Section 2.
Stellar structure and properties (I)

2.1 Hydrostatic equilibrium

2.2 Nuclear burning

Let's understand these questions with the words of physics

- Why are stars so luminous?
- Why do stars show $L \sim M^4$?
- Why do stars evolve?
- Why does the destiny of stars depend on the mass?
- Why do some stars explode?
- Why don't normal star explode?
- Why does stellar core collapses?
- Why is the energy of supernova so huge?
- …

Our sun

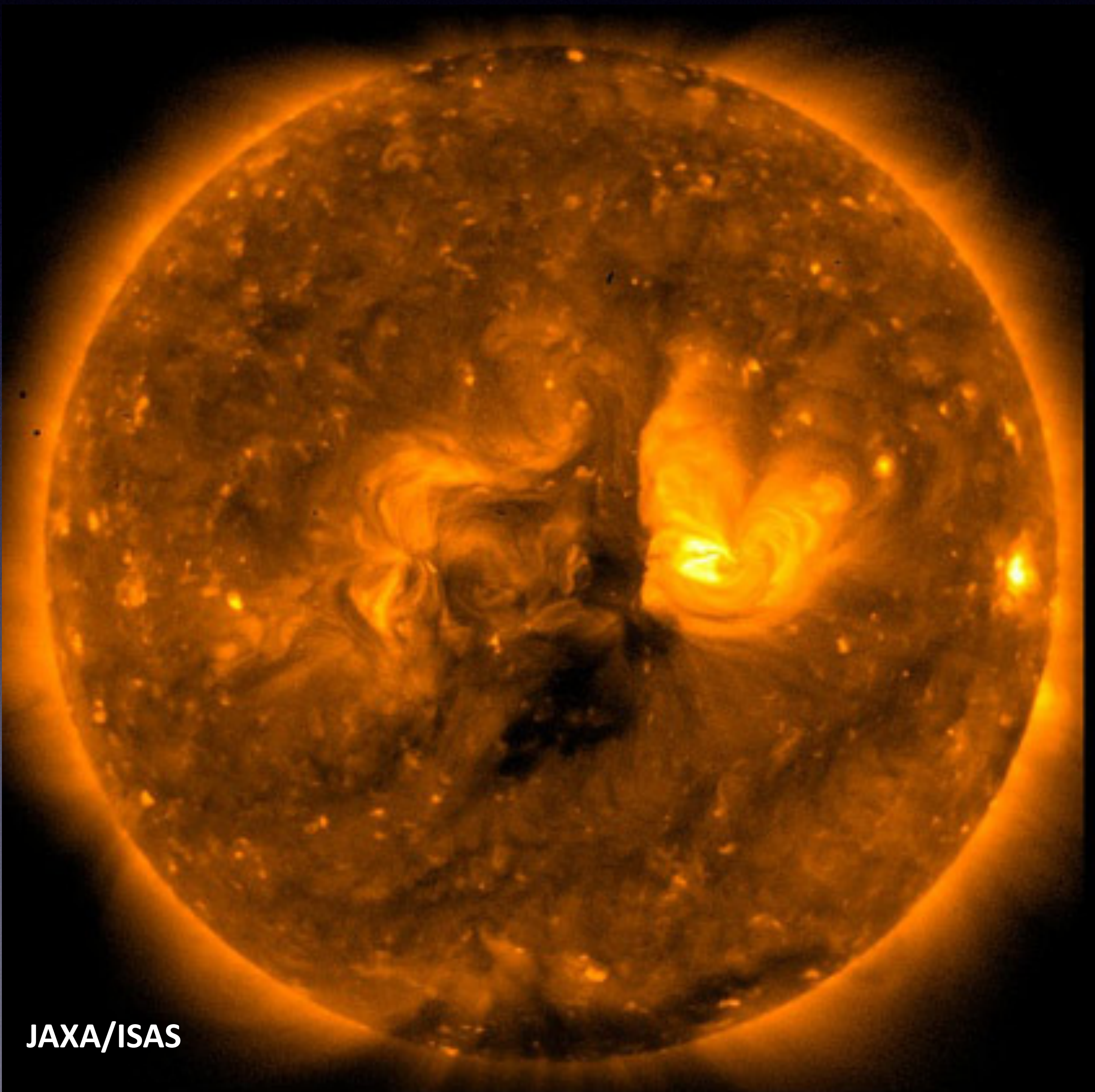
$L = 4 \times 10^{33}$ erg/s = 4×10^{26} J/s (W)

