Section 4.
Stellar evolution (I)

4.1 Virial theorem

4.2 Evolution of density and temperature

4.3 Burning stages

Let's understand these questions with the words of physics

- Why are stars so luminous?
- Why do stars show L ~ M^4?
- Why do stars evolve?
- Why does the destiny of stars depend on the mass?
- Why do some stars explode?
- Why don't normal star explode?
- Why does stellar core collapses?
- Why is the energy of supernova so huge?
- …

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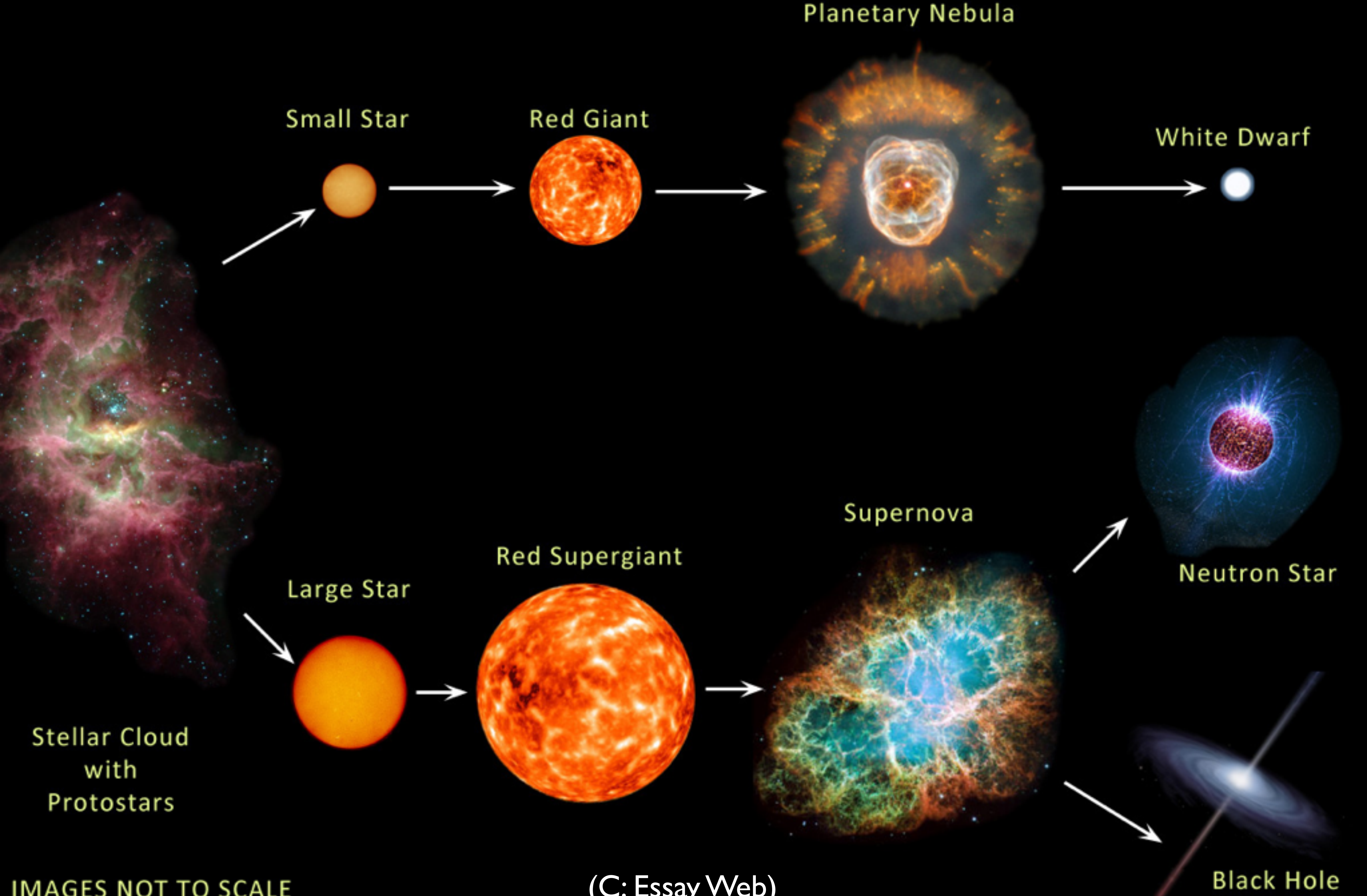
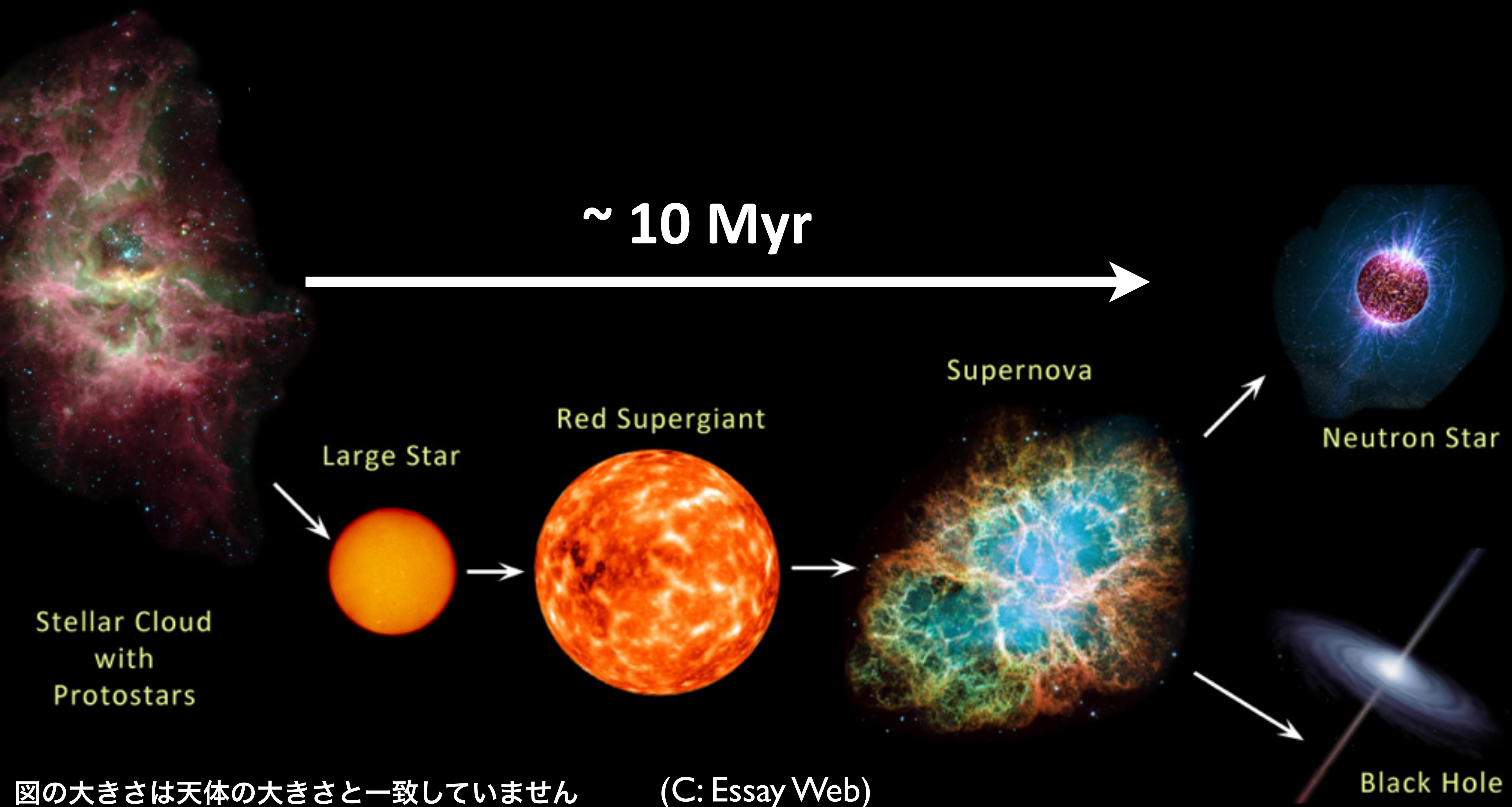
Stellar life



