

# A multiscale study of turbulent heating in hot accretion flows

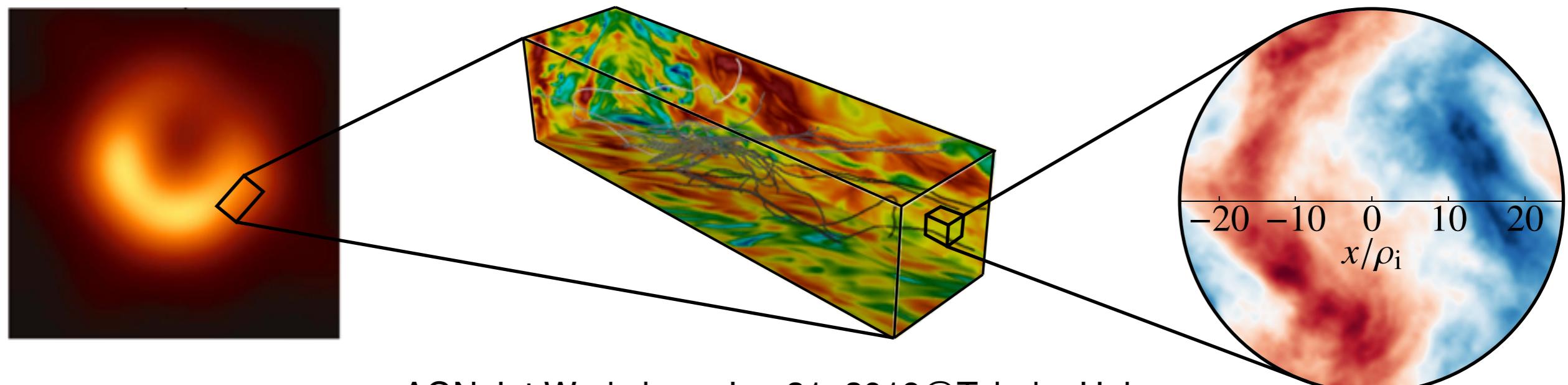
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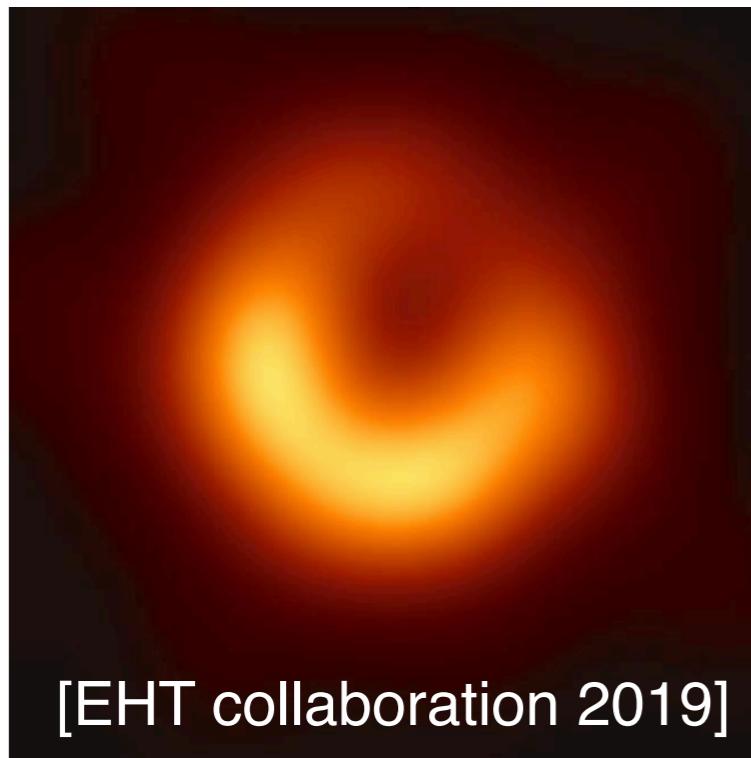
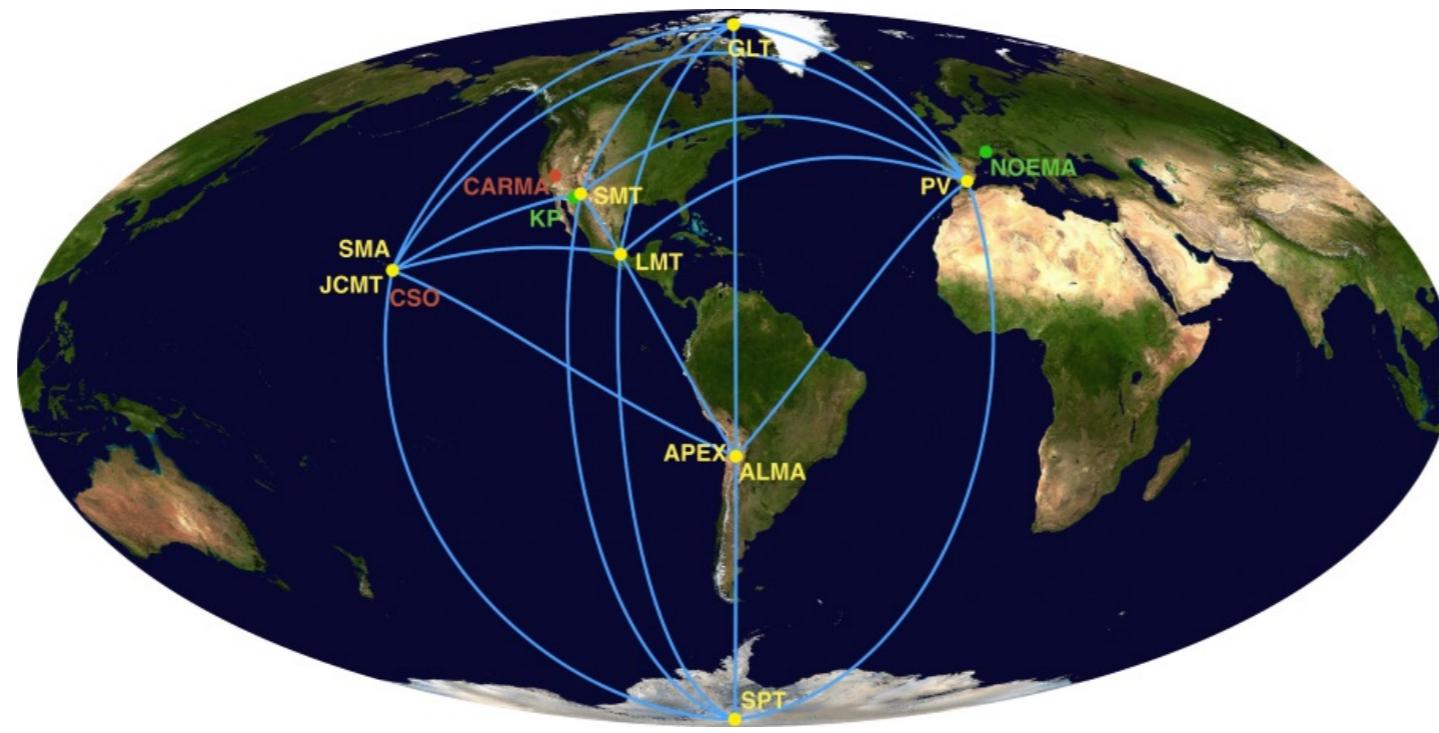
J. M. TenBarge (Princeton), Kris Klein (Arizona), Bill Dorland (Maryland)



# Plan of the talk

- ▶ Background and a research question  
*What sets the ion-to-electron heating ratio in hot accretion disks?*
- ▶ Possible mechanisms for ion and electron heating study (review)
- ▶ Gyrokinetic approach for turbulent heating
- ▶ Connection between gyrokinetics and magnetohydrodynamics
- ▶ Summary

# Event Horizon Telescope (EHT) took the “picture” of the accretion flows onto M87



How can we interpret this ?



How do we get the info of the BH ?

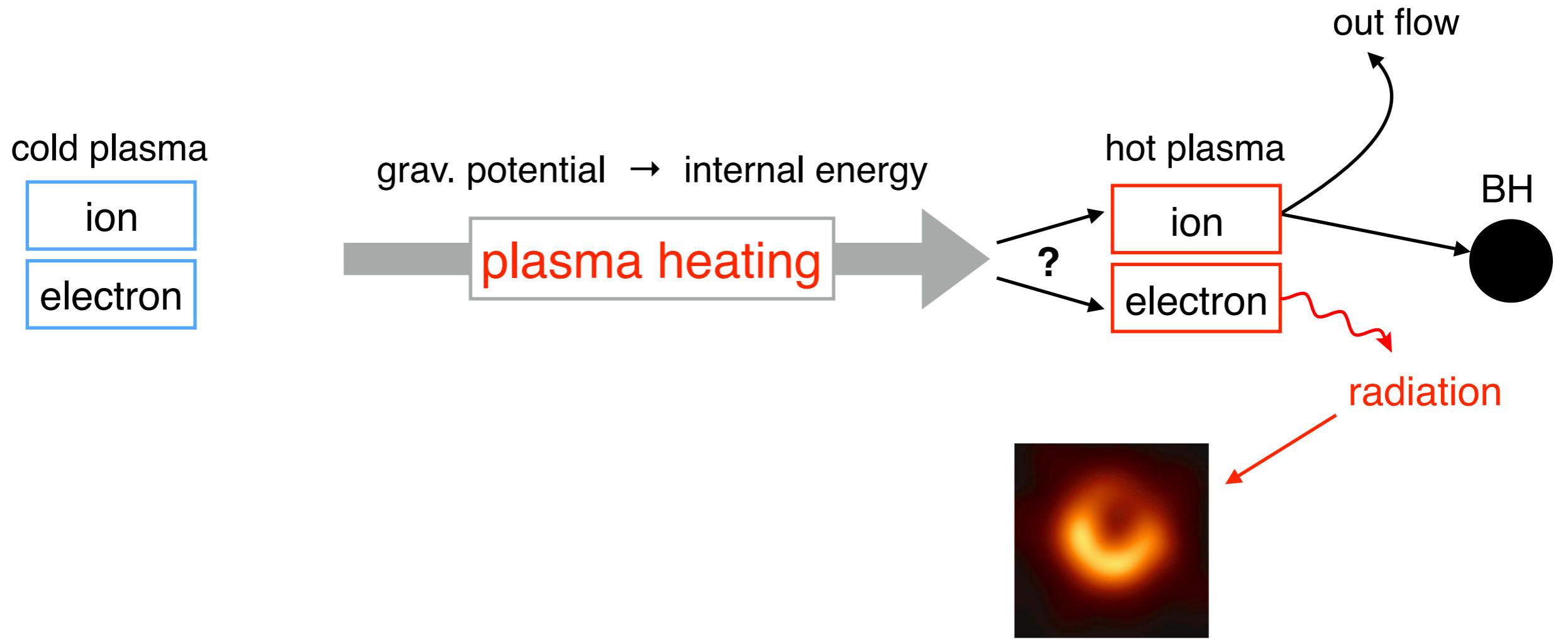
What kind of plasma is this ?



# Radiation mechanisms in RIAF

M87 is a radiationally inefficient accretion flow (RIAF)  
= collisionless plasma

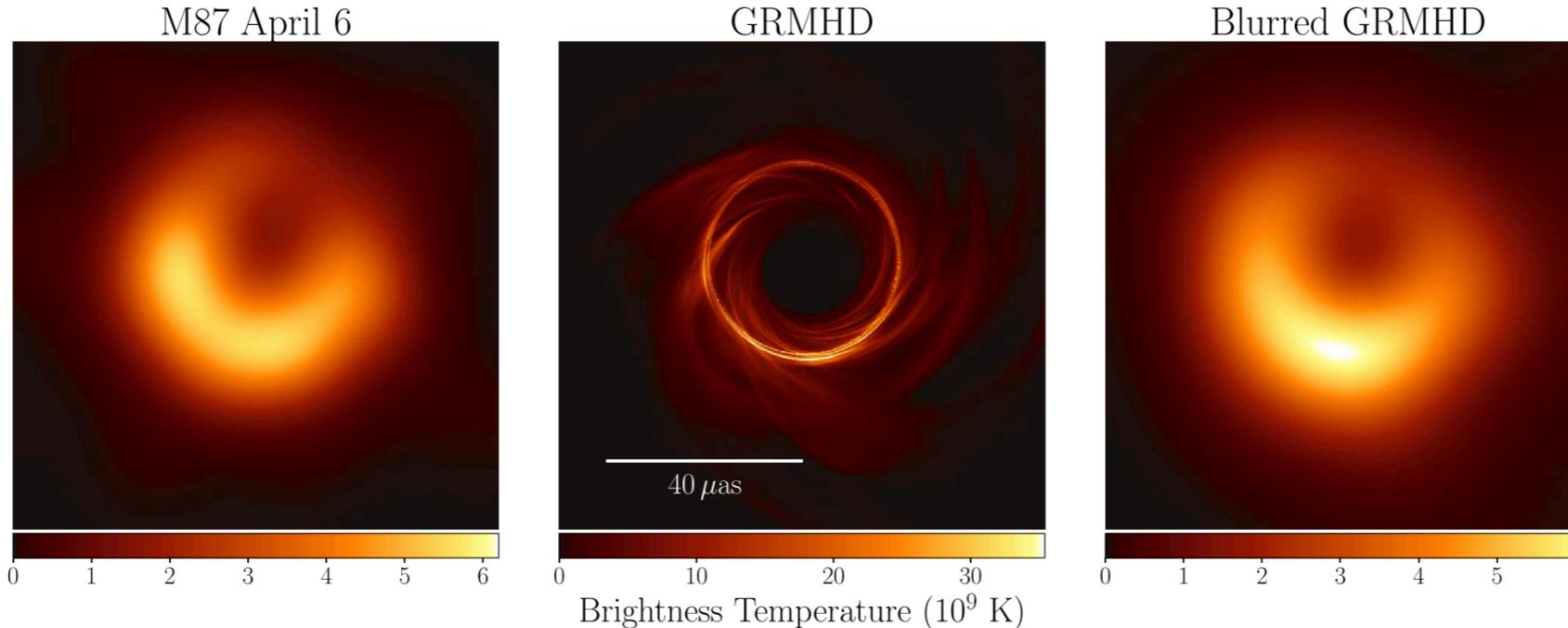
How does it radiate?



**Collisionless plasma → Ion heating ≠ electron heating**

To know the radiation, ion vs electron heating  $Q_i/Q_e$  is necessary.

# GRMHD simulation “reproduced” the observation



A prescription/assumption for is  $T_i/T_e$  important.

$$\frac{T_i}{T_e} = R_{\text{high}} \frac{\beta_p^2}{1 + \beta_p^2} + \frac{1}{1 + \beta_p^2}$$

If we get  $T_i/T_e$  physically,  
we can limit the possible  
scenarios!

Flux <sup>a</sup>	$a_*$ <sup>b</sup>	$R_{\text{high}}$ <sup>c</sup>	AIS <sup>d</sup>	$\epsilon$ <sup>e</sup>	$L_X$ <sup>f</sup>	$P_{\text{jet}}$ <sup>g</sup>	
SANE	-0.94	1	Fail	Pass	Pass	Pass	Fail
SANE	-0.94	10	Pass	Pass	Pass	Pass	Pass
SANE	-0.94	20	Pass	Pass	Pass	Pass	Pass
SANE	-0.94	40	Pass	Pass	Pass	Pass	Pass
SANE	-0.94	80	Pass	Pass	Pass	Pass	Pass
SANE	-0.94	160	Fail	Pass	Pass	Pass	Fail
SANE	-0.5	1	Pass	Pass	Fail	Fail	Fail
SANE	-0.5	10	Pass	Pass	Fail	Fail	Fail
SANE	-0.5	20	Pass	Pass	Pass	Fail	Fail
SANE	-0.5	40	Pass	Pass	Pass	Fail	Fail
SANE	-0.5	80	Fail	Pass	Pass	Fail	Fail
SANE	-0.5	160	Pass	Pass	Pass	Fail	Fail
SANE	+0.94	1	Pass	Fail	Pass	Fail	Fail
SANE	+0.94	10	Pass	Fail	Pass	Fail	Fail
SANE	+0.94	20	Pass	Pass	Pass	Fail	Fail
SANE	+0.94	40	Pass	Pass	Pass	Fail	Fail
SANE	+0.94	80	Pass	Pass	Pass	Pass	Pass
SANE	+0.94	160	Pass	Pass	Pass	Pass	Pass

# **Objective**

## **Understand heating in collisionless plasma**

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# Pressure anisotropy induced viscous heating

[Sharma+ 2007]

Kinetic MHD + Landau closure [Snyder+ 1997]  
Shearing box MRI

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B})$$

Valid at  $\ell_{\text{mfp}} \gtrsim \ell \gg \rho_i$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho (\mathbf{V} \cdot \nabla) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F}_g$$

$$\mathbf{P} = p_{\perp} \mathbf{I} + (p_{\parallel} - p_{\perp}) \hat{\mathbf{b}} \hat{\mathbf{b}} = p_{\perp} \mathbf{I} + \mathbf{\Pi}$$

$$\frac{\partial p_{\parallel,s}}{\partial t} + \nabla \cdot (p_{\parallel,s} \mathbf{V}) + \nabla \cdot \mathbf{q}_{\parallel,s} + 2p_{\parallel,s} \hat{\mathbf{b}} \hat{\mathbf{b}} : \nabla \mathbf{V} -$$

$$2q_{\perp,s} \nabla \cdot \hat{\mathbf{b}} = -\frac{2}{3} \nu_{\text{eff},s} (p_{\parallel,s} - p_{\perp,s})$$

$$\frac{\partial p_{\perp,s}}{\partial t} + \nabla \cdot (p_{\perp,s} \mathbf{V}) + \nabla \cdot \mathbf{q}_{\perp,s} + p_{\perp,s} \nabla \cdot \mathbf{V} - p_{\perp,s} \hat{\mathbf{b}} \hat{\mathbf{b}} : \nabla \mathbf{V}$$

