マルチスケール手法を用いた高温降着流における乱流加熱研究

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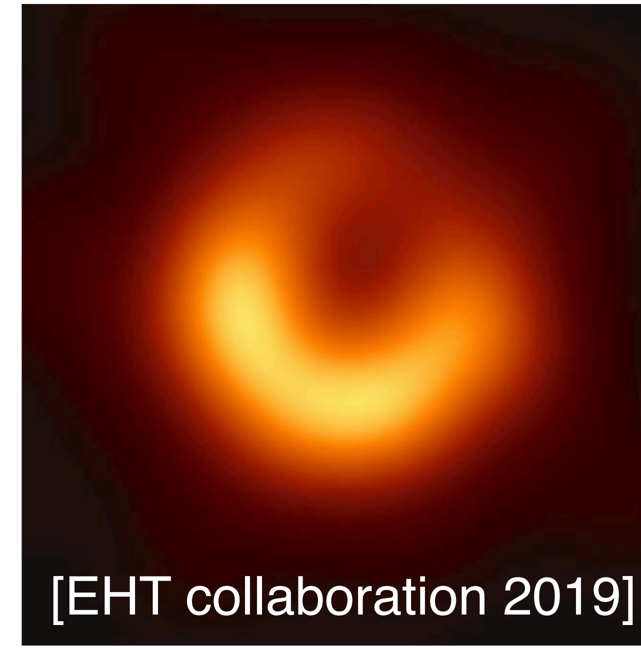
高エネルギー宇宙物理学研究会, 2019/12/6@蔵王アストリアホテル

Plan of the talk

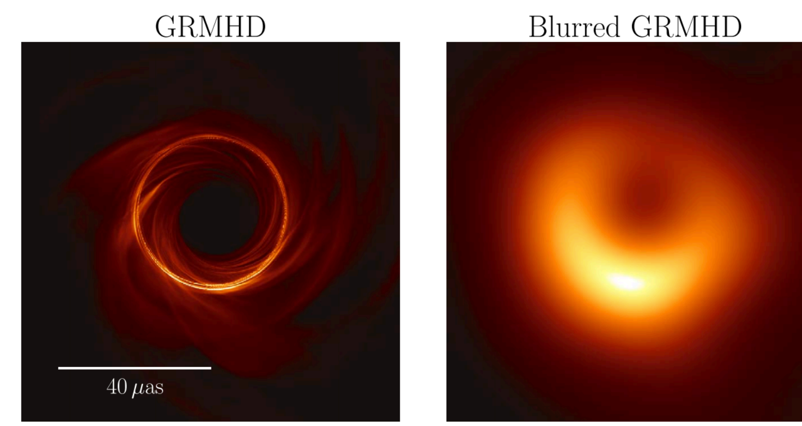
- Research background
- Three key questions of turbulent heating in collisionless plasma
  1. Partition of Alfvenic fluctuations between ion and electron heating [Kawazura+ PNAS 2019]
  2. Partition of compressive fluctuations between ion and electron heating [Kawazura+ in prep]
  3. Partition of Alfvenic and compressive fluctuation in MRI driven turbulence [Kawazura+ in prep]
- Summary

Ion vs electron heating in hot accretion disks

- Hot accretion disks (e.g M87, Sgr A*)
- Collisionless plasma → $T_\mathrm{i} \gg T_\mathrm{e}$
- Only electrons emit radiation
  → What we can measure is only T_e
  → Ion vs electrons heating ($Q_\mathrm{i}/Q_\mathrm{e}$) is important


[EHT collaboration 2019]

- GRMHD reproduced the observation [EHT 2019-V]
- MHD can't deal two temperature
  → They assumed $T_\mathrm{i}/T_\mathrm{e}$



$$\frac{T_\mathrm{i}}{T_\mathrm{e}} = R_\mathrm{high}\frac{\beta_\mathrm{p}^2}{1+\beta_\mathrm{p}^2} + \frac{1}{1+\beta_\mathrm{p}^2}$$

We need to determine $Q_\mathrm{i}/Q_\mathrm{e}$ physically

Flux[a]	a_*[b]	R_high[c]	AIS[d]	ϵ[e]	L_X[f]	P_jet[g]	
SANE	−0.94	1	Fail	Pass	Pass	Pass	Fail
SANE	−0.94	10	Pass	Pass	Pass	Pass	Pass
SANE	−0.94	20	Pass	Pass	Pass	Pass	Pass
SANE	−0.94	40	Pass	Pass	Pass	Pass	Pass
SANE	−0.94	80	Pass	Pass	Pass	Pass	Pass
SANE	−0.94	160	Fail	Pass	Pass	Pass	Fail
SANE	−0.5	1	Pass	Pass	Fail	Fail	Fail
SANE	−0.5	10	Pass	Pass	Fail	Fail	Fail
SANE	+0.94	20	Pass	Pass	Pass	Fail	Fail
SANE	+0.94	40	Pass	Pass	Pass	Fail	Fail
SANE	+0.94	80	Pass	Pass	Pass	Pass	Pass
SANE	+0.94	160	Pass	Pass	Pass	Pass	Pass

“Two temperature” MHD simulation

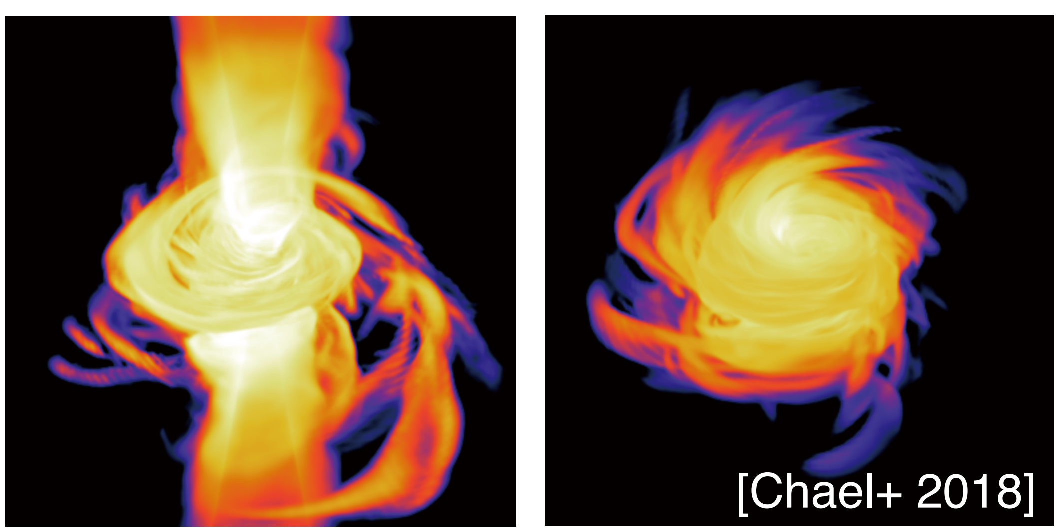
$$\nabla_\mu (\rho u^\mu) = 0$$
$$\nabla_\mu T^\mu{}_\nu = G_\nu$$
$$\nabla_\mu R^\mu{}_\nu = -G_\nu$$
$$\nabla_\mu F^{*\mu}{}_\nu = 0$$

Radiative GRMHD

$$T_\mathrm{e} \nabla_\mu (n_\mathrm{e} s_\mathrm{e} u^\mu) = \delta_\mathrm{e} q^\nu + q^\mathrm{C} + G^0$$
$$T_\mathrm{i} \nabla_\mu (n_\mathrm{i} s_\mathrm{i} u^\mu) = (1 - \delta_\mathrm{e})\, q^\nu - q^\mathrm{C}$$
$$\delta_\mathrm{e} = Q_\mathrm{e}/(Q_\mathrm{i} + Q_\mathrm{e})$$

Entropy evolution for ions & electrons, respectively

- How we prescribe $Q_\mathrm{i}/Q_\mathrm{e}$ changes the result



- left : $Q_\mathrm{i}/Q_\mathrm{e} =$ increasing func of β
- right : $Q_\mathrm{i}/Q_\mathrm{e} =$ decreasing func of β

Which is correct?

Kinetic theory is necessary to study plasma heating

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\nabla f_s + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\partial f_s}{\partial t}\right)_{\mathrm{c}} \qquad f_s = f_s(\mathbf{r},\mathbf{v}) \text{ : 6D phase space}$$

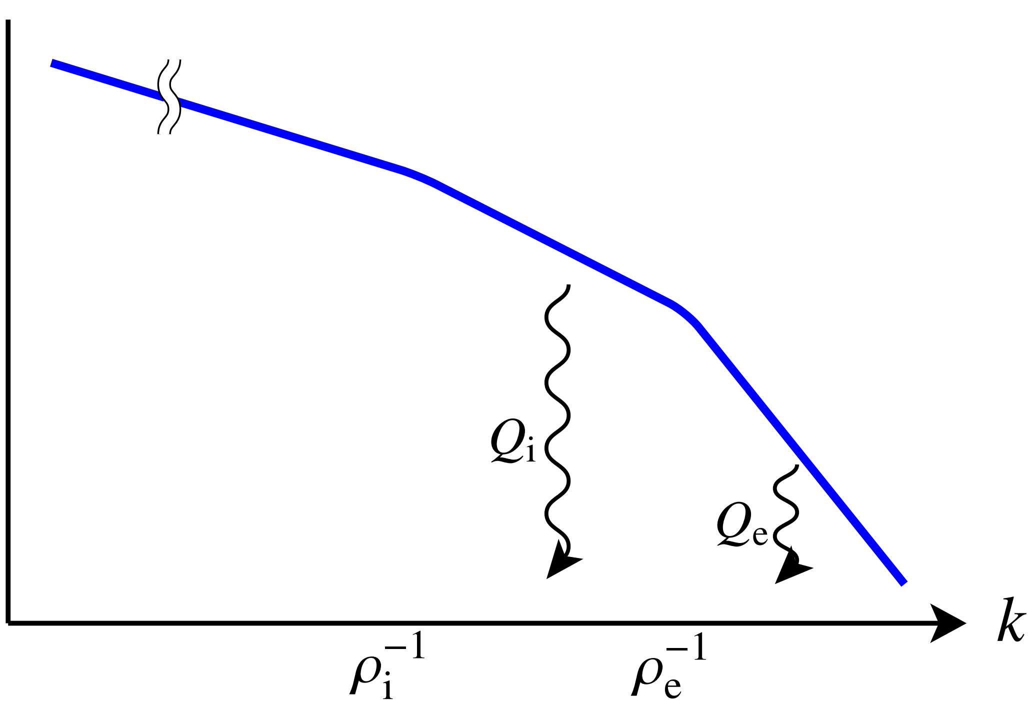
Vlasov eq

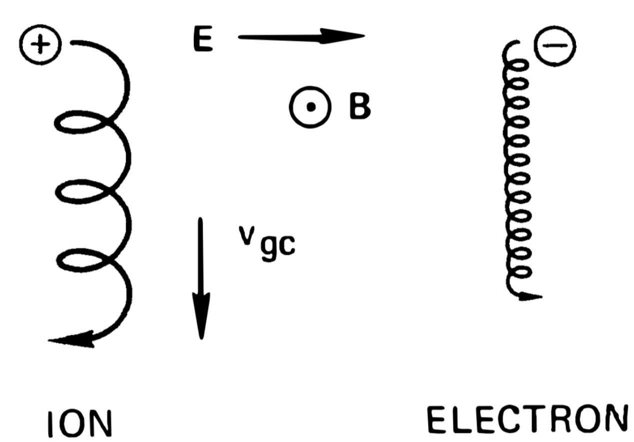
- To get turbulent heating, "in principle," we must resolve

Inertial range ~ ion gyroscale ~ electron gyroscale

ex. $0.1\rho_{\mathrm{i}}^{-1} \leqq k_\perp \leqq 2\rho_{\mathrm{e}}^{-1} \sim 100\rho_{\mathrm{i}}^{-1}$ → grid number ~ 1000

→ $n_x \times n_y \times n_z \times n_v^3 \sim 1000\times1000\times200\times16^3$ No way!

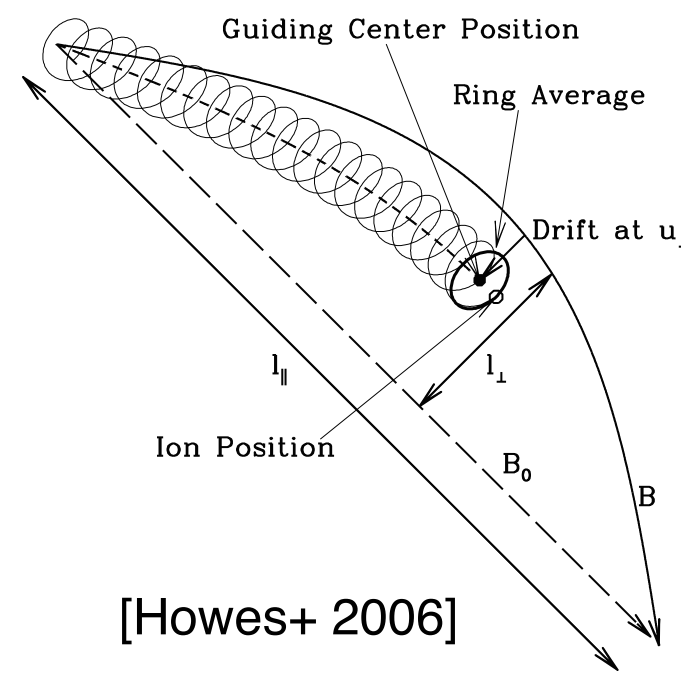




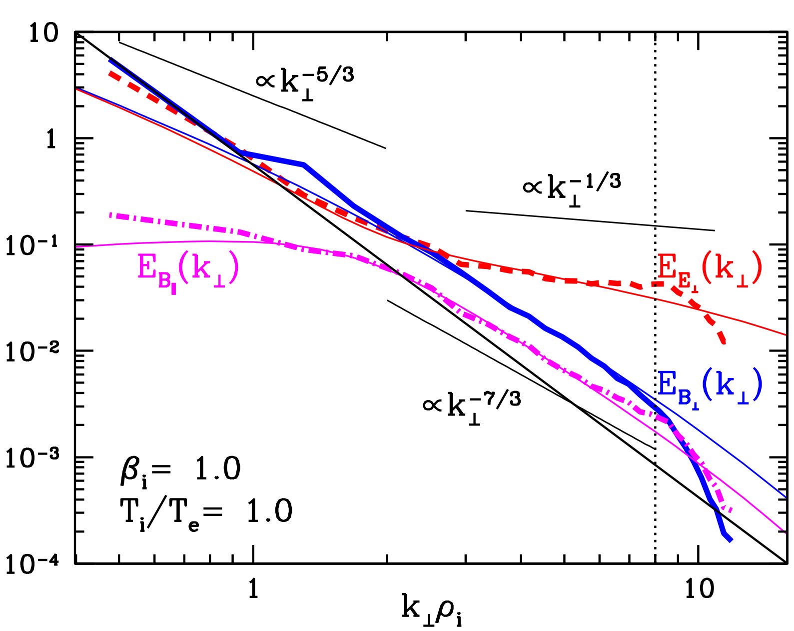
Gyrokinetics

[Rutherford & Frieman 1968; Catto 1982; Howes+ 2006; Brizard & Halm 2007]

