with increasing  $\Gamma_\beta$

$$\frac{1}{8\pi}\frac{\partial}{\partial t}(B^2 + E^2) = -\frac{c}{4\pi}\nabla \cdot (E \times B) - E \cdot J$$

$$\frac{\partial}{\partial t}\int \frac{B^2}{8\pi}dy + \int E \cdot J^{ad}dy = -\int E \cdot J^{res}dy$$

$$J^{ad} = \frac{c}{4\pi} \frac{A_{1z}}{\lambda^2 \cosh^2(\frac{y}{\lambda})}$$

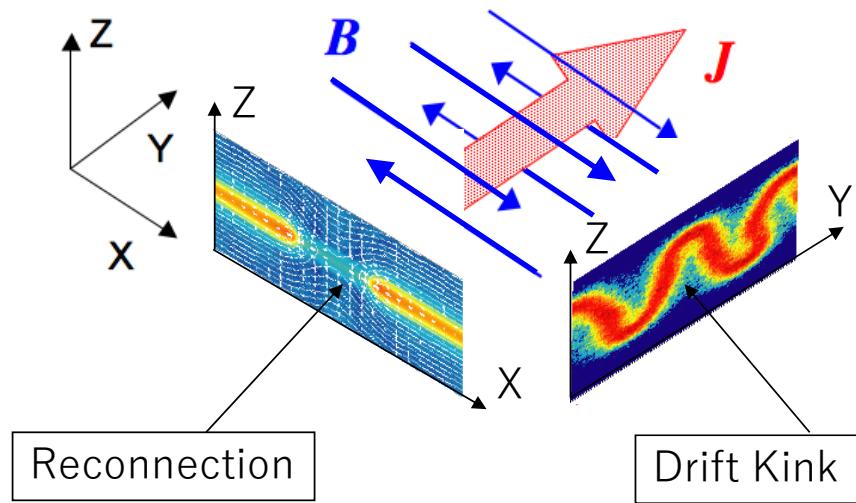
$$J^{res} = \sigma E_{1z} \quad \sigma = \left(\frac{ne^2}{m}\right) \left(\frac{\Gamma_\beta}{kc}\right) \left(\frac{mc^2}{T}\right)$$

$$\frac{\partial}{\partial t} \left[ \frac{1}{8\pi} \int_{-\infty}^{+\infty} \left( \left| \frac{d}{dy} A_{1z} \right|^2 + k^2 A_{1z}^2 - \frac{2A_{1z}^2}{\lambda^2 \cosh^2(\frac{y}{\lambda})} \right) dy \right]$$

$$= - \int_{-d}^{+d} \sigma E_{1z}^2 dy = \int_{-d}^{+d} \frac{\sigma}{c^2} \left( \frac{\partial}{\partial t} A_{1z} \right)^2 dy$$

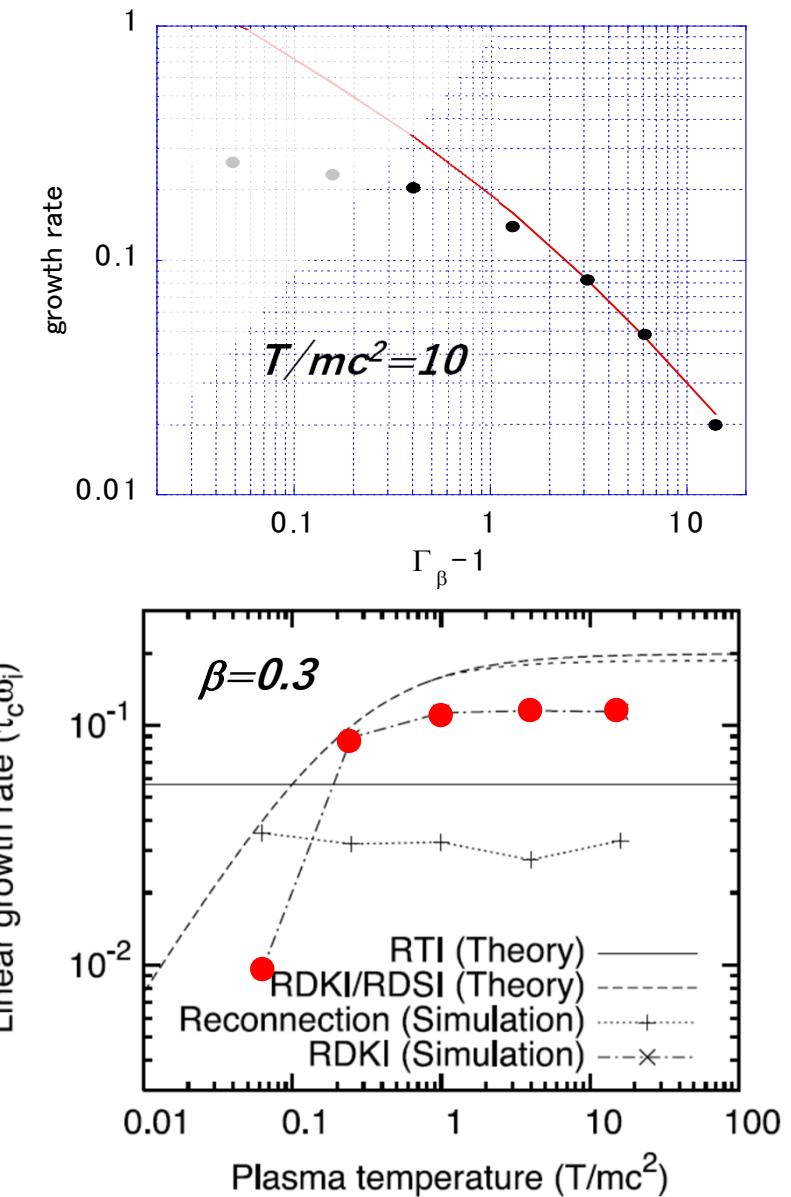
$$E = -\frac{1}{c} \frac{\partial}{\partial t} A \quad B = \nabla \times A$$