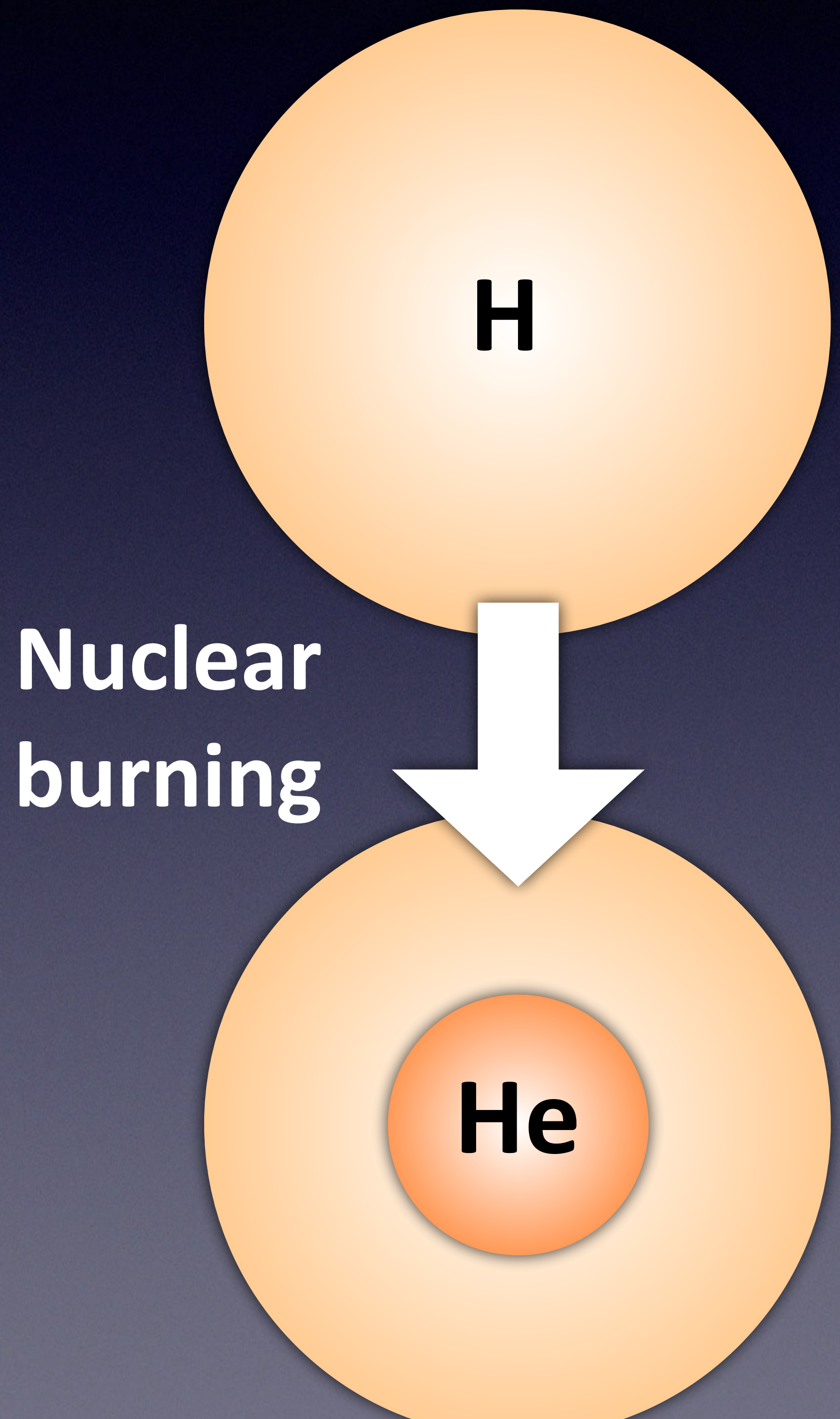
Electronic power consumption in Japan

1.5×10^{19} J / year

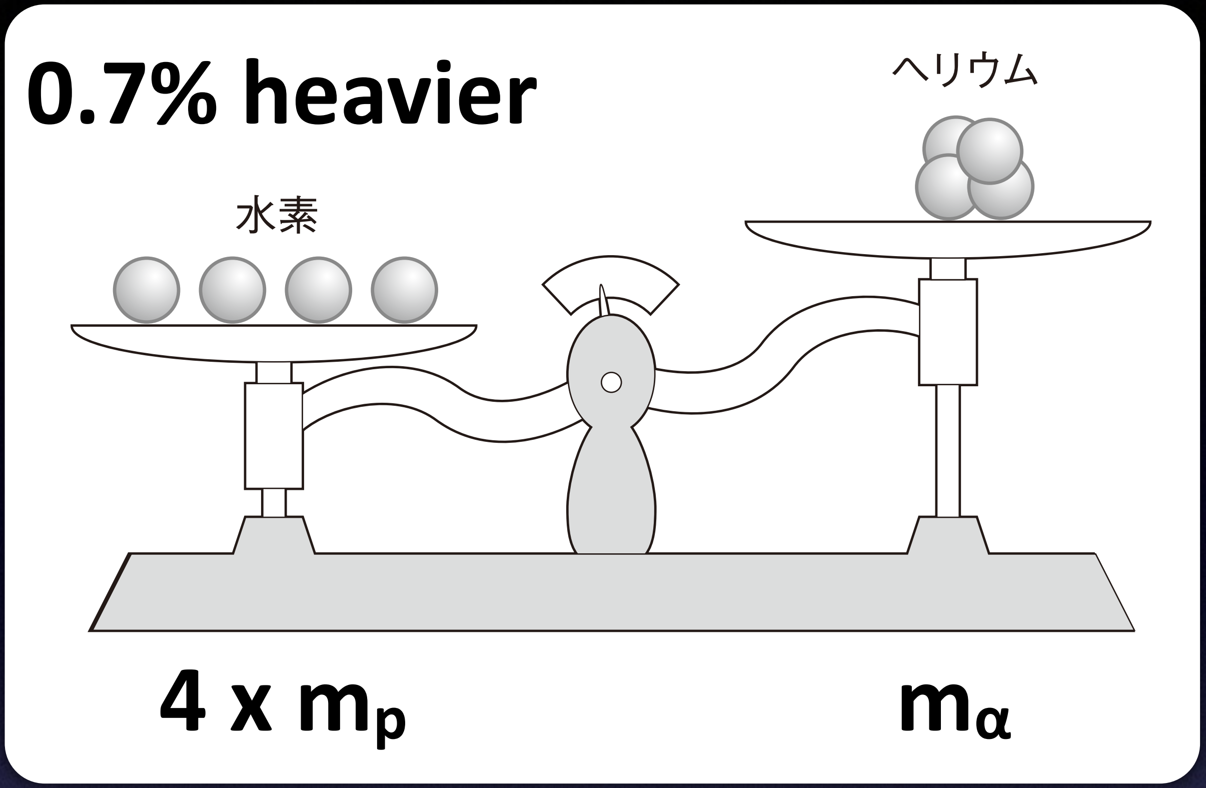
==> Japanese power consumption for 2×10^{7} yr

= solar radiation in 1 second





Energy source: $E = mc^2$



Q1: How much energy is released for one nucleon? (核子あたり)

$E = \Delta mc^2$

Solar mass：2×10^{33} g

Q2: How much energy does the Sun can produce?
(assume 10% of solar mass can be used for nuclear burning)

Q3: How many years the sun can keep shining?

$L = 4 \times 10^{33}$ erg/s　　1 yr ~ 3×10^7 sec

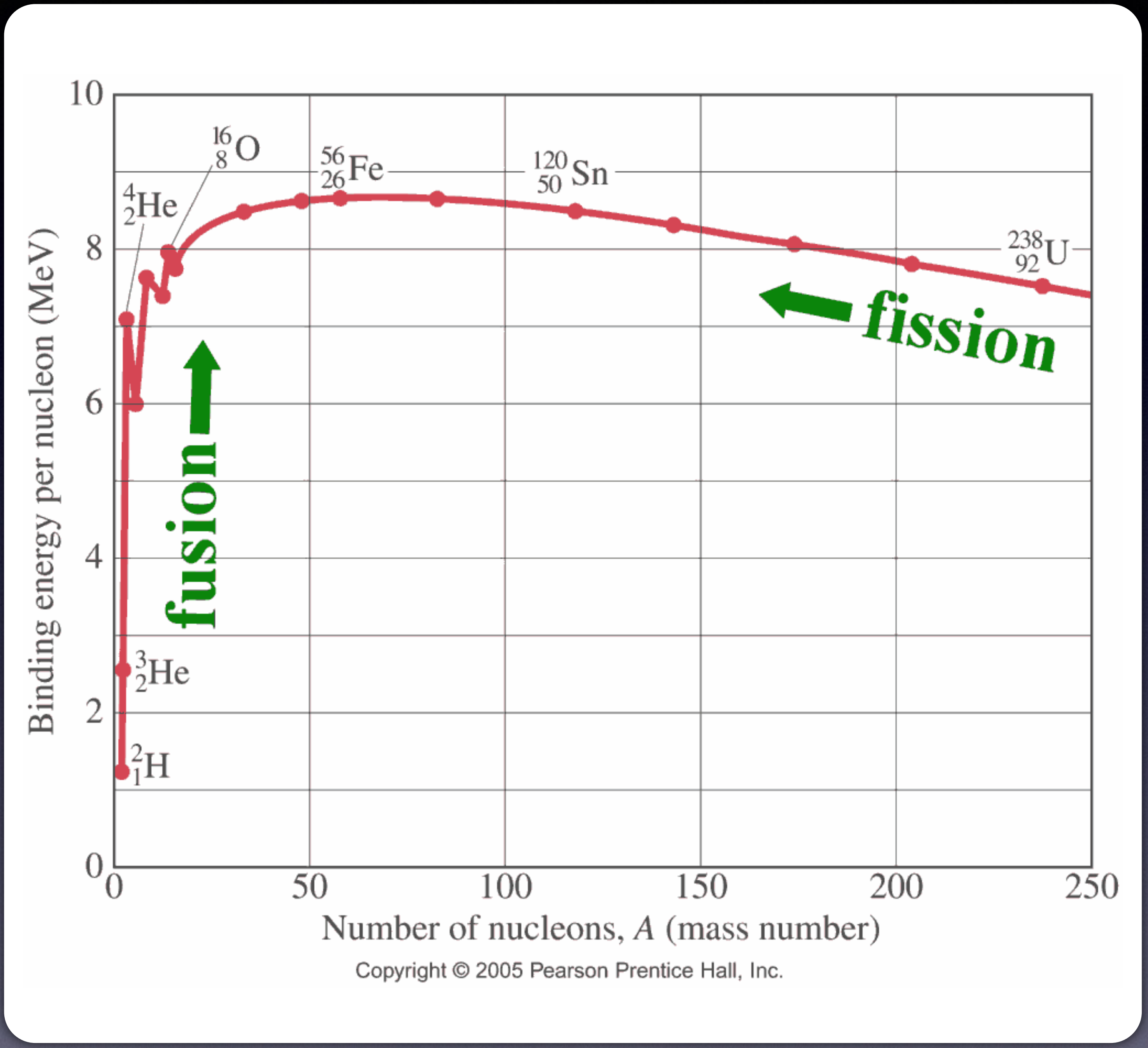
Binding energy of nuclei

$$E_b = [Nm_N + Zm_p - m_i]\,c^2$$

$$\underbrace{Nm_N + Zm_p}_{P\,+\,n} \qquad \underbrace{m_i}_{\text{Nuclei}}$$

Higher binding energy
= strongly "bound"
= more stable
= "lighter"

