Impact of massive neutrinos on nonlinear matter power spectrum

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We present an approach to describe the nonlinear matter power spectrum for a mixed dark matter (cold dark matter plus neutrinos having total mass of ~ 0.1eV) model based on cosmological perturbation theory. The suppression of the power spectrum amplitude caused by massive neutrinos is enhanced in the weakly nonlinear regime where standard linear theory ceases to be accurate. Due to this enhanced effect and the gain in the applicable range of the model prediction, the nonlinear model may enable a precision of $\sigma(m_{\nu,\mathrm{tot}}) \sim 0.09$eV in constraining the total neutrino mass for the planned galaxy redshift survey, a factor 2.5 improvement compared to the linear regime. The refined model prescription thus offers a vital opportunity to determine the neutrino masses.

Introduction: The relic neutrinos having finite masses cause a characteristic suppression of the growth of structure formation on scales below the neutrino free-streaming scale [1]. Exploring this suppression signature from a galaxy redshift survey, most conventionally via the galaxy power spectrum, can be a powerful way of constraining or potentially *determining* the neutrino masses [2]. In fact, the existing galaxy surveys such as the Sloan Digital Sky Survey (SDSS) and the 2dF Galaxy Redshift Survey, combined with the Wilkinson Microwave Anisotropy Probe (WMAP), have already provided a more stringent upper bound on the total neutrino mass, $m_{\nu,\mathrm{tot}} \lesssim 0.6$eV [3, 4] than the terrestrial experiments, $m_{\nu,\mathrm{tot}} \lesssim 2$eV [5]. Planned high-redshift galaxy surveys such as the Wide-Field Multi-Object Spectrograph (WFMOS) survey [6] conducted with the 8.2m Subaru Telescope further allow a more precise measurement of the galaxy power spectrum and therefore will continue to improve the cosmological sensitivity to the neutrino masses (e.g., [7]).

However, most of the previous work on the subject has been based on linear perturbation theory for a mixed dark matter (MDM) model (see [8] for a review). Even at scales as large as $\sim 100h^{-1}$Mpc relevant for the neutrino free-streaming scale, recent studies based on numerical techniques or perturbation theory have shown that the impact of nonlinear clustering cannot be ignored for high-precision future surveys, where these studies focused on the nonlinear effect on the baryon acoustic oscillations (BAOs) in the power spectrum [9, 10]. Yet, the effects of massive neutrinos are ignored in these studies, even though the neutrinos with total mass $\gtrsim 0.06$eV (implied from the oscillation experiments) make a $\gtrsim 5\%$ suppression in the power spectrum amplitude that surpasses the expected measurement accuracy ($\sim 1\%$) at each wavenumber band for future surveys. The neutrino suppression may also degrade the ability of BAO experiments for constraining the nature of dark energy as the neutrino effect appears at very similar scales to BAOs.

In this *Letter*, we develop a new approach to study the nonlinear power spectrum for the MDM model, based on perturbation theory (PT). We will then study the impact of massive neutrinos on nonlinear clustering, and discuss how the use of the PT model may allow an improved constraint on the neutrino masses for future surveys, particularly focused on the WFMOS survey.

Methodology: We will throughout focus on the evolution of total matter density perturbations: $\delta_{\mathrm{m}} \equiv (\delta\rho_{\mathrm{c}} + \delta\rho_{\mathrm{b}} + \delta\rho_\nu)/\bar{\rho}_{\mathrm{m}} = f_{\mathrm{cb}}\delta_{\mathrm{cb}} + f_\nu\delta_\nu$, where the subscripts 'm', 'c', 'b', 'ν' and 'cb' stand for total matter, cold dark matter (CDM), baryons, massive neutrinos, and CDM plus baryons, respectively, and δ_{cb} and δ_ν denote their density perturbations. The coefficients, f_{cb} and f_ν, are the fractional contributions to the matter density, Ω_{m0}: $f_\nu \equiv \Omega_{\nu 0}/\Omega_{\mathrm{m0}} = m_{\nu,\mathrm{tot}}/(94.1\Omega_{\mathrm{m0}}h^2 \text{ eV})$ and $f_{\mathrm{cb}} = 1 - f_\nu$. Then the total matter power spectrum, $P_{\mathrm{m}}(k)$, is defined as