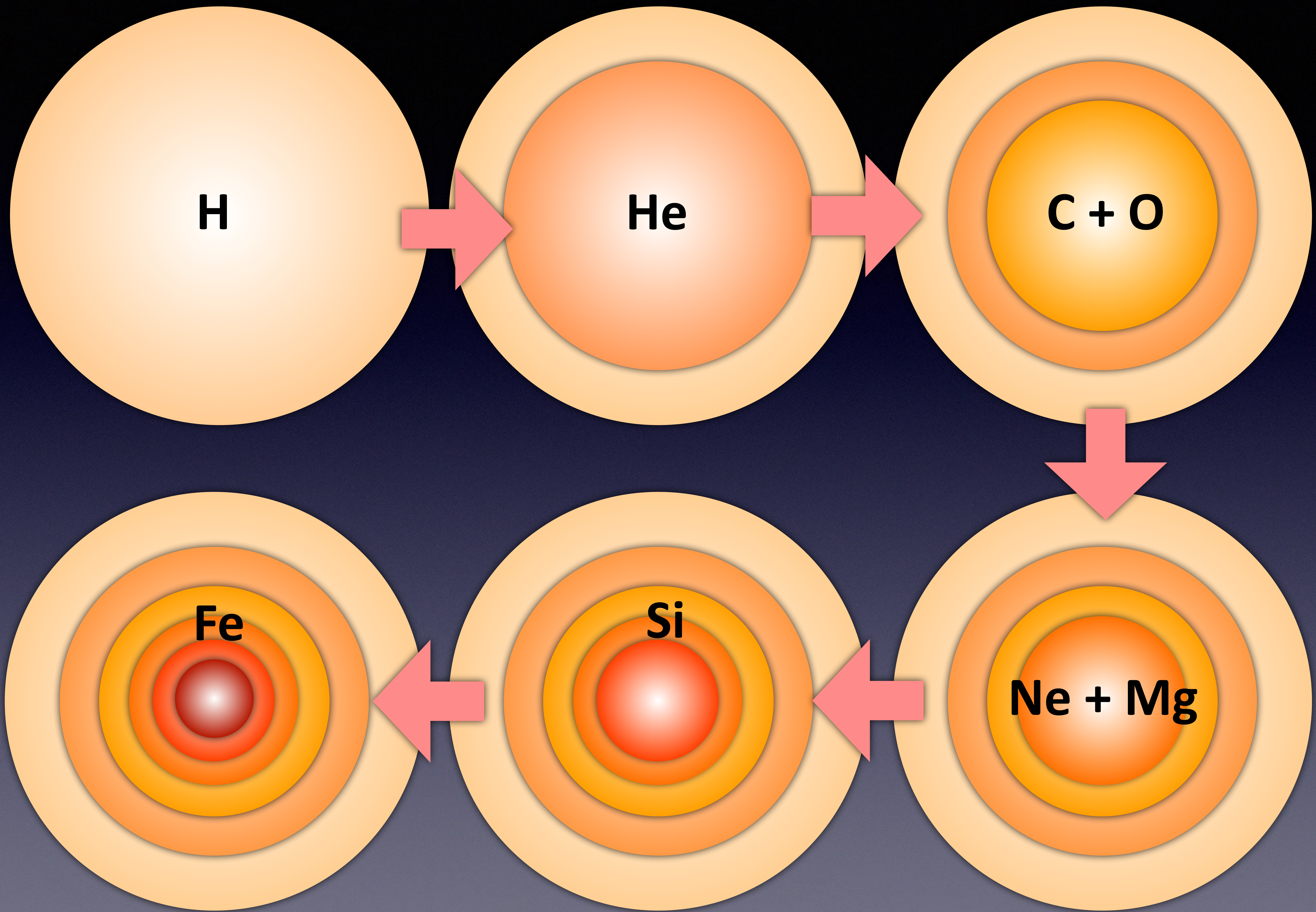
(C: Essay Web)