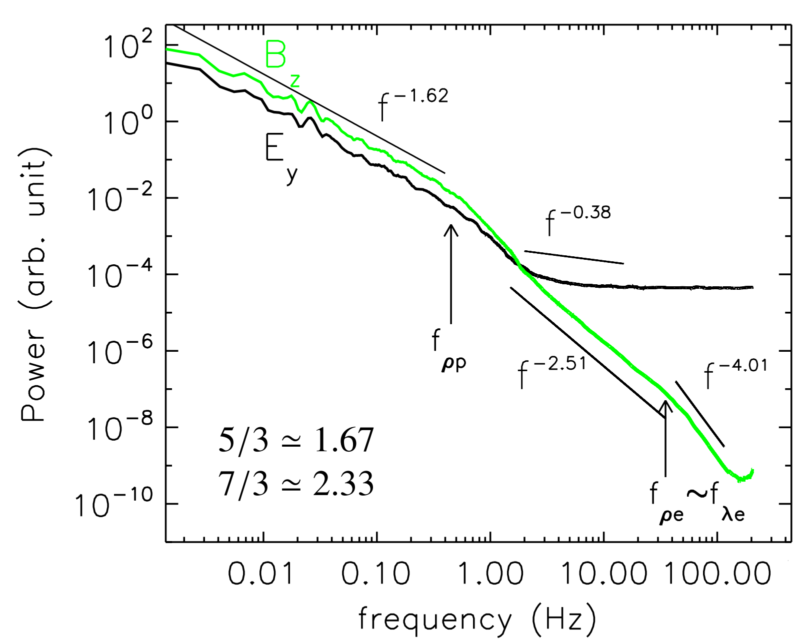
- Average gyro-motions
- Solve distribution func of gyro-centers
- $(\mathbf{r}, \mathbf{v}) \rightarrow (\mathbf{R}, v_\perp, v_\parallel)$: 5D phase space
- Popular in magnetic confinement fusion
- Has been used in space plasma
- **Never been used for accretion disks**



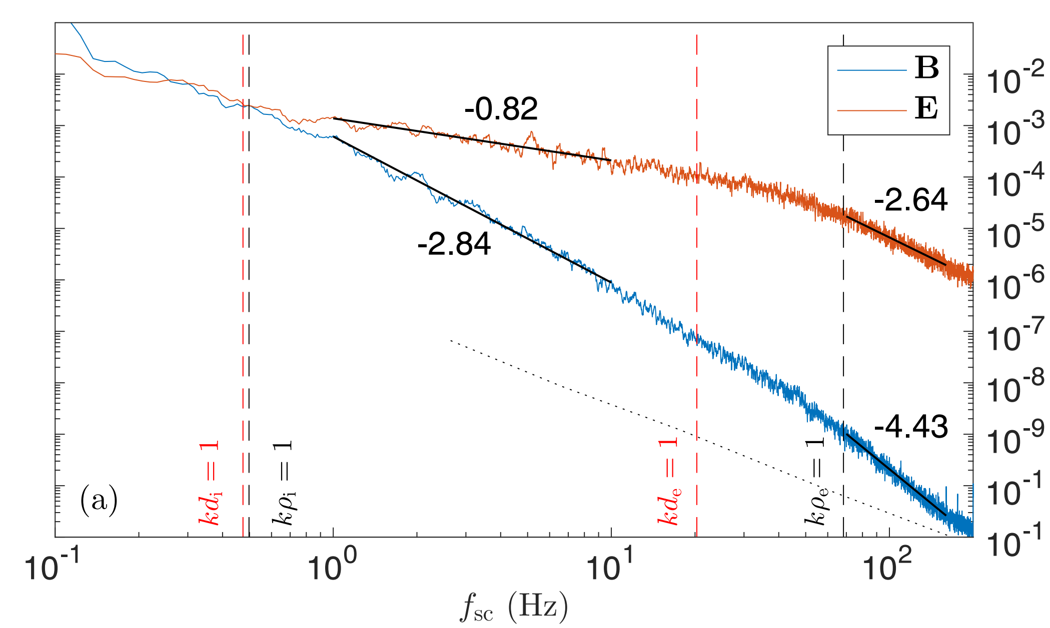
[Howes+ 2006]



GK simulation



Solar wind (Cluster)



Earth's magnetosphere (MMS)

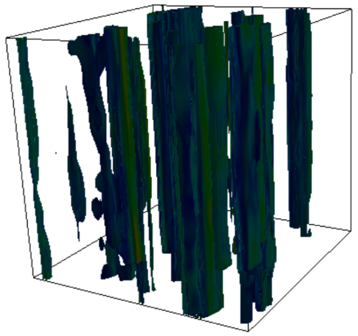
Gyrokinetic ordering

1. Cyclotron motion is much faster than turbulent fluctuations

$$\frac{\omega}{\Omega} \ll 1$$

2. Anisotropic [Goldreich & Sridhar 1995]

$$\frac{k_{\parallel}}{k_{\perp}} \ll 1$$



[Sundar+ 2017]

3. Small fluctuation amplitude

$$\frac{\delta \mathbf{B}}{B_0} \sim \frac{\delta f_s}{F_s} \ll 1$$

$$\frac{\partial h_s}{\partial t} + v_{\parallel} \frac{\partial h_s}{\partial z} + \frac{c}{B_0} \{\langle \chi \rangle_{\mathbf{R}_s}, h_s\} = \frac{q_s}{T_s} \frac{\partial \langle \chi \rangle_{\mathbf{R}_s}}{\partial t} F_s + \langle C\,[h_s] \rangle_{\mathbf{R}_s}$$

+ Maxwell's equations

h_s : gyro-center dist func

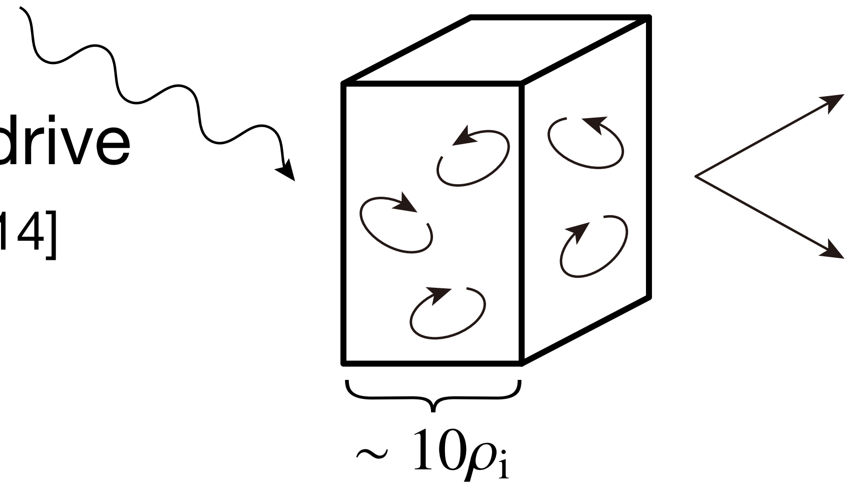
$\chi = \varphi - \mathbf{v} \cdot \mathbf{A}/c$

$C[h_s]$: collision operator

$\langle \ldots \rangle_{R_s}$: gyro-average

- This ordering is OK for solar wind[Howes+ 2008], not sure for accretion disks

Simulation setting



Externally inject AWs to drive turbulence [Tenbarge+ 2014]

$$\frac{k_{\perp 0}A_{\parallel 0}}{B_0} \sim \frac{k_{\parallel 0}}{k_{\perp 0}}$$

Q_{i} ⟵ Ion collisions

Q_{e} ⟵ Hyper dissipation

Hybrid GK (Ion: GK, electron: fluid)

$$\frac{\partial h_{\mathrm{i}}}{\partial t} + v_{\parallel}\frac{\partial h_{\mathrm{i}}}{\partial z} + \frac{c}{B_0}\{\langle\chi\rangle_{\mathbf{R}_{\mathrm{i}}}, h_{\mathrm{i}}\} = \frac{Ze}{T_{\mathrm{i}}}\frac{\partial\langle\chi\rangle_{\mathbf{R}_{\mathrm{i}}}}{\partial t}F_{\mathrm{i}} - \frac{ZeF_{\mathrm{i}}}{T_{\mathrm{i}}}\frac{\partial}{\partial t}\left\langle\frac{v_{\parallel}A_{\parallel}^{\mathrm{a}}}{c}\right\rangle_{\mathbf{R}_{\mathrm{i}}} + \langle C[h_{\mathrm{i}}]\rangle_{\mathbf{R}_{\mathrm{i}}} \quad \left(J_{\parallel}^{\mathrm{a}} = \frac{c}{4\pi}\nabla_{\perp}^2 A_{\parallel}^{\mathrm{a}}\right)$$

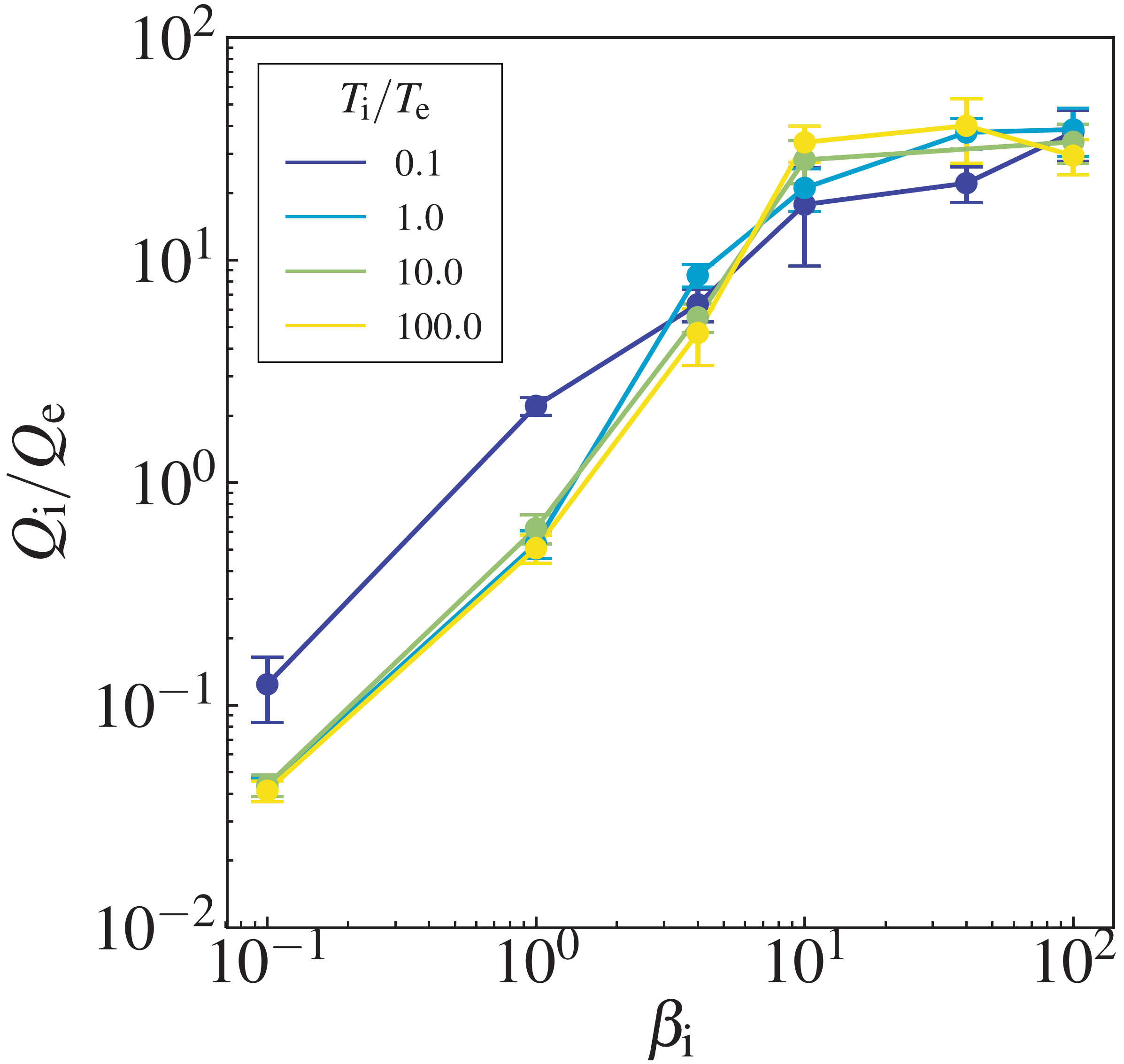
$$\frac{1}{c}\frac{\partial A_{\parallel}}{\partial t} + \nabla_{\parallel}\phi = \nabla_{\parallel}\frac{T_{0\mathrm{e}}}{\mathrm{e}}\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}$$ **+ hyper resistivity**

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}} - \frac{\delta B_{\parallel}}{B_0}\right) + \nabla_{\parallel}u_{\parallel\mathrm{e}} + \frac{cT_{0\mathrm{e}}}{eB_0}\left\{\frac{\delta n_{\mathrm{e}}}{n_{0\mathrm{e}}}, \frac{\delta B_{\parallel}}{B_0}\right\} = 0$$ **+ hyper viscosity**

$$\longrightarrow \frac{\mathrm{d}W}{\mathrm{d}t} = \underbrace{\int \frac{\mathrm{d}^3\mathbf{r}}{V}\frac{J_{\parallel}^{\mathrm{a}}}{c}\frac{\partial A_{\parallel}}{\partial t}}_{=:P_{\mathrm{AW}}} + \underbrace{\int \mathrm{d}^3\mathbf{v}\int\frac{\mathrm{d}^3\mathbf{R}_{\mathrm{i}}}{V}\frac{T_{0\mathrm{i}}}{F_{0\mathrm{i}}}\langle h_{\mathrm{i}}C[h_{\mathrm{i}}]\rangle_{\mathbf{R}_{\mathrm{i}}}}_{=:Q_{\mathrm{i}}} - \underbrace{(\text{hyper res} + \text{hyper vis})}_{=:Q_{\mathrm{e}}}$$