$$P_{\mathrm{m}}(k) = f_{\mathrm{cb}}^2 P_{\mathrm{cb}}(k) + 2f_{\mathrm{cb}}f_\nu P^L_{\mathrm{cb},\nu}(k) + f_\nu^2 P^L_\nu(k), \quad (1)$$

where the power spectra with the superscript 'L' denote the linear-order spectra and $P^L_{\mathrm{cb},\nu}$ is the cross spectrum between δ_{cb} and δ_ν. Mixture of the neutrinos in total matter affects the nonlinear power spectrum as follows. The neutrinos would tend to remain in the linear regime rather than going into the nonlinear stage together with CDM and baryon, due to the large free-streaming. In addition, the prefactor f_ν is likely to be small for a realistic model (e.g. $f_\nu \lesssim 0.07$ in [4]), allowing the nonlinear corrections of the neutrino perturbations to be approximately ignored. In the following we will thus include only the linear-order neutrino perturbations, which can be accurately computed by solving the linearized Boltzmann equation [11]. The validity of our assumption will be demonstrated in [12], in which the correction arising from nonlinear neutrino clustering is shown to be very small, $\lesssim 0.02\%$ in the power spectrum amplitude on scales of interest.

Following the standard PT approach [13], the CDM

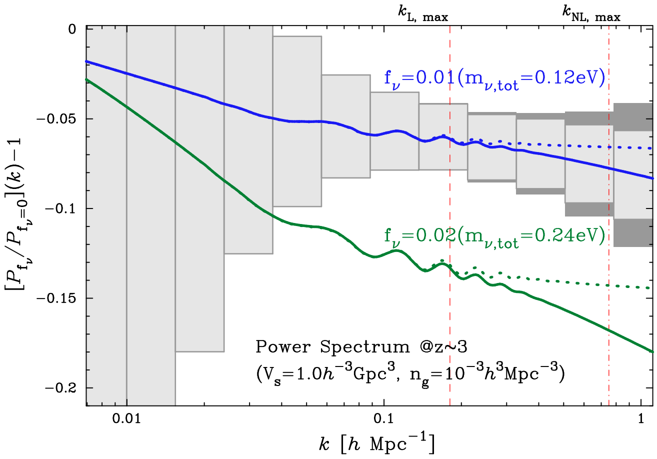

FIG. 1: Fractional difference between the mass power spectra at redshift $z = 3$ with and without the massive neutrino contributions, where the two cases $f_\nu = 0.01$ and 0.02 are considered. The solid and dotted curves show the PT and linear theory results, respectively. The two vertical lines indicate a maximum wavenumber limit $k_{\rm max}$ up to which the two models are expected to be valid (see text). The shaded boxes show the expected 1σ errors on the power spectrum measurement for the $z \sim 3$ WFMOS survey and the case of $f_\nu = 0.01$.

plus baryon component can be treated as a pressure-less and irrotational fluid for the scales of interest. Then the fluid equations for mass and momentum conservation and the Poisson equation fully describe the dynamics of the density perturbation field, $\delta_{\rm cb}$, and the velocity divergence field, $\theta_{\rm cb} \equiv \nabla\cdot\boldsymbol{v}_{\rm cb}/(aH)$. The solutions to this system can be obtained by making a perturbative expansion, $\delta_{\rm cb} = \delta_{\rm cb}^{(1)} + \delta_{\rm cb}^{(2)} + \delta_{\rm cb}^{(3)} + \cdots$ and $\theta = \theta^{(1)} + \theta^{(2)} + \theta^{(3)} + \cdots$, where the superscript '(i)' denotes the i-th order perturbation. In our setting, the nonlinear correction to the total matter power spectrum $P_{\rm m}(k)$ arises only through $P_{\rm cb}(k)$ in Eq. (1). The nonlinear $P_{\rm cb}$ including the next-to-leading order corrections is expressed as

$$P_{\rm cb}(k;z) = P_{\rm cb}^L + P_{\rm cb}^{(13)} + P_{\rm cb}^{(22)}\ , \qquad (2)$$

where the last two terms describe the nonlinear corrections, the so-called one-loop corrections, that include contributions up to the third-order perturbations.