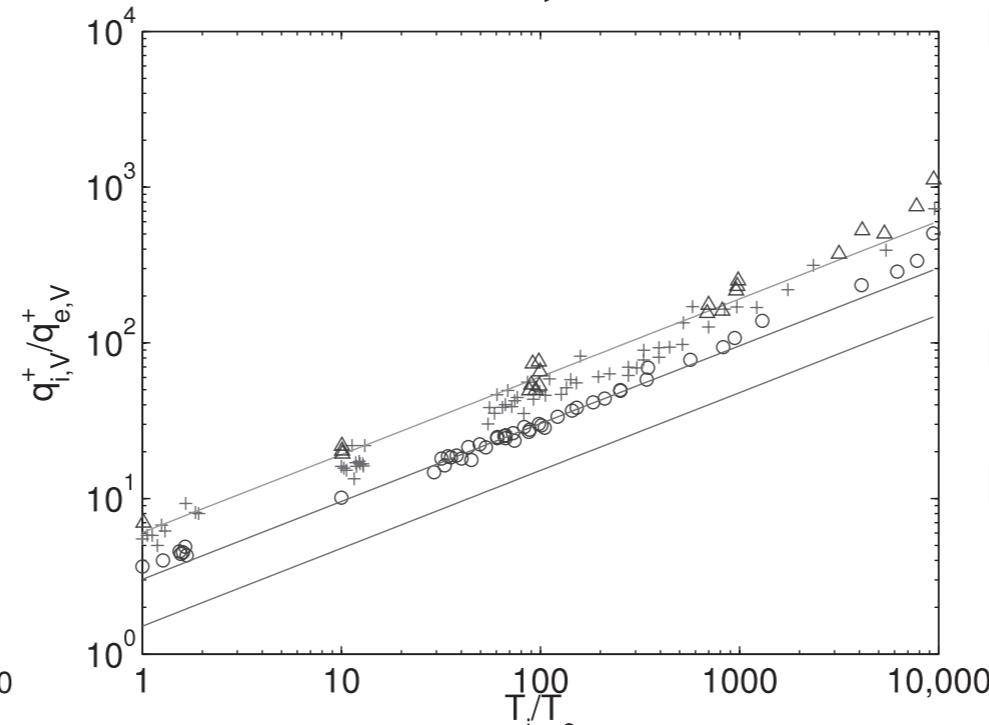
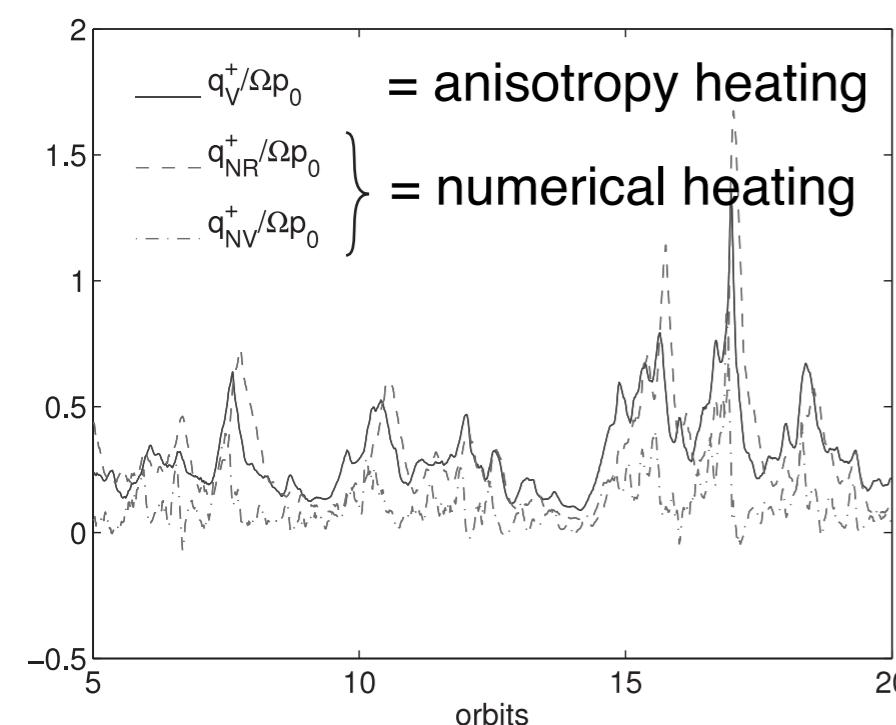
$$+ q_{\perp,s} \nabla \cdot \hat{\mathbf{b}} = -\frac{1}{3} \nu_{\text{eff},s} (p_{\perp,s} - p_{\parallel,s})$$

$$\left. \begin{aligned} \frac{\partial e_s}{\partial t} + \nabla \cdot (e_s \mathbf{V} + \mathbf{q}_s) + p_{\perp,s} \nabla \cdot \mathbf{V} &= -\Delta p_s \hat{\mathbf{b}} \hat{\mathbf{b}} : \nabla \mathbf{V} \\ \Delta p_s &= p_{\parallel,s} - p_{\perp,s} \end{aligned} \right\}$$

Heating  
↑

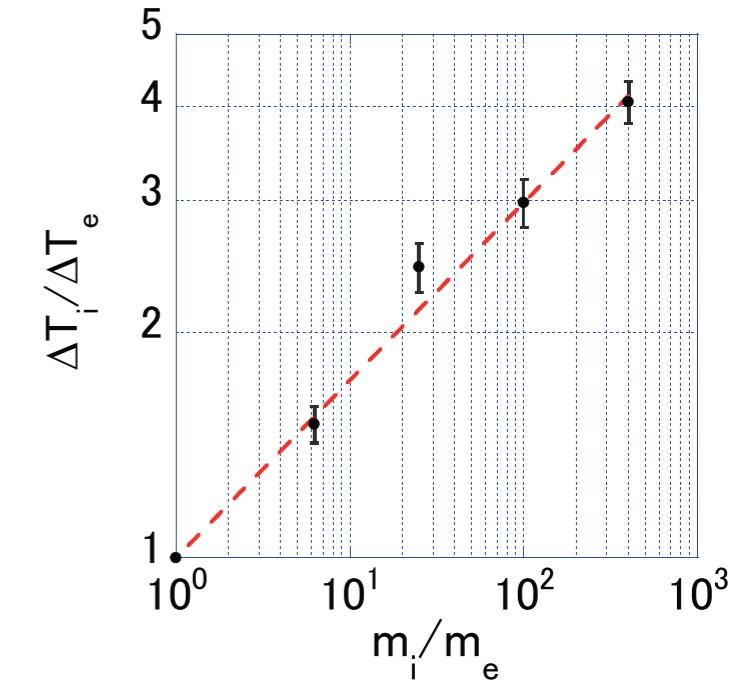
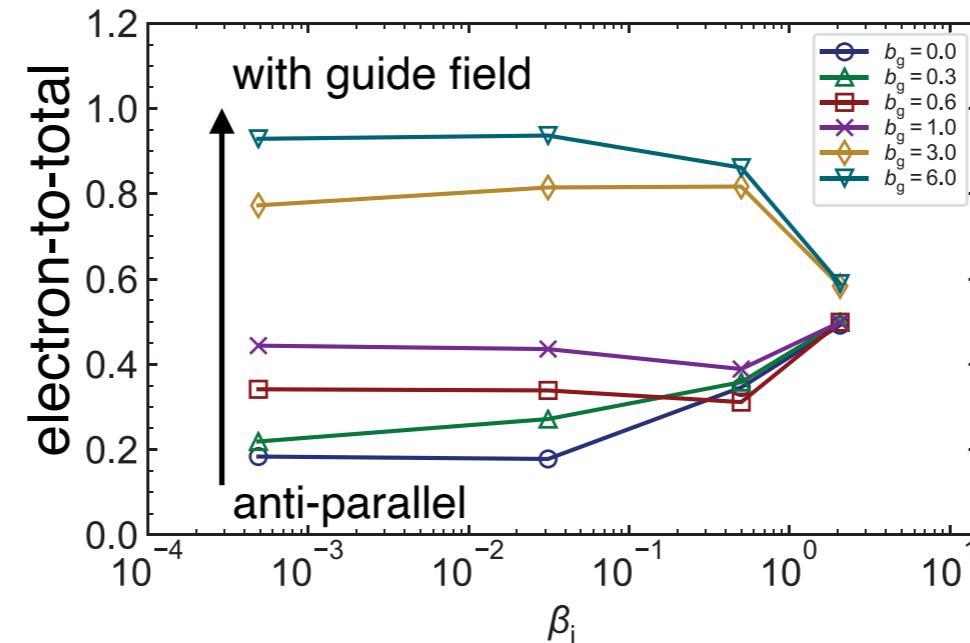
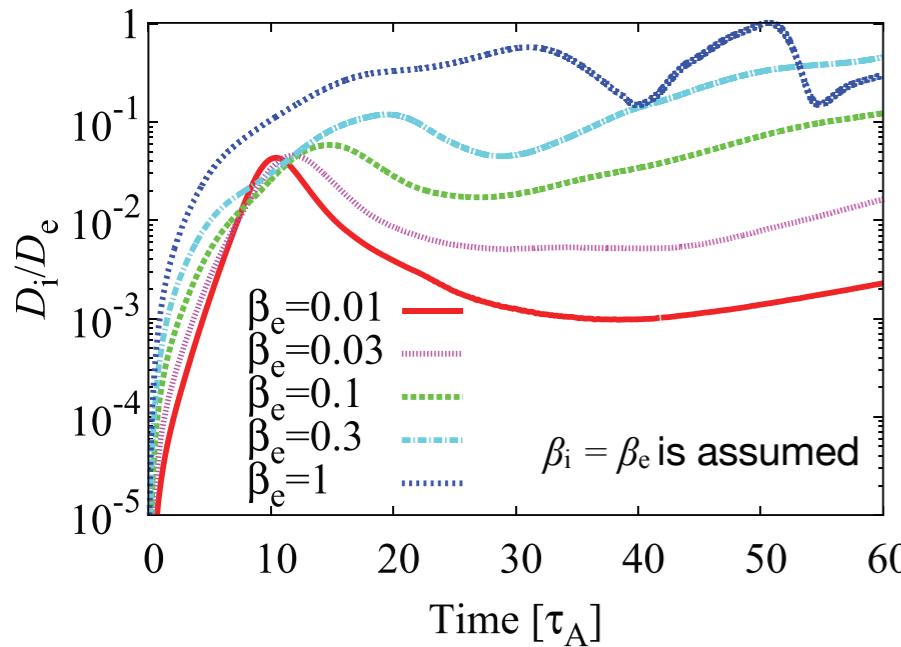
- ~50% of MRI power dissipated via anisotropy heating.

- $Q_i/Q_e \propto (T_i/T_e)^{1/2}$
- Remaining 50% should be dissipated at  $\sim \rho_i$  and smaller.



# Reconnection

[Numata & Loureiro 2015; Rowan+ 2017, 2018; Hoshino 2018]



Numata & Loureiro 2015

- Used gyrokinetics = strong guide field
- $Q_i/Q_e$  increases as  $\beta$  increases (up to  $\beta = 1$ ).

Rowan+ 2018

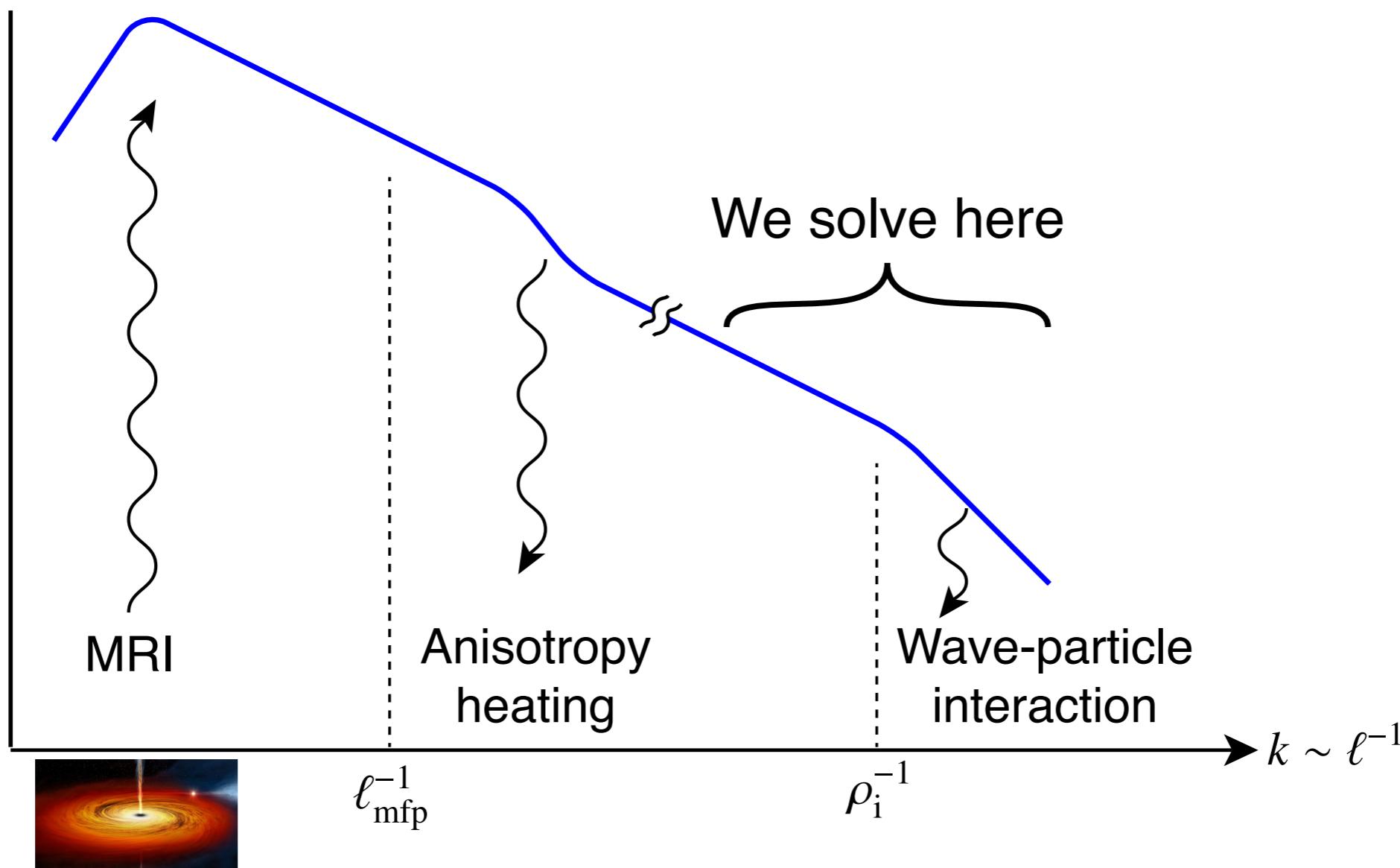
- Used trans-relativistic PIC
- $Q_i/Q_e$  decreases as  $\beta$  increases for an anti-parallel reconnection.
- $Q_i/Q_e$  increases as  $\beta$  increases for a guide-field reconnection.

Hoshino 2018

- Used PIC for anti-parallel reconnection
- $Q_i/Q_e \propto (m_i/m_e)^{1/4}$

# Kinetic turbulence

- ▶ This is what we are interested in.
  - Turbulence is necessary for accretion.
  - Turbulent fluctuations ultimately turn into heat.
  - Reconnection and etc. are included.



# Plan of the talk

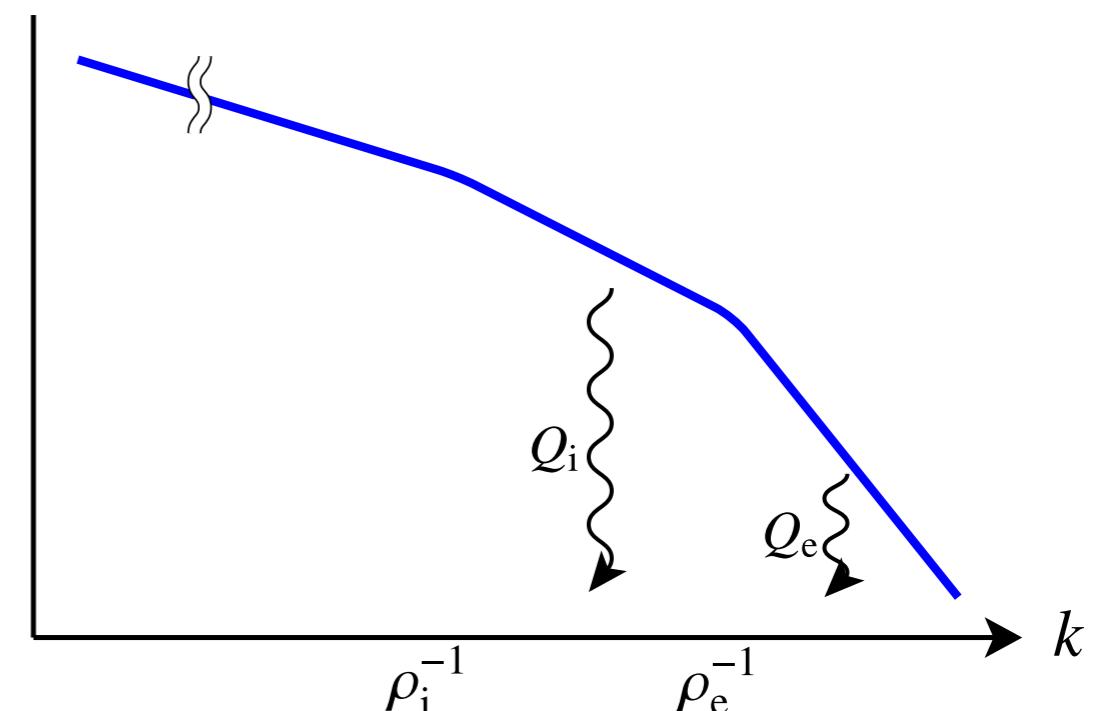
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# Kinetic turbulence is challenging

- ▶ There are some analytical calculations  
[e.g., Quataert & Gruzinov 1999; Howes 2010].
- ▶ Direct numerical simulations are promising.
- ▶ Challenging because
  - One needs to solve 6D phase space.
  - “in principle,” we must resolve  
inertial range  $\sim$  ion gyroscale  $\sim$  electron gyroscale.

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s}{m_s} \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left( \frac{\partial f_s}{\partial t} \right)_c$$

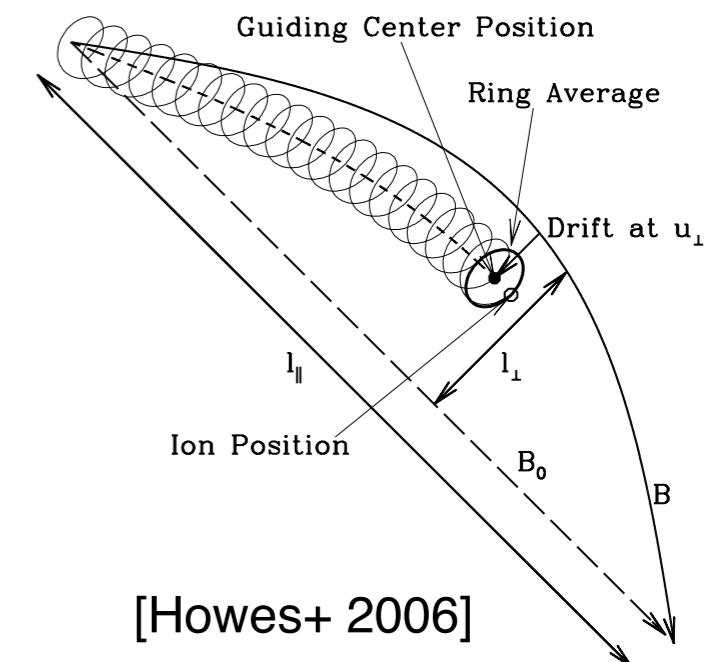
$f_s = f_s(\mathbf{r}, \mathbf{v})$  : 6D phase space



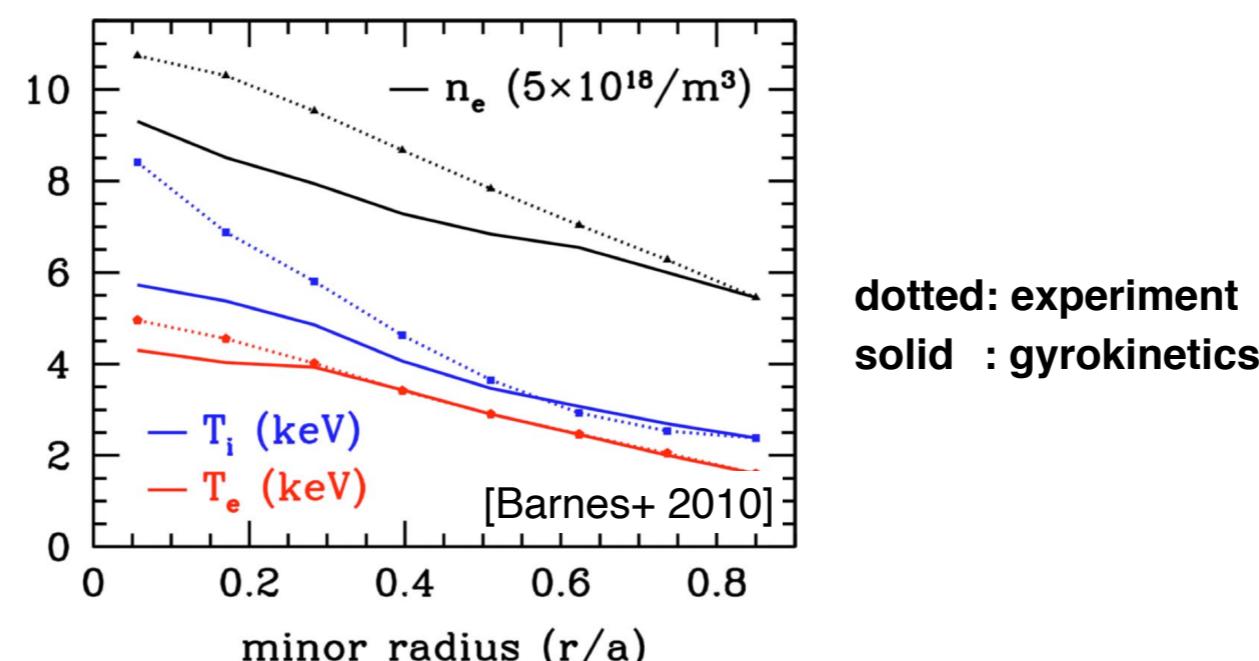
# Gyrokinetics

[Rutherford & Frieman 1968; Catto 1982; Howes+ 2006; Brizard & Halm 2007]