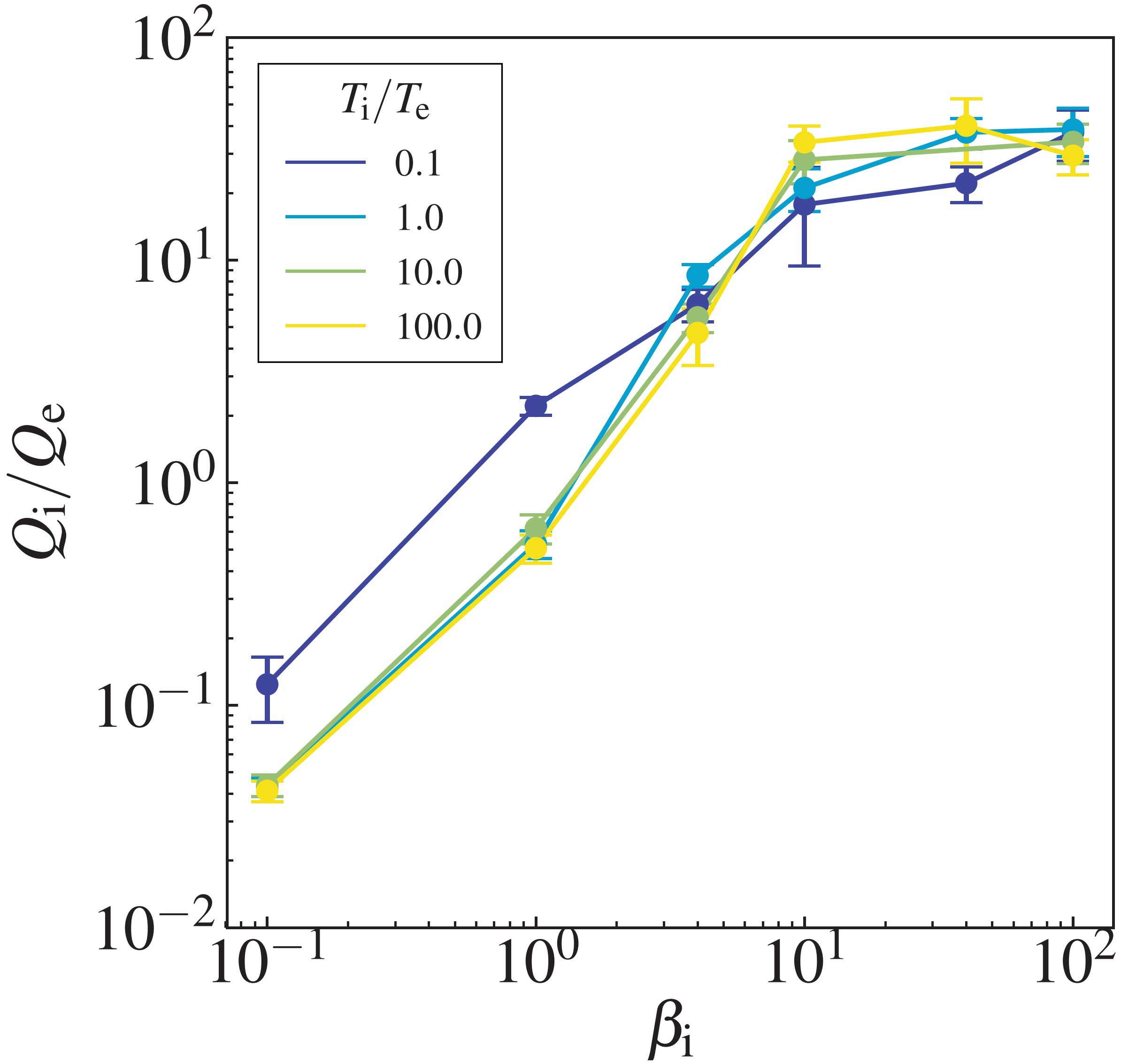
- Parameter scan $0.1 \leqq \beta_{\mathrm{i}} \leqq 100$ & $0.1 \leqq T_{\mathrm{i}}/T_{\mathrm{e}} \leqq 100$

Result : Q_i/Q_e



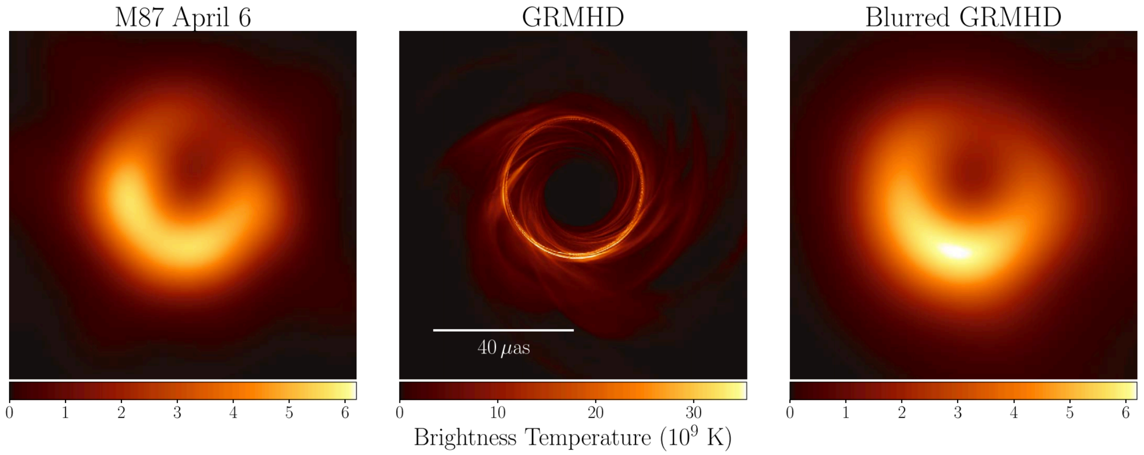
- An incresing function of β_i. But there is upper limit < 50
- At low β_i, $Q_i/Q_e \to 0$: Consistent with theoretical prediction [Schekochihin, Kawazura, & Barnes 2019]
- At the Hall regime ($\beta_i \ll 1$, $\beta_e \sim 1$), $Q_i/Q_e \sim 0.5$: Consistent with the theoretical prediction [Schekochihin, Kawazura, & Barnes 2019]

Result : Q_i/Q_e



- A prescription of EHT is not really bad :)

$$\frac{T_i}{T_e} = R_{high}\frac{\beta_p^2}{1+\beta_p^2} + \frac{1}{1+\beta_p^2}$$



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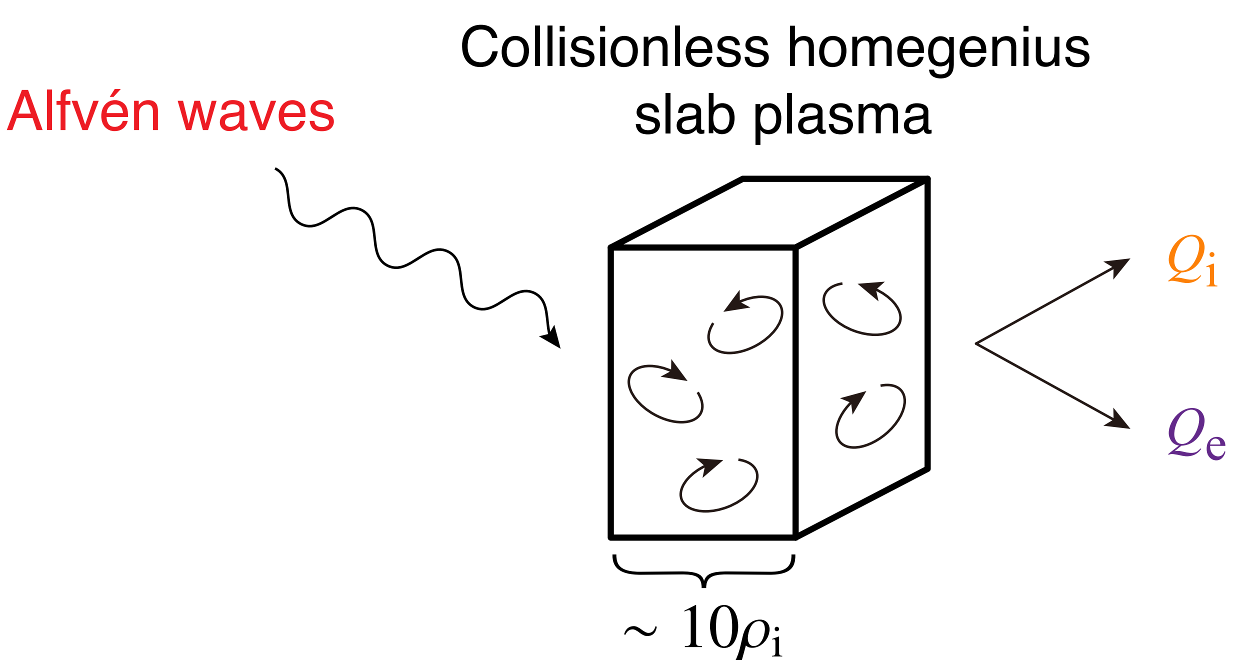
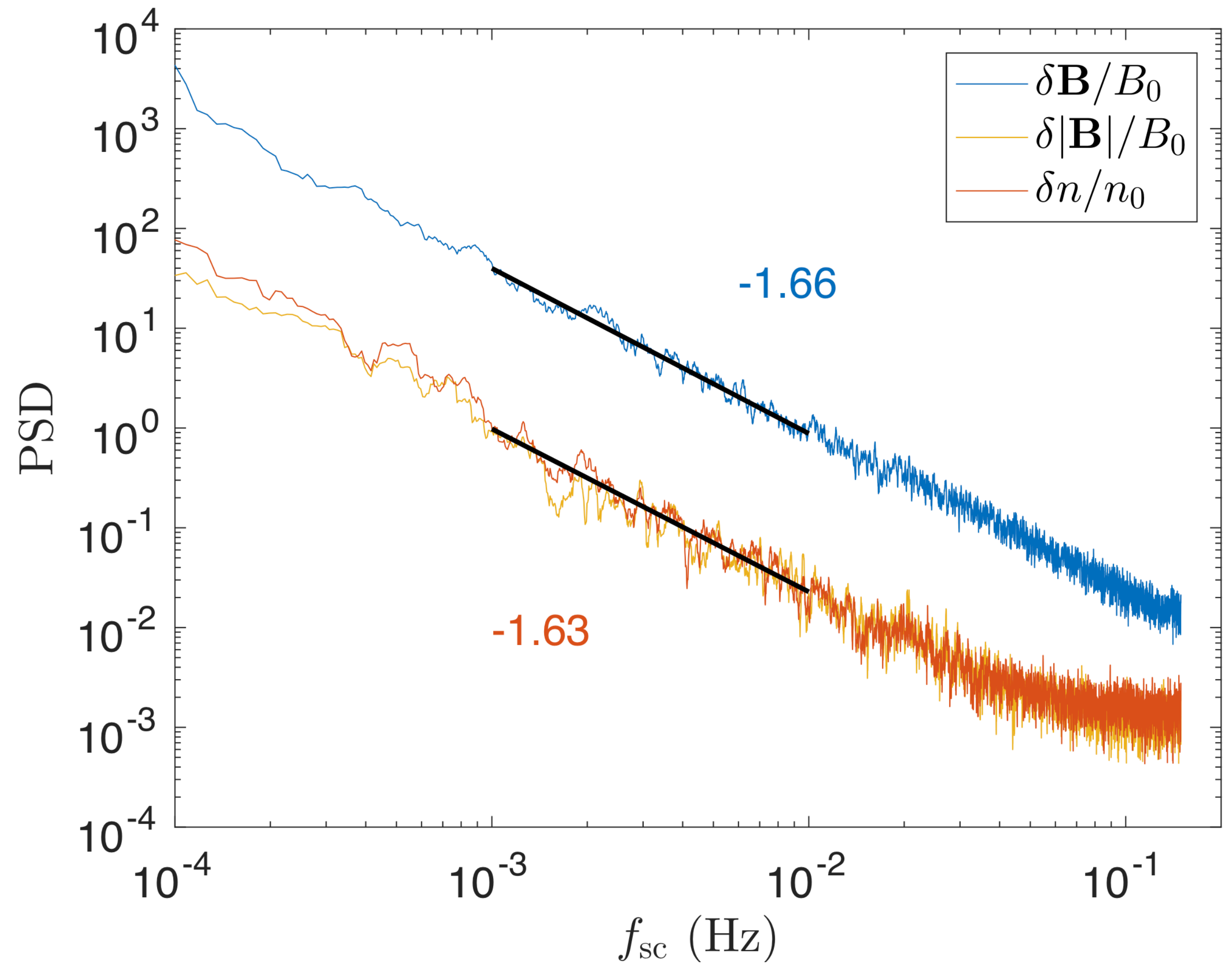
- Summary

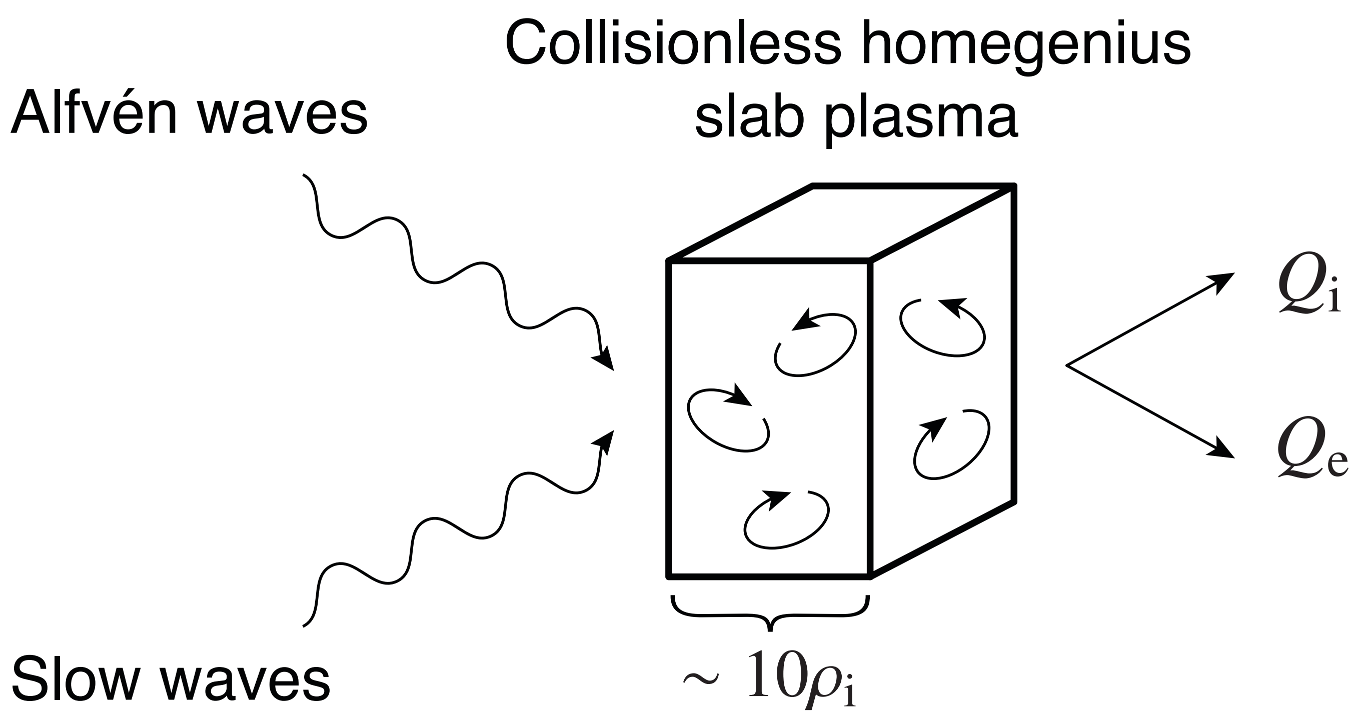
Is accretion turbulence Alfvenic or compressive?