Fe is the most stable nucleus



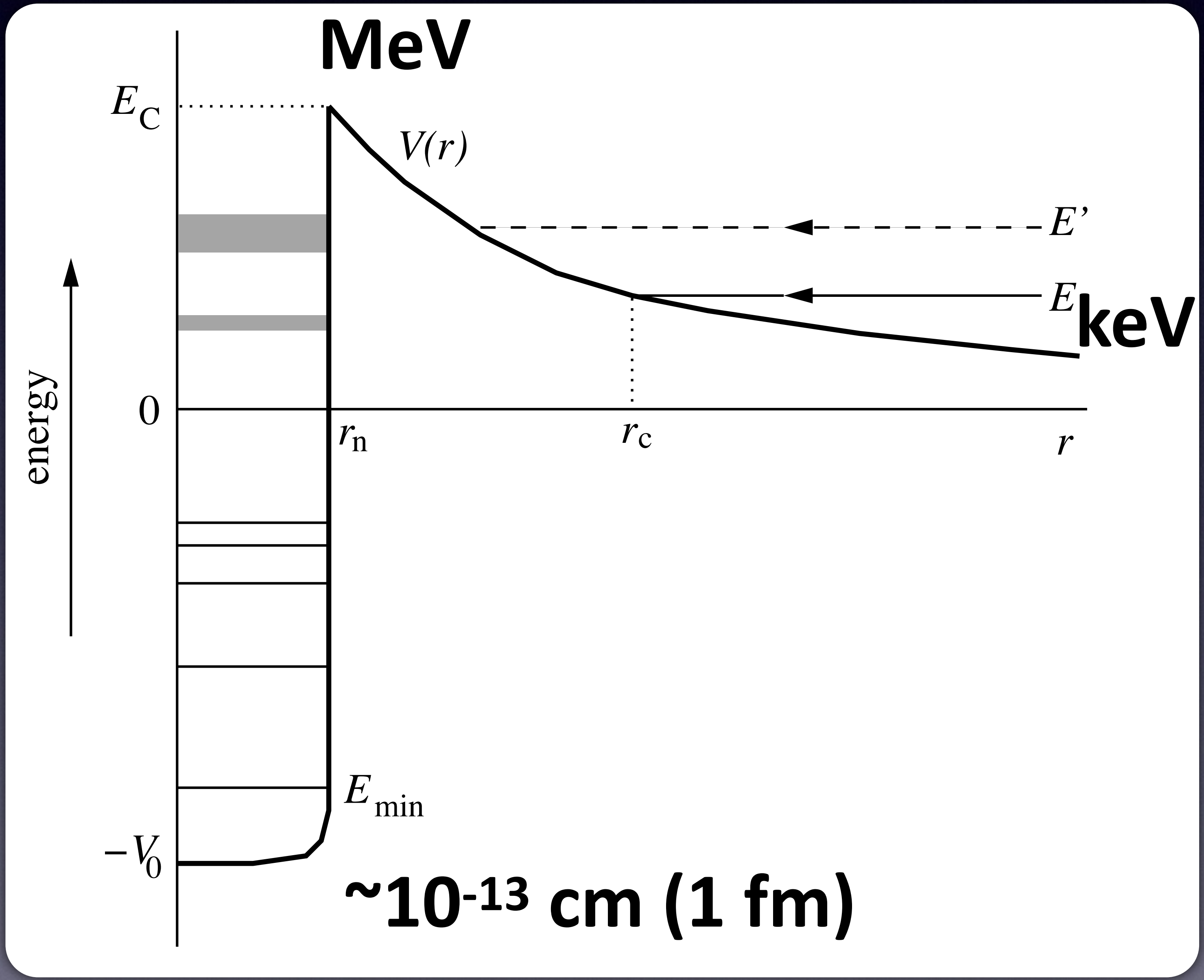
What is going on at the center of the star?
How does nuclear burning occurs?

Nuclear burning

Coulomb barrier $E \sim (Z_1Z_2e^2)/r \sim 10^6$ eV (MeV)