- ▶ Average gyro-motions
- ▶ Solve distribution func of gyro-centers
- ▶  $(r, v) \rightarrow (R, v_\perp, v_\parallel)$  : 5D phase space
- ▶ Popular in magnetic confinement fusion



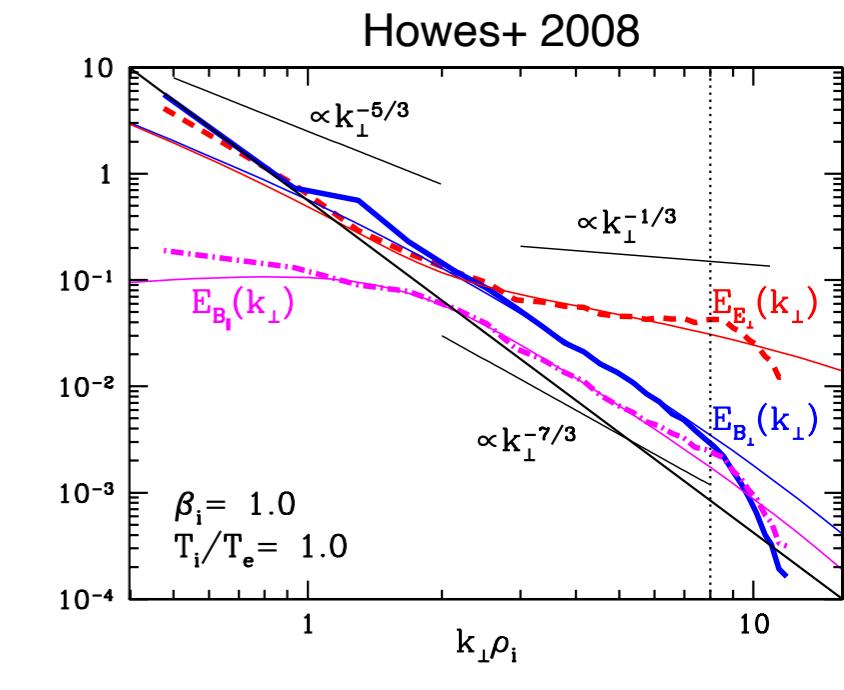
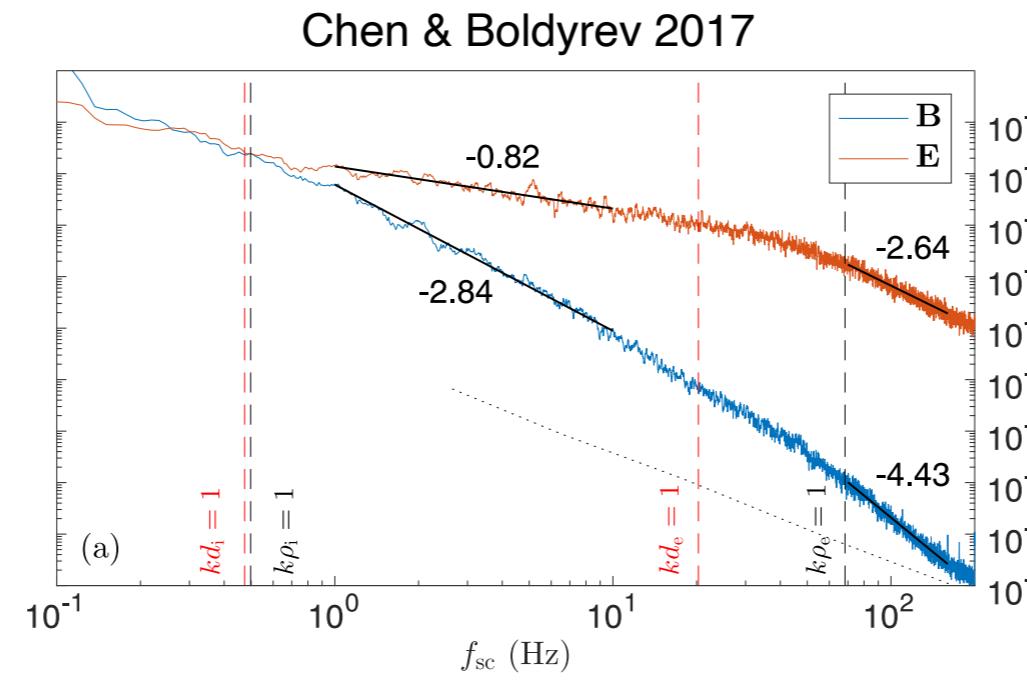
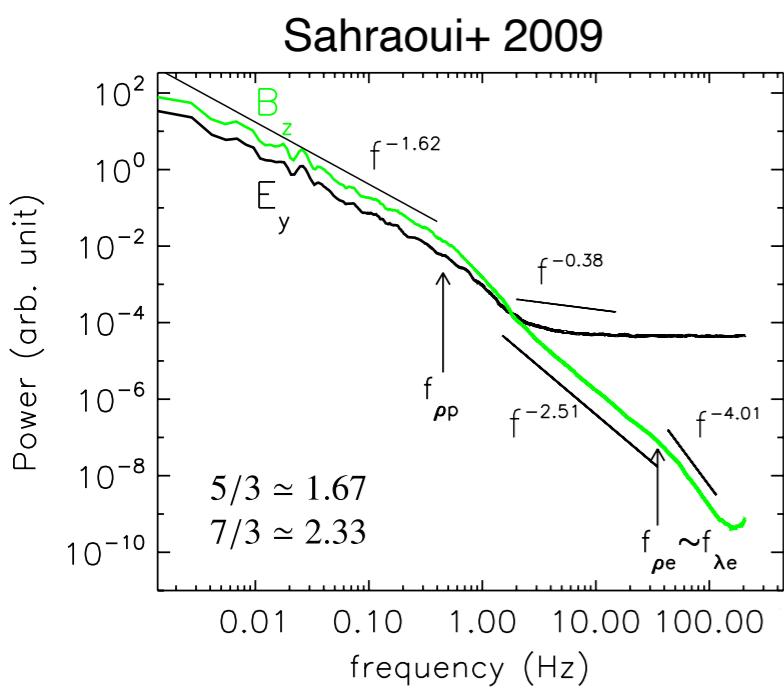
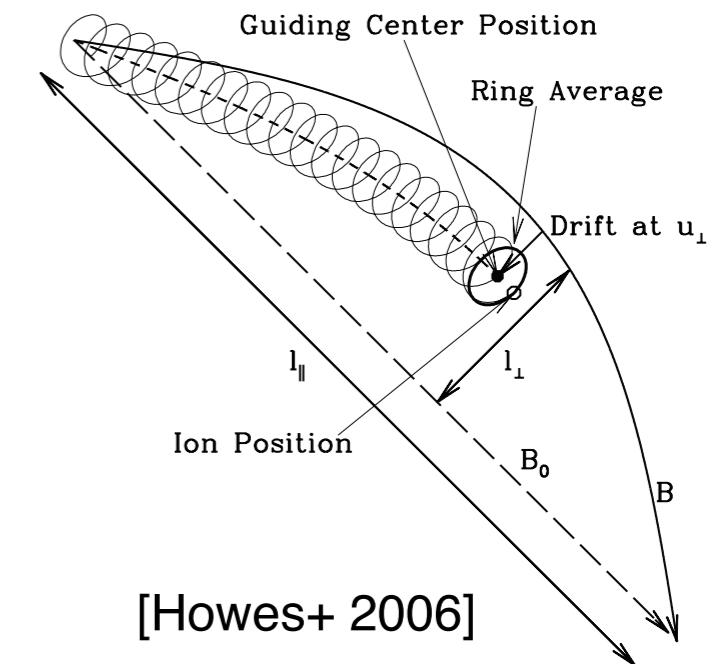
[Howes+ 2006]



# Gyrokinetics

[Rutherford & Frieman 1968; Catto 1982; Howes+ 2006; Brizard & Halm 2007]

- ▶ Average gyro-motions
- ▶ Solve distribution func of gyro-centers
- ▶  $(r, v) \rightarrow (R, v_\perp, v_\parallel)$  : 5D phase space
- ▶ Popular in magnetic confinement fusion
- ▶ Has been used in space plasma
- ▶ **Never been used for accretion disks.**



**Solar wind (Cluster)**

**Earth's magnetosphere (MMS)**

**GK simulation**

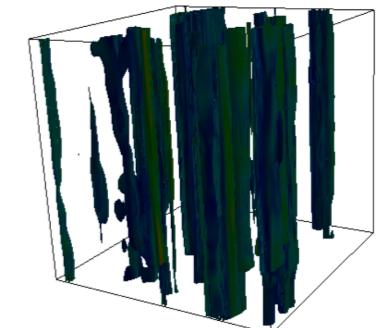
# Gyrokinetic ordering

1. Cyclotron motion is much faster than turbulent fluctuations.

$$\frac{\omega}{\Omega} \ll 1$$

2. Anisotropic

Goldreich & Sridhar 1995:  $k_{\parallel} \sim k_{\perp}^{2/3} \Rightarrow \frac{k_{\parallel}}{k_{\perp}} \ll 1$  at  $k_{\perp} \gg 1$



[Sundar+ 2017]

3. Small fluctuation amplitude

$$\frac{\delta \mathbf{B}}{B_0} \sim \frac{\delta f_s}{F_s} \ll 1$$



**Caution!**  
In GK,  $\omega/\Omega \ll 1$  abandons  
cyclotron heating, and  $\delta B/B_0 \ll 1$   
abandons stochastic heating.

$$\frac{\partial h_s}{\partial t} + v_{\parallel} \frac{\partial h_s}{\partial z} + \frac{c}{B_0} \{ \langle \chi \rangle_{\mathbf{R}_s}, h_s \} = \frac{q_s}{T_s} \frac{\partial \langle \chi \rangle_{\mathbf{R}_s}}{\partial t} F_s + \langle C[h_s] \rangle_{\mathbf{R}_s}$$

$h_s$  : gyro-center dist func

$\chi = \phi - \mathbf{v} \cdot \mathbf{A}/c$

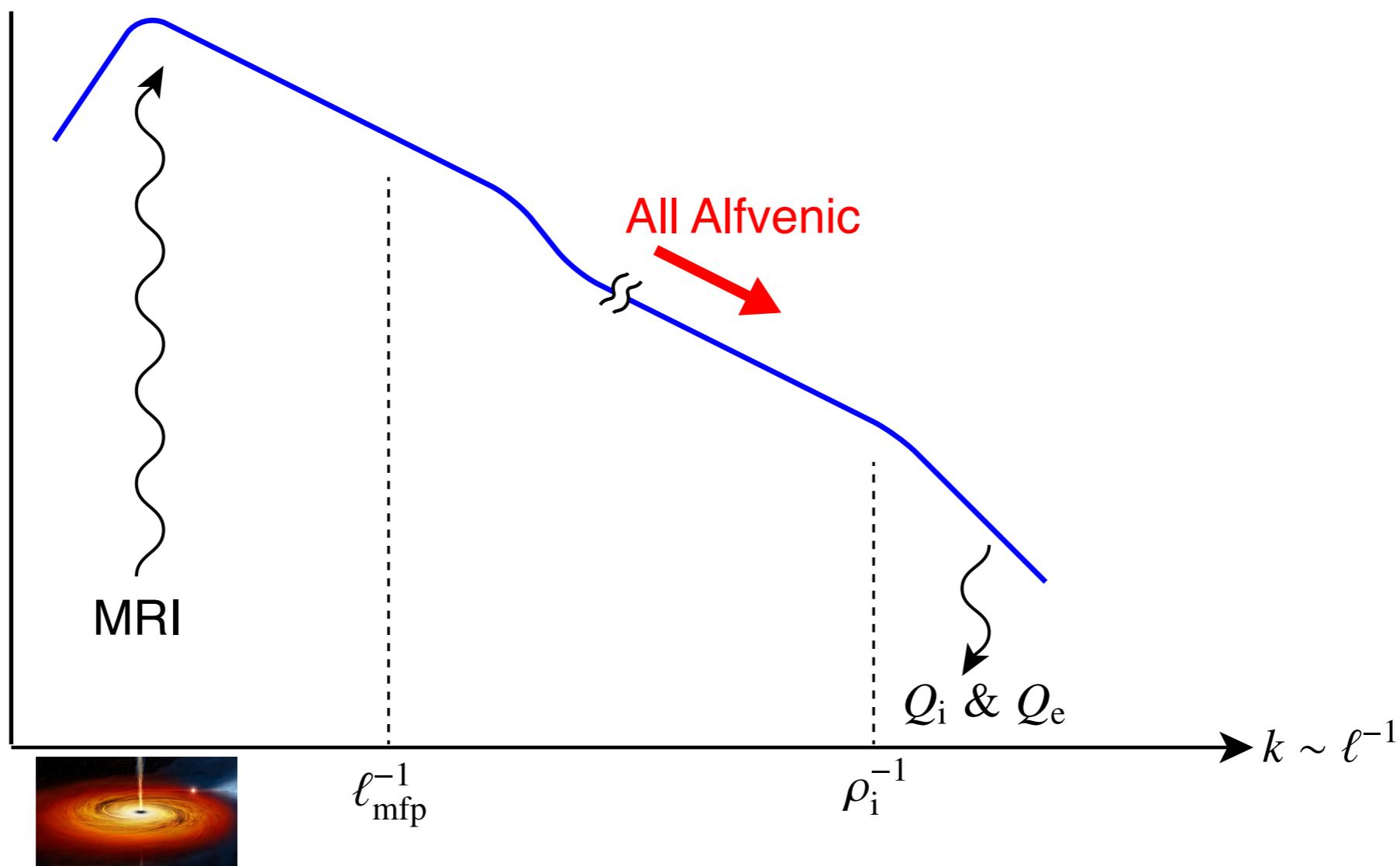
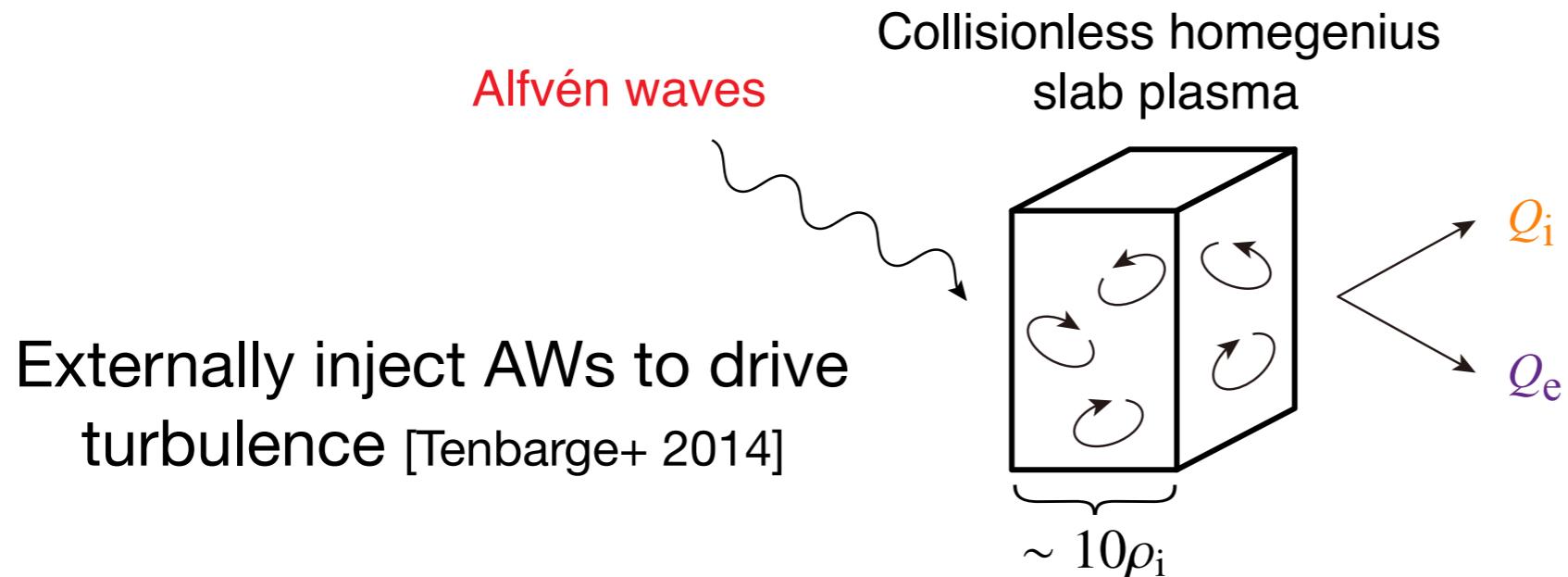
+ Maxwell's equations

$C[h_s]$  : collision operator

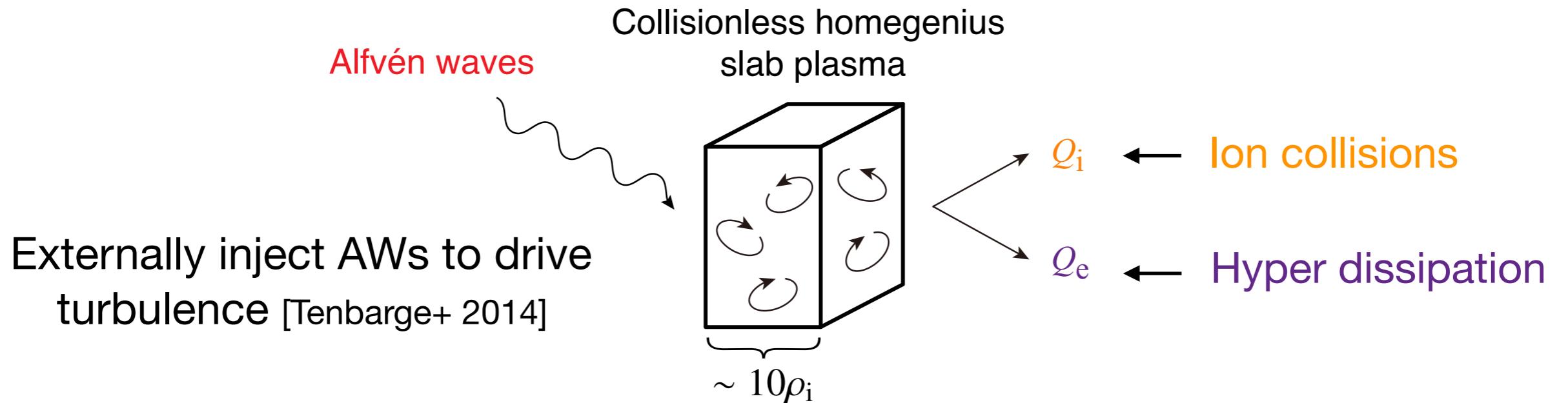
$\langle \dots \rangle_{\mathbf{R}_s}$  : gyro-average

- ▶ This ordering is OK for solar wind [Howes+ 2008], not sure for accretion disks.