- In the previous GK simulation, turbulence was Alfvenically driven
- This is based on the fact that solar wind is Alfvenic
- But, we do not know if accretion disks are Alfvenic
- We need to drive turbulence compressively





Alfvenic & compressive forcing



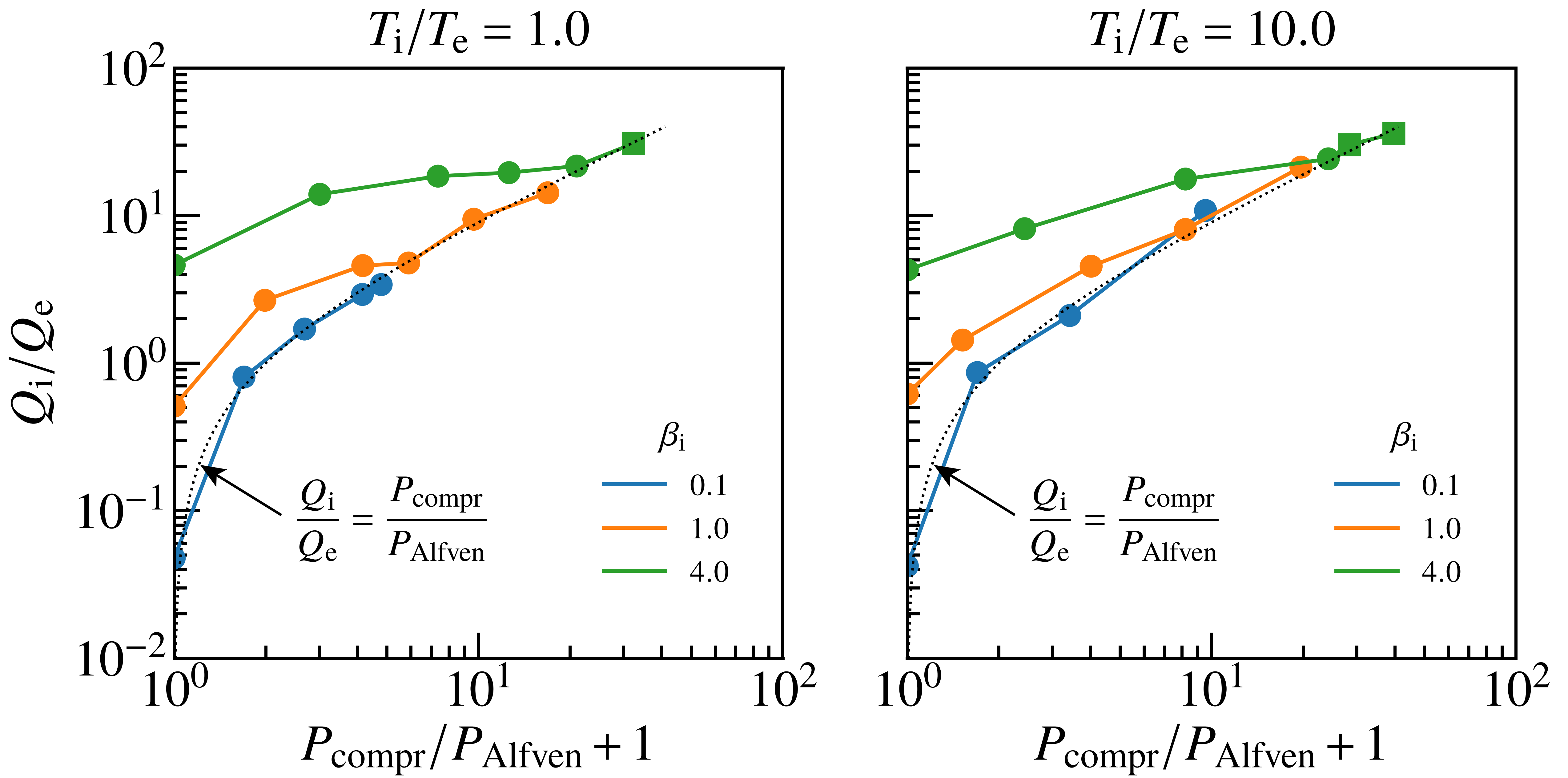
$$\frac{\partial h_\mathrm{i}}{\partial t} + v_\parallel \frac{\partial h_\mathrm{i}}{\partial z} + \frac{c}{B_0}\left\{\langle\chi\rangle_{\mathbf{R}_\mathrm{i}}, h_\mathrm{i}\right\} = \frac{Ze}{T_\mathrm{i}}\frac{\partial\langle\chi\rangle_{\mathbf{R}_\mathrm{i}}}{\partial t}F_\mathrm{i} - \frac{ZeF_\mathrm{i}}{T_\mathrm{i}}\frac{\partial}{\partial t}\left\langle\frac{v_\parallel A_\parallel^\mathrm{a}}{c}\right\rangle_{\mathbf{R}_\mathrm{i}} + \frac{v_\parallel\langle a_\mathrm{ext}\rangle_{\mathbf{R}_\mathrm{i}}}{v_\mathrm{thi}^2}F_\mathrm{i} + \langle C[h_\mathrm{i}]\rangle_{\mathbf{R}_\mathrm{i}}$$

$$\frac{1}{c}\frac{\partial A_\parallel}{\partial t} + \nabla_\parallel\phi = \nabla_\parallel\frac{T_\mathrm{e}}{\mathrm{e}}\frac{\delta n_\mathrm{e}}{n_\mathrm{e}}$$ **+ hyper resistivity** $$\left(J_\parallel^\mathrm{a} = \frac{c}{4\pi}\nabla_\perp^2 A_\parallel^\mathrm{a}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\delta n_\mathrm{e}}{n_\mathrm{e}} - \frac{\delta B_\parallel}{B_0}\right) + \nabla_\parallel u_{\parallel\mathrm{e}} + \frac{cT_\mathrm{e}}{eB_0}\left\{\frac{\delta n_\mathrm{e}}{n_\mathrm{e}}, \frac{\delta B_\parallel}{B_0}\right\} = 0$$ **+ hyper viscosity**

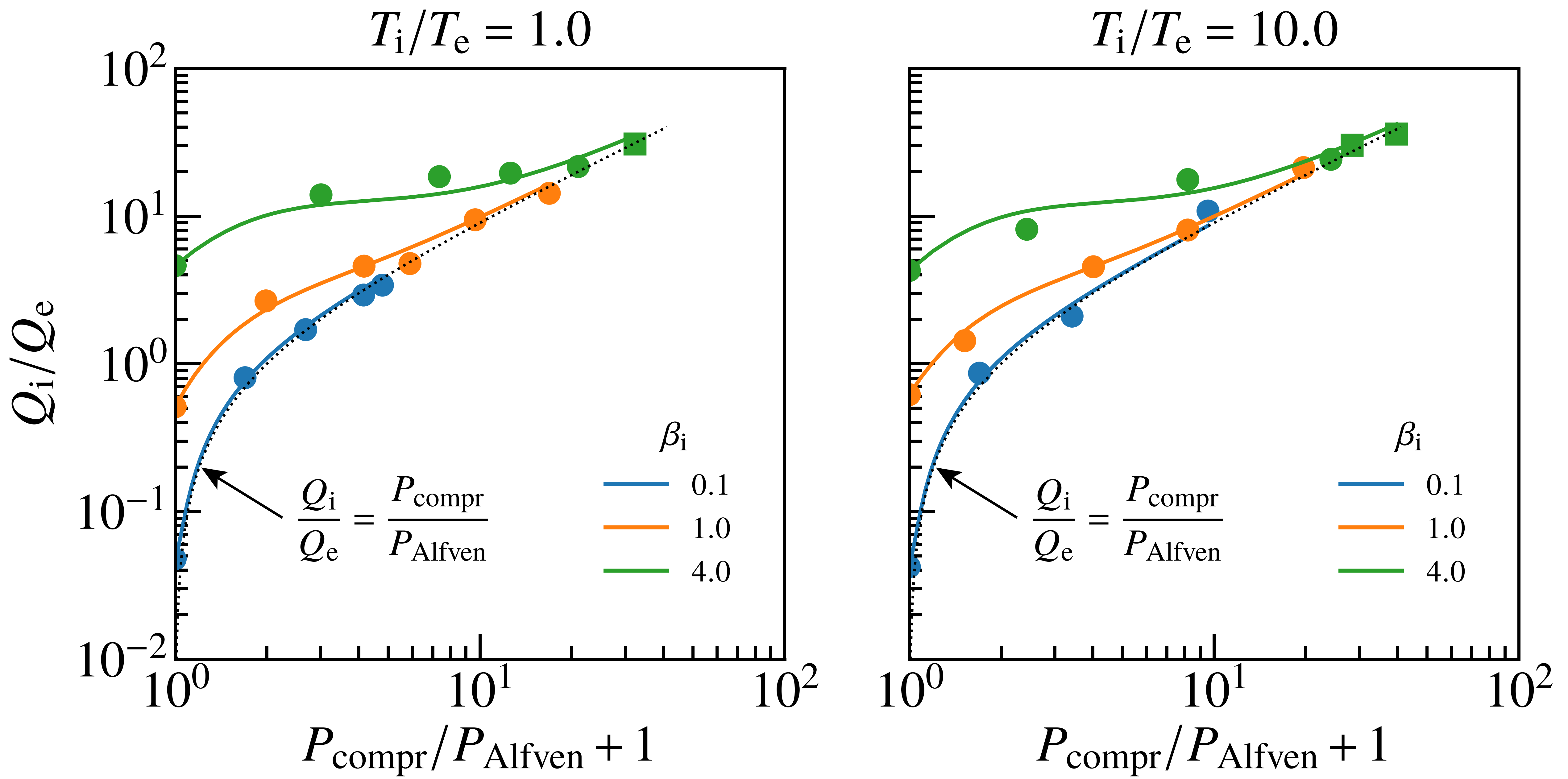
$$\frac{\mathrm{d}W}{\mathrm{d}t} = \underbrace{\int\frac{\mathrm{d}^3\mathbf{r}}{V}\frac{J_\parallel^\mathrm{a}}{c}\frac{\partial A_\parallel}{\partial t}}_{=:P_\mathrm{AW}} + \underbrace{\int\mathrm{d}^3\mathbf{v}\int\frac{\mathrm{d}^3\mathbf{R}_\mathrm{i}}{V}T_\mathrm{i}\frac{v_\parallel h_\mathrm{i}\langle a_\mathrm{ext}\rangle_{\mathbf{R}_\mathrm{i}}}{v_\mathrm{thi}^2}}_{=:P_\mathrm{compr}} + \underbrace{\int\mathrm{d}^3\mathbf{v}\int\frac{\mathrm{d}^3\mathbf{R}_\mathrm{i}}{V}\frac{T_\mathrm{i}}{F_\mathrm{i}}\langle h_\mathrm{i}C[h_\mathrm{i}]\rangle_{\mathbf{R}_\mathrm{i}}}_{=:Q_\mathrm{i}} - \underbrace{(\text{hyper res} + \text{hyper vis})}_{=:Q_\mathrm{e}}$$

Result : Q_i/Q_e



- Q_i/Q_e is an increasing function of P_{compr}/P_{Alfven}
- $Q_i/Q_e \simeq P_{compr}/P_{Alfven}$ for $\beta_i = 0.1$ ← Consistent with a theory [Schekochihin+ 2019]
- When $P_{compr}/P_{Alfven} \gg 1$, $Q_i/Q_e \simeq P_{compr}/P_{Alfven}$ for any β_i

Result : Q_i/Q_e



$$\frac{Q_i}{Q_e}(\beta_i,\ \tau,\ \wp) = \frac{35}{1 + (\beta_i/15)^{-1.4} e^{-0.1/\tau}} + \wp\left(\frac{\wp}{\wp + \beta_i} + \frac{2\beta_i}{1 + \wp^{1.5}/2}\right)$$

$$\left(\tau \equiv T_i/T_e,\quad \wp \equiv P_{compr}/P_{AW}\right)$$

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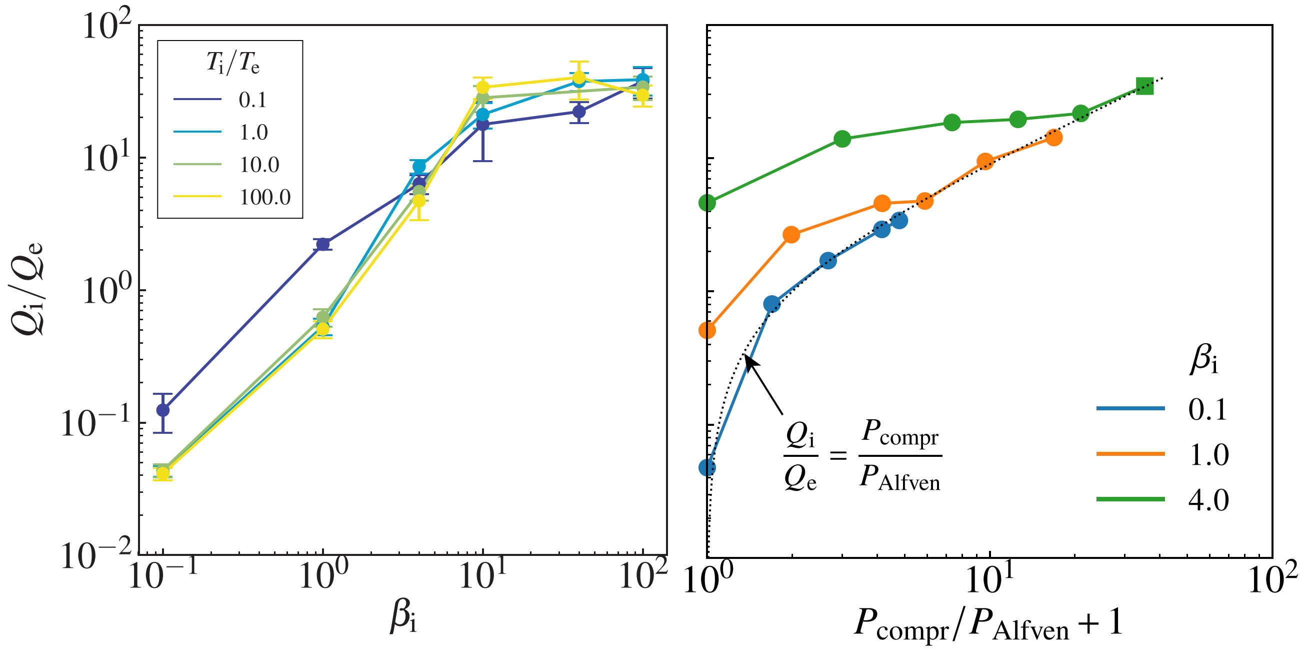
How do we apply Q_i/Q_e to actual disks?