# drift kink instability



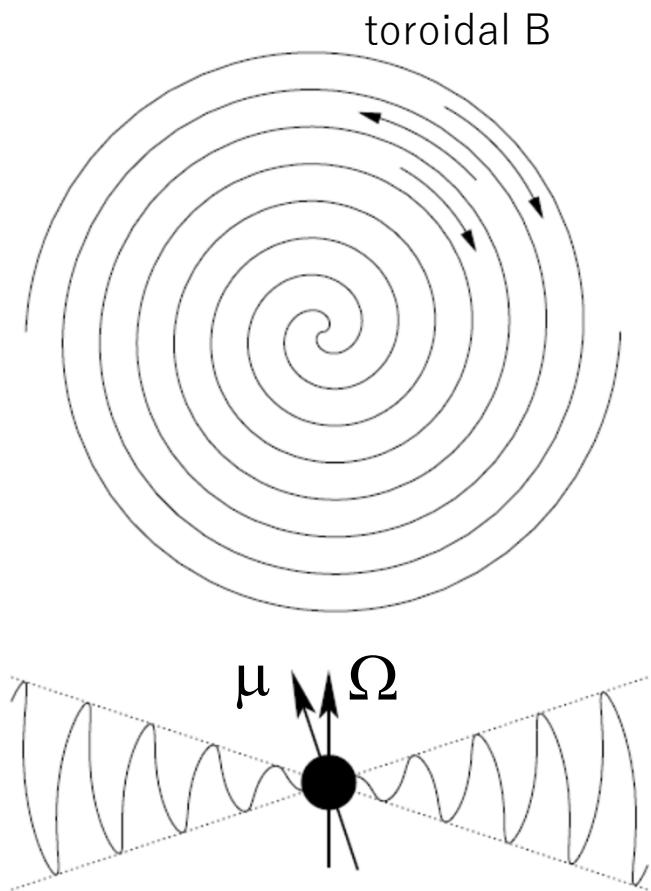
$$\gamma_{DK} \tau_c = \begin{cases} \frac{1}{4\Gamma_\beta \beta} \frac{T}{mc^2}, & T \ll mc^2 \\ \frac{1}{16\Gamma_\beta \beta}, & T \gg mc^2 \end{cases}$$

(Zenitani & MH, ApJ, 2007)



# Summary

## Pulsar Wind



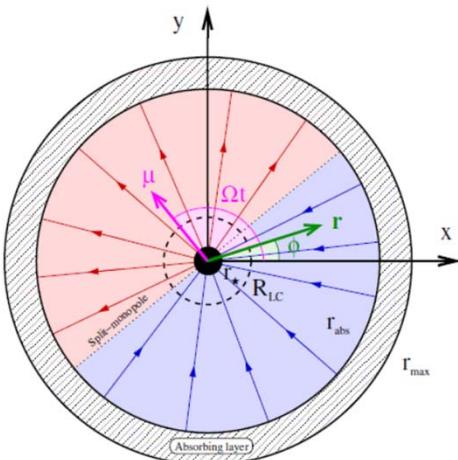
relativistic drift velocity

$$v_d \approx c$$

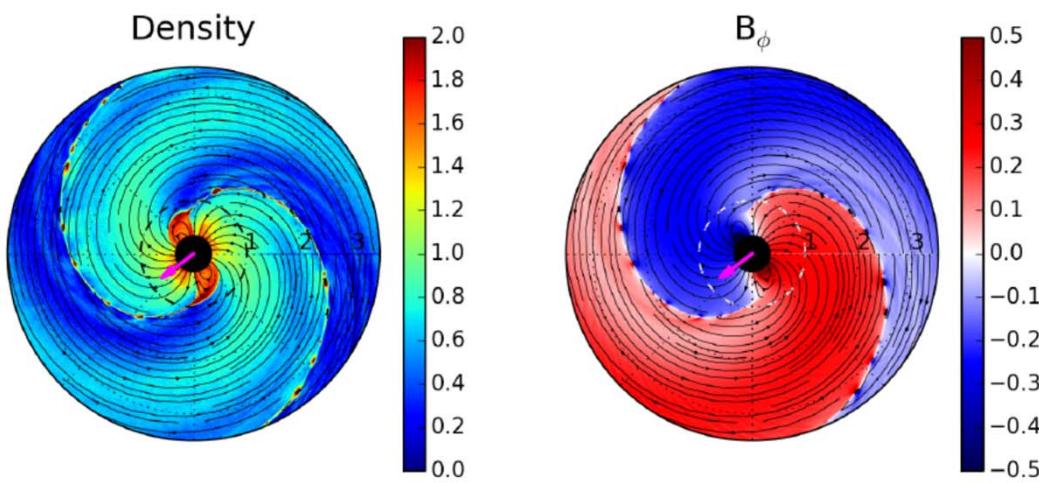
- slow reconnection & slow drift-kink
- slow magnetic energy release

magnetic energy dissipation is not easy in pulsar wind, and current starvation may happen...

# pulsar wind evolution (PIC)



Split-monopole,  
2d-PIC simulation,  
equatorial plane  
(Cerutti & Philippov, A&A 2017)



Poynting flux toward sheet  
reconnection rate  
drift velocity current sheet thickness