# Simulation setting



# Simulation setting



## Hybrid GK (Ion: GK, electron: fluid)

$$\frac{\partial h_i}{\partial t} + v_{\parallel} \frac{\partial h_i}{\partial z} + \frac{c}{B_0} \{ \langle \chi \rangle_{R_i}, h_i \} = \frac{Ze}{T_i} \frac{\partial \langle \chi \rangle_{R_i}}{\partial t} F_i - \frac{ZeF_i}{T_i} \frac{\partial}{\partial t} \left\langle \frac{v_{\parallel} A_{\parallel}^a}{c} \right\rangle_{R_i} + \langle C[h_i] \rangle_{R_i} \quad \left( J_{\parallel}^a = \frac{c}{4\pi} \nabla_{\perp}^2 A_{\parallel}^a \right)$$

$$\frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} + \nabla_{\parallel} \phi = \nabla_{\parallel} \frac{T_{0e}}{e} \frac{\delta n_e}{n_{0e}} \quad \text{+ hyper resistivity}$$

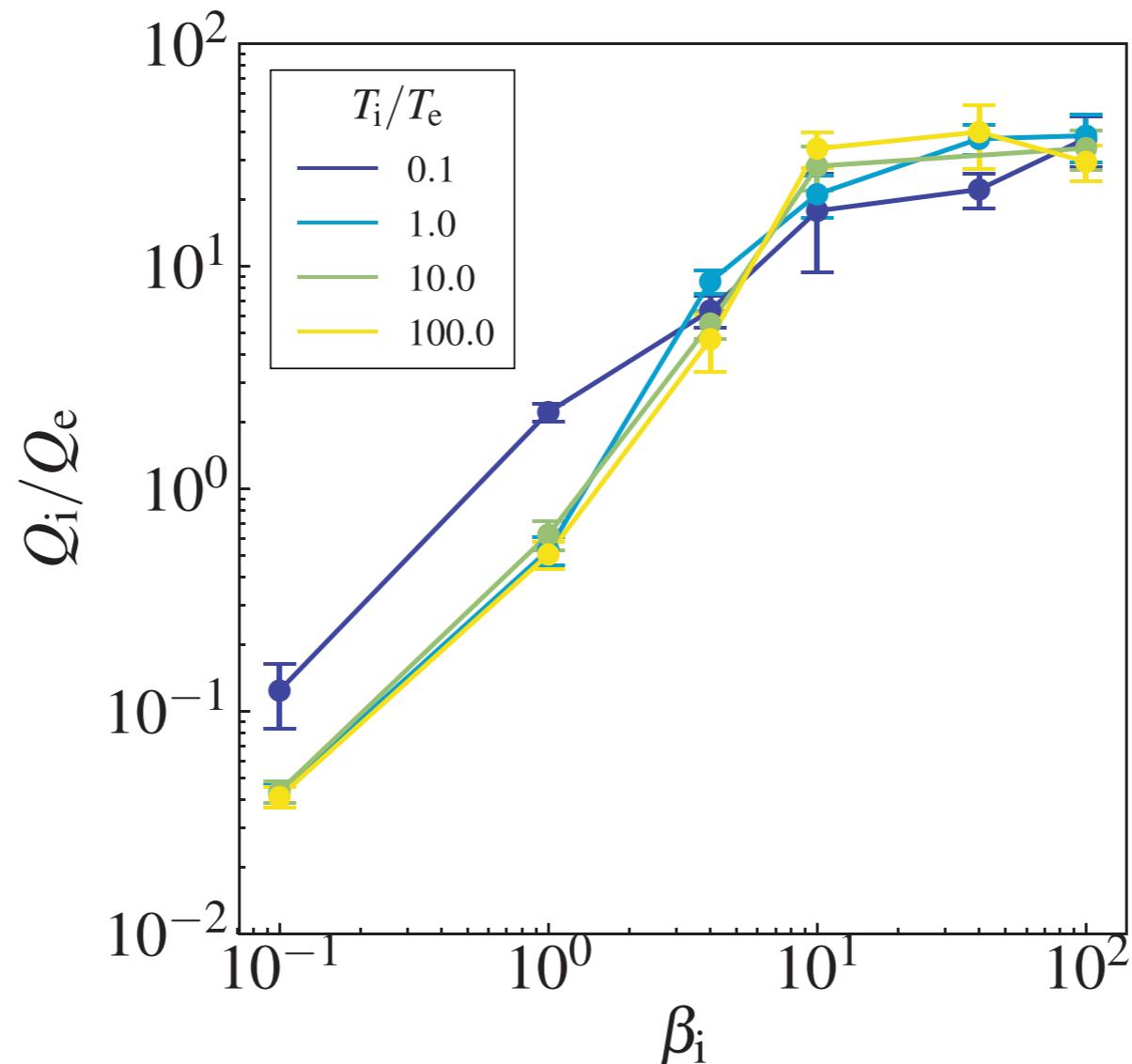
$$\frac{d}{dt} \left( \frac{\delta n_e}{n_{0e}} - \frac{\delta B_{\parallel}}{B_0} \right) + \nabla_{\parallel} u_{\parallel e} + \frac{c T_{0e}}{e B_0} \left\{ \frac{\delta n_e}{n_{0e}}, \frac{\delta B_{\parallel}}{B_0} \right\} = 0 \quad \text{+ hyper viscosity}$$

$$\rightarrow \frac{dW}{dt} = \underbrace{\int \frac{d^3 r}{V} \frac{J_{\parallel}^a}{c} \frac{\partial A_{\parallel}}{\partial t}}_{=: P_{AW}} + \underbrace{\int d^3 v \int \frac{d^3 R_i}{V} \frac{T_{0i}}{F_{0i}} \langle h_i C[h_i] \rangle_{R_i}}_{=: Q_i} - \underbrace{(\text{hyper res} + \text{hyper vis})}_{=: Q_e}$$

- Parameter scan  $0.1 \leq \beta_i \leq 100$  &  $0.1 \leq T_i/T_e \leq 100$

# Result : $Q_i/Q_e$

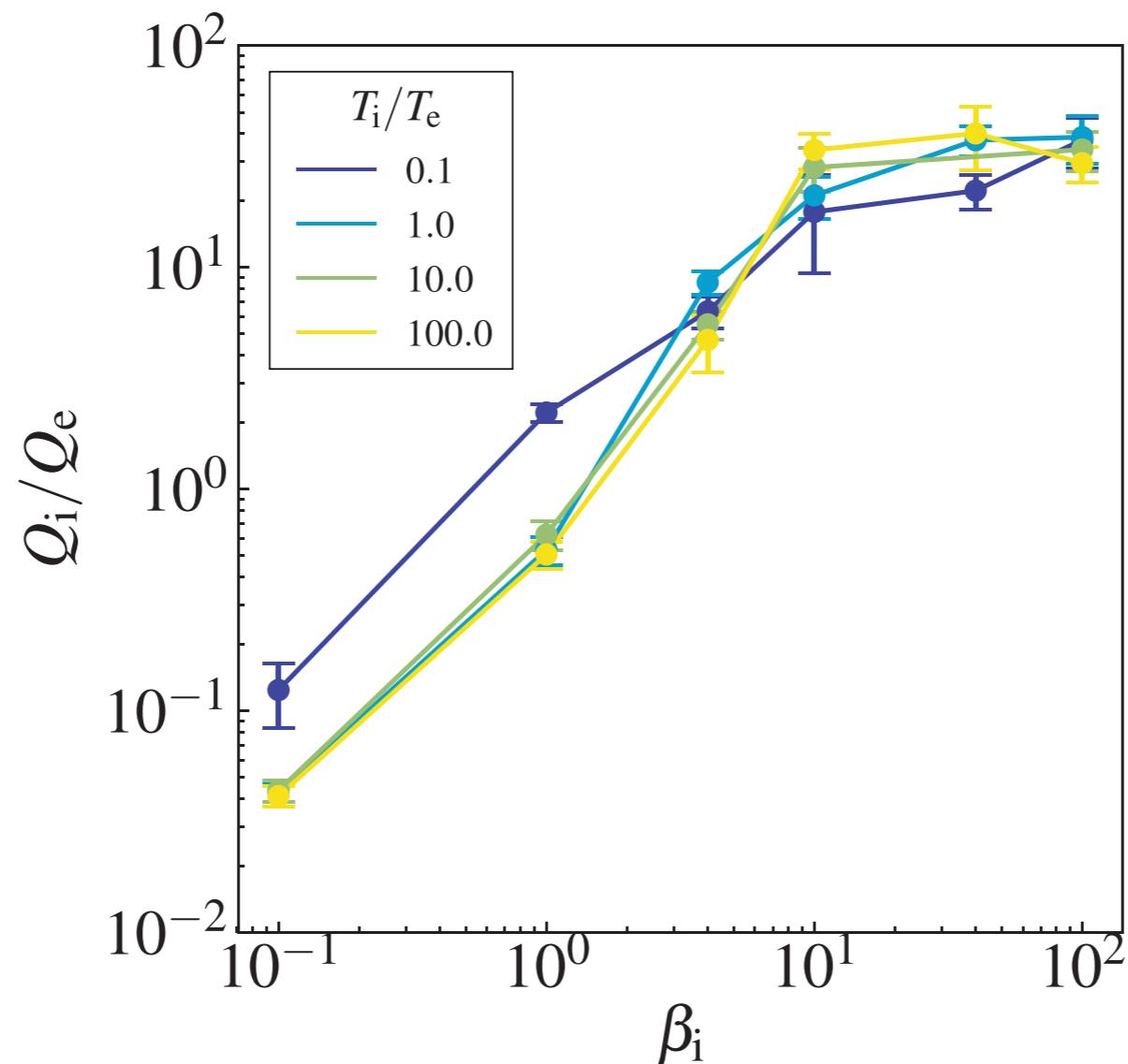
[YK+ 2019]



- ▶ An increasing function of  $\beta_i$ . But there is upper limit  $< 50$ .
- ▶ At low  $\beta_i$ ,  $Q_i/Q_e \rightarrow 0$  : Consistent with theoretical prediction [Schekochihin, YK, & Barnes 2019]
- ▶ At the Hall regime ( $\beta_i \ll 1$ ,  $\beta_e \sim 1$ ),  $Q_i/Q_e \sim 0.5$  : Consistent with theoretical prediction [Schekochihin, YK, & Barnes 2019]

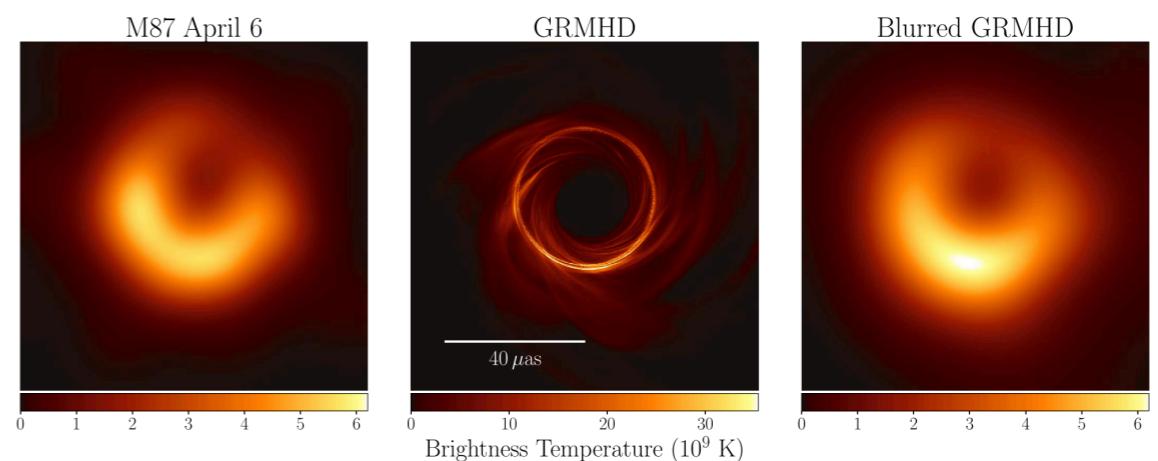
# Result : $Q_i/Q_e$

[YK+ 2019]



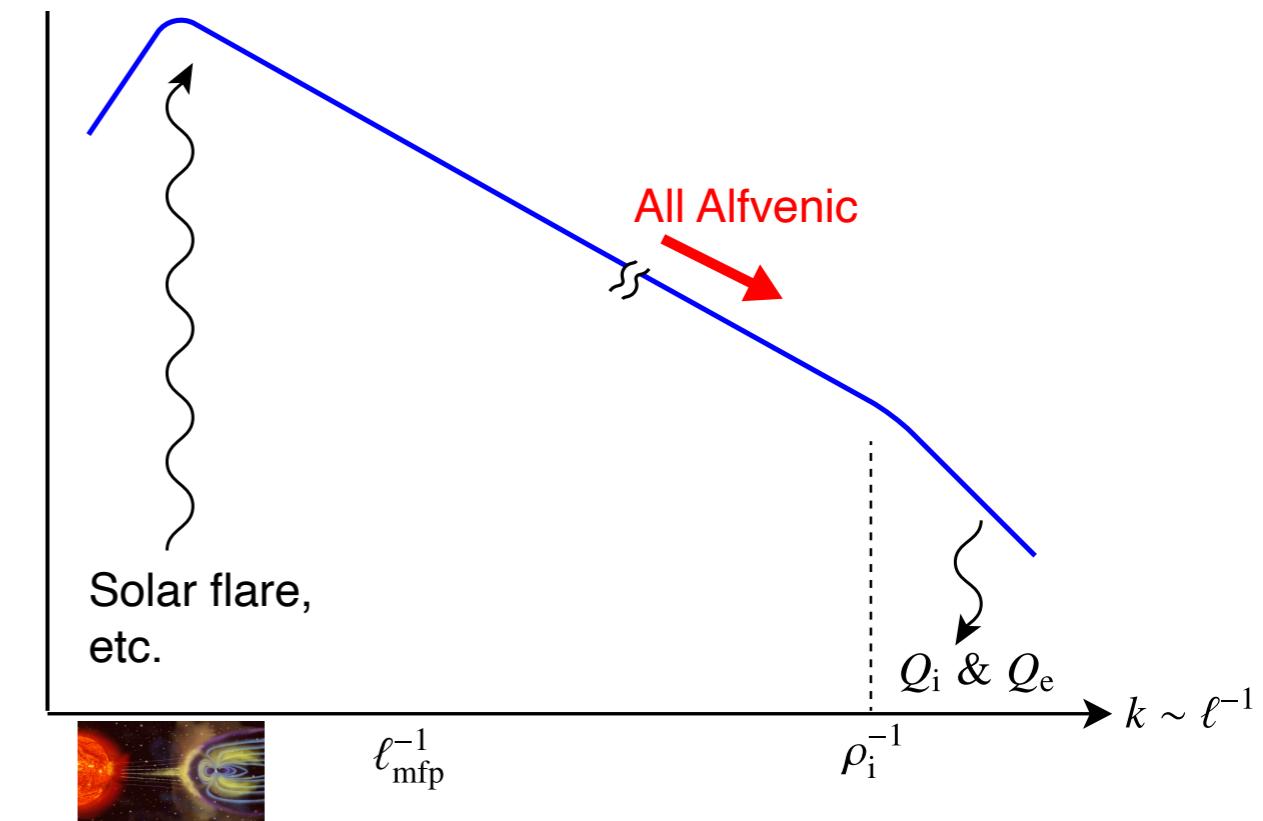
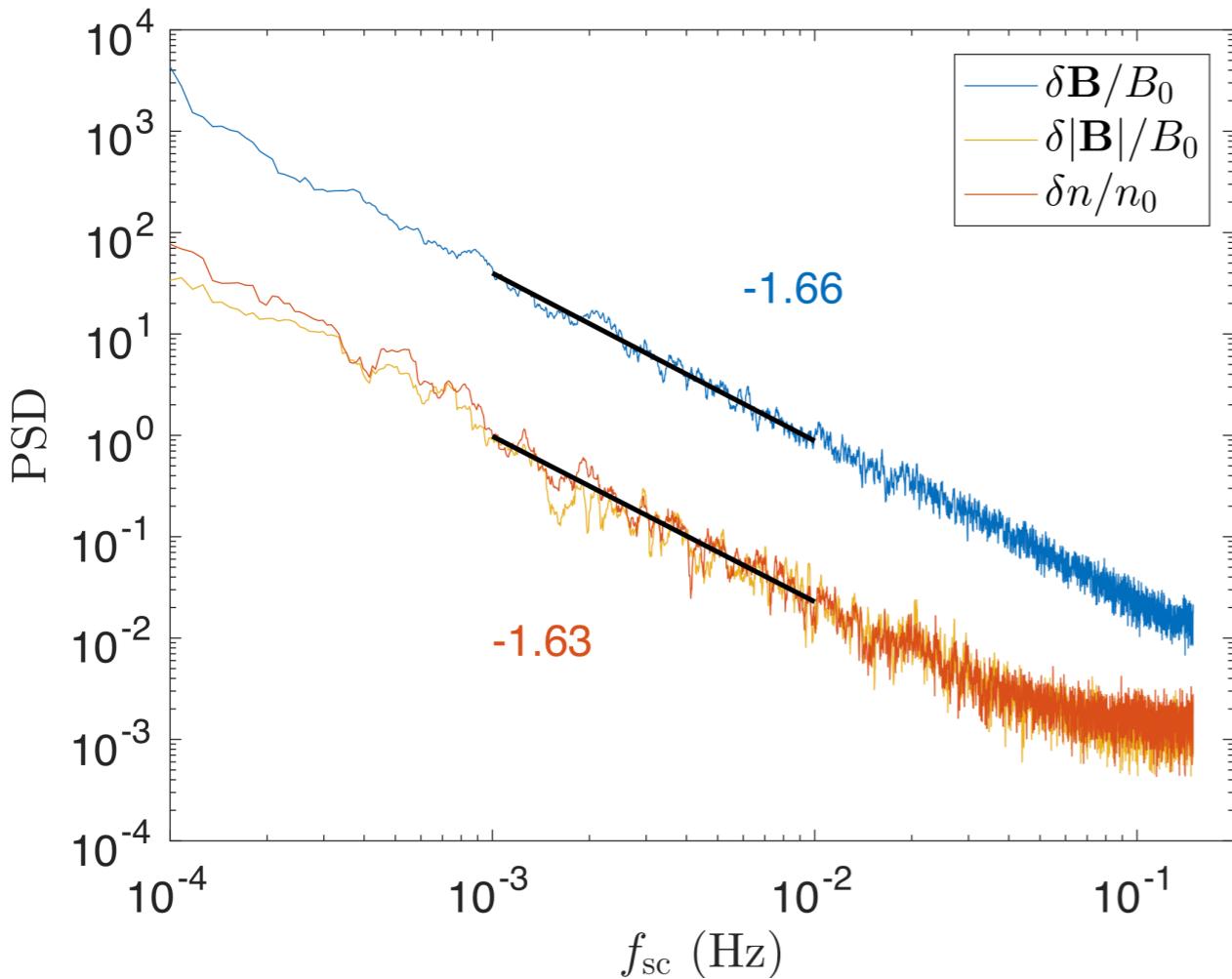
- A prescription of EHT is not really bad :)

$$\frac{T_i}{T_e} = R_{\text{high}} \frac{\beta_p^2}{1 + \beta_p^2} + \frac{1}{1 + \beta_p^2}$$