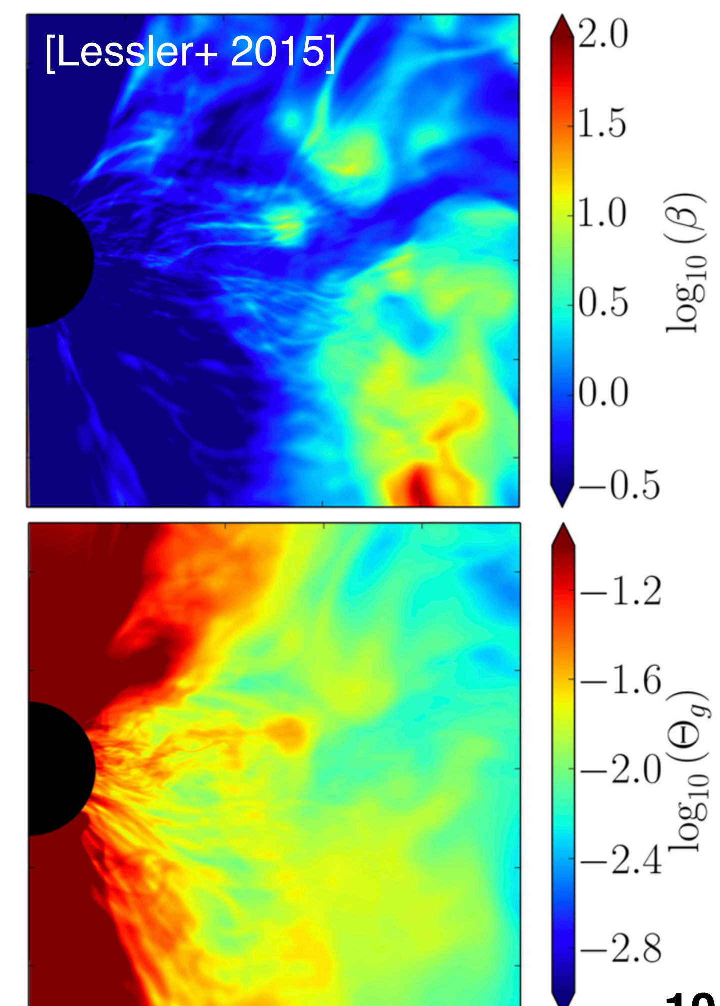


$$Q_i/Q_e = f(\beta_i, T_i/T_e, P_{compr}/P_{AW})$$

1. Solve Global MHD and calculate B, $T = T_i + T_e$, P_{compr}/P_{AW} at each grid
2. Use $Q_i/Q_e = f(\beta_i, T_i/T_e, P_{compr}/P_{AW})$ to solve the transport equations for ions and electrons
3. One obtains T_i and T_e respectively

How should we get P_{compr}/P_{AW} ?

→ A question of MRI driven MHD turbulence



Compressive—Alfvenic decoupling at $k_\parallel/k_\perp \ll 1$

- In smaller scales, Alfven and slow fluctuations satisfy $k_\parallel/k_\perp \ll 1$
- Ideal MHD + $k_\parallel/k_\perp \ll 1$ = Reduced MHD

→ Alfven and slow decouples [Schekochihin+ 2009]

$$\frac{d\Psi}{dt} = v_A \frac{\partial \Phi}{\partial z}, \quad \frac{d}{dt}\nabla_\perp^2 \Phi = v_A \nabla_\parallel \nabla_\perp^2 \Psi$$

$$\frac{du_\parallel}{dt} = v_A^2 \nabla_\parallel \left(\frac{\delta B_\parallel}{B_0}\right), \quad \frac{d}{dt}\frac{\delta B_\parallel}{B_0} = \left(1 + \frac{v_A^2}{c_s^2}\right)^{-1} \nabla_\parallel u_\parallel$$

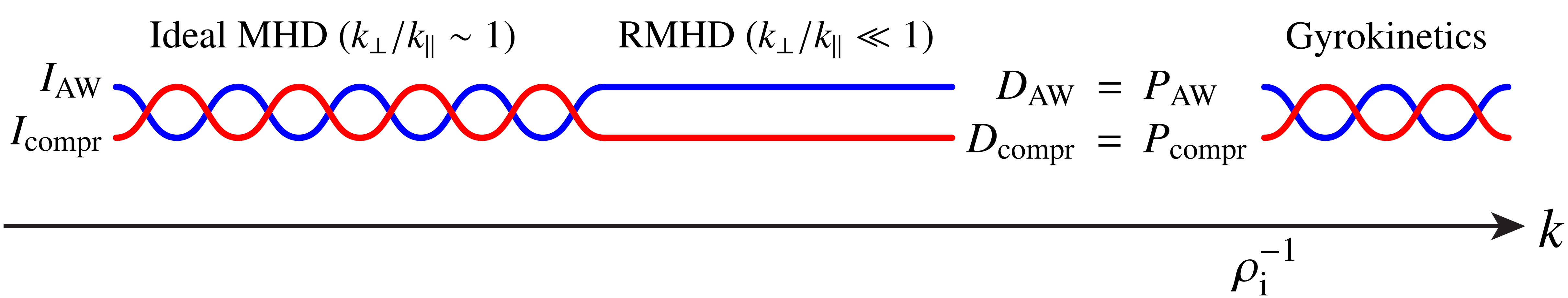
$$\mathbf{u} = \hat{\mathbf{z}} \times \nabla_\perp \Phi + u_\parallel \hat{\mathbf{z}}_\perp$$

$$\delta \mathbf{B} = \hat{\mathbf{z}} \times \nabla_\perp \Psi + \delta B_\parallel \hat{\mathbf{z}}_\perp$$

Φ, Ψ : Alfven

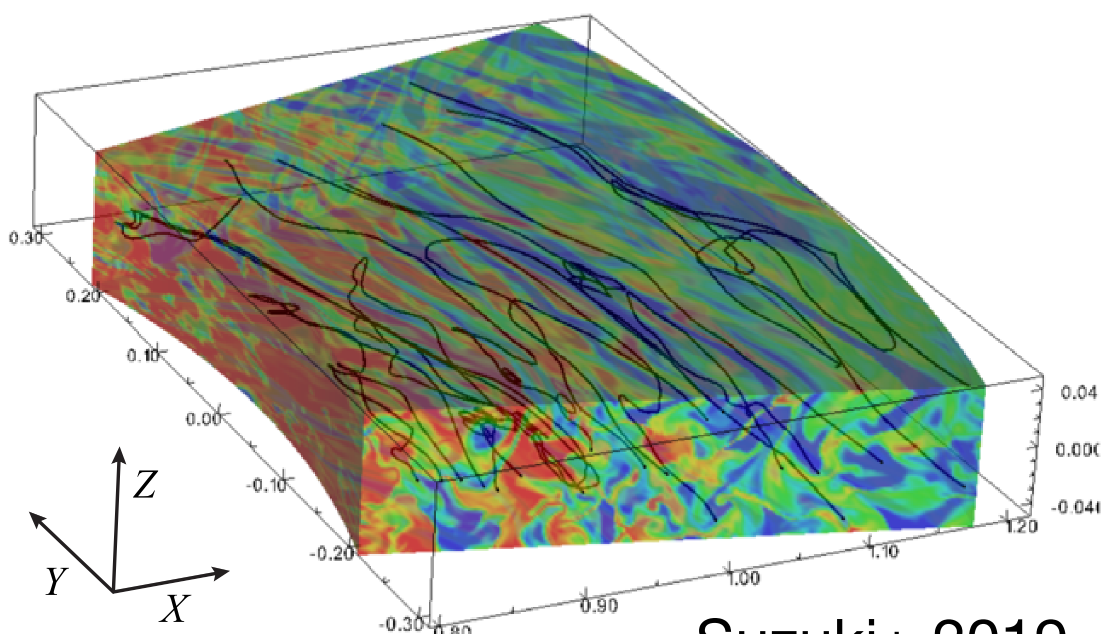
$u_\parallel, \delta B_\parallel$: Slow

- The ratio of dissipation D_{compr}/D_{AW} at RMHD = P_{compr}/P_{AW} for GK

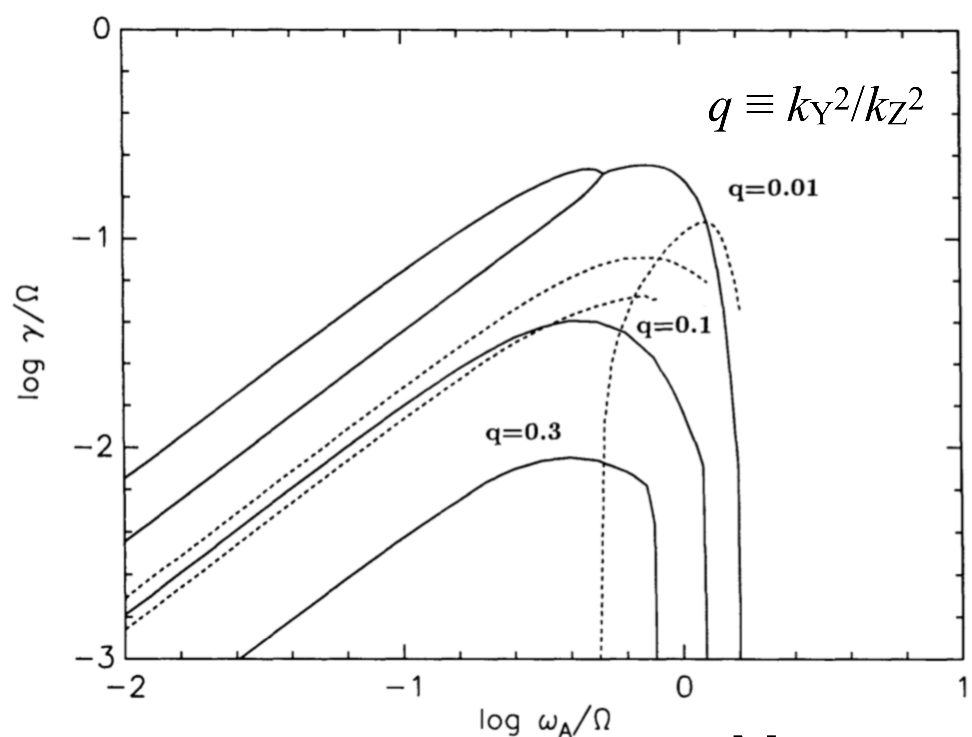


$k_{\|}/k_{\perp} \ll 1$ is difficult to be achieved in MRI turb

- $\boldsymbol{B}$ tends to be azimuthal in disks
- The fastest growing mode resides in $k_Z \rightarrow \infty$ ($\mathbf{Z}$: rotational axis)
  [Balbus&Hawley 1992, Hawley+ 1995, Matsumoto & Tajima 1995]
- We cannot reach the compressive—Alfvenic decoupling scale



Suzuki+ 2019



Matsumoto & Tajima

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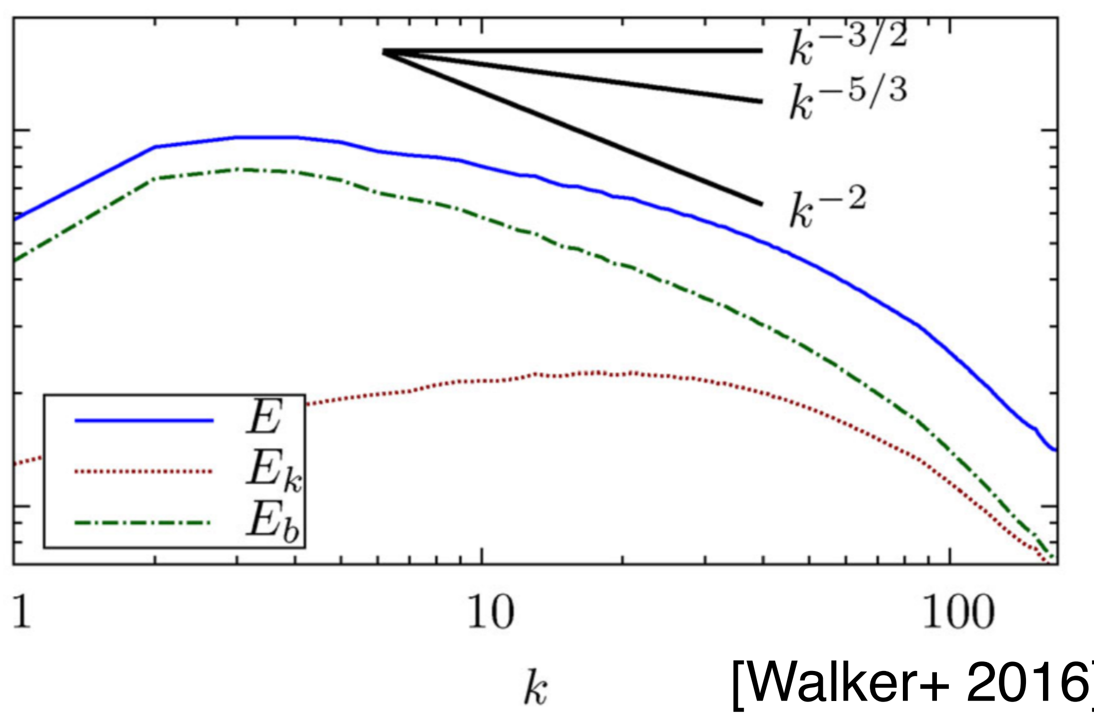
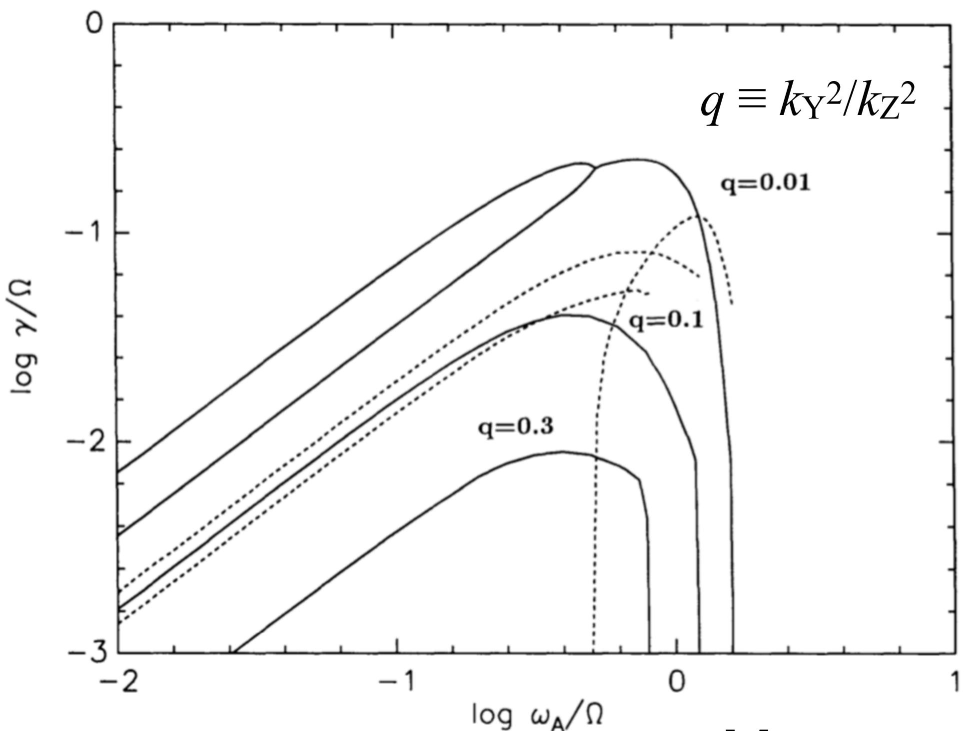