1. Massive stars

M > 10 Msun



図の大きさは天体の大きさと一致していません (C: Essay Web)



Images are not to scale

Why do stars evolve??

"Evolution" = Changes in the state with time

What happens when there is no more fuel for nuclear burning

E_{tot}: Total energy

Ω: Gravitational energy

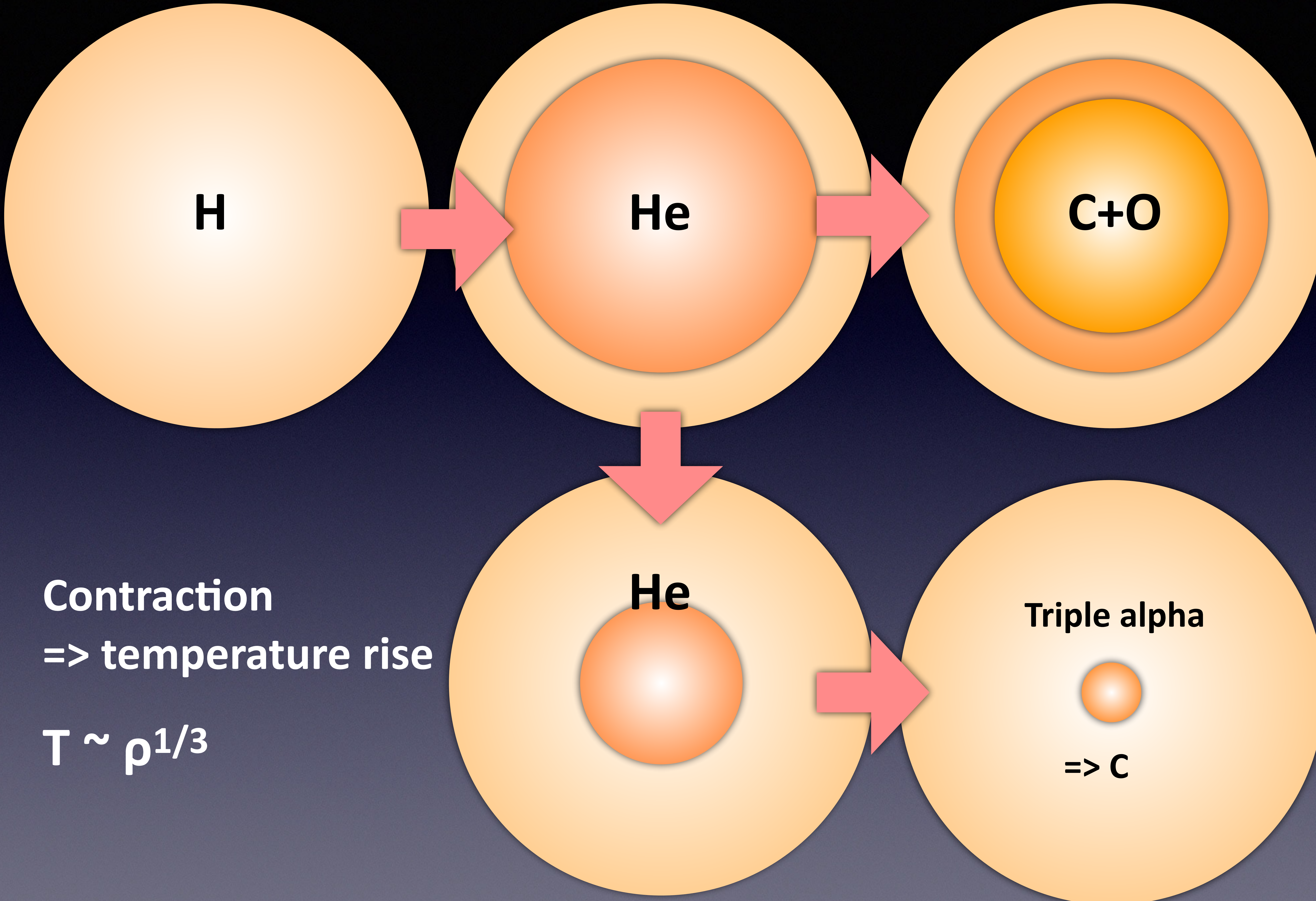
U: Internal energy

$$U = -\frac{1}{2}\Omega$$

$$E_{\rm tot} = U + \Omega = \frac{1}{2}\Omega = -U$$