The neutrinos affect the spectrum $P_{\rm cb}$ through the effect on the linear growth rate [14]. At wavenumbers smaller than the neutrino free-streaming scale, $k_{\rm fs}(z) \simeq 0.023(m_\nu/0.1{\rm eV})[2/(1+z)]^{1/2}(\Omega_{\rm m0}/0.23)^{1/2}\,h{\rm Mpc}^{-1}$, the neutrinos can cluster together with CDM and baryon. Conversely, at $k > k_{\rm fs}$, the growth rate of CDM perturbations is suppressed due to the weaker gravitational force caused by the lack of neutrino perturbations. Thus the growth rate, $D_{\rm cb}(z,k)$, has a characteristic scale-dependence for the MDM model. This fact causes one complication in computing the second- and third-order solutions for $\delta_{\rm cb}$ and $\theta_{\rm cb}$. The k-dependence of $D_{\rm cb}$ causes an additional mode-coupling between the perturbations of different wavenumbers. Interestingly, we have found that, using the analytic fitting formula for $D_{\rm cb}$ in [14], this additional mode-coupling can be safely ignored for the expected small value of f_ν [12]. As a result, the nonlinear spectra, $P_{\rm cb}^{(22)}$ and $P_{\rm cb}^{(13)}$, are simply expressed as

$$P_{\rm cb}^{(22)}(k;z) = \frac{k^3}{98(2\pi)^2}\int_0^\infty dr P_{\rm cb}^L(kr;z) \times \int_{-1}^{1} d\mu P_{\rm cb}^L(k\sqrt{1+r^2-2r\mu};z)\frac{(3r+7\mu-10r\mu^2)^2}{(1+r^2-2r\mu)^2},$$
$$P_{\rm cb}^{(13)}(k;z) = \frac{k^3 P_{\rm cb}^L(kr;z)}{252(2\pi)^2}\int_0^\infty dr P_{\rm cb}^L(kr;z)\left[\frac{12}{r^2} - 158 + 100r^2 - 42r^4 + \frac{3}{r^2}(r^2-1)^3(7r^2+2)\ln\left|\frac{1+r}{1-r}\right|\right].(3)$$

Note that $P_{\rm cb}^{(22)}$ and $P_{\rm cb}^{(13)}$ are roughly proportional to the square of $P_{\rm cb}^L$, which enhances the neutrino effect in the nonlinear regime, compared to the linear case, $P_{\rm cb}^L$.

Results: Eqs. (1) and (3) show that the PT prediction for $P_{\rm m}(k)$ at a given redshift can be computed once the linear spectra, P_{cb}^L, $P_{cb,\nu}^L$ and P_ν^L, are specified. We use the CAMB code [15] to compute the input linear spectra for a given MDM model.

Fig. 1 shows the fractional difference between the power spectra at redshift $z = 3$ with and without massive neutrino contributions, where the two cases $f_\nu = 0.01$ and 0.02 are considered and other parameters are fixed to their fiducial values (see below). Several interesting results can be found from this plot. First, the massive neutrinos induce a characteristic k-dependent suppression in the spectrum amplitude. For the case of linear theory, the suppression becomes nearly independent of k at small scales, $k \gg k_{\rm fs}$, as roughly given by $\Delta P/P \sim -8f_\nu$ [2]. In contrast the PT result demonstrates that the neutrino suppression is *enhanced* in the nonlinear regime, yielding a new k-dependence in the spectrum shape.