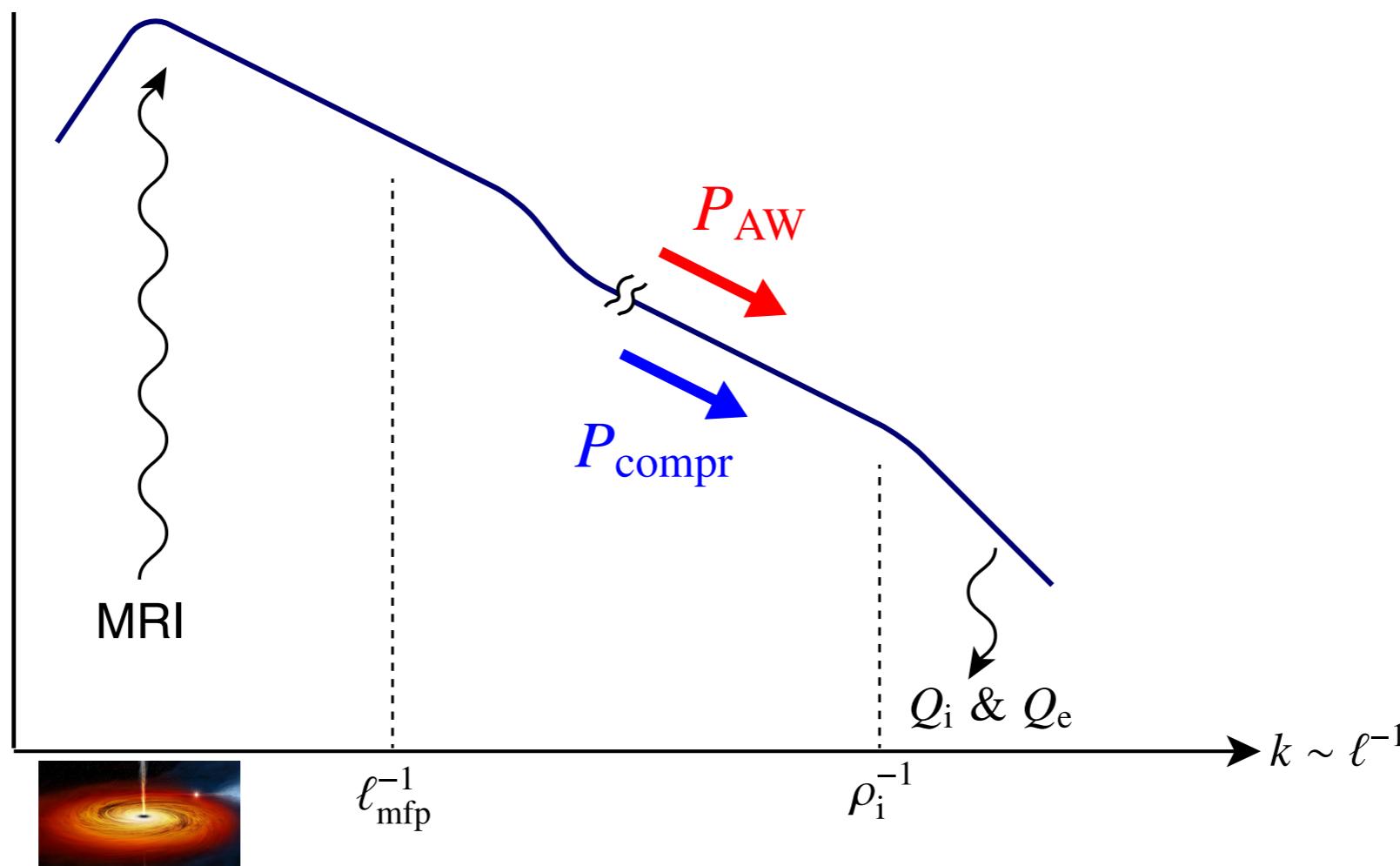
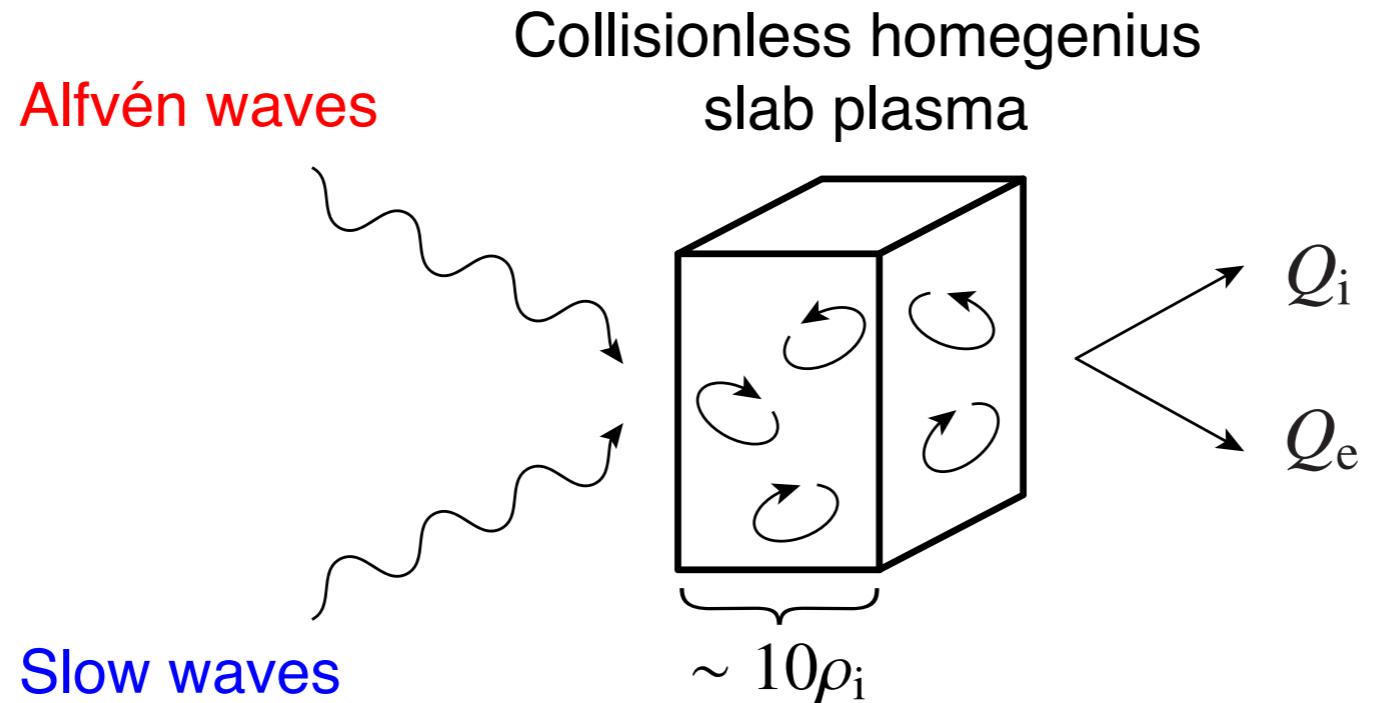


# Is accretion turbulence Alfvénic or compressive?

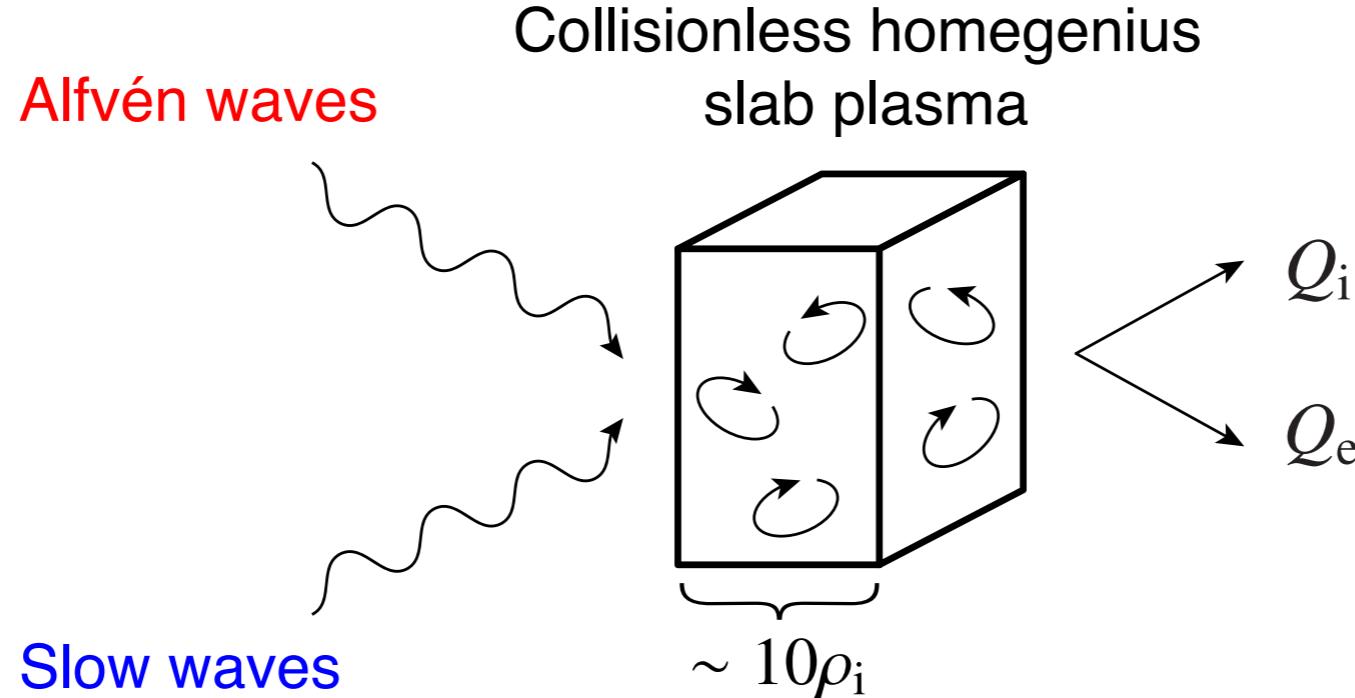
- ▶ In our GK simulation, turbulence was Alfvénically driven.
- ▶ This is based on the fact that solar wind is Alfvénic.
- ▶ But, we do not know if accretion disks are Alfvénic.
- ▶ We need to drive turbulence compressively.



# Alfvenic & compressive forcing



# Alfvenic & compressive forcing



$$\frac{\partial h_i}{\partial t} + v_{\parallel} \frac{\partial h_i}{\partial z} + \frac{c}{B_0} \{ \langle \chi \rangle_{R_i}, h_i \} = \frac{Ze}{T_i} \frac{\partial \langle \chi \rangle_{R_i}}{\partial t} F_i - \frac{ZeF_i}{T_i} \frac{\partial}{\partial t} \left\langle \frac{v_{\parallel} A_{\parallel}^a}{c} \right\rangle_{R_i} + \frac{v_{\parallel} \langle a_{\text{ext}} \rangle_{R_i}}{v_{\text{thi}}^2} F_i + \langle C[h_i] \rangle_{R_i}$$

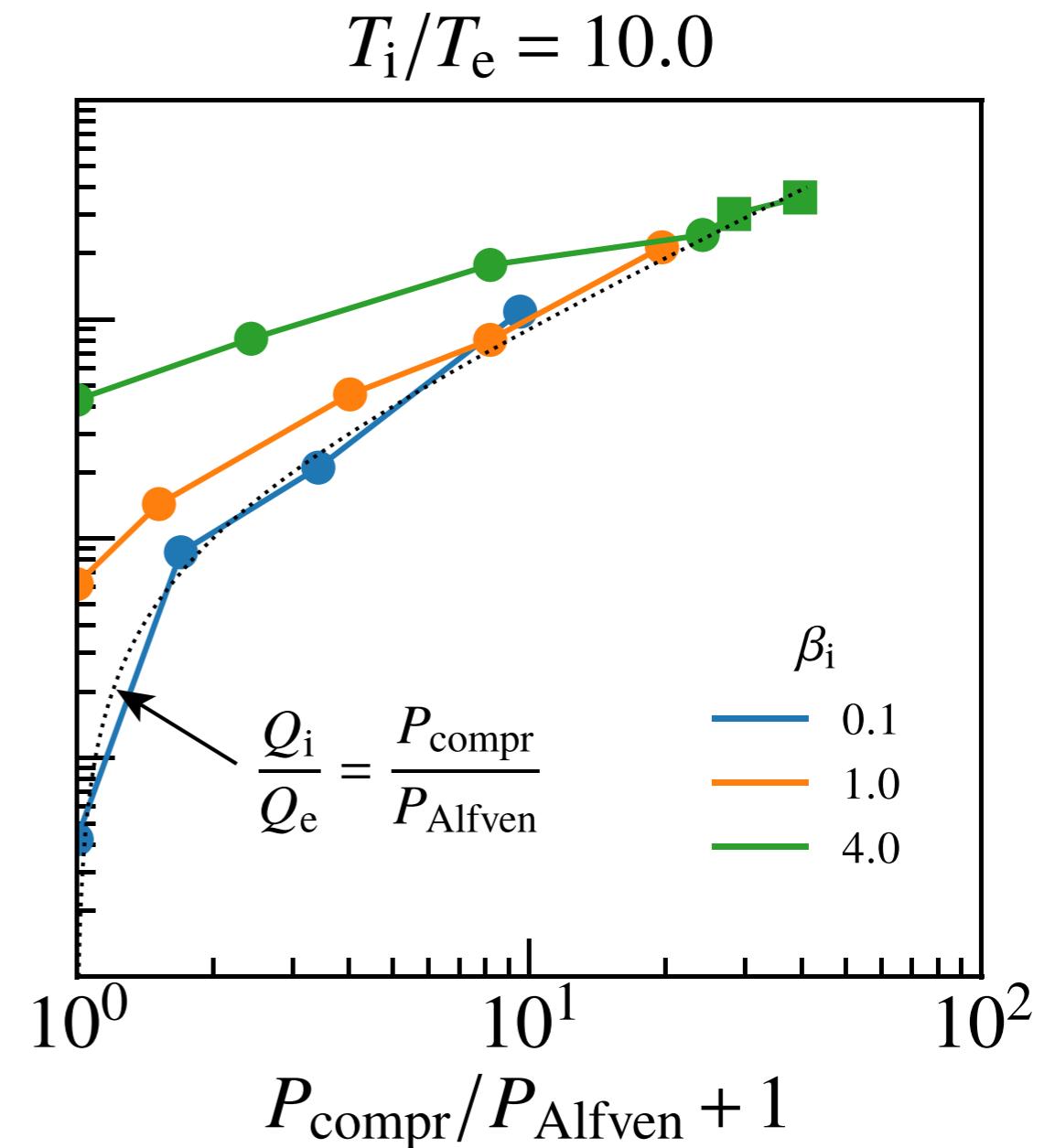
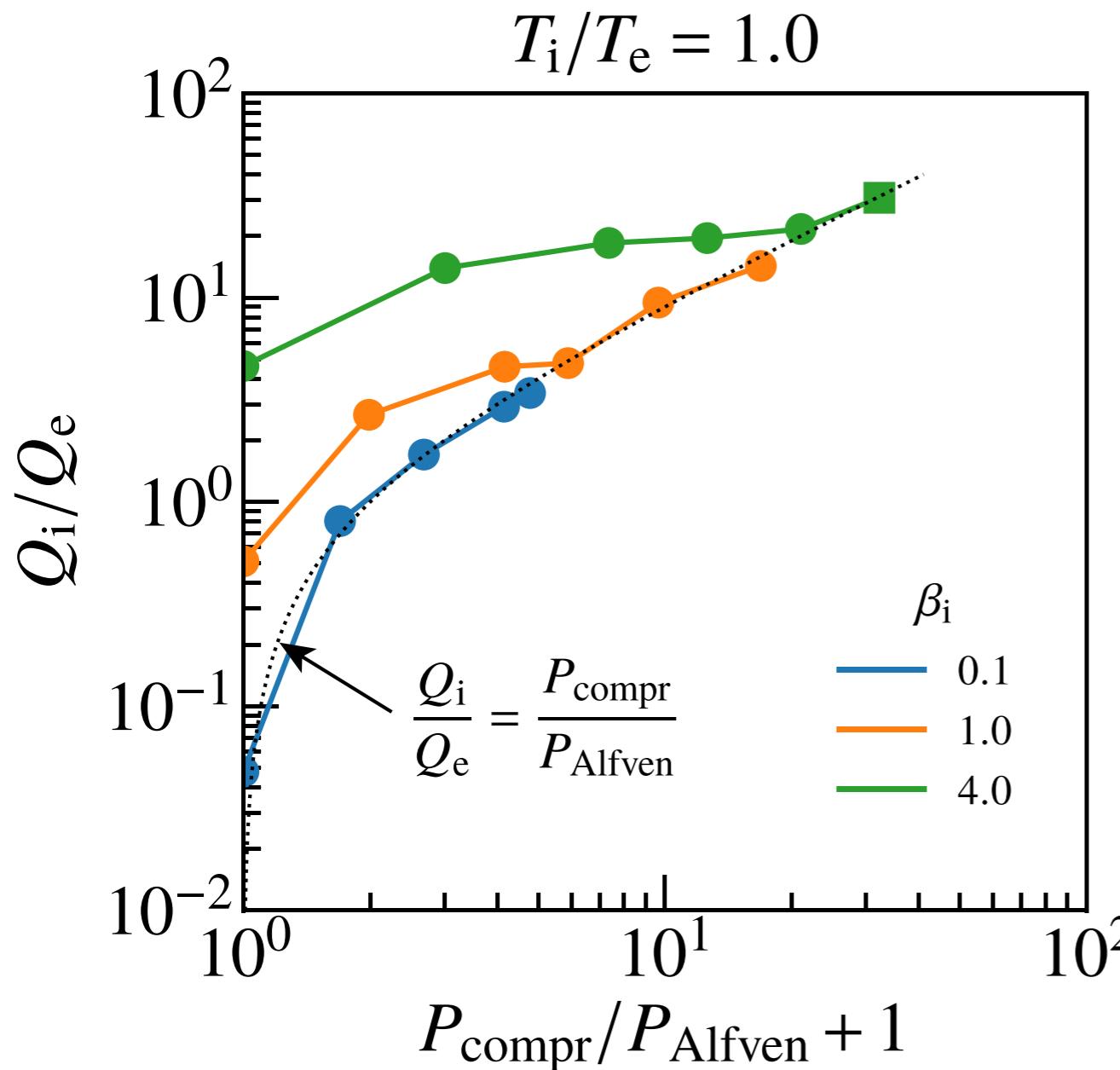
$$\frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} + \nabla_{\parallel} \phi = \nabla_{\parallel} \frac{T_e}{e} \frac{\delta n_e}{n_e} \quad + \text{hyper resistivity} \quad \left( J_{\parallel}^a = \frac{c}{4\pi} \nabla_{\perp}^2 A_{\parallel}^a \right)$$

$$\frac{d}{dt} \left( \frac{\delta n_e}{n_e} - \frac{\delta B_{\parallel}}{B_0} \right) + \nabla_{\parallel} u_{\parallel e} + \frac{c T_e}{e B_0} \left\{ \frac{\delta n_e}{n_e}, \frac{\delta B_{\parallel}}{B_0} \right\} = 0 \quad + \text{hyper viscosity}$$

$$\frac{dW}{dt} = \underbrace{\int \frac{d^3 \mathbf{r}}{V} \frac{J_{\parallel}^a}{c} \frac{\partial A_{\parallel}}{\partial t}}_{=: P_{\text{AW}}} + \underbrace{\int d^3 \mathbf{v} \int \frac{d^3 \mathbf{R}_i}{V} T_i \frac{v_{\parallel} h_i \langle a_{\text{ext}} \rangle_{R_i}}{v_{\text{thi}}^2}}_{=: P_{\text{compr}}} + \underbrace{\int d^3 \mathbf{v} \int \frac{d^3 \mathbf{R}_i}{V} \frac{T_i}{F_i} \langle h_i C[h_i] \rangle_{R_i}}_{=: Q_i} - \underbrace{(\text{hyper res} + \text{hyper vis})}_{=: Q_e}$$