[Walker+ 2016]



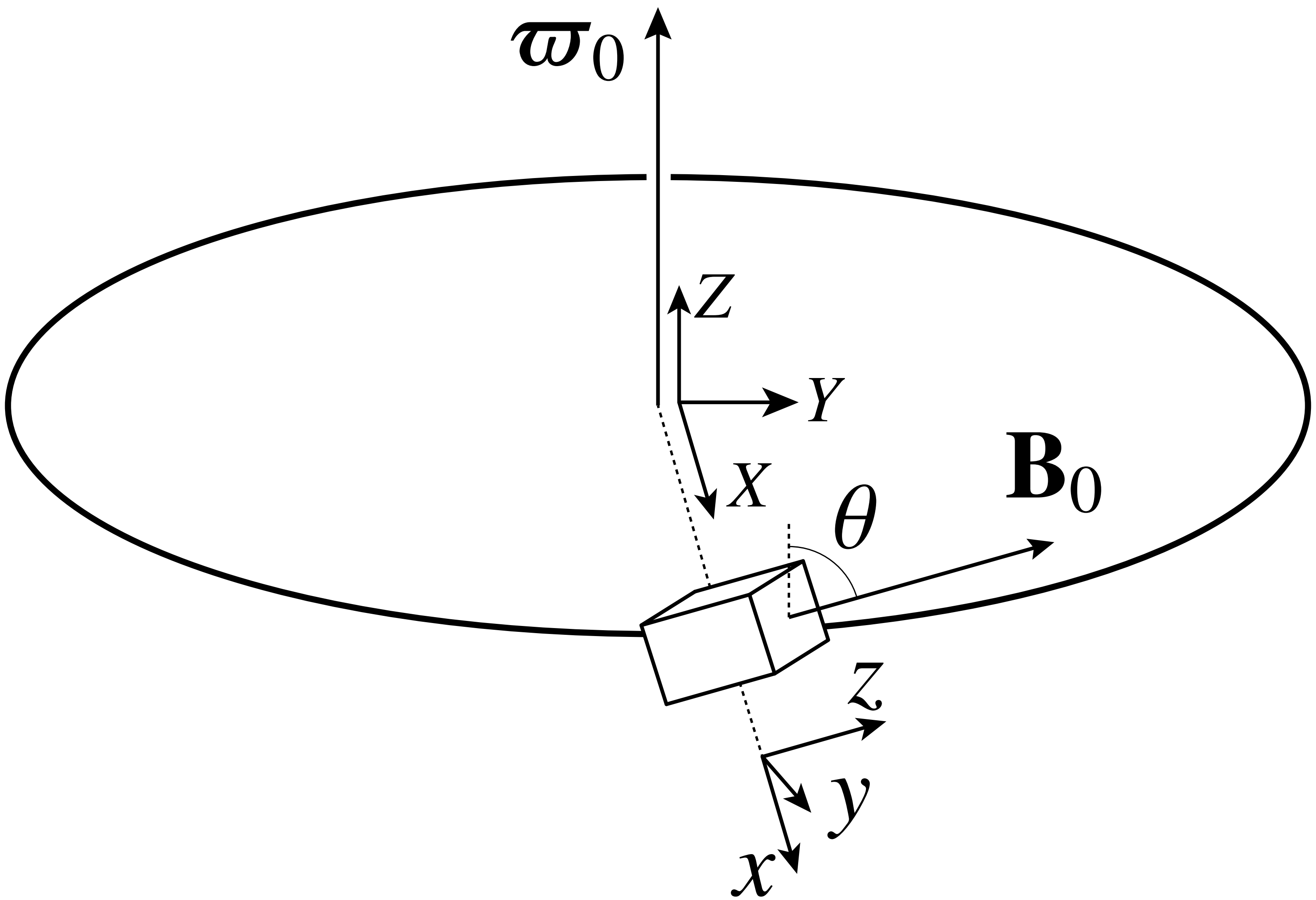
Matsumoto & Tajima

- When $\mathbf{B}$ is nearly azimuthal, $k_Z \sim k_{\perp}$, $k_Y \sim k_{\parallel}$
- $k_Z/k_Y \to \infty \iff k_{\perp}/k_{\parallel} \to \infty$ Reduced MHD limit

Our conjecture : $P_{\rm compr}/P_{\rm AW}$ can be obtained by RMHD + shear

Tilted coordinate

- Let θ be angle between $\boldsymbol{B}_0$ and $\boldsymbol{\varpi}_0$ (azimuthal $\Leftrightarrow$ $\theta \simeq \pi/2$)
- Convenient to take $\mathbf{z}$ axis along $\boldsymbol{B}_0$ for considering $k_\parallel/k_\perp \ll 1$



Shearing RMHD

$$\frac{\partial \rho}{\partial t} + \mathbf{u}\cdot\nabla\rho + \mathbf{u}_0\cdot\nabla\rho = -\rho(\nabla\cdot\mathbf{u}),$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla + \mathbf{u}_0\cdot\nabla\right)\mathbf{u} = -\nabla\left(p + \frac{B^2}{8\pi}\right) + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi} - 2\rho\boldsymbol{\varpi}_0\times\mathbf{u} - \rho\mathbf{u}\cdot\nabla\mathbf{u}_0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{B} + \mathbf{u}_0\cdot\nabla\mathbf{B} + \mathbf{B}(\nabla\cdot\mathbf{u}) = \mathbf{B}\cdot\nabla\mathbf{u} + \mathbf{B}\cdot\nabla\mathbf{u}_0,$$

$$\frac{\partial p}{\partial t} + \mathbf{u}\cdot\nabla p + \mathbf{u}_0\cdot\nabla p + \gamma p\nabla\cdot\mathbf{u} = 0.$$

$$\nabla\cdot\mathbf{B} = 0$$

RMHD ordering $\frac{k_\parallel}{k_\perp} \sim \frac{\delta\mathbf{B}}{B_0} \sim \frac{u_\perp}{v_\mathrm{A}} \sim \epsilon$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)\Psi = v_\mathrm{A}\frac{\partial\Phi}{\partial z}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)\nabla_\perp^2\Phi = v_\mathrm{A}\nabla_\parallel\nabla_\perp^2\Psi - 2\varpi_0\sin\theta\frac{\partial u_\parallel}{\partial y}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)u_\parallel = v_\mathrm{A}^2\nabla_\parallel\left(\frac{\delta B_\parallel}{B_0}\right) + (2-q)\varpi_0\sin\theta\frac{\partial\Phi}{\partial y}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)\left(1 + \frac{v_\mathrm{A}^2}{c_\mathrm{s}^2}\right)\frac{\delta B_\parallel}{B_0} = \nabla_\parallel u_\parallel + \frac{q\varpi_0\sin\theta}{v_\mathrm{A}}\frac{\partial\Psi}{\partial y},$$

$$\mathbf{u} = \hat{\mathbf{z}}\times\nabla_\perp\Phi + u_\parallel\hat{\mathbf{z}}_\perp$$

$$\delta\mathbf{B} = \hat{\mathbf{z}}\times\nabla_\perp\Psi + \delta B_\parallel\hat{\mathbf{z}}_\perp$$

Φ, Ψ : Alfvenic

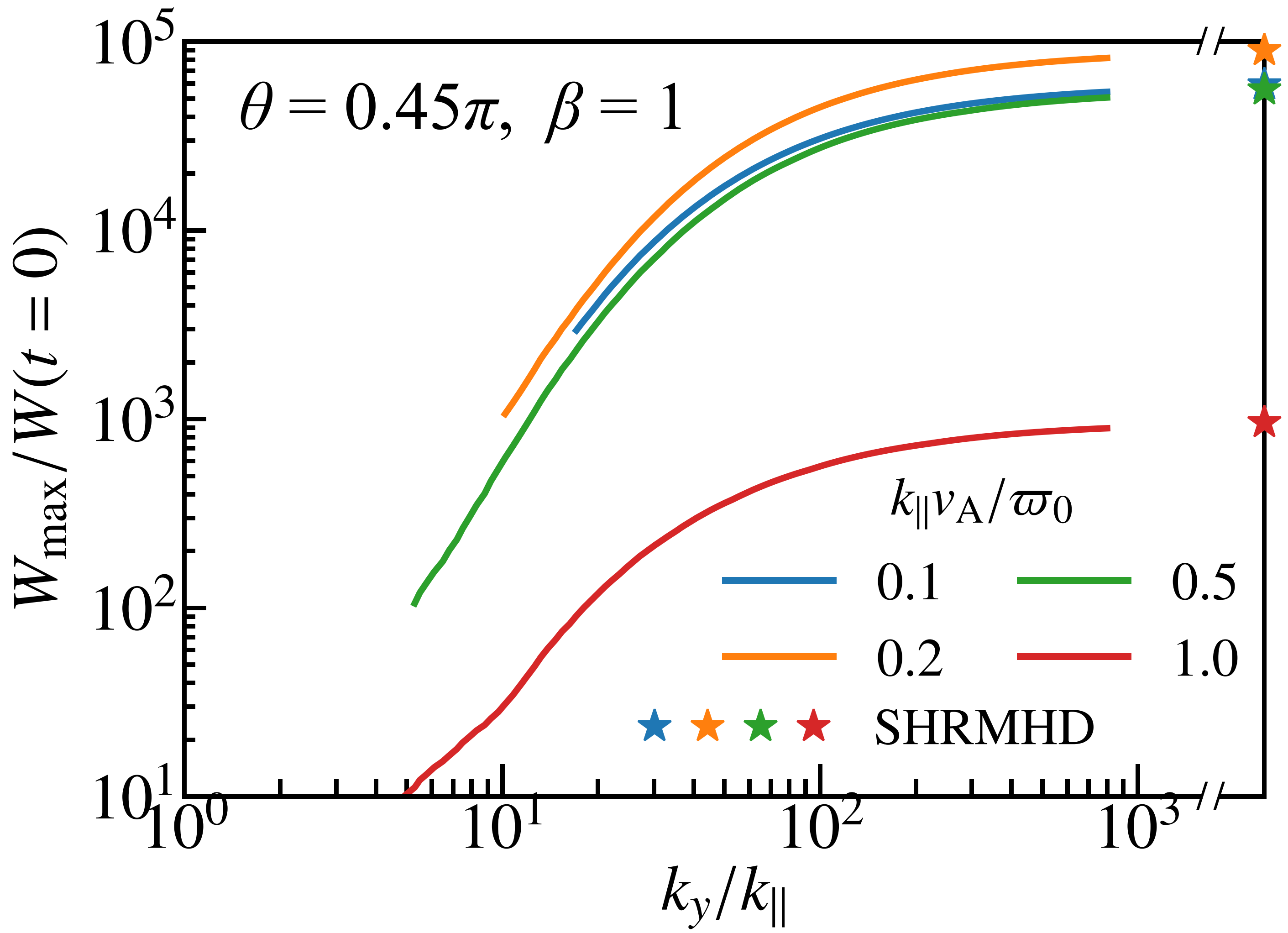
$u_\parallel, \delta B_\parallel$: compressive

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \{\Phi, \dots\}$$

$$\nabla_\parallel = \frac{\partial}{\partial z} + v_\mathrm{A}^{-1}\{\Psi, \dots\}$$

Linear analysis (MRI growth rate vs $k_y/k_\parallel$)

- Compare the growth rate between SHRMHD and ideal MHD to check if SHRMHD includes the fastest growing mode of ideal MHD



- Growth rate of ideal MHD : an increase func of $k_y/k_\parallel$
- Saturate at the growth rate of SHRMHD
  - → When $\theta \simeq \pi/2$, it is enough to solve SHRMHD