Second, comparing the linear theory and PT results explicitly tells the limitation of the linear theory: the linear theory no longer gives an accurate prediction at $k \gtrsim 0.2h{\rm Mpc}^{-1}$. More precisely, the linear theory result starts to deviate from the PT result at $k \gtrsim k_{\rm L,max} = 0.18h{\rm Mpc}^{-1}$ by $\gtrsim 1\%$ in the amplitude, as denoted by the vertical dotted line[1]. However PT also breaks down at scales greater than a certain maximum wavenumber limit, $k_{\rm NL,max}$, due to a stronger mode-coupling arising from the higher-order perturbations ignored here. Using N-body simulations for a CDM model, [9] showed that the one-loop PT well matches the simulation results

[1] In Fig. 1, the deviation of dashed curve (linear) and solid curve (PT) around $k_{\rm L,max}$ looks seemingly small due to the fact that the plot shows the fractional difference $P_{f_\nu}/P_{f_\nu=0}$ and the denominator $P_{f_\nu=0}$ is also computed from the PT.

up to $k_{\rm NL,max}$ given by the condition $\Delta^2(k_{\rm NL,max},z) \equiv k^3 P_m(k,z)/2\pi^2|_{k=k_{\rm NL,max}} \simeq 0.4$. We simply adopt this criterion for a MDM model and the resulting $k_{\rm NL,max}$ is indicated by the vertical dot-dashed line. Thus, in the case of $z\sim 3$, the PT model may allow a factor 4 gain in $k_{\rm max}$; observationally, this is roughly equivalent to a factor $64(=4^3)$ gain in independent Fourier modes of the density perturbations probed for a fixed survey volume, which in turn improves the precision of the power spectrum measurement if the measurement errors are limited by the sampling variance rather than the shot noise.

Can a future survey be precise enough to measure the neutrino effect? This question is partly answered in Fig. 1. The light-gray shaded boxes around the solid curve show the 1-σ measurement errors on $P(k)$ at each k bin, expected for the $z\sim 3$ WFMOS survey (see below). The neutrino suppression appears to be greater than the errors at $k\gtrsim 0.03h{\rm Mpc}^{-1}$. Another intriguing consequence of the nonlinear clustering is that the amplified power of $P_m(k)$ reduces the relative importance of the shot noise contamination to the measurement errors, leading the errors to be more in the sampling variance limited regime. This can be seen by the dark-gray shaded boxes showing the 1-σ errors for the linear spectrum.

Finally it will be worth noting that the wiggles in the curves reflect shifts in the BAO peak locations caused by the scale-dependent suppression effect due to the neutrinos. The amount of the modulations is smaller than the measurement errors. Hence the uncertainty in neutrino mass is unlikely to largely degrade the power of BAO experiments, at least for an expected small f_ν [12].

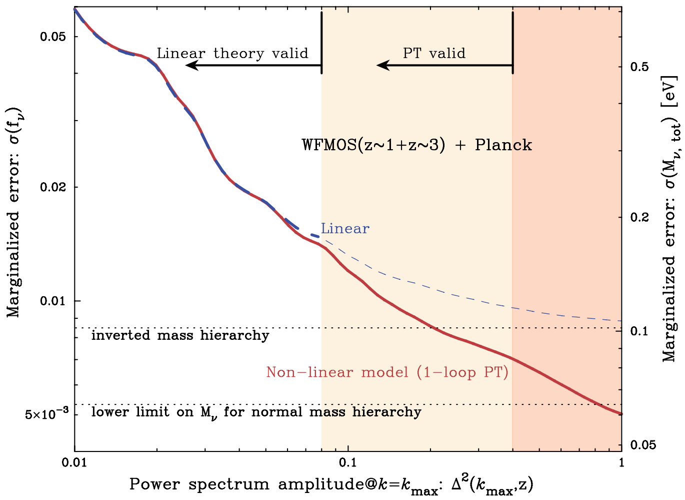

FIG. 2: The marginalized 1σ error on the total neutrino mass as a function of the maximum wavenumber $k_{\rm max}$ used in each redshift slice (see text), for the WFMOS survey combined with the minimal CMB constraints. The fiducial value of $f_\nu=0.01$ is assumed. The solid and dashed curves show the results for the PT and linear theory models, respectively. The light and dark shaded regions represent the range of k where the linear theory and the one-loop PT break down due to the stronger nonlinearities.