Typical energy of the gas $E \sim kT \sim 10^3$ eV (keV) <= 10^7 K

=> Tunnel effects



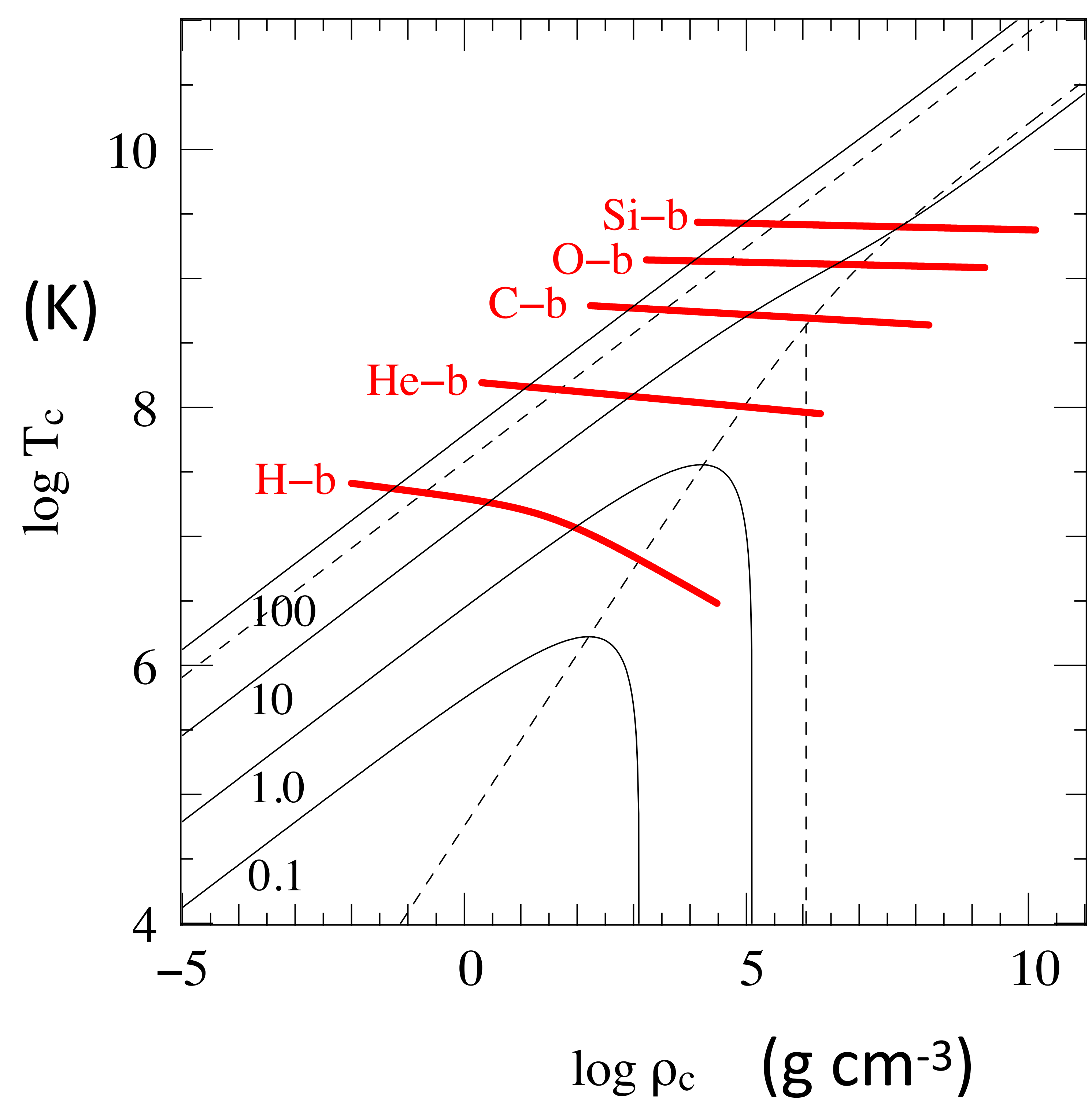
Textbook by Pols

Condition of H-burning

Fusion reactor

~10^8 K

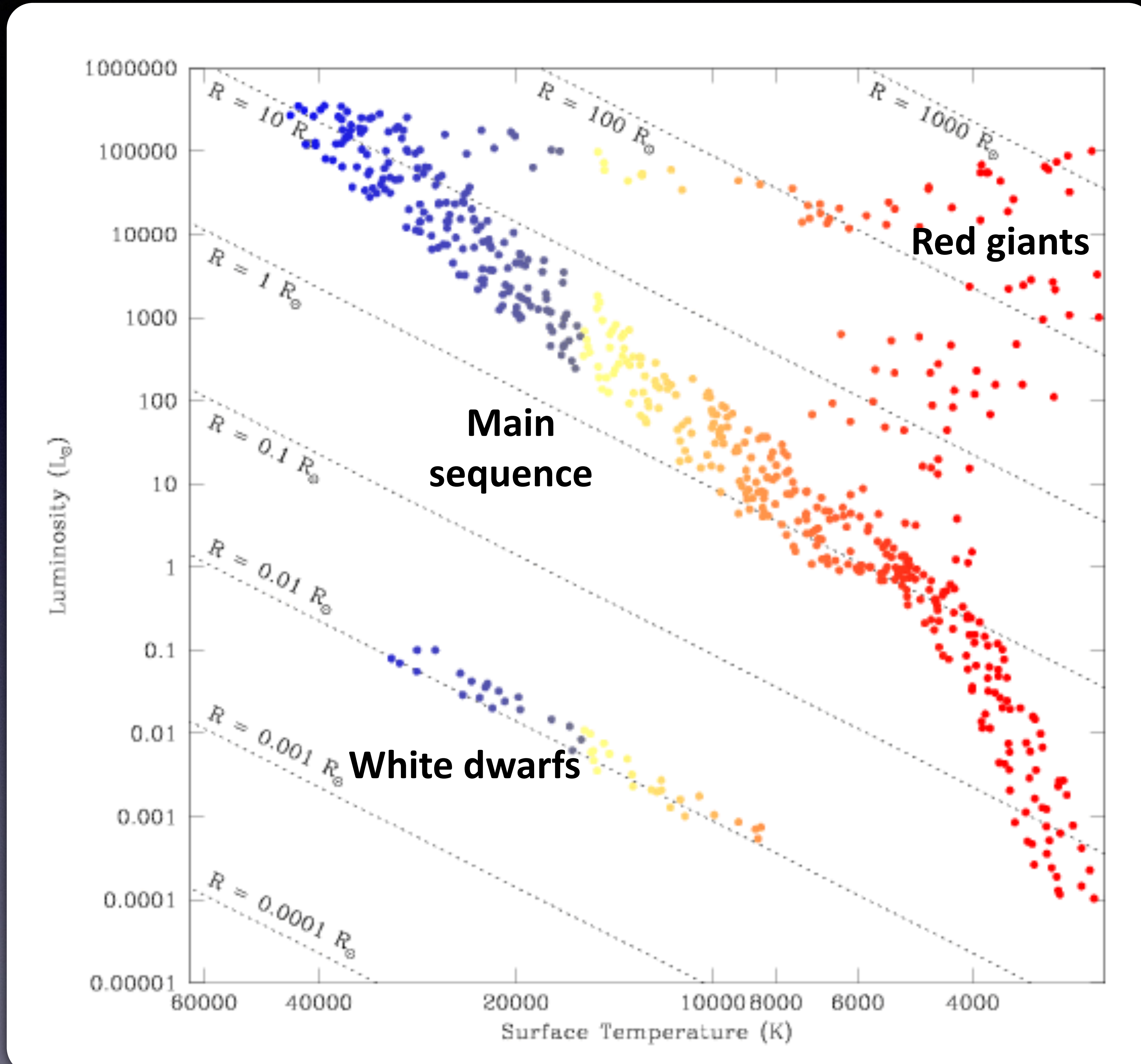
-10



Lecture Note by Pols

Hertzsprung-Russel diagram

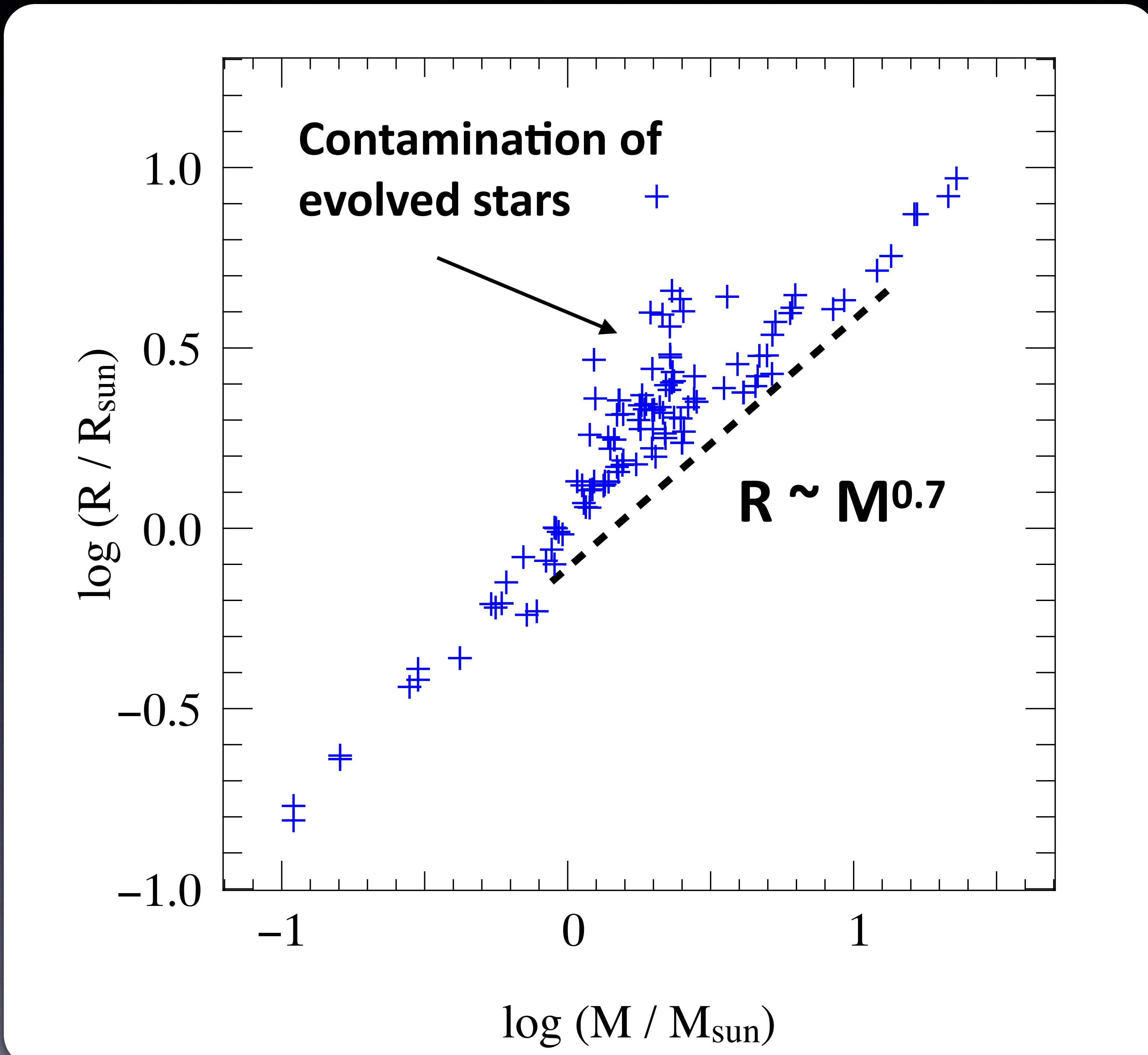
Luminosity



Temperature (K)

http://astronomy.nmsu.edu/geas/lectures/lecture23/slide04.html

Mass - radius relation for the main sequence



Outcome of the central property of the star

Lecture Note by Pols