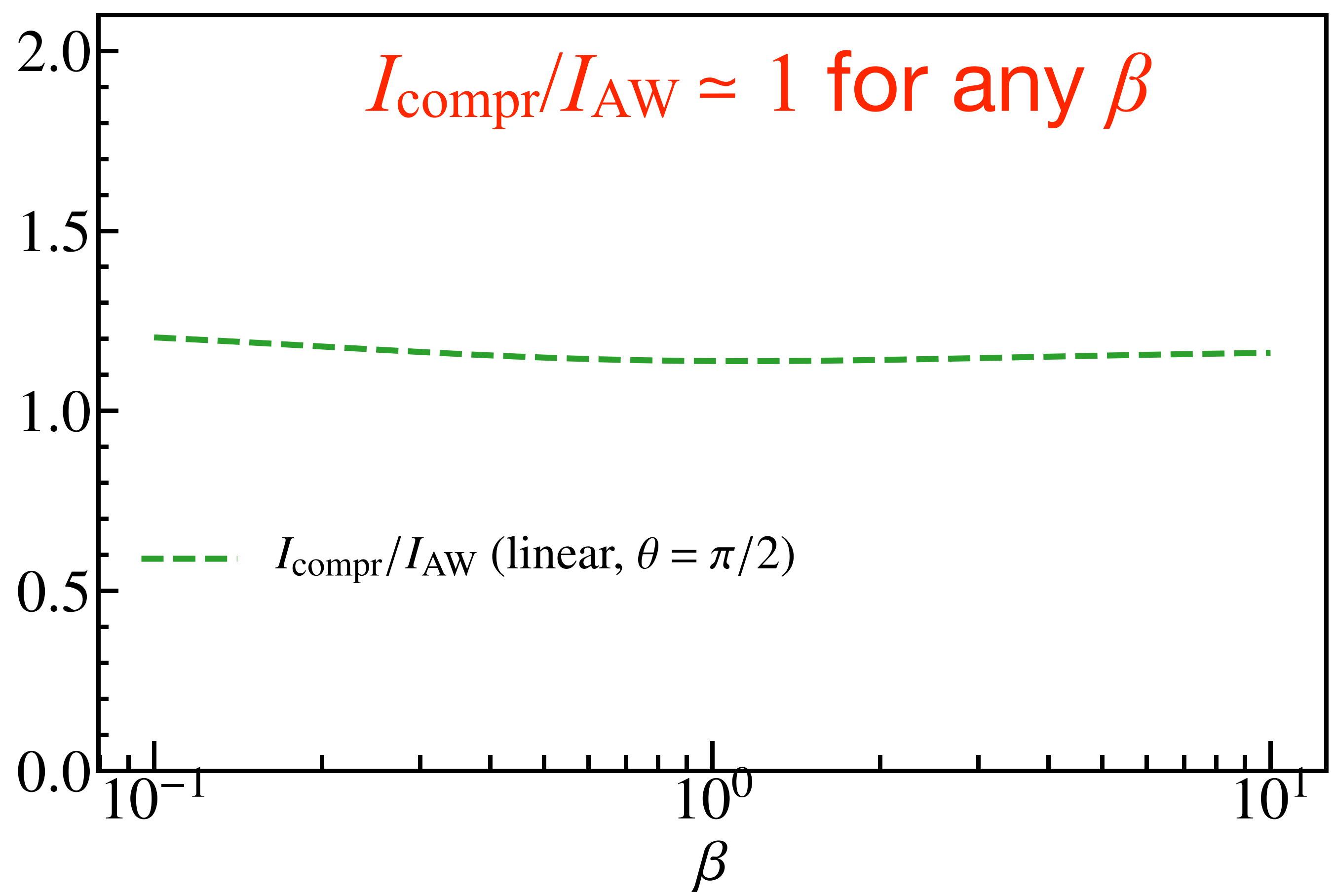
Linear analysis ($I_{\rm compr}/I_{\rm AW}$)

- Substituting the eigenfunctions into

$$I_{\rm AW} = \int {\rm d}^3\mathbf{r}\left[q\varpi_0\cos\theta\left(u_x u_y - v_{\rm A}^2\frac{\delta B_x}{B_0}\frac{\delta B_y}{B_0}\right) + 2\varpi_0\sin\theta\, u_\parallel u_x\right],$$

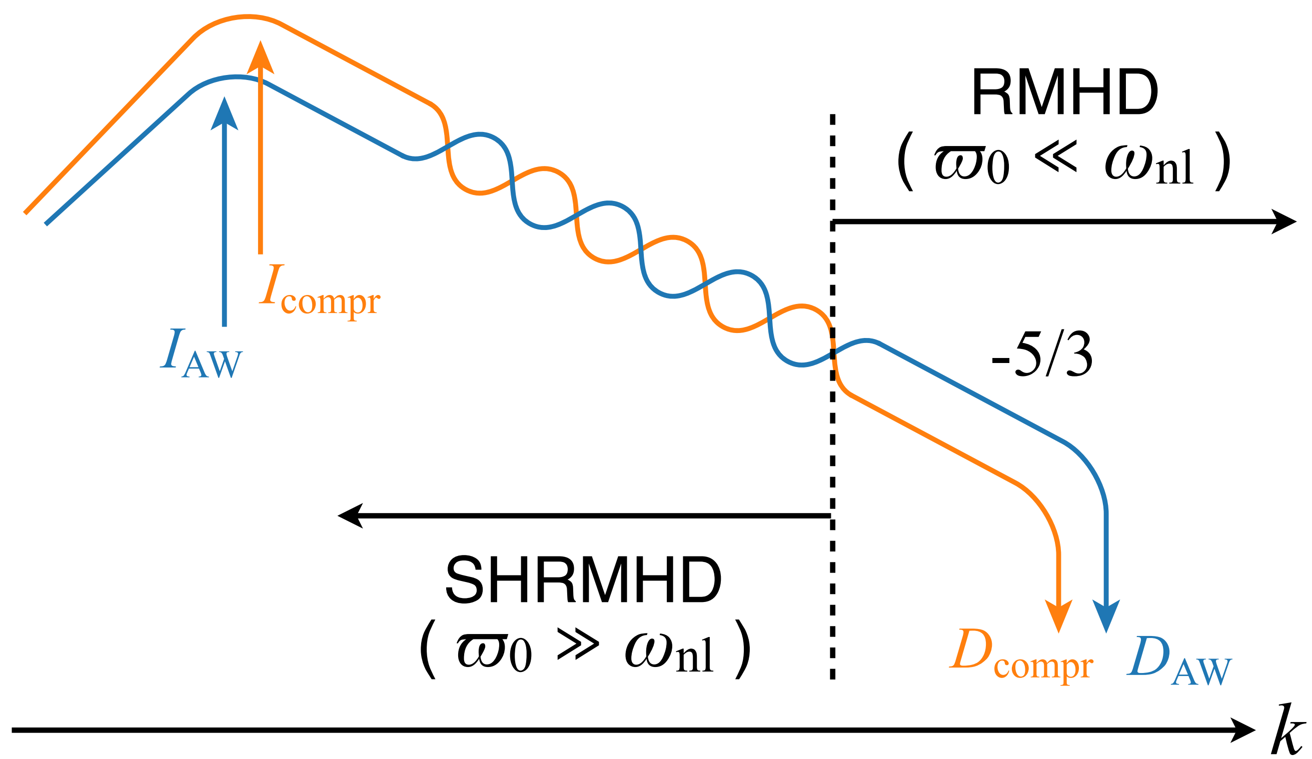
$$I_{\rm compr} = \int {\rm d}^3\mathbf{r}\left[q\varpi_0\sin\theta\left(u_x u_\parallel - v_{\rm A}^2\frac{\delta B_x}{B_0}\frac{\delta B_\parallel}{B_0}\right) - 2\varpi_0\sin\theta\, u_\parallel u_x\right]$$

we obtaine the compressive-to-Alfven MRI injection power ratio $I_{\rm compr}/I_{\rm AW}$



Nonlinear simulation of SHRMHD

- Injection ratio $I_{\text{compr}}/I_{\text{AW}}$ and dissipation ratio $D_{\text{compr}}/D_{\text{AW}}$ shall be different



$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)\Psi = v_{\mathrm{A}}\frac{\partial\Phi}{\partial z} \quad + \text{perp resitivity}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)\nabla_\perp^2\Phi = v_{\mathrm{A}}\nabla_\parallel\nabla_\perp^2\Psi - 2\varpi_0\sin\theta\frac{\partial u_\parallel}{\partial y} \quad + \text{perp viscosity}$$

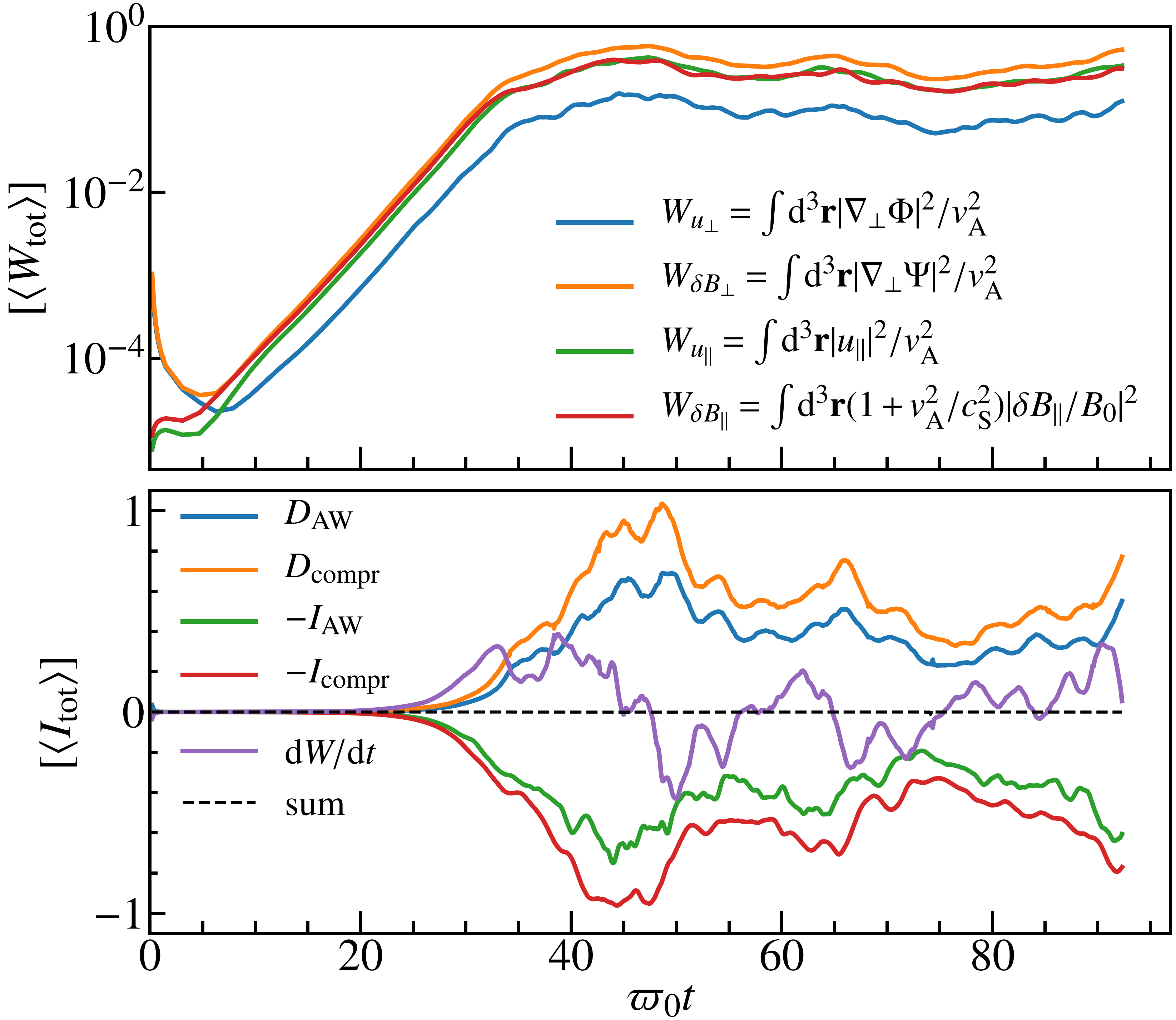
$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)u_\parallel = v_{\mathrm{A}}^2\nabla_\parallel\left(\frac{\delta B_\parallel}{B_0}\right) + (2-q)\varpi_0\sin\theta\frac{\partial\Phi}{\partial y} \quad + \text{para resitivity}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} - q\varpi_0 x\cos\theta\frac{\partial}{\partial y}\right)\left(1 + \frac{v_{\mathrm{A}}^2}{c_{\mathrm{s}}^2}\right)\frac{\delta B_\parallel}{B_0} = \nabla_\parallel u_\parallel + \frac{q\varpi_0\sin\theta}{v_{\mathrm{A}}}\frac{\partial\Psi}{\partial y} \quad + \text{para viscosity}$$

$$\Rightarrow \quad \frac{\mathrm{d}W}{\mathrm{d}t} = I_{\text{AW}} + I_{\text{compr}} - D_{\text{AW}} - D_{\text{compr}}$$

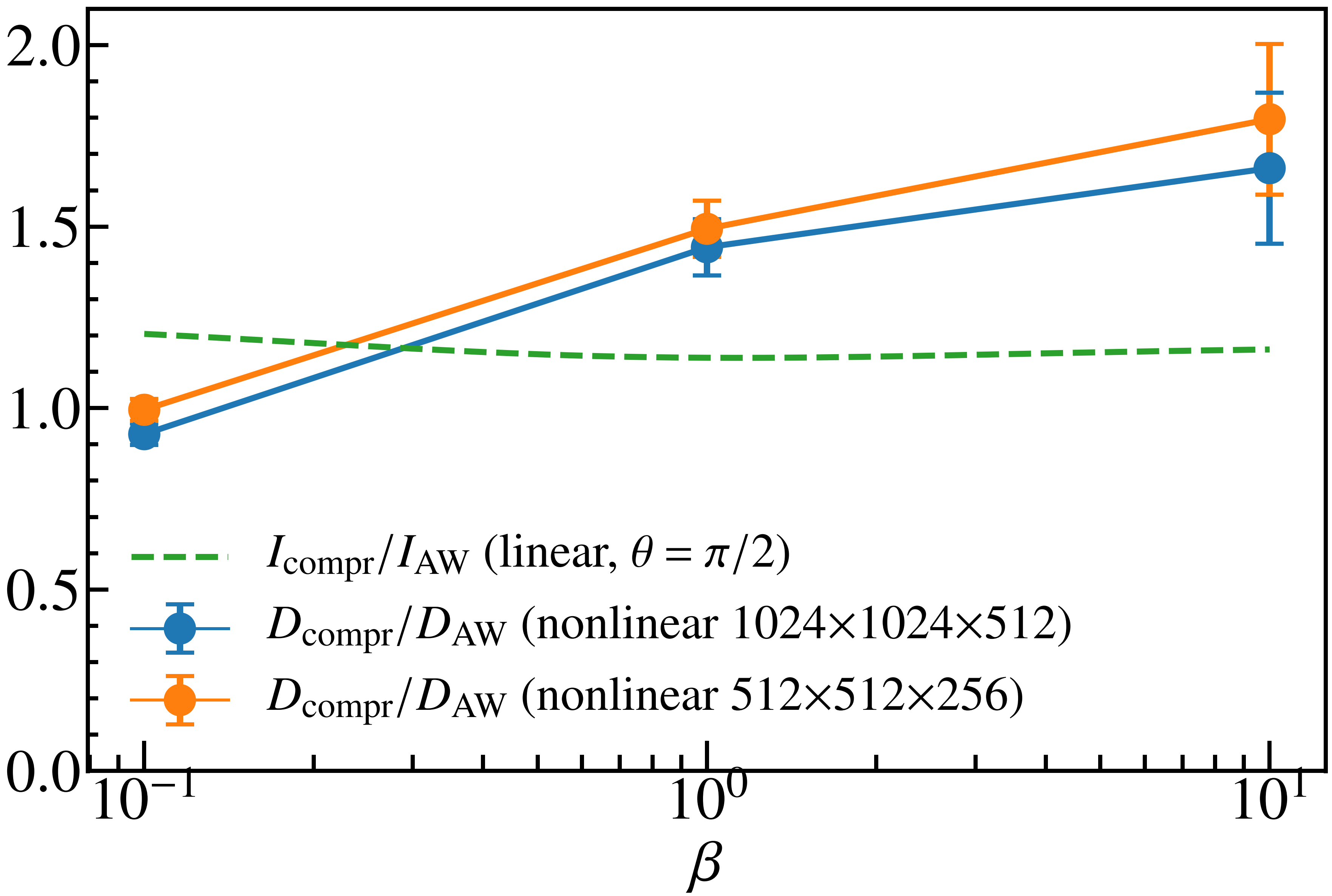
- Compute $D_{\text{compr}}/D_{\text{AW}}$ at the MRI saturated state
- Fix $\theta = 0.45\pi$ and scan $\beta = 0.1,\ 1,\ 10$