Parameter forecasts: To realize the genuine power of future surveys for constraining the neutrino masses, we have to carefully take into account parameter degeneracies [7]. Here, we estimate accuracies of the neutrino mass determination using the Fisher matrix formalism.

The observable we consider is the two-dimensional galaxy power spectrum given as a function of $k_\parallel$ and $k_\perp$, the wavenumbers parallel and perpendicular to the line-of-sight direction [16]:

$$P_s(k_{{\rm fid}\parallel},k_{{\rm fid}\perp}) = \frac{D_A(z)^2_{\rm fid}H(z)}{D_A(z)^2H(z)_{\rm fid}}\left[1+\beta\mu^2\right]^2 b_1^2 P_{\rm m}(k,z) \tag{4}$$

where $k=(k_\perp^2+k_\parallel^2)^{1/2}$ and $\mu=k_\parallel/k$. Here, $k_\perp = [D_A(z)_{\rm fid}/D_A(z)]k_{{\rm fid}\perp}$ and $k_\parallel = [H(z)_{\rm fid}/H(z)]k_{{\rm fid}\parallel}$, where $D_A(z)$ and $H(z)$ are the comoving angular diameter distance and Hubble parameter, respectively. The quantities with the subscript 'fid' denote the quantities estimated assuming a fiducial cosmological model, which generally differs from the underlying true model. In the equation above, we simply assumed the linear galaxy bias b_1 and the linear redshift distortion β. However we will instead treat b_1 and β as free parameters for the parameter forecasts shown below, in order not to derive too optimistic forecasts. This treatment would be adequate for our current purpose, which is to estimate how PT allows an improvement in the parameter constraints mainly caused by the gain in $k_{\rm max}$. A more careful analysis will be presented in detail in [12].

Following [16], the Fisher matrix for the galaxy power spectrum measurement is computed as $F_{\alpha\beta} = \int_{-1}^{1} d\mu \int_{k_{\rm min}}^{k_{\rm max}} 2\pi k^2 dk (\partial P_s/\partial p_\alpha){\rm Cov}^{-1}(\partial P_s/\partial p_\beta)$, where p_α represents a set of parameters and ${\rm Cov}^{-1}$ is the inverse of the covariance matrix that depends on the power spectrum itself and on survey parameters, the comoving survey volume and the number density of galaxies. To compute $F_{\alpha\beta}$ we need to specify the integration range $k_{\rm min}$ and $k_{\rm max}$; we will throughout employ $k_{\rm min} = 10^{-4}h{\rm Mpc}^{-1}$ and below discuss for the choice of $k_{\rm max}$. Note that, for several redshift slices, we simply add the Fisher matrix for each slice to obtain the total Fisher matrix. Then, the 1σ error on a certain parameter p_α marginalized over other parameters is given by $\sigma^2(p_\alpha) = ({\bf F}^{-1})_{\alpha\alpha}$, where ${\bf F}^{-1}$ is the inverse of Fisher matrix. We assume the WFMOS survey parameters given in [16] consisting of two types of redshift surveys: the $z\sim 1$ survey covering $0.5\le z\le 1.3$ with 2000 deg^2 and the $z\sim 3$ survey covering $2.5\le z\le 3.5$ with 300 deg^2. We consider 5 redshift slices. The choice of free parameters is also important for the Fisher matrix formalism: we include a fairly broad range of the model parameters given by $p_\alpha = \{\Omega_{\rm m0},\Omega_{\rm m0}h^2,\Omega_{\rm b0}h^2,f_\nu,n_s,\alpha_s,\Delta^2_{\mathcal R},w,\beta(z_i),b_1(z_i)\}$, where n_s, α_s and $\Delta^2_{\mathcal R}$ are the primordial power spectrum parameters (tilt, running and the normalization parameter) and w is the dark energy equation of state. Note that we assume three neutrino species that are totally mass degenerate. As for the fiducial model we assume $f_\nu = 0.01$ and adopt the WMAP results for a flat ΛCDM model to fix other parameters. The fiducial $\beta(z_i)$ and $b_1(z_i)$ for the i-th redshift slice are computed following [16]. In total we include 18 free parameters.