Summary: Stellar structure and properties (I)

- Energy source of the stars
  - Nuclear burning
  - $E = mc^2$
- Stellar structure
  - Hydrostatic Equilibrium
  - Central temperature of the stars $T \sim 10^7$ K
  - Require tunnel effects for nuclear burning
- Stellar properties
  - Almost constant central T => $R \sim M$
  - Observed mass-radius relation ($R \sim M^{0.7}$)

Appendix

Virial theorem
(galaxy clusters)

Velocity of galaxies in Virgo Cluster

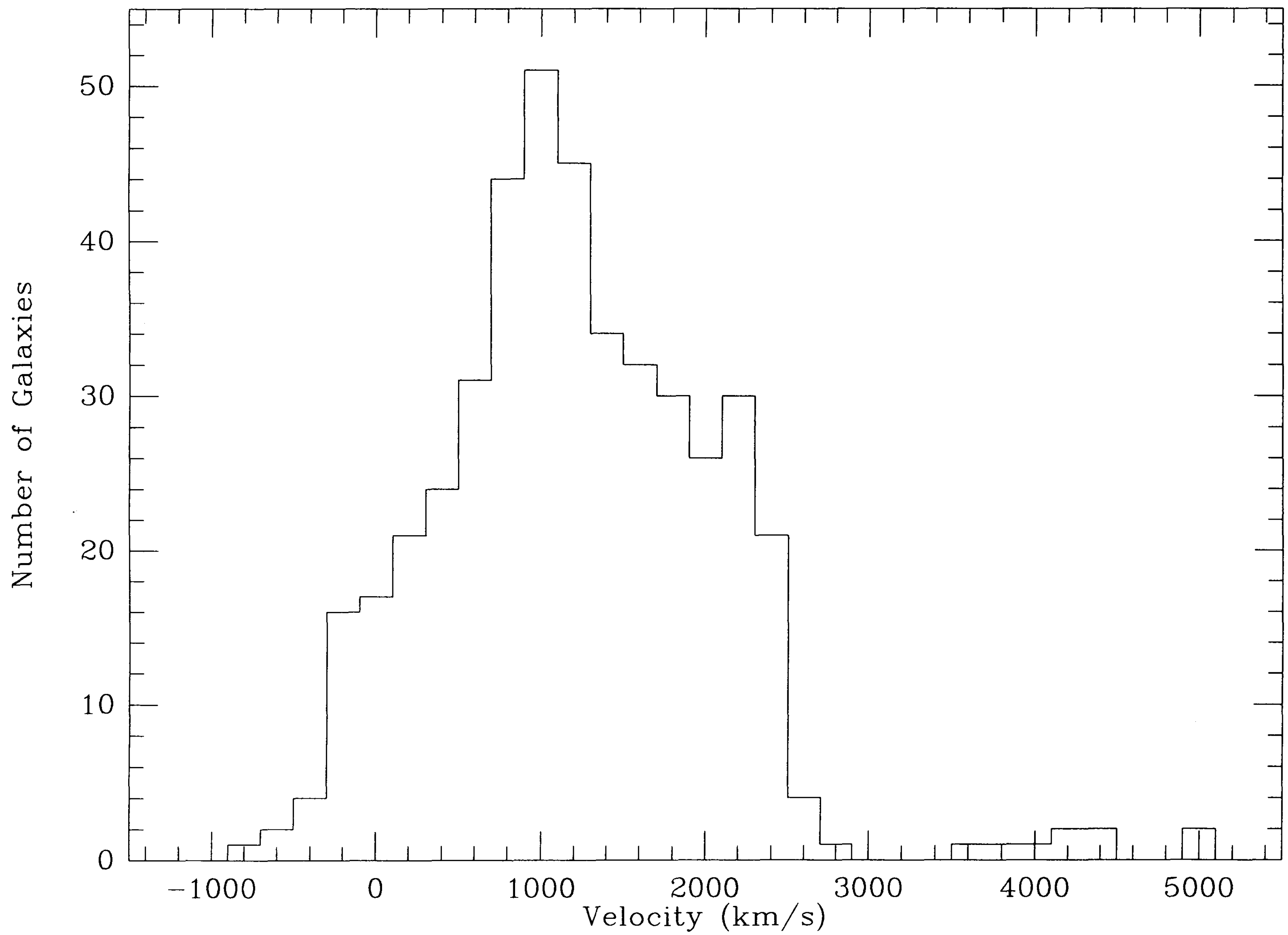
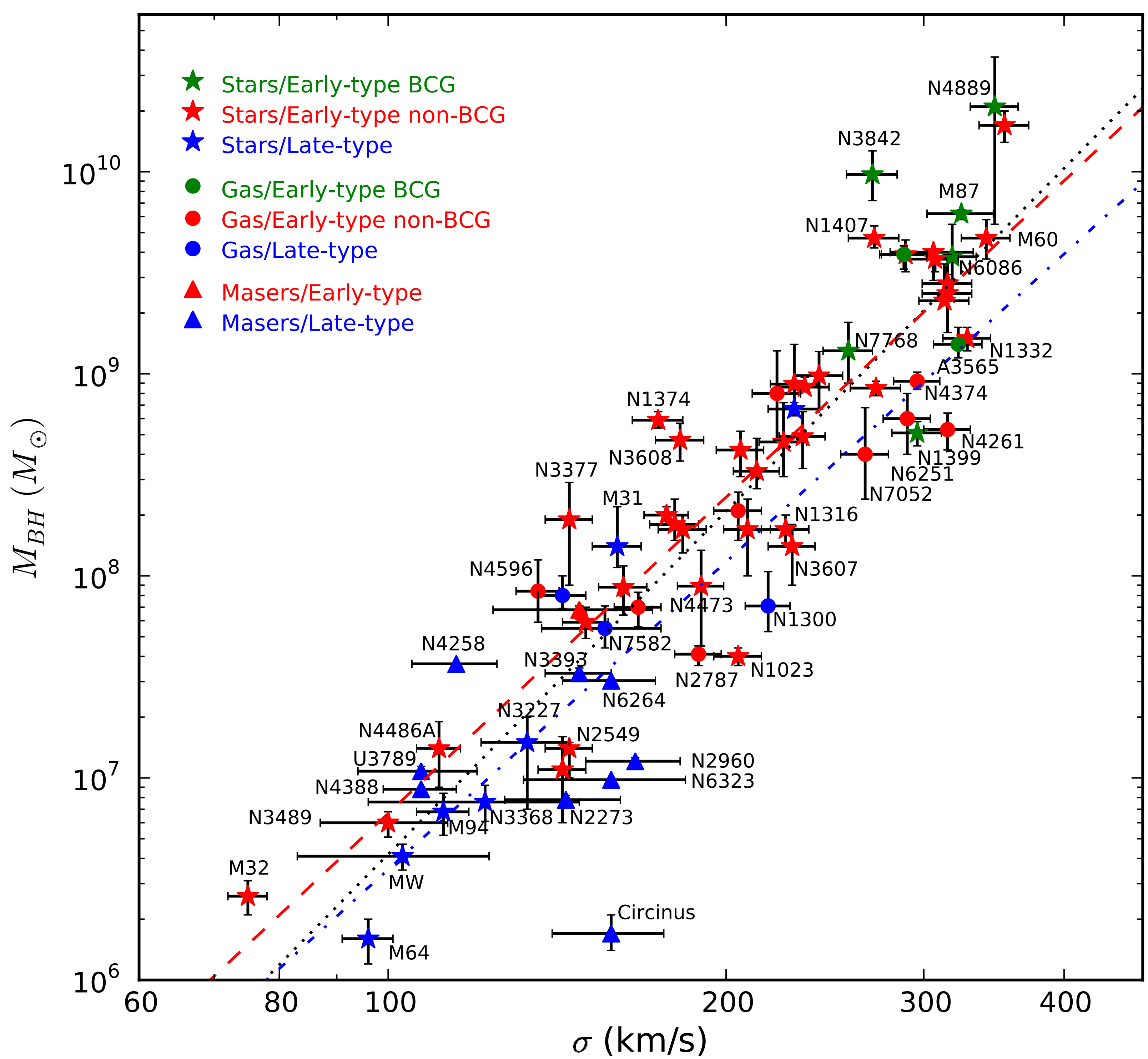