Time evolution



$$\frac{dW}{dt} = I_{\rm AW} + I_{\rm compr} - D_{\rm AW} - D_{\rm compr}$$

$D_{\mathrm{compr}}/D_{\mathrm{AW}}$ vs β

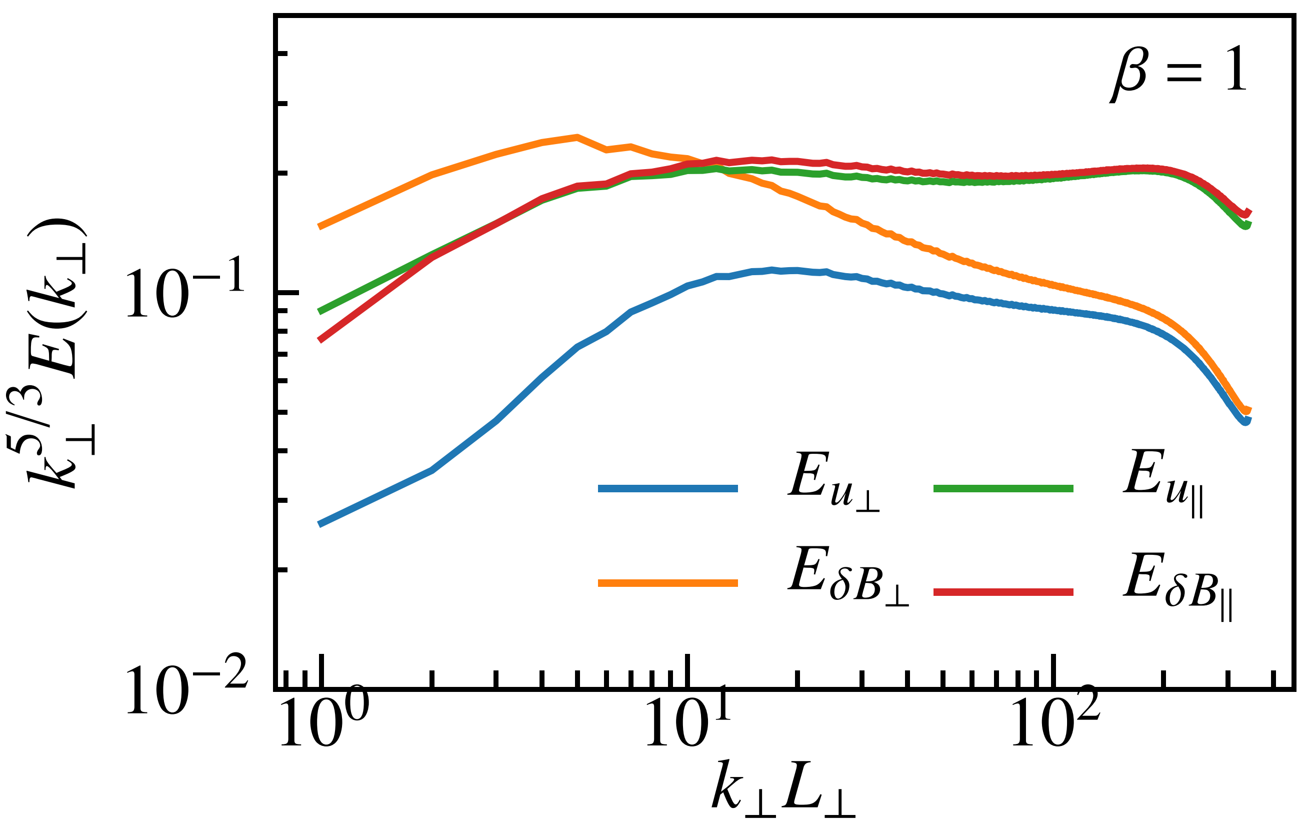


Linear & nonlinear calculations show $D_{\mathrm{compr}}/D_{\mathrm{AW}} \simeq 1$

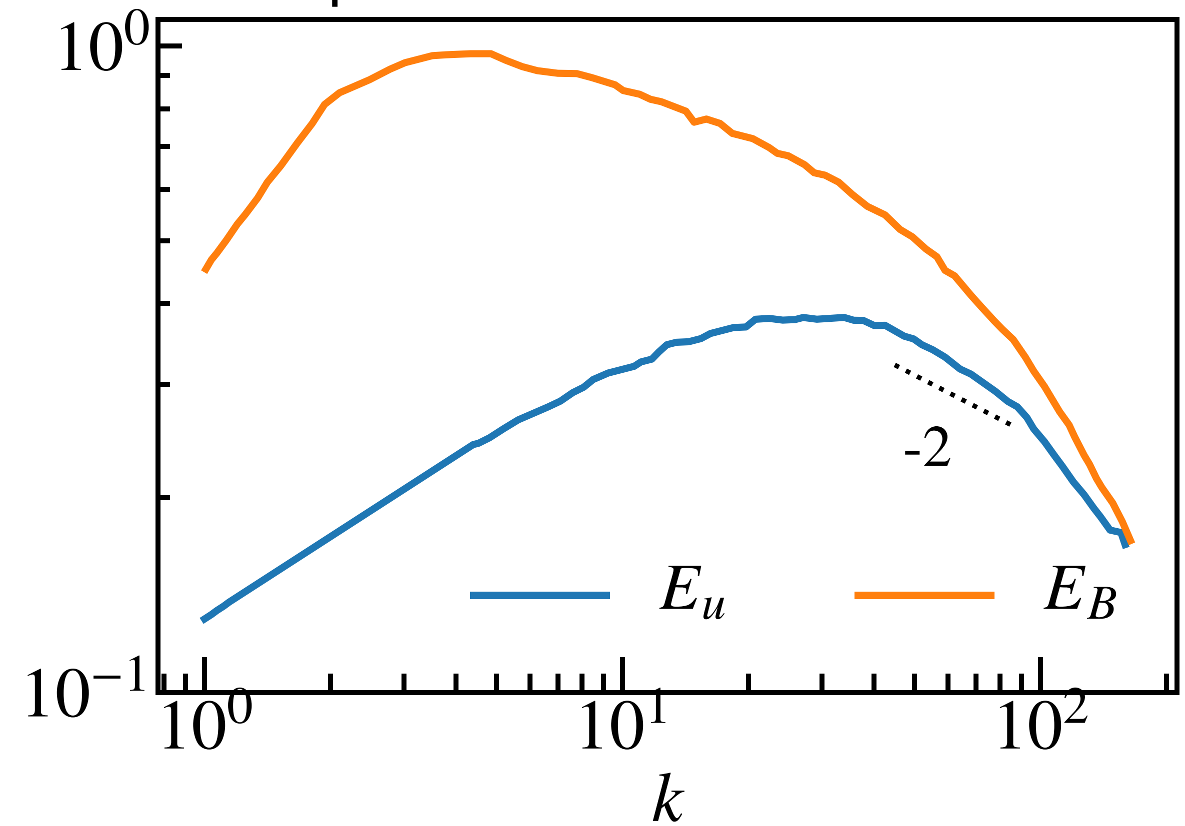
$$\frac{Q_{\mathrm{i}}}{Q_{\mathrm{e}}}(\beta_{\mathrm{i}},\, \tau,\, \wp) = \frac{35}{1 + (\beta_{\mathrm{i}}/15)^{-1.4} \mathrm{e}^{-0.1/\tau}} + \wp\left(\frac{\wp}{\wp + \beta_{\mathrm{i}}} + \frac{2\beta_{\mathrm{i}}}{1 + \wp^{1.5}/2}\right) \quad \text{with} \quad \wp = 1 \quad \text{is the answer!}$$

$k_\perp$-spectrum

Our SHRMHD (1024x1024x512 grids)

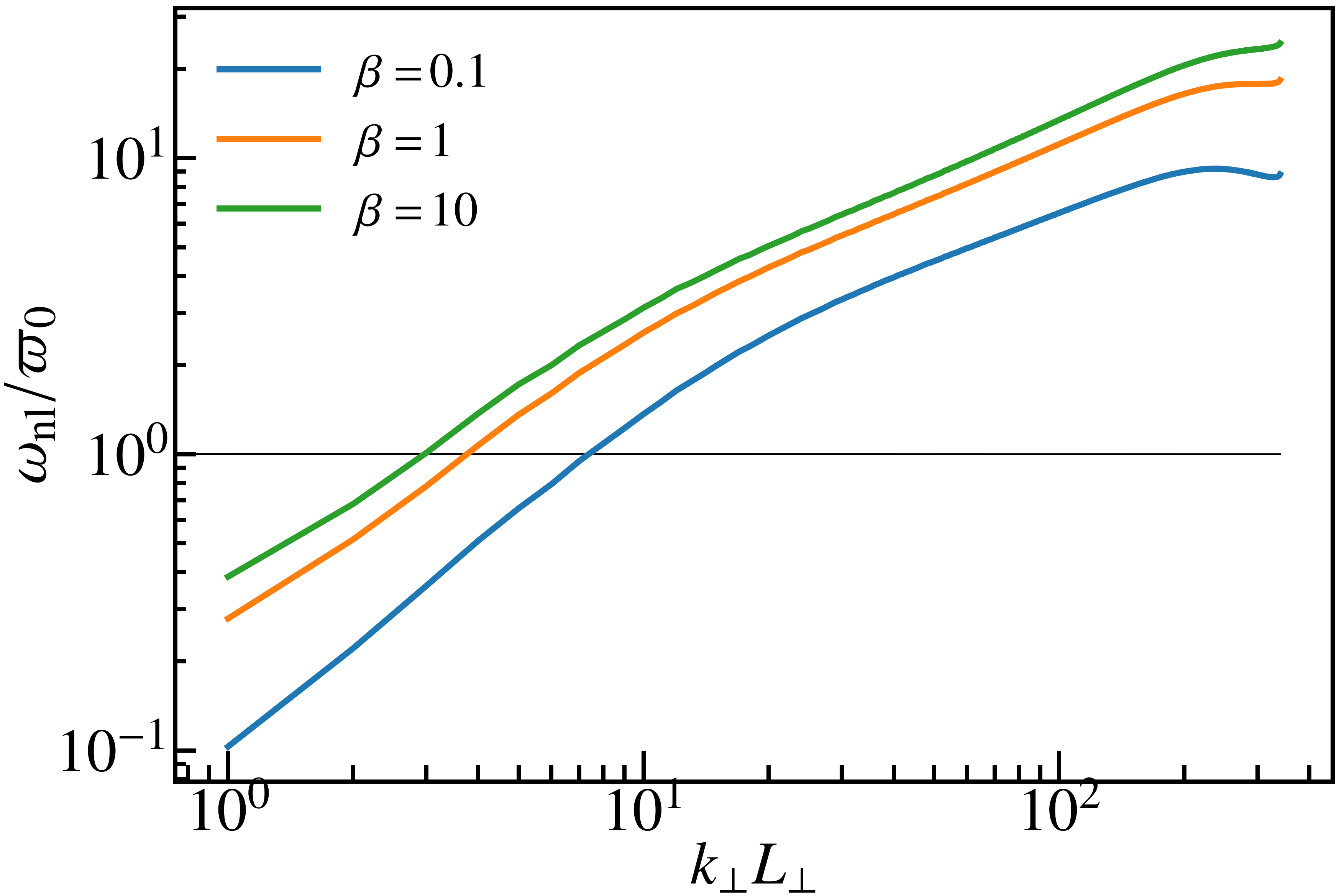


Full-MHD (1024x1024x512 grids)
Reproduced from Walker+ 2016



- Both Alfven & slow look $k_\perp^{-5/3}$
  - → RMHD regime is reached
  - → Alfven & slow are decoupled
- In full-MHD, $k_\perp^{-5/3}$ is not achieved

Shear rotation vs nonlinear cascade



RMHD ($\varpi_0 \ll \omega_{\rm nl}$)

AW and compressive are decoupled

SHRMHD ($\varpi_0 \gg \omega_{\rm nl}$)

AW and compressive are coupled

Summary

- Ion vs electron heating is critical for interpreting the EHT data.
- In collisionless plasma, turbulent heating is multiscale in nature.
- We split the problem into three pieces

① What is $P_{\mathrm{compr}}/P_{\mathrm{AW}}$ for given P_{MRI}?

② What is $Q_{\mathrm{i}}/Q_{\mathrm{e}}$ for given P_{AW}?

③ What is $Q_{\mathrm{i}}/Q_{\mathrm{e}}$ for given P_{compr}?

- For ② & ③, we used gyrokinetics
  - GK allows us to solve kinetic turbulence with reasonable cost
  - Obtained a heating prescription

$$\frac{Q_{\mathrm{i}}}{Q_{\mathrm{e}}}(\beta_{\mathrm{i}},\,\tau,\,\wp)=\frac{35}{1+(\beta_{\mathrm{i}}/15)^{-1.4}\mathrm{e}^{-0.1/\tau}}+\wp\left(\frac{\wp}{\wp+\beta_{\mathrm{i}}}+\frac{2\beta_{\mathrm{i}}}{1+\wp^{1.5}/2}\right)$$

- For ①, we developed SHRMHD because full-MHD is difficult to use
  - When θ≃π/2 (toroidal magnetic field), $P_{\mathrm{compr}}/P_{\mathrm{AW}}\simeq 1$