No nuclear burning

- Total energy decreases
- Contraction (gravitational energy decreases)
- Temperature rises



Contraction
=> temperature rise

$T \sim \rho^{1/3}$

Images are not to scale

Heated iron | **stars**

↓ Gets colder

↓ Gets hotter

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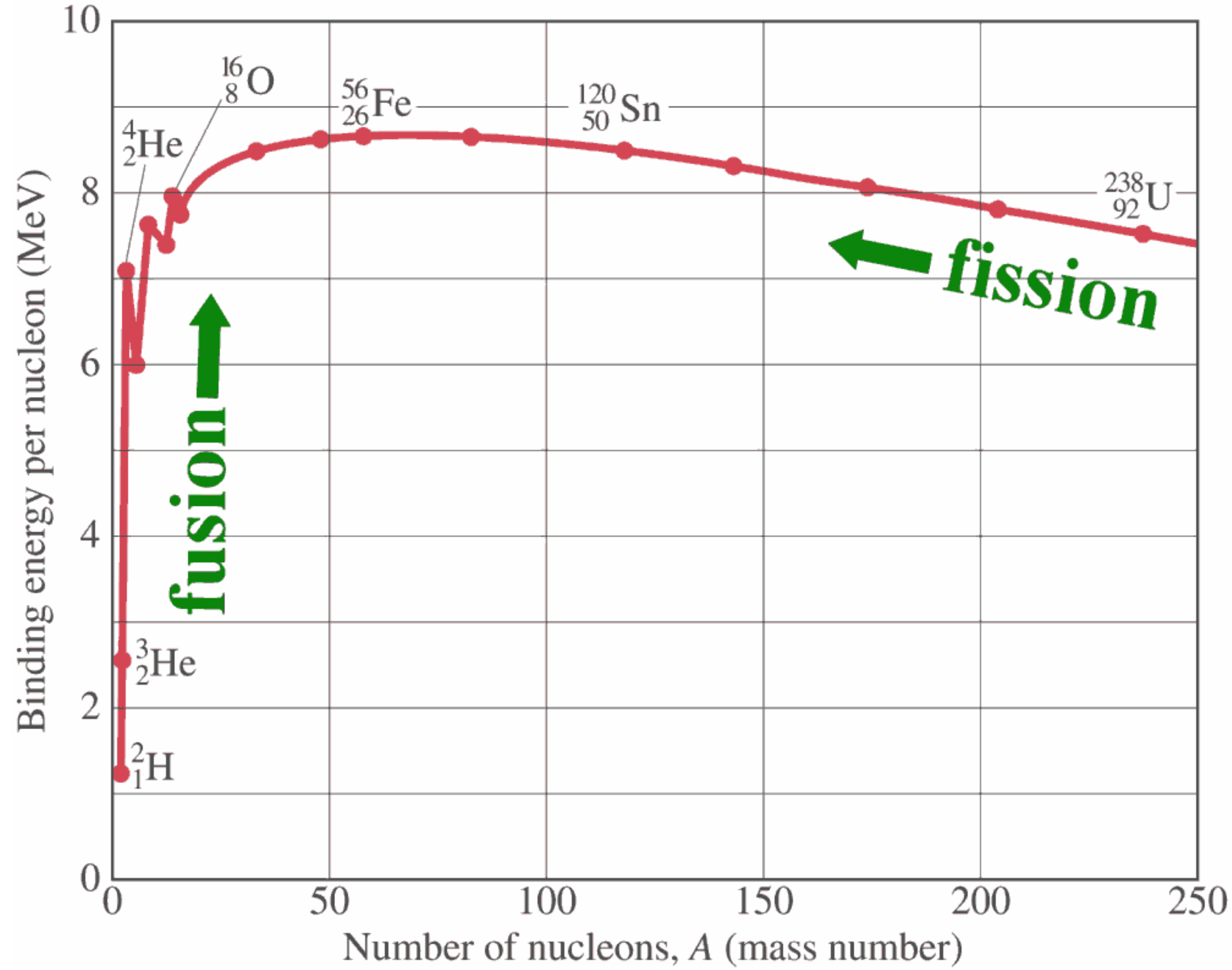
Nuclear binding energy