FIG. 1.—Velocity histogram for the Virgo Cluster. Notice the obvious background group at $\sim$4000 km s^{-1}.

Girardi+1993

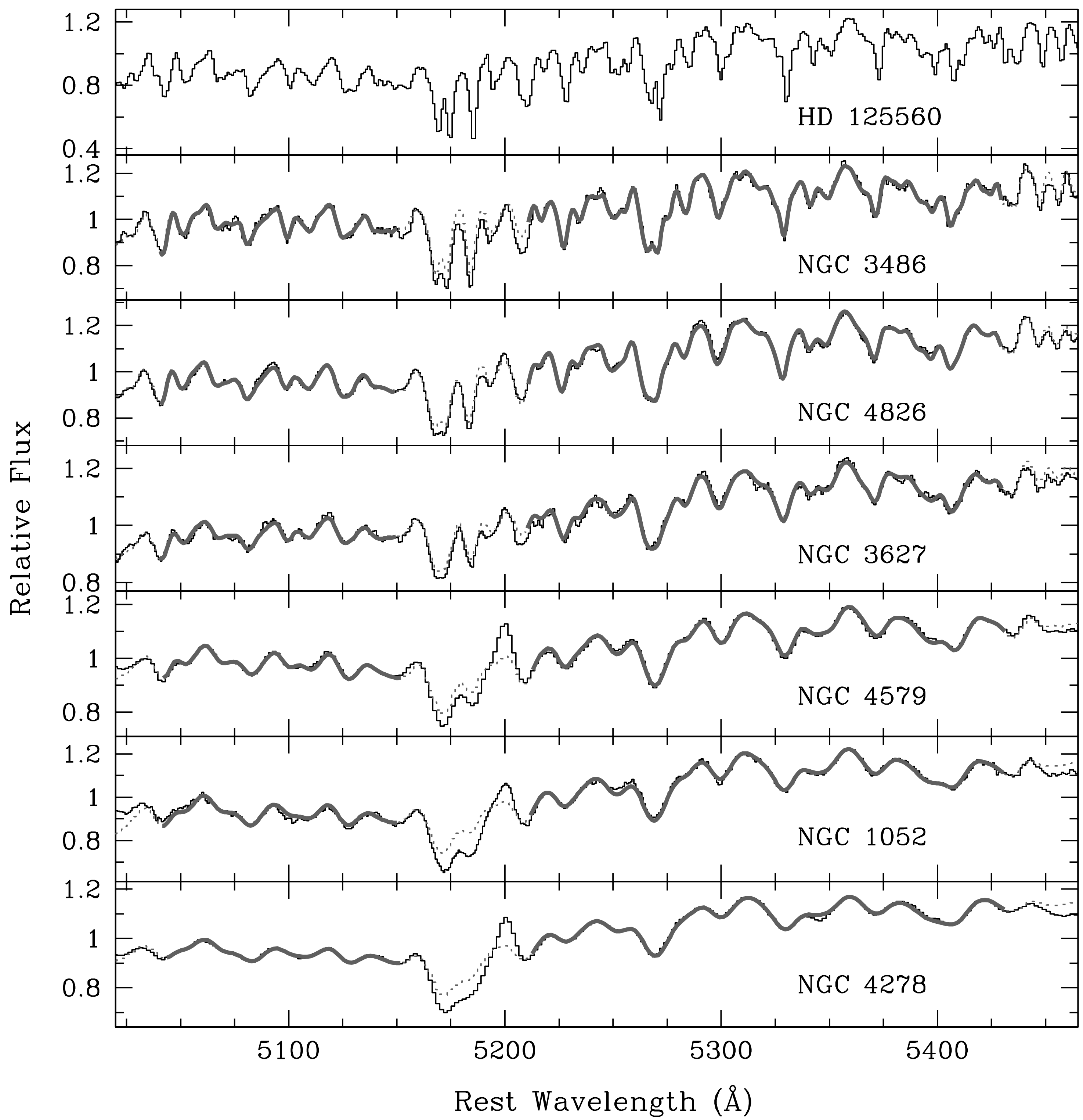
Virial theorem
(galaxies)