# Result : $Q_i/Q_e$

[YK+ in prep]



- ▶  $Q_i/Q_e$  is an increasing function of  $P_{\text{compr}}/P_{\text{Alfven}}$ .
- ▶  $Q_i/Q_e \approx P_{\text{compr}}/P_{\text{Alfven}}$  for  $\beta_i = 0.1$  ← Consistent with a theory [Schekochihin+ 2019]
- ▶ When  $P_{\text{compr}}/P_{\text{Alfven}} \gg 1$ ,  $Q_i/Q_e \approx P_{\text{compr}}/P_{\text{Alfven}}$  for any  $\beta_i$

# Heating prescription

$$\frac{Q_i}{Q_e}(\beta_i, \tau, \varphi) = \frac{35}{1 + (\beta_i/15)^{-1.4} e^{-0.1/\tau}} + \varphi \left( 1 + \frac{1}{1 + \varphi + \beta_i^{-1}} \right)$$

$$\tau \equiv \frac{T_i}{T_e}, \quad \varphi = \frac{P_{\text{compr}}}{P_{\text{AW}}}$$

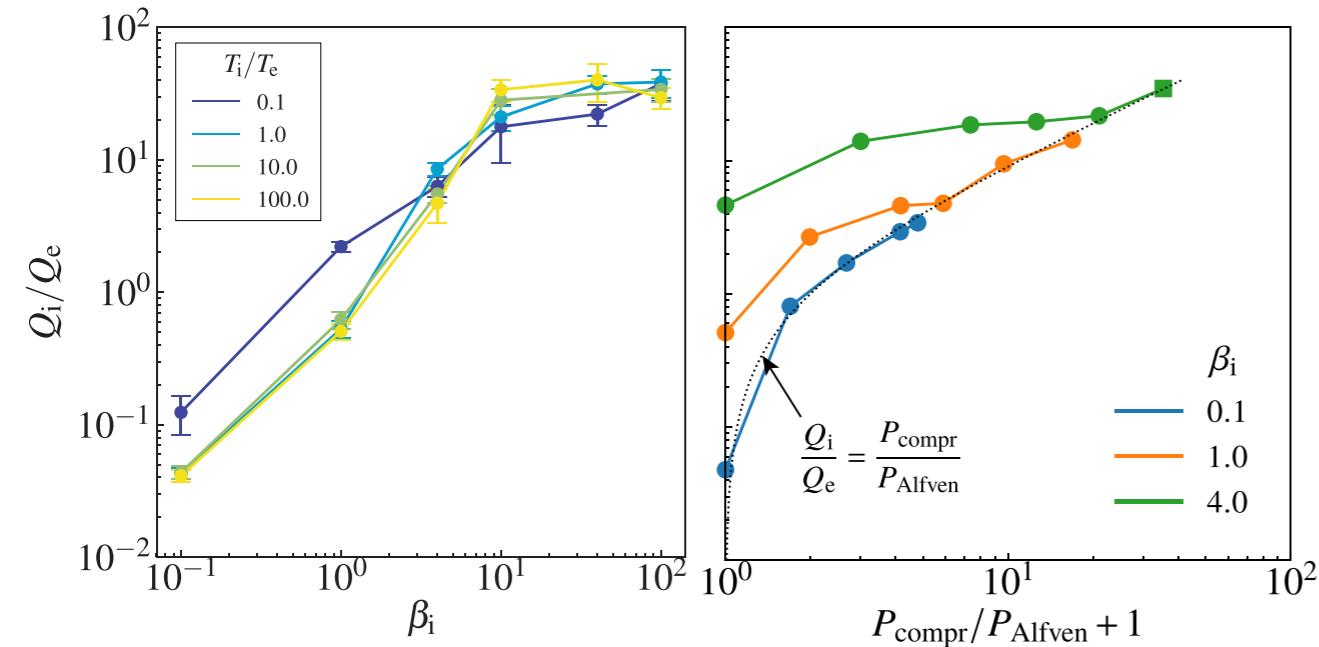
- ▶ Can be used for *any* collisionless turbulence wherein the GK approximation holds.
- ▶  $Q_i/Q_e > 1$  when  $P_{\text{compr}}/P_{\text{AW}} > 1$ 
  - Preferential electron heating is a very limited case, viz., low  $\beta$  pure Alvenic.

**For more details (e.g., the mechanism of heating and power spectra), please see YK+ PNAS 2019.**

# Plan of the talk

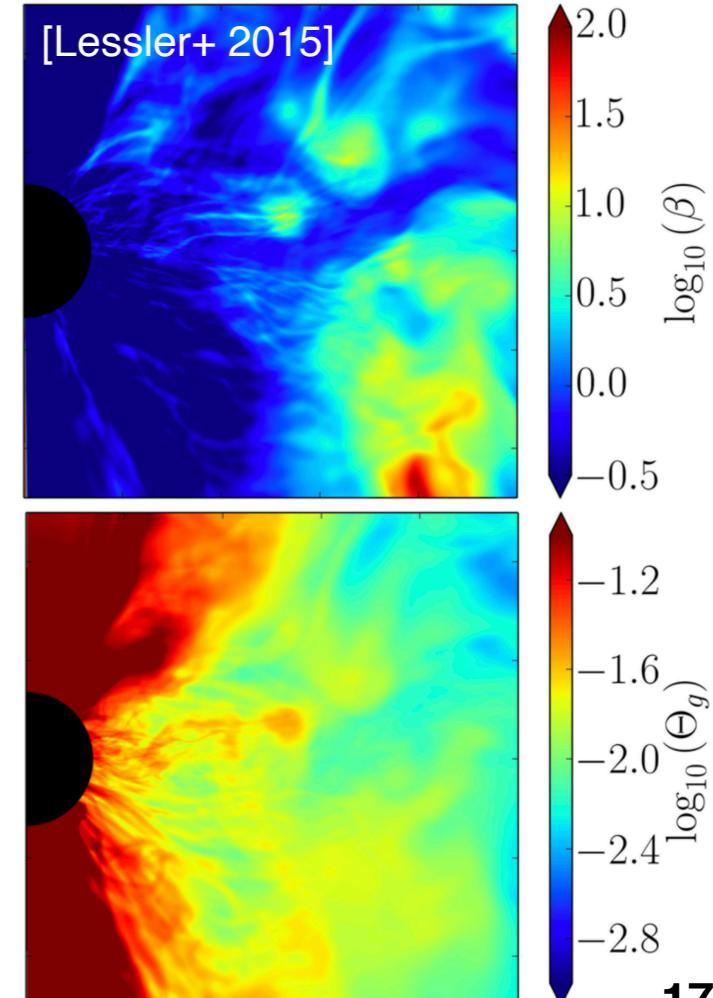
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# How do we apply $Q_i/Q_e$ to actual disks?

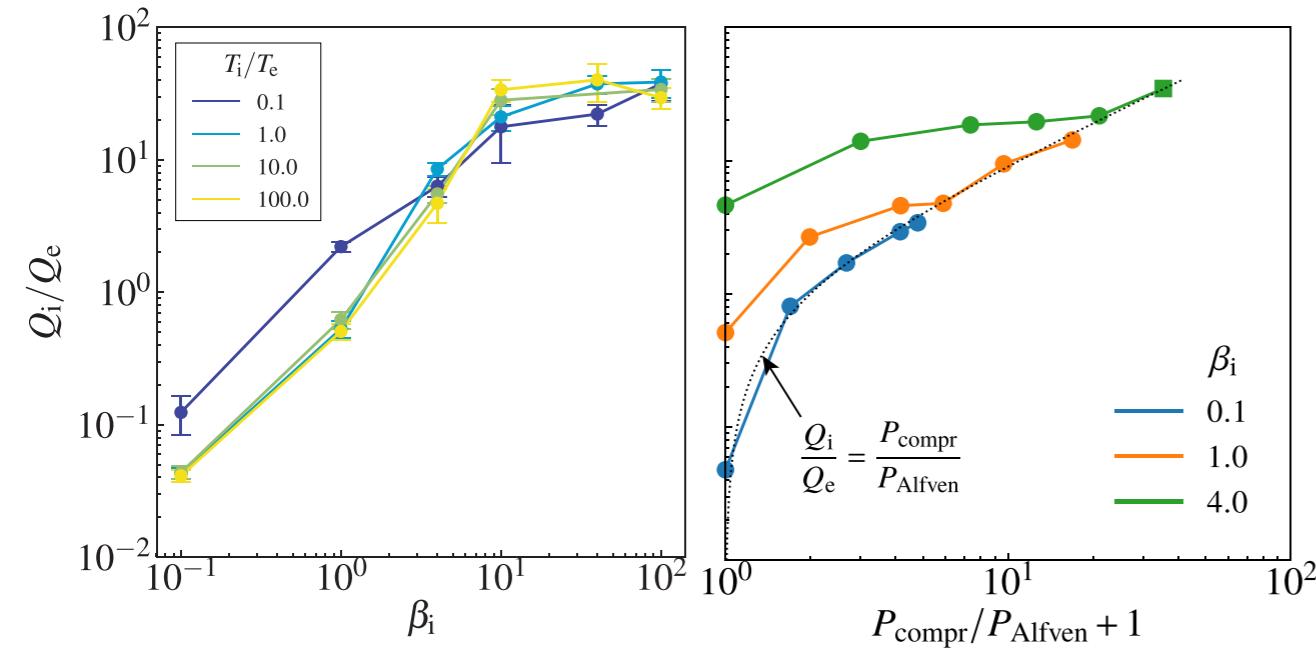


1. Solve Global MHD and calculate  $B$ ,  $T = T_i + T_e$ ,  $P_{\text{compr}}/P_{\text{AW}}$  at each grid.
2. Use  $Q_i/Q_e = f(\beta_i, T_i/T_e, P_{\text{compr}}/P_{\text{AW}})$  to solve the transport equations for ions and electrons.
3. One obtains  $T_i$  and  $T_e$  respectively.

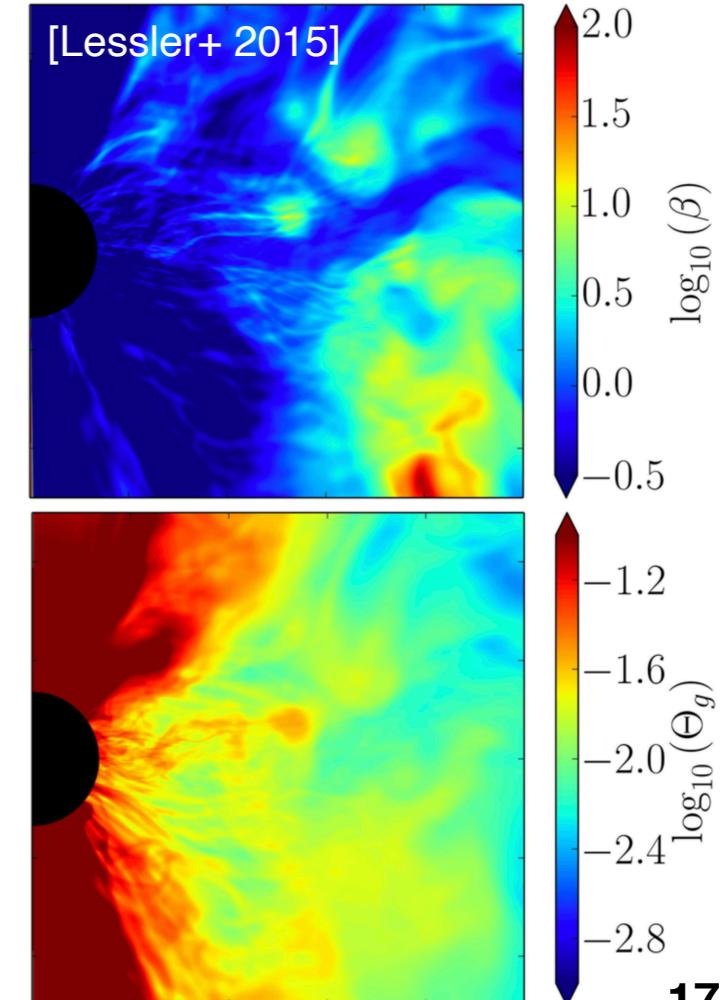
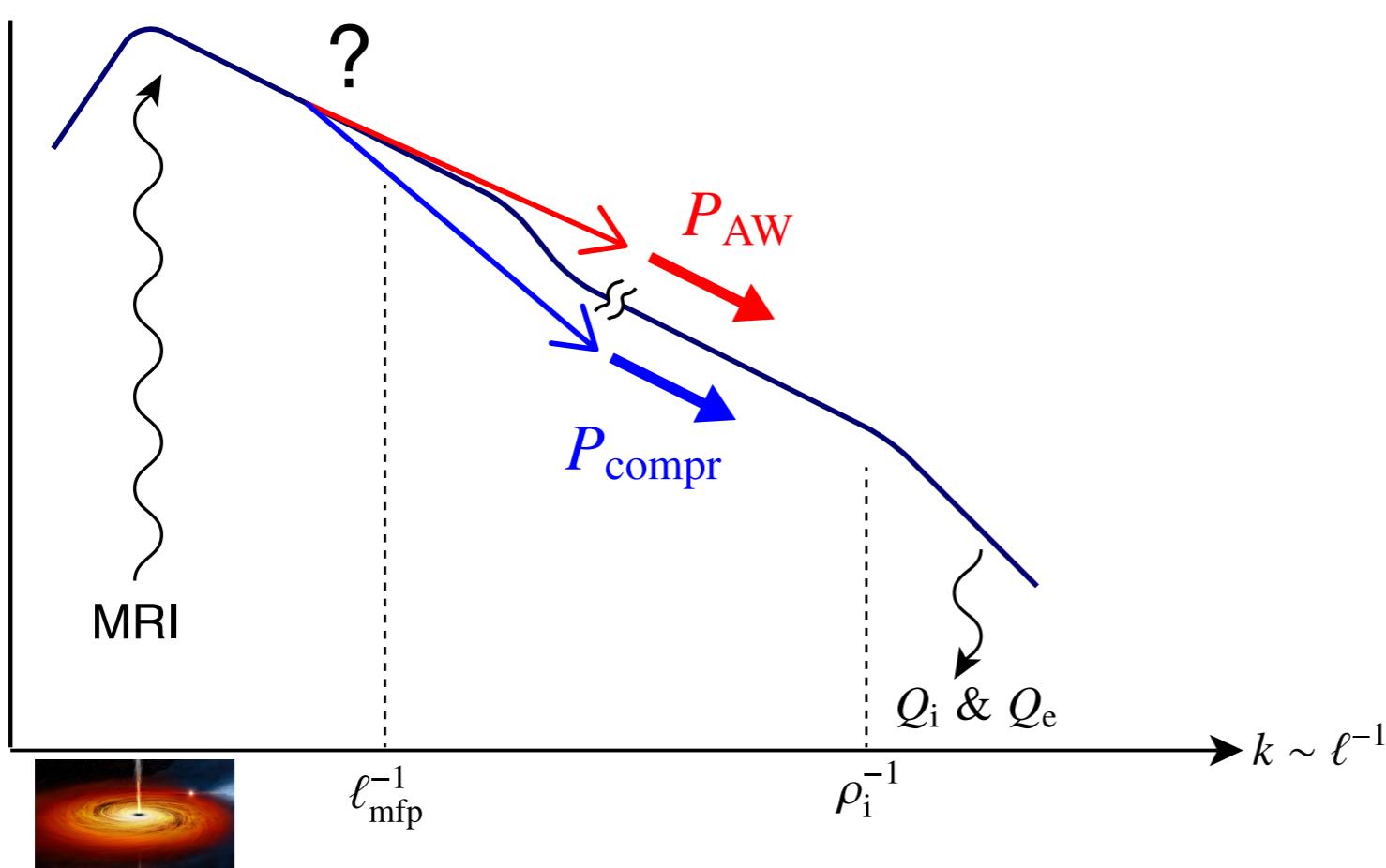
**How should we get  $P_{\text{compr}}/P_{\text{AW}}$  ?**  
 → A question of MRI driven MHD turbulence



# How do we apply $Q_i/Q_e$ to actual disks?

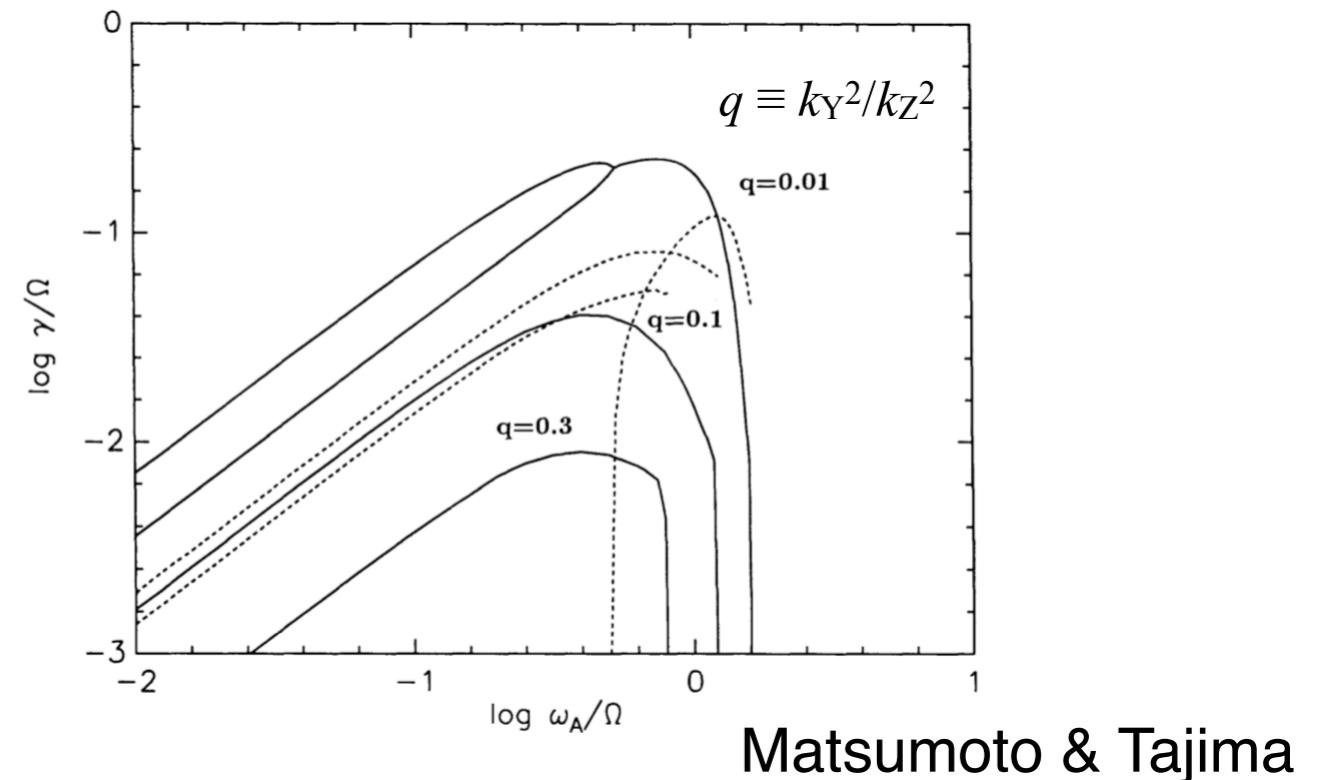
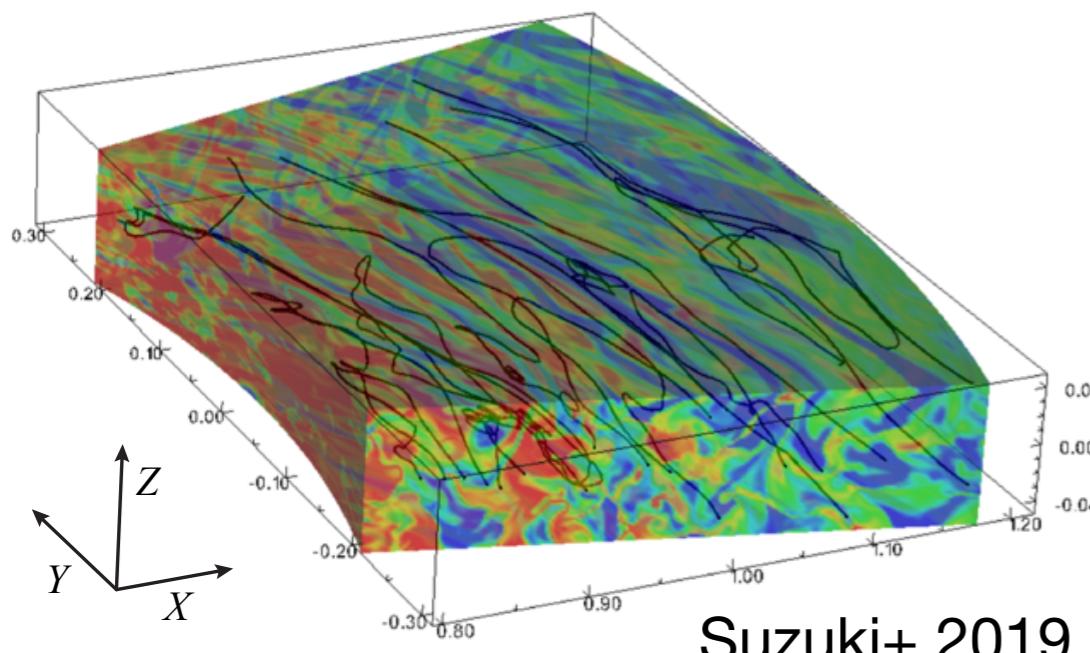