$$Eb = [Nm_N + Zm_p - m_i]\, c^2 > 0$$



Larger binding energy
= more stable

Fe has the largest
Eb/nucleon

Then, all the stars produce Fe? => No
Stellar material does not always behave as ideal gas

Phase 燃焼段階	Main reactions おもな反応	Products おもな生成物	T 温度（10^8 K）
H	pp チェイン CNO サイクル	^4He ^{14}N	0.15-0.2
He	$3\,^4He \longrightarrow {}^{12}C$ $^{12}C + {}^4He \longrightarrow {}^{16}O + \gamma$	^{12}C ^{16}O	1.5
C	$^{12}C + {}^{12}C \longrightarrow \begin{cases} ^{23}Na + p \\ ^{20}Ne + \alpha \end{cases}$	Ne,Na Mg,Al	7
Ne	$^{20}Ne + \gamma \longrightarrow {}^{16}O + \alpha$ $^{20}Ne + \alpha \longrightarrow {}^{24}Mg + \gamma$	O Mg	15
O	$^{16}O + {}^{16}O \longrightarrow \begin{cases} ^{28}Si + \alpha \\ ^{31}P + p \end{cases}$	Si,P,S, Cl,Ar,Ca	30
Si	$^{28}Si + \gamma \longrightarrow {}^{24}Mg + \alpha$ $^{24}Mg + \gamma \longrightarrow \begin{cases} ^{23}Na + p \\ ^{20}Ne + \alpha \end{cases}$ 多くの反応 ⟶ 統計平衡	Cr,Mn, Fe,Co, Ni,Cu	40

Nuclear statistical equilibrium

元素はいかにつくられたか（岩波書店）

ρ-T plane

密度 - 温度平面

Summary: Stellar evolution (I)

- Virial theorem (for ideal gas case)
  - Internal energy always relates with gravitational energy
  - When stars lose energy, they contract
  - Temperature rises ("negative heat capacity")
- Evolution of density and temperature
  - Rise in temperature due to contraction $T \sim \rho^{1/3}$
  - Next burning stages => Onion-like structure
  - Do all the stars produce Fe?? => No.
    Equation of states plays an important role

Appendix

1a. H-burning (pp chain)

$$4\,{}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2\,e^{+} + 2\,\nu$$

$${}^{1}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{2}\mathrm{H} + e^{+} + \nu$$

$${}^{2}\mathrm{H} + {}^{1}\mathrm{H} \rightarrow {}^{3}\mathrm{He} + \gamma$$

$${}^{3}\mathrm{He} + {}^{3}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + 2\,{}^{1}\mathrm{H}$$

pp1

$${}^{3}\mathrm{He} + {}^{4}\mathrm{He} \rightarrow {}^{7}\mathrm{Be} + \gamma$$

$${}^{7}\mathrm{Be} + e^{-} \rightarrow {}^{7}\mathrm{Li} + \nu$$

$${}^{7}\mathrm{Li} + {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

pp2

$${}^{7}\mathrm{Be} + {}^{1}\mathrm{H} \rightarrow {}^{8}\mathrm{B} + \gamma$$

$${}^{8}\mathrm{B} \rightarrow {}^{8}\mathrm{Be} + e^{+} + \nu$$

$${}^{8}\mathrm{Be} \rightarrow {}^{4}\mathrm{He} + {}^{4}\mathrm{He}$$

pp3

Textbook by Pols

Energy production rate (per gram)