M-sigma relation



McConnell & Ma 2013

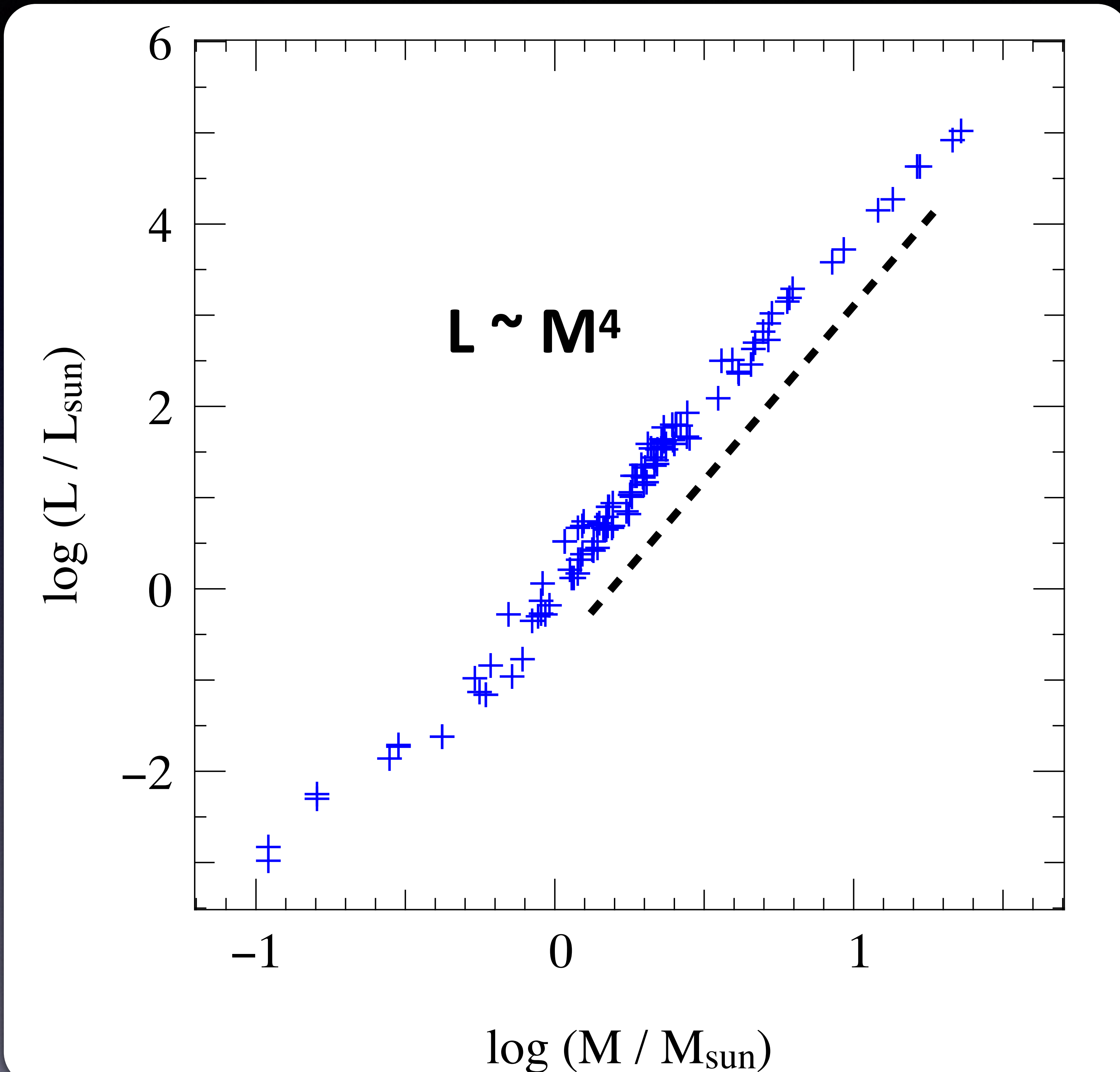
Velocity dispersion



Barth+2002

Central temperature of stars

Mass - luminosity relation of the main sequence stars



Star with M = 10 Msun

=> L ~ 10^4 Lsun

=> Lifetime

~ $1/10^3$ of the Sun

~ 10^{10} yr (100億年)/10^3

~ 10^7 yr (1000万年)

More massive stars have shorter lifetime

Lecture Note by Pols

1a. H-burning (pp chain)

$$4\,{}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2\,\mathrm{e}^{+} + 2\,\nu$$

$${}^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + \mathrm{e}^{+} + \nu$$

$${}^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma$$

$${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2\,{}^{1}\mathrm{H}$$

pp1

$${}^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma$$

$${}^{7}\mathrm{Be} + \mathrm{e}^{-} \rightarrow {}^{7}\mathrm{Li} + \nu$$

$${}^{7}\mathrm{Li} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

pp2

$${}^{7}\mathrm{Be} + {}^{1}\mathrm{H} \rightarrow {}^{8}\mathrm{B} + \gamma$$

$${}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be} + \mathrm{e}^{+} + \nu$$