$$Q_i/Q_e = f(\beta_i, T_i/T_e, P_{\text{compr}}/P_{\text{AW}})$$



# Compressive & Alfvénic fluctuations are coupled even at small scales in MRI-turbulence

- ▶  $B$  tends to be azimuthal in disks.
- ▶ The fastest growing mode resides in  $k_z \rightarrow \infty$  ( $Z$  : rotational axis).  
[Balbus&Hawley 1992, Hawley+ 1995, Matsumoto & Tajima 1995]
- ▶ Compressive & Alfvénic are coupled via linear MRI even at the small scales.



# Shearing Reduced MHD

[YK+ in prep]

- Since the fastest-growing MRI mode is at  $k_{\parallel}/k_{\perp} \ll 1$ , we impose this ordering in the first place i.e., ignore all the MRI modes slower than the fastest one.

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \mathbf{u}_0 \cdot \nabla \rho = -\rho(\nabla \cdot \mathbf{u}),$$

$$\rho \left( \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla + \mathbf{u}_0 \cdot \nabla \right) \mathbf{u} = -\nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi} - 2\rho \boldsymbol{\varpi}_0 \times \mathbf{u} - \rho \mathbf{u} \cdot \nabla \mathbf{u}_0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} + \mathbf{u}_0 \cdot \nabla \mathbf{B} + \mathbf{B}(\nabla \cdot \mathbf{u}) = \mathbf{B} \cdot \nabla \mathbf{u} + \mathbf{B} \cdot \nabla \mathbf{u}_0,$$

$$\begin{aligned} \frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p + \mathbf{u}_0 \cdot \nabla p + \gamma p \nabla \cdot \mathbf{u} &= 0, \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned}$$



RMHD ordering

$$\frac{k_{\parallel}}{k_{\perp}} \sim \frac{\delta \mathbf{B}}{B_0} \sim \frac{u_{\perp}}{v_A} \sim \epsilon$$

$$\left( \frac{d}{dt} + q\boldsymbol{\varpi}_0 x \cos \theta \frac{\partial}{\partial z} \right) \Psi = v_A \frac{\partial \Phi}{\partial y}$$

$$\left( \frac{d}{dt} + q\boldsymbol{\varpi}_0 x \cos \theta \frac{\partial}{\partial z} \right) \nabla_{\perp}^2 \Phi = v_A \nabla_{\parallel} \nabla_{\perp}^2 \Psi + 2\boldsymbol{\varpi}_0 \sin \theta \frac{\partial u_{\parallel}}{\partial z}$$

$$\left( \frac{d}{dt} + q\boldsymbol{\varpi}_0 x \cos \theta \frac{\partial}{\partial z} \right) u_{\parallel} = v_A^2 \nabla_{\parallel} \left( \frac{\delta B_{\parallel}}{B_0} \right) - (2-q)\boldsymbol{\varpi}_0 \sin \theta \frac{\partial \Phi}{\partial z}$$

$$\left( \frac{d}{dt} + q\boldsymbol{\varpi}_0 x \cos \theta \frac{\partial}{\partial z} \right) \left( 1 + \frac{v_A^2}{c_s^2} \right) \frac{\delta B_{\parallel}}{B_0} = \nabla_{\parallel} u_{\parallel} - \frac{q\boldsymbol{\varpi}_0 \sin \theta}{v_A} \frac{\partial \Psi}{\partial z},$$

$$\mathbf{u} = \hat{\mathbf{y}} \times \nabla_{\perp} \Phi + u_{\parallel} \hat{\mathbf{y}}$$

$$\delta \mathbf{B} = \hat{\mathbf{y}} \times \nabla_{\perp} \Psi + \delta B_{\parallel} \hat{\mathbf{y}}$$

$\Phi, \Psi$  : Alfvénic

$u_{\parallel}, \delta B_{\parallel}$  : compressive

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \{\Phi, \dots\}, \quad \nabla_{\parallel} = \frac{\partial}{\partial y} + v_A^{-1} \{\Psi, \dots\}$$

# Shearing Reduced MHD

[YK+ in prep]

- ▶ Since the fastest-growing MRI mode is at  $k_{\parallel}/k_{\perp} \ll 1$ , we impose this ordering in the first place
  - i.e., ignore all the MRI modes slower than the fastest one.
- ▶ Valid only for nearly (or completely) toroidal  $\mathbf{B}_0$  (otherwise the fastest mode is not in  $k_{\parallel}/k_{\perp} \ll 1$  regime).
- ▶ When *eddy turn over time*  $\ll \omega_0^{-1}$ , compressive and Alfvénic fluctuations *perfectly* decouples.
- ▶ Once decoupled, they keep decoupled until  $\rho_i$  scale.

$$\left( \frac{d}{dt} + q\omega_0 x \cos \theta \frac{\partial}{\partial z} \right) \Psi = v_A \frac{\partial \Phi}{\partial y}$$

$$\left( \frac{d}{dt} + q\omega_0 x \cos \theta \frac{\partial}{\partial z} \right) \nabla_{\perp}^2 \Phi = v_A \nabla_{\parallel} \nabla_{\perp}^2 \Psi + 2\omega_0 \sin \theta \frac{\partial u_{\parallel}}{\partial z}$$

$$\left( \frac{d}{dt} + q\omega_0 x \cos \theta \frac{\partial}{\partial z} \right) u_{\parallel} = v_A^2 \nabla_{\parallel} \left( \frac{\delta B_{\parallel}}{B_0} \right) - (2-q)\omega_0 \sin \theta \frac{\partial \Phi}{\partial z}$$

$$\left( \frac{d}{dt} + q\omega_0 x \cos \theta \frac{\partial}{\partial z} \right) \left( 1 + \frac{v_A^2}{c_s^2} \right) \frac{\delta B_{\parallel}}{B_0} = \nabla_{\parallel} u_{\parallel} - \frac{q\omega_0 \sin \theta}{v_A} \frac{\partial \Psi}{\partial z},$$

$$\mathbf{u} = \hat{\mathbf{y}} \times \nabla_{\perp} \Phi + u_{\parallel} \hat{\mathbf{y}}$$

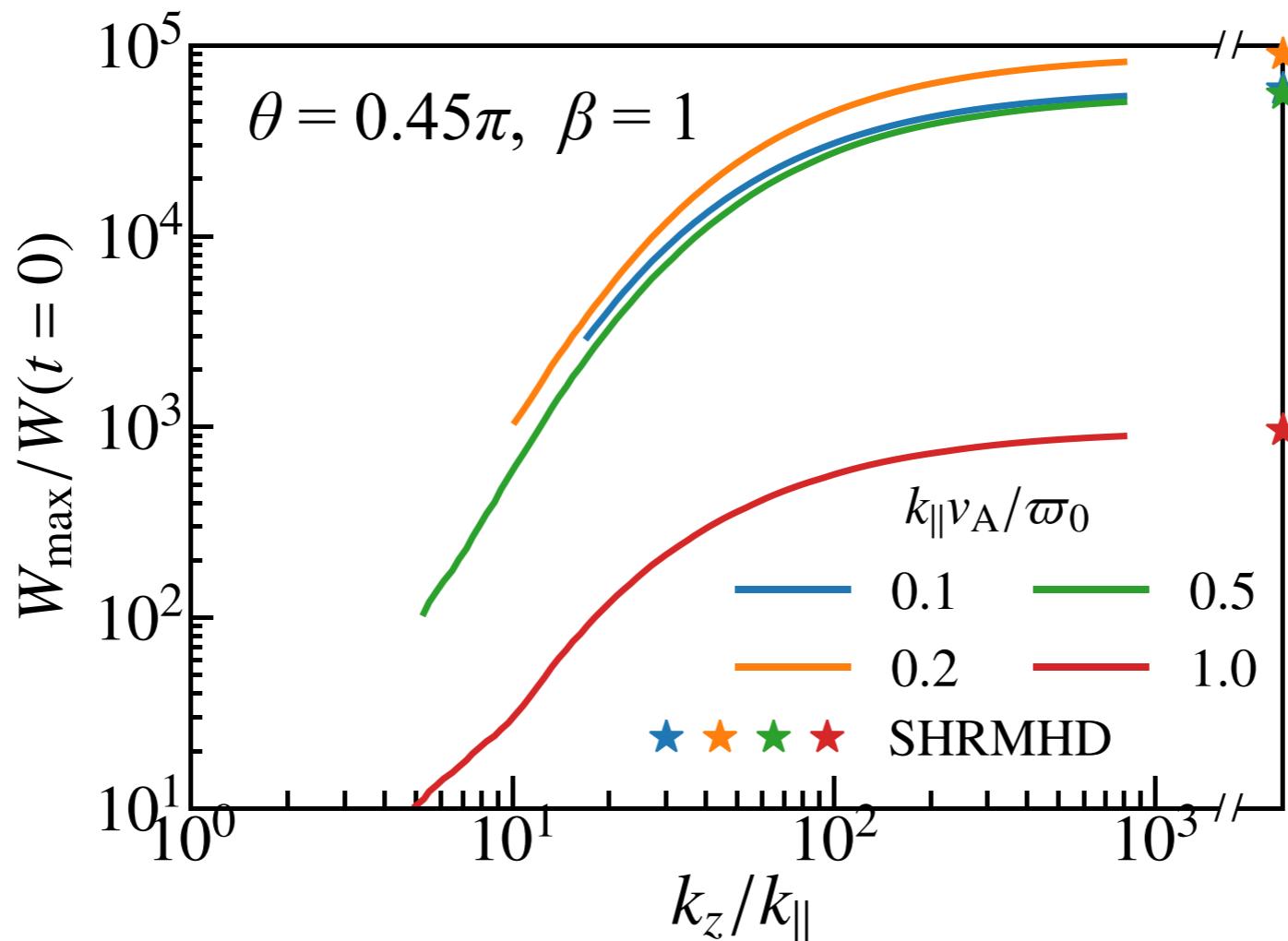
$$\delta \mathbf{B} = \hat{\mathbf{y}} \times \nabla_{\perp} \Psi + \delta B_{\parallel} \hat{\mathbf{y}}$$

$\Phi, \Psi$  : Alfvénic

$u_{\parallel}, \delta B_{\parallel}$  : compressive

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \{\Phi, \dots\}, \quad \nabla_{\parallel} = \frac{\partial}{\partial y} + v_A^{-1} \{\Psi, \dots\}$$

# Linear analysis (MRI growth rate vs $k_z/k_{\parallel}$ )



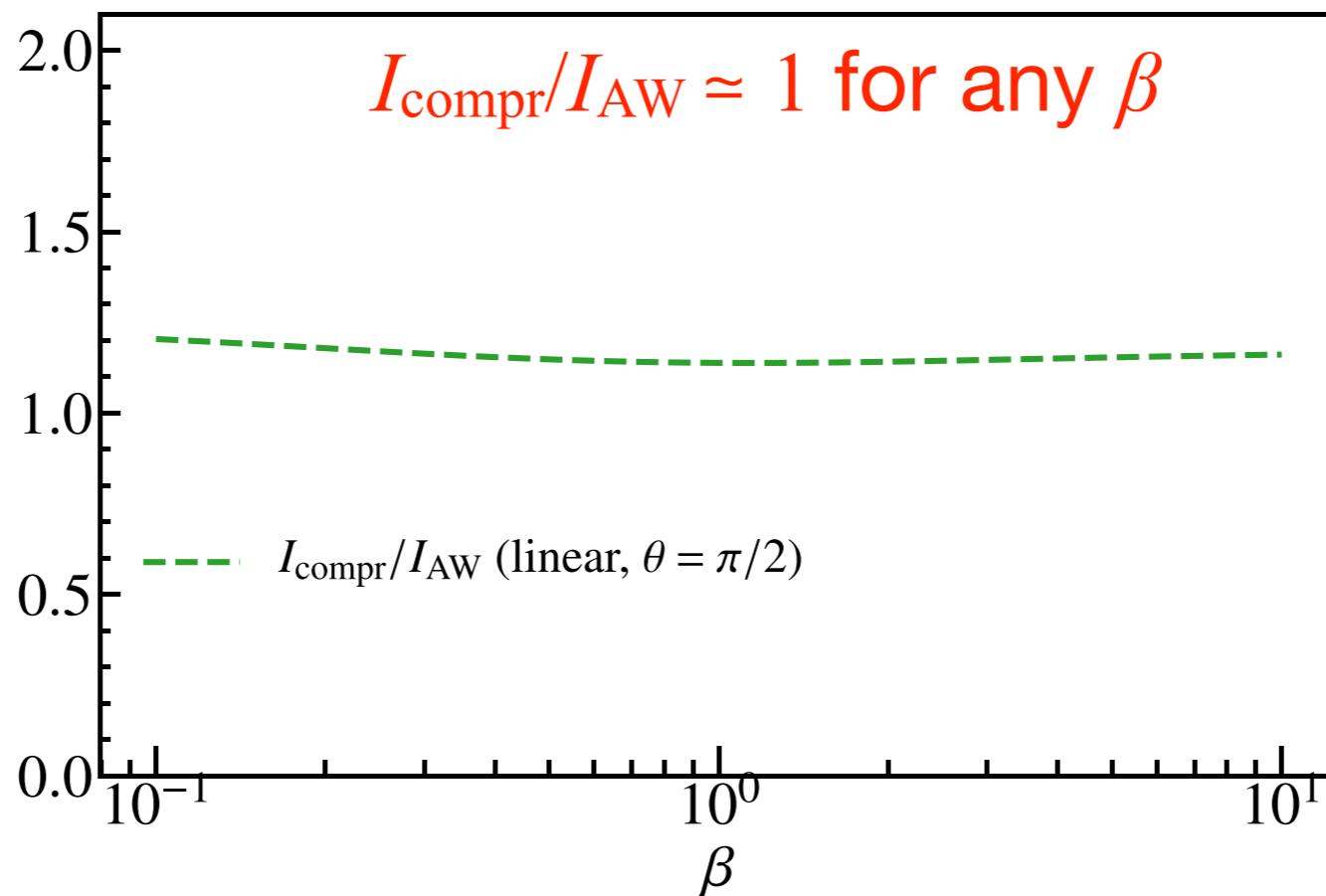
- ▶ Growth rate of ideal MHD : an increase func of  $k_z/k_{\parallel}$
- ▶ Saturate at the growth rate of SHRMHD.
- ▶ As expected, SHRMHD captures the fastest growing mode.

# Linear analysis ( $I_{\text{compr}}/I_{\text{AW}}$ )

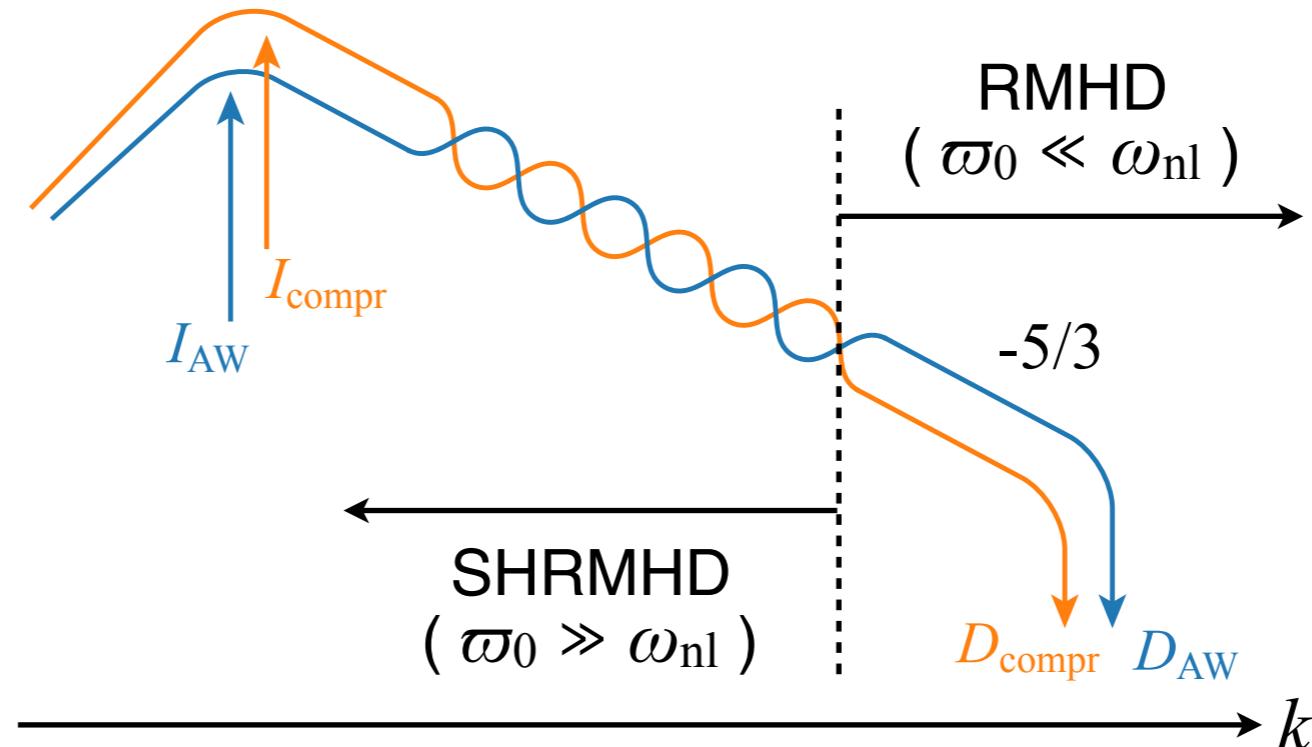
- ▶ Substituting the eigenfunctions into

$$I_{\text{AW}} = \int d^3\mathbf{r} \left[ q\varpi_0 \cos \theta \left( u_x u_y - v_A^2 \frac{\delta B_x}{B_0} \frac{\delta B_y}{B_0} \right) + 2\varpi_0 \sin \theta u_{\parallel} u_x \right],$$
$$I_{\text{compr}} = \int d^3\mathbf{r} \left[ q\varpi_0 \sin \theta \left( u_x u_{\parallel} - v_A^2 \frac{\delta B_x}{B_0} \frac{\delta B_{\parallel}}{B_0} \right) - 2\varpi_0 \sin \theta u_{\parallel} u_x \right]$$

we obtain the compressive-to-Alfven MRI injection power ratio  $I_{\text{compr}}/I_{\text{AW}}$ .