Fig. 2 demonstrates the marginalized 1-σ errors on the total neutrino mass as a function of $k_{\rm max}$, where the galaxy power spectrum over a range of $k_{\rm min} \leq k \leq k_{\rm max}$ is included. The value of $k_{\rm max}$ for each redshift slice is specified by inverting $\Delta^2(k_{\rm max}; z_i)$ for the given value in the horizontal axis. The errors shown here are for the WFMOS survey combined with the CMB information on cosmological parameters except for the neutrino masses, f_ν, and the dark energy parameter, w. The solid and dashed curves show the results for the PT and linear theory, respectively. If the linear theory is employed, a reliable accuracy to be obtained is $\sigma(m_{\nu,{\rm tot}}) \simeq 0.2$eV in order not to have the biased constraint due to the inaccurate model prediction [12]. On the other hand, if the PT prediction is valid up to $\Delta^2(k_{\rm max}) \simeq 0.4$ as discussed in Fig. 1, the accuracy of $\sigma(m_{\nu,{\rm tot}}) \simeq 0.086$eV may be attainable, a factor 2.5 improvement. Encouragingly, this improved constraint is between the two lower limits of the normal and inverted mass hierarchies, as denoted by the horizontal dotted lines.

It should be also noted that a wide redshift coverage for the planned WFMOS survey is very efficient in breaking parameter degeneracies, especially between the neutrino mass and the dark energy parameters [17, 18], because the dark energy is likely to affect gravitational clustering only at low redshifts, $z \lesssim 1$. To be more precise, the correlation coefficient $r(f_\nu, w) \equiv (\mathbf{F^{-1}})_{f_\nu w}/\sqrt{(\mathbf{F^{-1}})_{f_\nu f_\nu}(\mathbf{F^{-1}})_{ww}} \simeq -0.003$ for the PT result. This small correlation is partly due to the precise determination of the BAO peak locations in the measured power spectrum, allowing tight constraints on the dark energy parameters without using information on the amplitude. The neutrino mass is most degenerate with the bias parameter b_1, as given by $r(f_\nu, b_1) \sim 0.8$.

Discussion: It is of great importance to carefully study nonlinear structure formation for a most realistic model, i.e. a MDM model including ~ 0.1eV neutrinos, in preparation for future galaxy surveys. While the PT model developed in this *Letter* gives the first step in this direction, another complement to the analytic approach is to implement a hybrid N-body simulation consisting of cold and hot particles, which seems feasible with the advent of current numerical resources, by extending the pioneering work [19] for a model with ~ 10eV neutrinos. PT will also play a useful role in calibrating/checking the simulations results as done in the early stage of the simulation based studies for a CDM model.

We have demonstrated that the use of PT may enable an improvement in the neutrino mass constraint by a factor 2.5 compared to the case that linear theory is used, for the planned WFMOS survey. However our study involves several idealizations: most importantly we assumed the linear galaxy bias and the linear redshift distortion. At least for the large scales ~ 100Mpc considered here, it seems feasible to develop a self-consistent, accurate model to describe galaxy clustering observables including the non-linear effects on the galaxy bias and redshift distortions for a MDM model, by using the perturbation theory approach [20] and/or the halo model approach and by combining with simulations. Given such a model is obtained, including the large-scale redshift distortions whose strength varies with the galaxy bias may help break the degeneracies between the galaxy bias and the power spectrum amplitude [7, 21]. For a similar reason, combining the galaxy power spectrum and bispectrum may be another useful way to determine the galaxy bias [22]. Such a refined model to describe galaxy clustering observables in the weakly nonlinear regime would be worth exploring in order to exploit the full potential of the forthcoming galaxy surveys for constraining or even *determining* the neutrino masses.

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