$${}^{8}\mathrm{Be} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

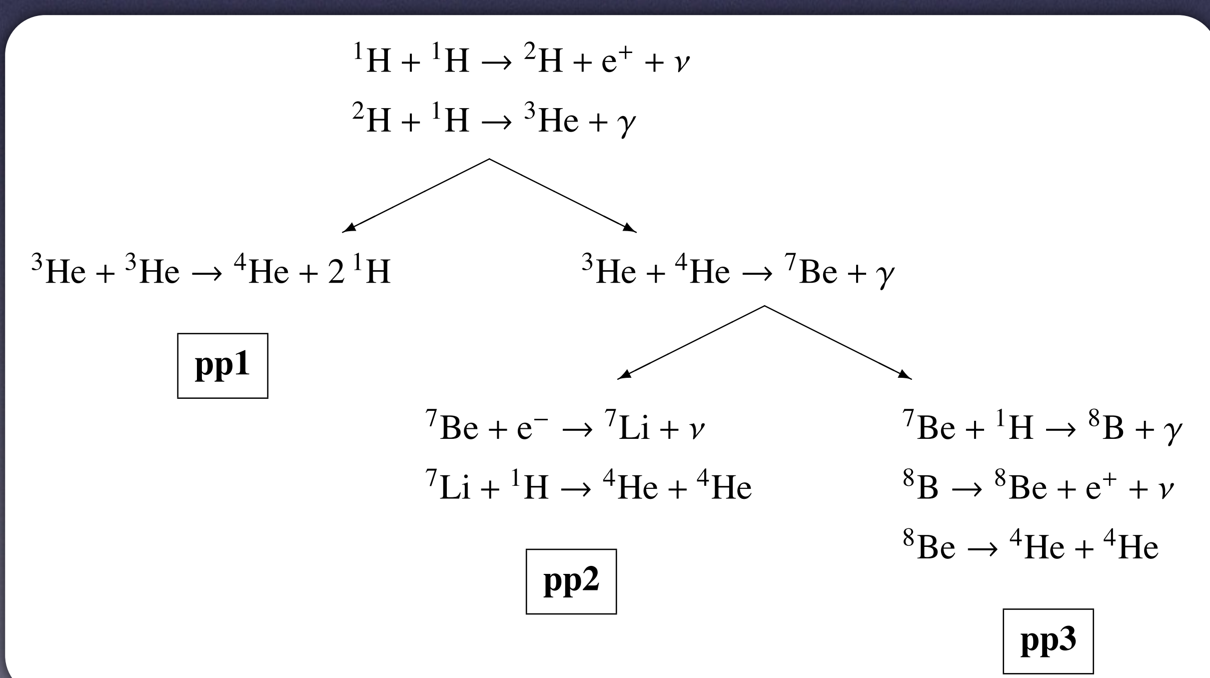
pp3

Textbook by Pols

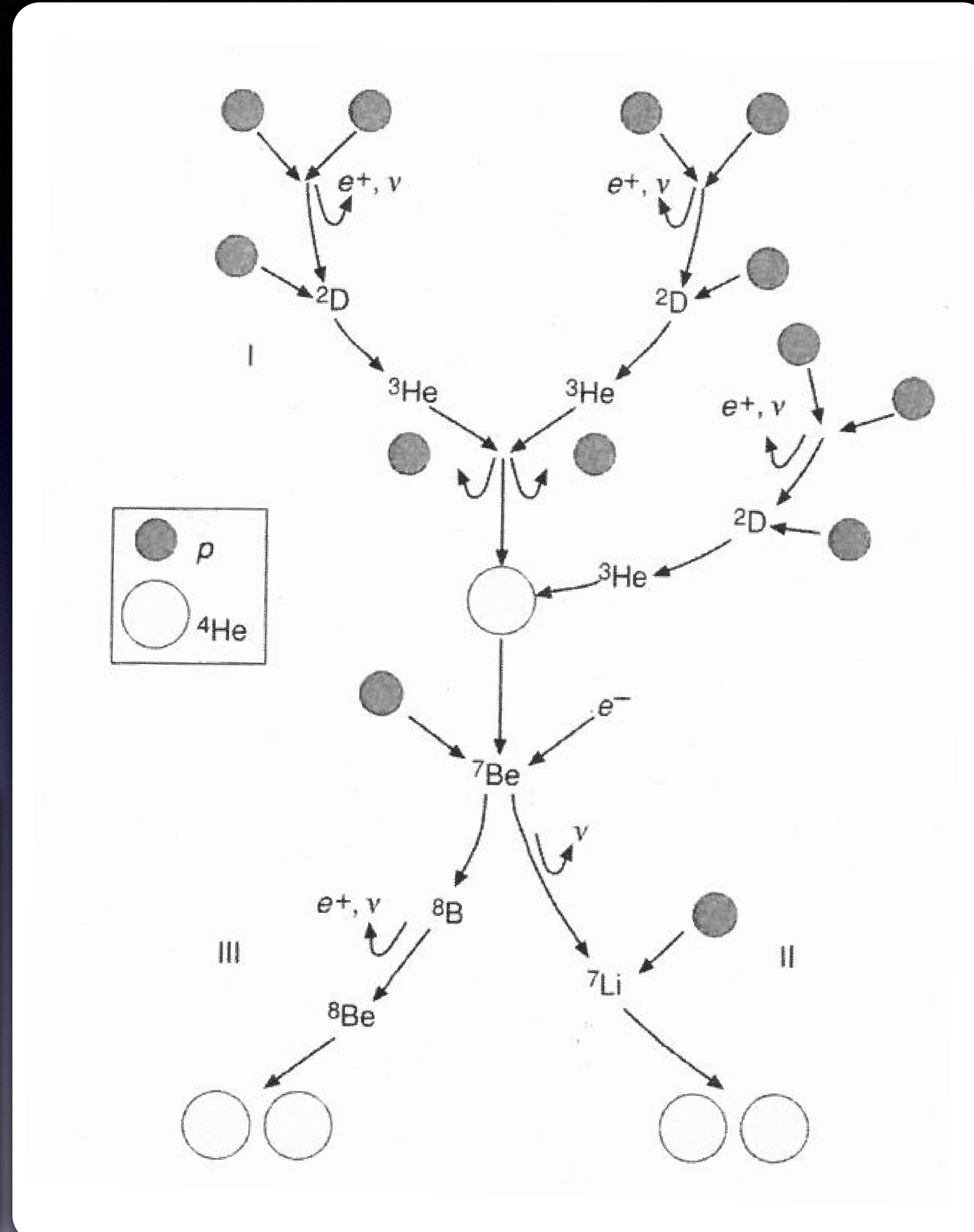
Energy production rate (per gram)

$q \sim \rho T^4$

$T \sim 4 \times 10^6$ K



Textbook by Pols



Textbook by Prialnik

1b. H burning (CNO cycle)

E production rate $q \sim \rho T^{16}$

$T \sim 1.5 \times 10^7$ K

$$^{12}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{13}\mathrm{N} + \gamma$$

$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + e^+ + \nu$$

$$^{13}\mathrm{C} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + \gamma$$

$$^{14}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{15}\mathrm{O} + \gamma$$

$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + e^+ + \nu$$

$$^{15}\mathrm{N} + {}^{1}\mathrm{H} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$

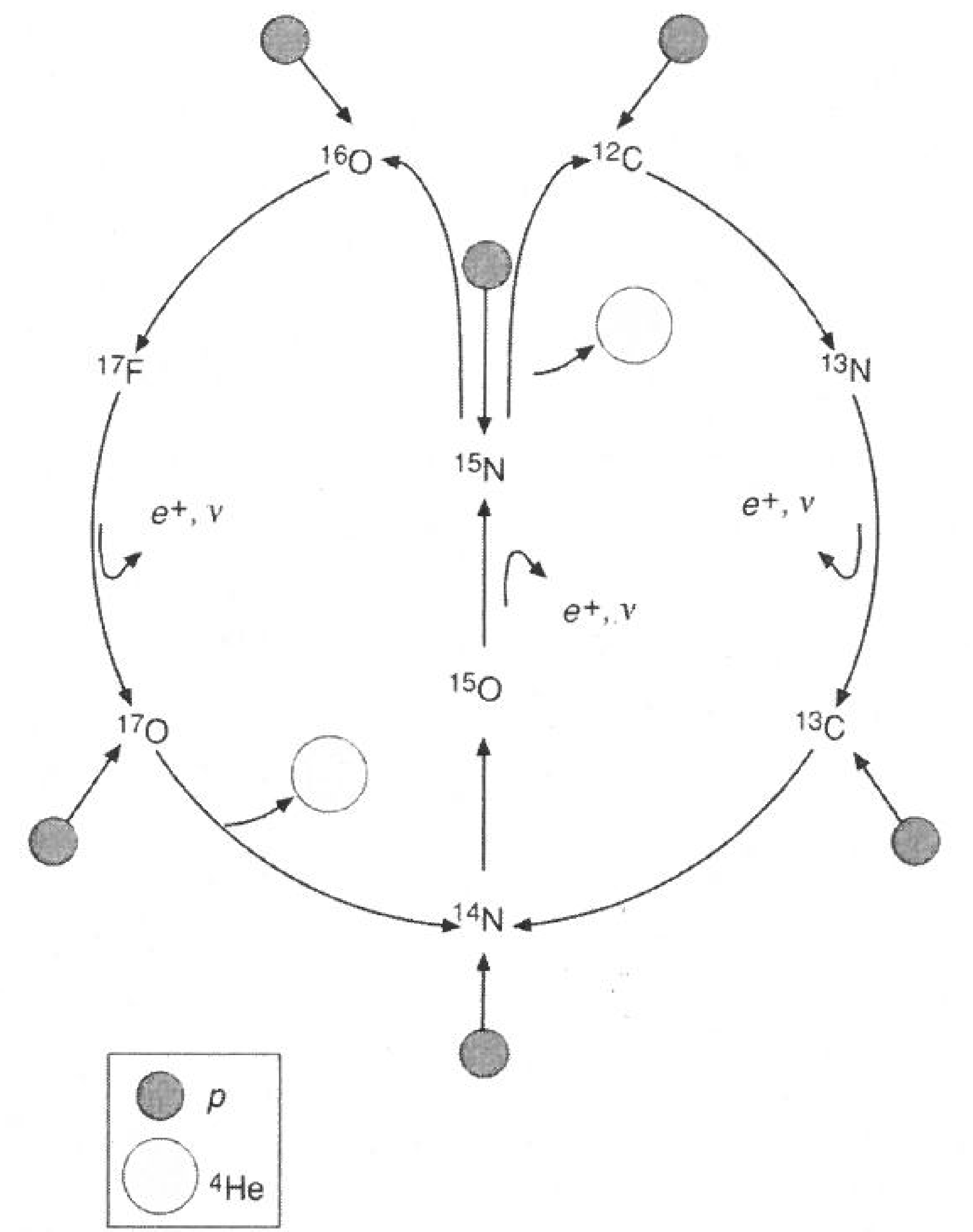
$$\rightarrow {}^{16}\mathrm{O} + \gamma$$

$$^{16}\mathrm{O} + {}^{1}\mathrm{H} \rightarrow {}^{17}\mathrm{F} + \gamma$$

$$^{17}\mathrm{F} \rightarrow {}^{17}\mathrm{O} + e^+ + \nu$$

$$^{17}\mathrm{O} + {}^{1}\mathrm{H} \rightarrow {}^{14}\mathrm{N} + {}^{4}\mathrm{He}$$

Textbook by Pols



Textbook by Prialnik