# Nonlinear simulation of SHRMHD



$$\left( \frac{d}{dt} - q\varpi_0 x \cos \theta \frac{\partial}{\partial y} \right) \Psi = v_A \frac{\partial \Phi}{\partial z} \quad + \text{perp resistivity}$$

$$\left( \frac{d}{dt} - q\varpi_0 x \cos \theta \frac{\partial}{\partial y} \right) \nabla_{\perp}^2 \Phi = v_A \nabla_{\parallel} \nabla_{\perp}^2 \Psi - 2\varpi_0 \sin \theta \frac{\partial u_{\parallel}}{\partial y} \quad + \text{perp viscosity}$$

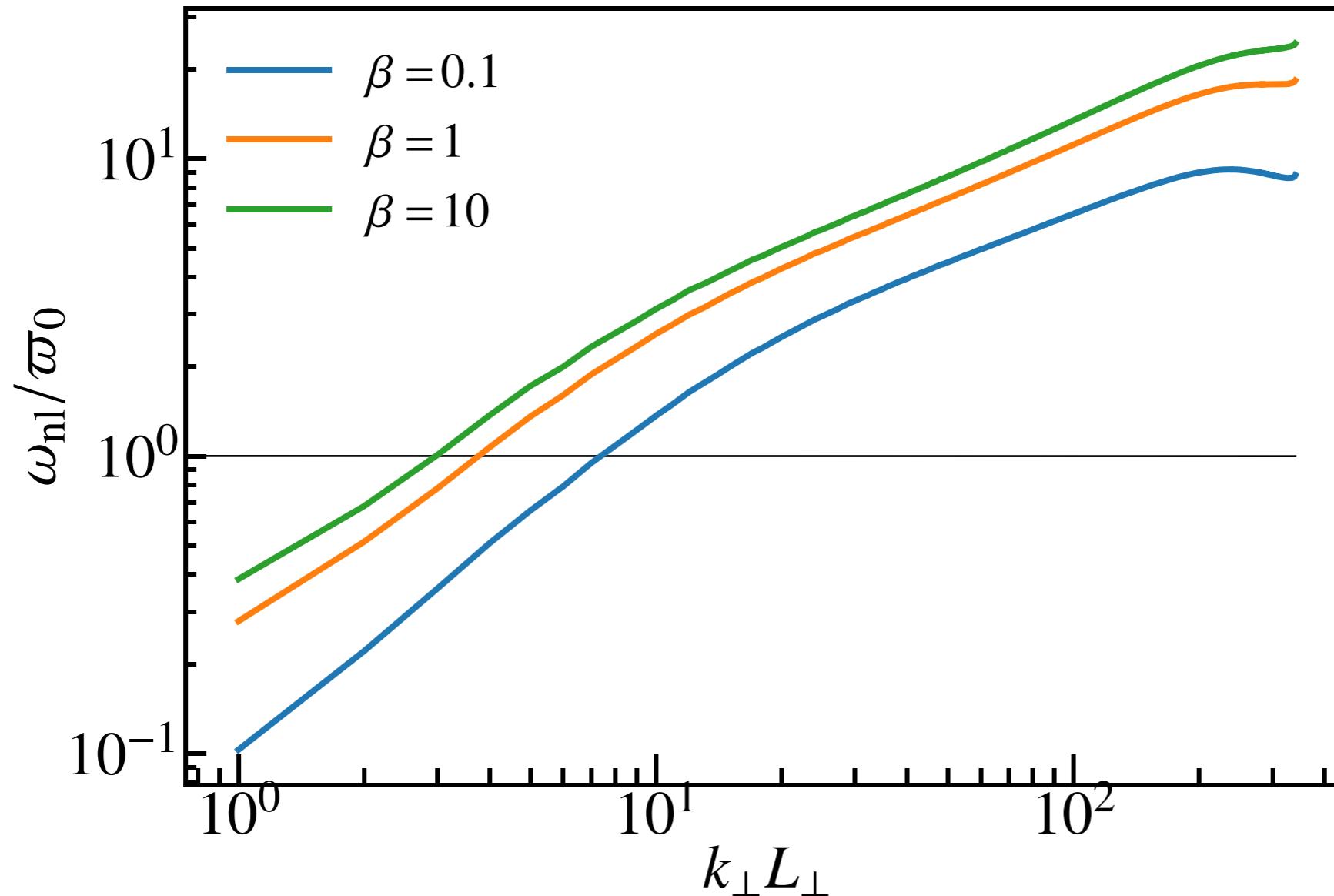
$$\left( \frac{d}{dt} - q\varpi_0 x \cos \theta \frac{\partial}{\partial y} \right) u_{\parallel} = v_A^2 \nabla_{\parallel} \left( \frac{\delta B_{\parallel}}{B_0} \right) + (2-q)\varpi_0 \sin \theta \frac{\partial \Phi}{\partial y} \quad + \text{para resistivity}$$

$$\left( \frac{d}{dt} - q\varpi_0 x \cos \theta \frac{\partial}{\partial y} \right) \left( 1 + \frac{v_A^2}{c_s^2} \right) \frac{\delta B_{\parallel}}{B_0} = \nabla_{\parallel} u_{\parallel} + \frac{q\varpi_0 \sin \theta}{v_A} \frac{\partial \Psi}{\partial y} \quad + \text{para viscosity}$$

$$\rightarrow \frac{dW}{dt} = I_{AW} + I_{compr} - D_{AW} - D_{compr}$$

- ▶ Compute  $D_{\text{compr}}/D_{\text{AW}}$  at the MRI saturated state
- ▶ Fix  $\theta = 0.45\pi$  and scan  $\beta = 0.1, 1, 10$

# Shear rotation vs nonlinear cascade



RMHD ( $\varpi_0 \ll \omega_{\text{nl}}$ )  
AW and compressive are decoupled

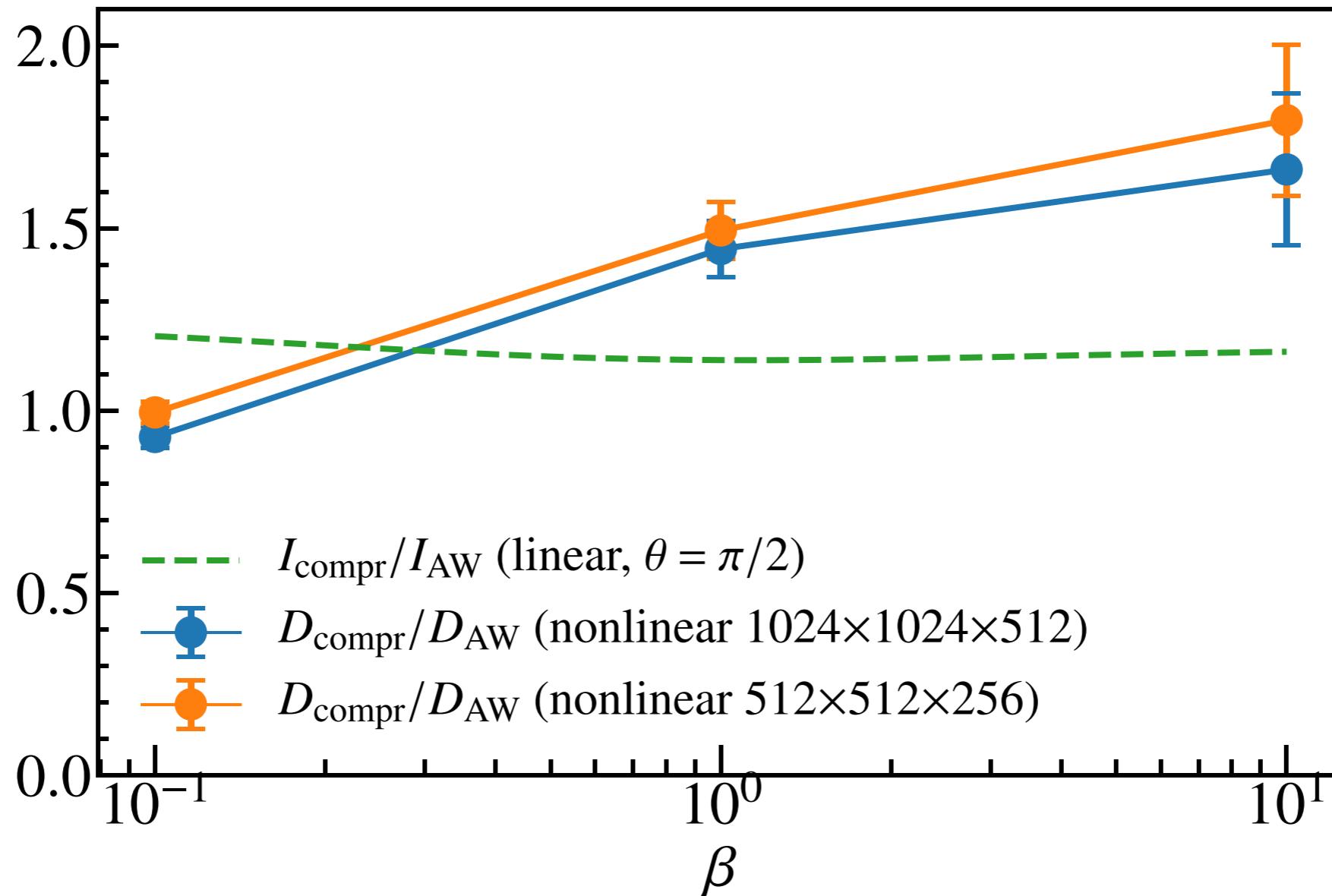
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SHRMHD ( $\varpi_0 \gg \omega_{\text{nl}}$ )  
AW and compressive are coupled

- Our simulations are resolving the scale at which  $\varpi_0 \ll \omega_{\text{nl}}$  holds.
- Therefore, compressive and Alfvénic fluctuations are decoupled.

# $D_{\text{compr}}/D_{\text{AW}}$ vs $\beta$

[YK+ in prep]

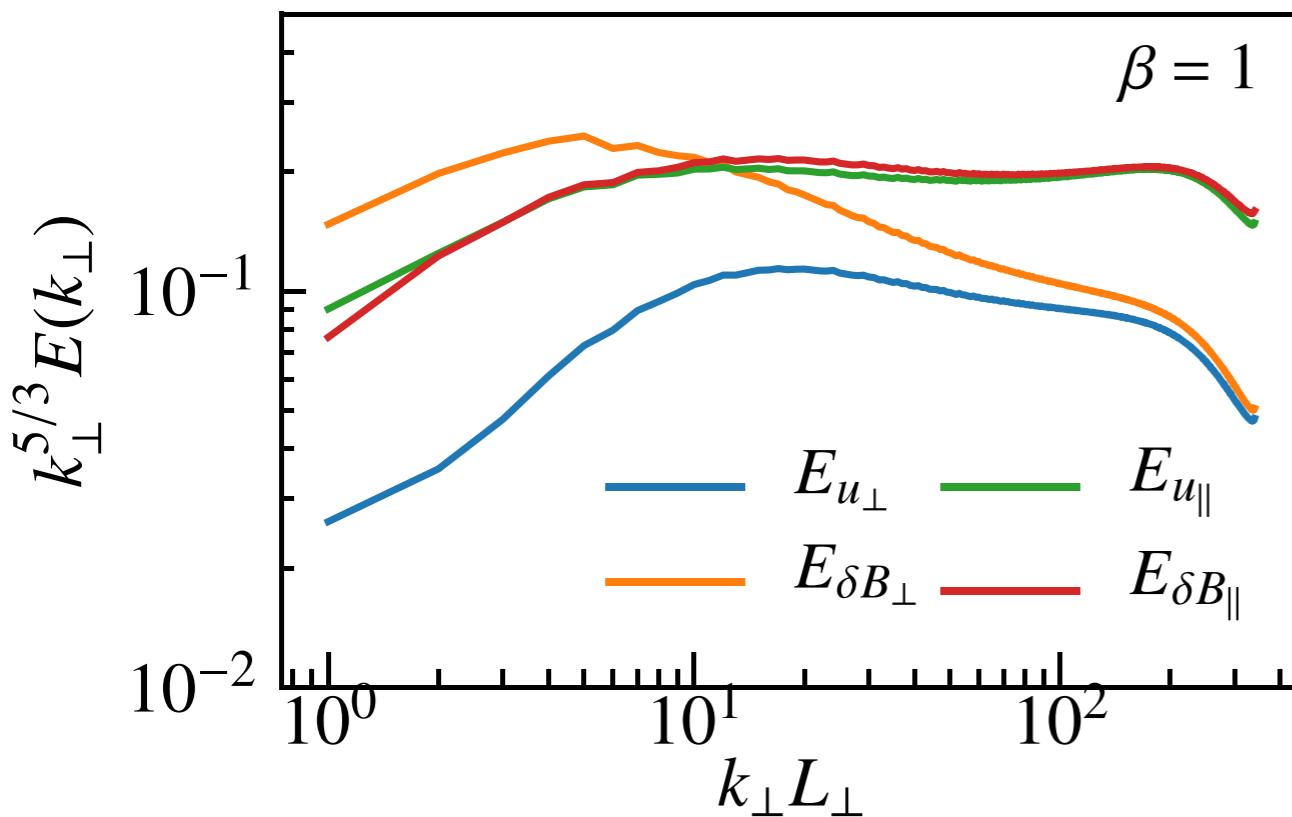


Linear & nonlinear calculations show  $D_{\text{compr}}/D_{\text{AW}} \approx 1$ .

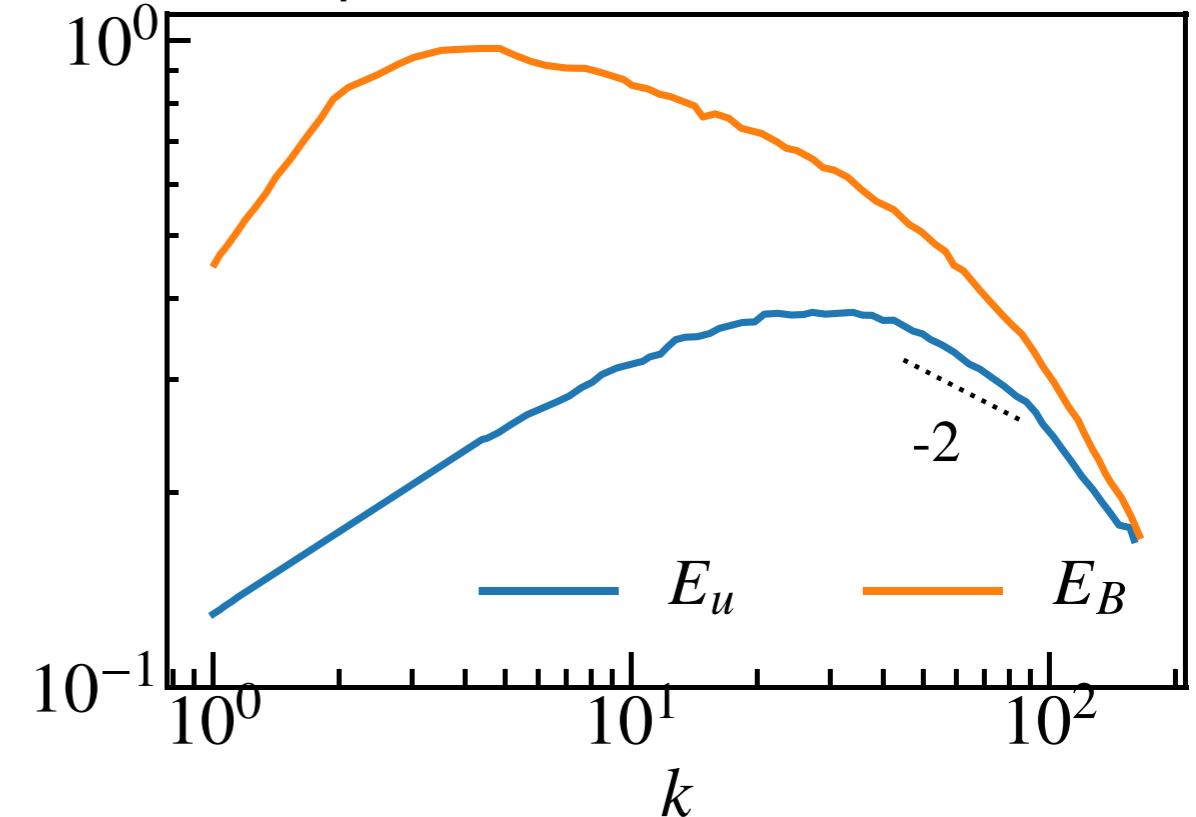
$$\frac{Q_i}{Q_e}(\beta_i, \tau, \varphi) = \frac{35}{1 + (\beta_i/15)^{-1.4} e^{-0.1/\tau}} + \varphi \left( \frac{\varphi}{\varphi + \beta_i} + \frac{2\beta_i}{1 + \varphi^{1.5}/2} \right) \quad \text{with } \varphi = 1 \text{ is the answer!}$$

# $k_{\perp}$ -spectrum

Our SHRMHD (1024x1024x512 grids)



Full-MHD (1024x1024x512 grids)  
Reproduced from Walker+ 2016



- Both Alfvén & Slow looks  $k_{\perp}^{-5/3}$   
→ RMHD regime is reached.

- In full-MHD,  $k_{\perp}^{-5/3}$  is not achieved.

If you want a -5/3 spectrum, 1024 grid is not enough, i.e., calculating  $D_{\text{compr}}/D_{\text{AW}}$  for a general setting (e.g.,  $\theta \sim 0$ ) using full-MHD is difficult.

# Plan of the talk

- ▶ Background and a research question  
*What sets the ion-to-electron heating ratio in hot accretion disks?*
- ▶ Possible mechanisms for ion and electron heating study (review)
- ▶ Gyrokinetic approach for turbulent heating
- ▶ Connection between gyrokinetics and magnetohydrodynamics
- ▶ Summary

# Summary

- ▶ Ion vs electron heating is critical for interpreting the EHT data.
- ▶ We used gyrokinetics.
  - GK allows us to solve kinetic turbulence with reasonable cost.
  - Obtained a heating prescription:

$$\frac{Q_i}{Q_e}(\beta_i, \tau, \varphi) = \frac{35}{1 + (\beta_i/15)^{-1.4} e^{-0.1/\tau}} + \varphi \left( \frac{\varphi}{\varphi + \beta_i} + \frac{2\beta_i}{1 + \varphi^{1.5}/2} \right)$$

- ▶ To get  $P_{\text{compr}}/P_{\text{AW}}$ , we developed SHRMHD, and only for toroidal MRI,  $P_{\text{compr}}/P_{\text{AW}} \approx 1$ .

**Interdisciplinarity (lab – space – astro) is the key!**