$q \sim \rho T^4$

$T \sim 4 \times 10^6$ K

$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$

$^2H + {}^1H \rightarrow {}^3He + \gamma$

$^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$

pp1

$^3He + {}^4He \rightarrow {}^7Be + \gamma$

$^7Be + e^- \rightarrow {}^7Li + \nu$

$^7Li + {}^1H \rightarrow {}^4He + {}^4He$

pp2

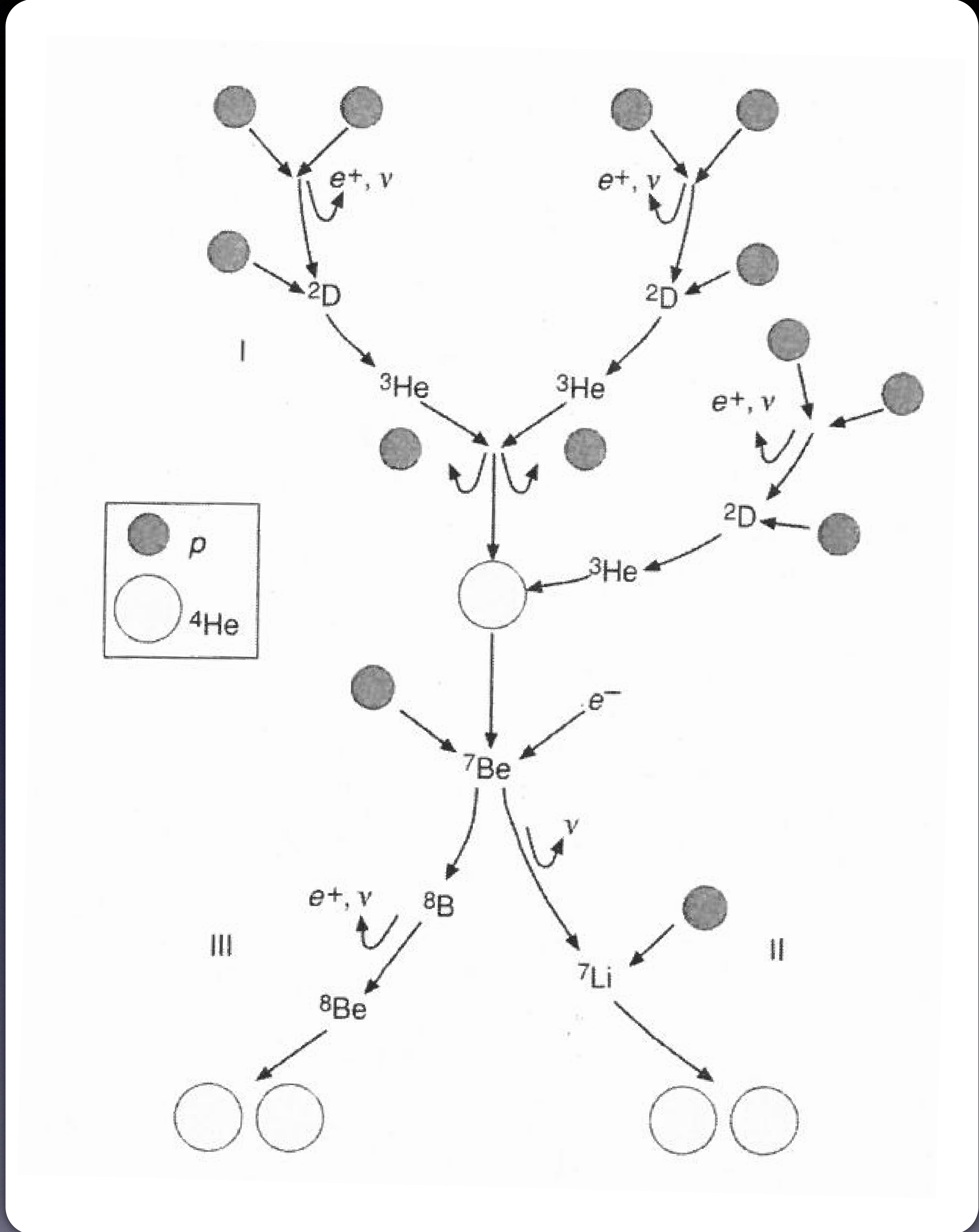
$^7Be + {}^1H \rightarrow {}^8B + \gamma$

$^8B \rightarrow {}^8Be + e^+ + \nu$

$^8Be \rightarrow {}^4He + {}^4He$

pp3

Textbook by Pols



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1b. H burning (CNO cycle)

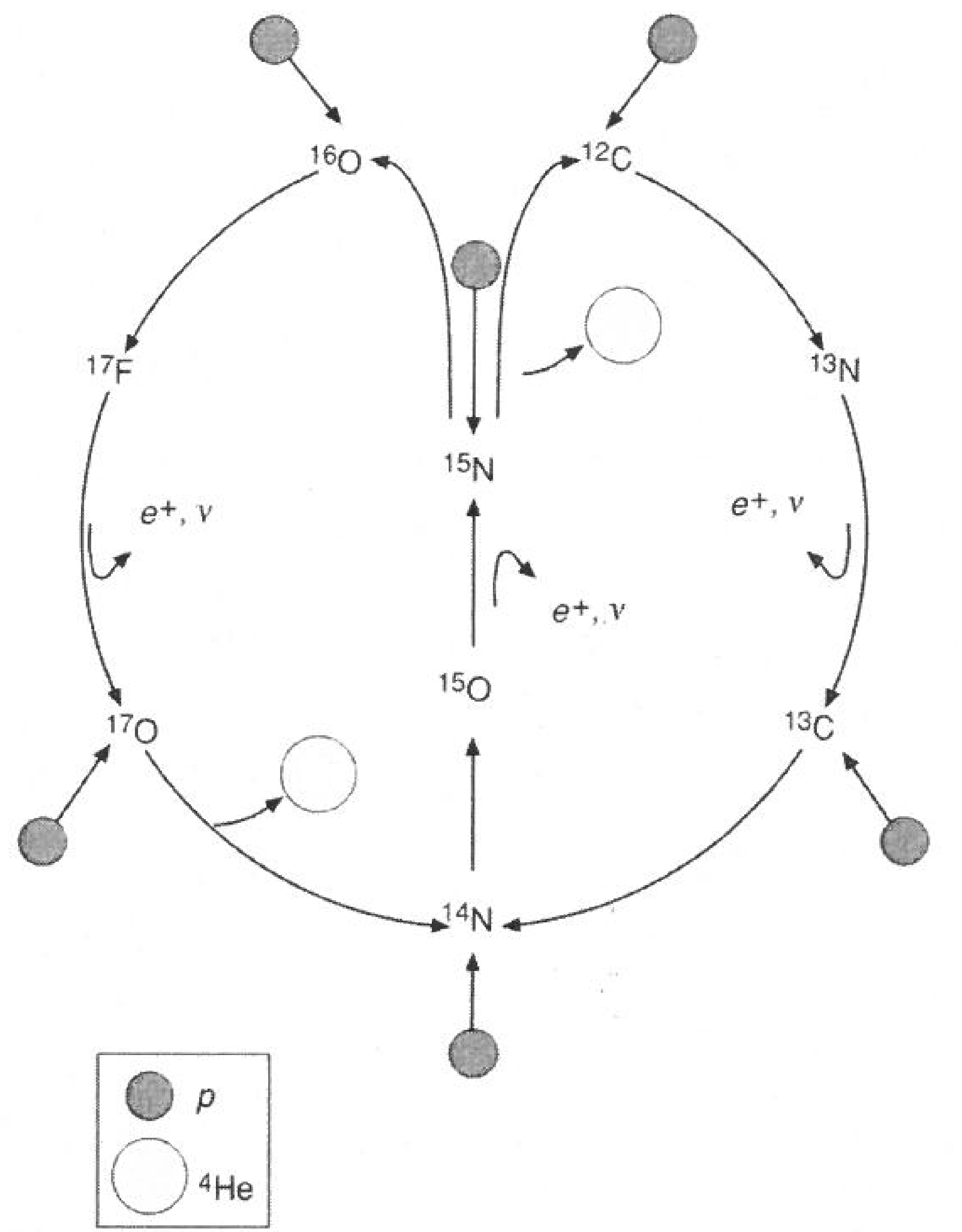
E production rate $q \sim \rho T^{16}$

$T \sim 1.5 \times 10^7$ K

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$
$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$$
$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$
$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$
$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$$
$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

$$\rightarrow {}^{16}O + \gamma$$
$$^{16}O + {}^{1}H \rightarrow {}^{17}F + \gamma$$
$$^{17}F \rightarrow {}^{17}O + e^{+} + \nu$$
$$^{17}O + {}^{1}H \rightarrow {}^{14}N + {}^{4}He$$

Textbook by Pols



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2. He-burning (triple alpha)

$$^4He + ^4He \leftrightarrow ^8Be$$

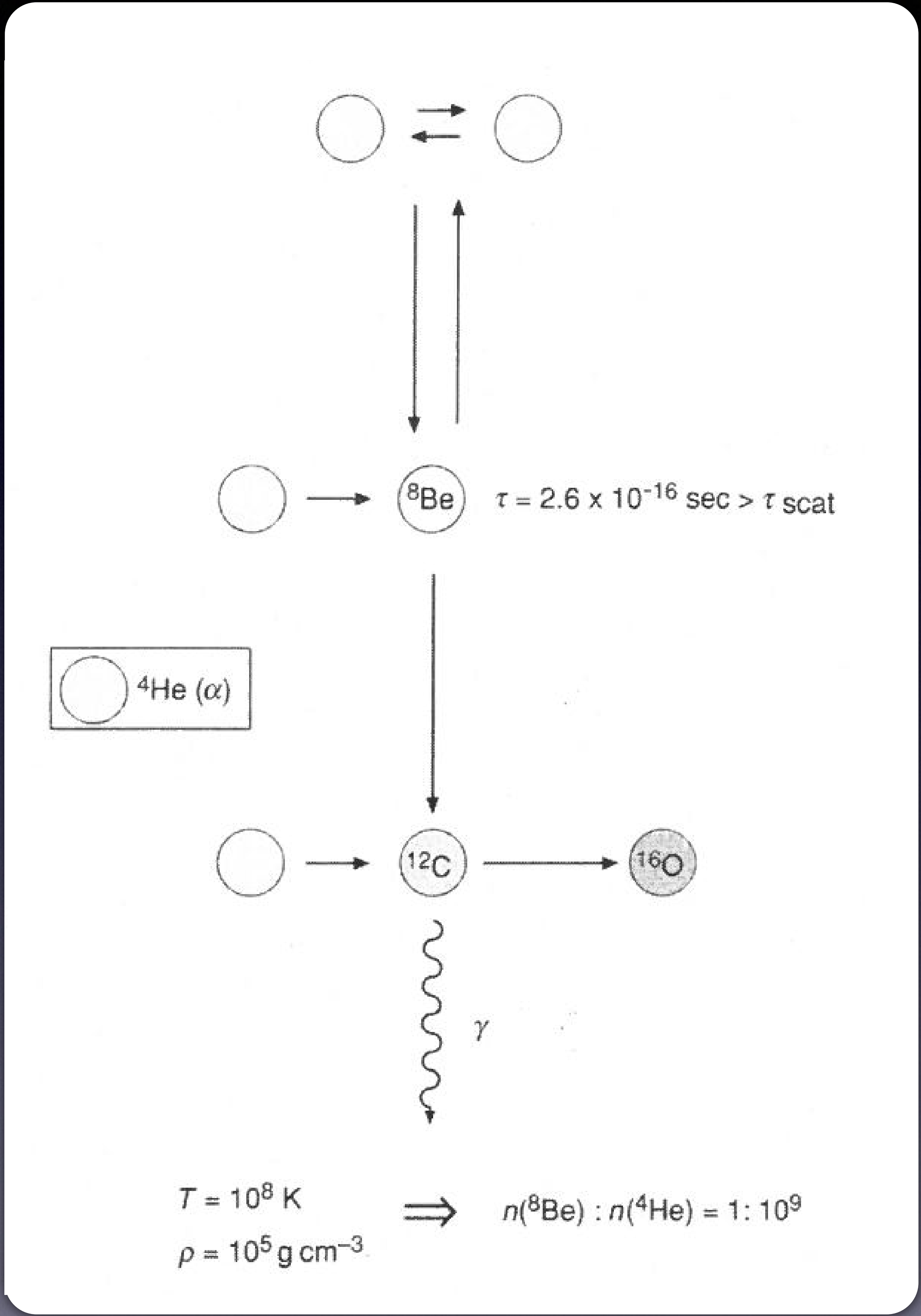
$$^8Be + ^4He \rightarrow ^{12}C^* \rightarrow ^{12}C + \gamma$$

$$^{12}C + ^4He \rightarrow ^{16}O + \gamma,$$

Energy production rate (per gram)

$q \sim \rho^2 T^{40}$

$T \sim 1.5 \times 10^8$ K



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3. C-burning

$$^{12}\mathrm{C} + {}^{12}\mathrm{C} \rightarrow {}^{24}\mathrm{Mg}^{*} \rightarrow {}^{20}\mathrm{Ne} + \alpha$$
$$\rightarrow {}^{23}\mathrm{Na} + \mathrm{p}$$

T ~ 7 x 10^8 K

4. Ne-burning

$$^{20}\mathrm{Ne} + \gamma \leftrightarrow {}^{16}\mathrm{O} + \alpha$$
$$^{20}\mathrm{Ne} + \alpha \rightarrow {}^{24}\mathrm{Mg} + \gamma$$

T ~ 1.5 x 10^9 K

5. O-burning

$$^{16}\mathrm{O} + {}^{16}\mathrm{O} \rightarrow {}^{32}\mathrm{S}^{*} \rightarrow {}^{28}\mathrm{Si} + \alpha$$
$$\rightarrow {}^{31}\mathrm{P} + \mathrm{p}$$

T ~ 2-3 x 10^9 K

6. Si-burning（Nuclear statistical equilibrium）

T > 4 x 10⁹ K

High temperature => photo-dissociation

$$^{28}\mathrm{Si}\,(\gamma,\alpha)\,^{24}\mathrm{Mg}\,(\gamma,\alpha)\,^{20}\mathrm{Ne}\,(\gamma,\alpha)\,^{16}\mathrm{O}\,(\gamma,\alpha)\,^{12}\mathrm{C}\,(\gamma,\alpha)\,2\alpha$$

He capture

$$^{28}\mathrm{Si}\,(\alpha,\gamma)\,^{32}\mathrm{S}\,(\alpha,\gamma)\,^{36}\mathrm{Ar}\,(\alpha,\gamma)\,^{40}\mathrm{Ca}\,(\alpha,\gamma)\,^{44}\mathrm{Ti}\,(\alpha,\gamma)\,\ldots\,^{56}\mathrm{Ni}$$

=> equilibrium of many reactions

(Ex.)

$$^{28}\mathrm{Si} + \gamma \leftrightarrow\ ^{24}\mathrm{Mg} + \alpha,$$

Nuclei with high binding energy tend to be produced (Fe, Co, Ni)