Frontier Research Institute for Interdisciplinary Sciences
Tohoku University

Theoretical Astrophysics
Tohoku University

Current driving mechanism and role of the negative energies in Blandford-Znajek process

Toma & Takahara 2016, PTEP, 3E01 (arXiv:1605.03659)

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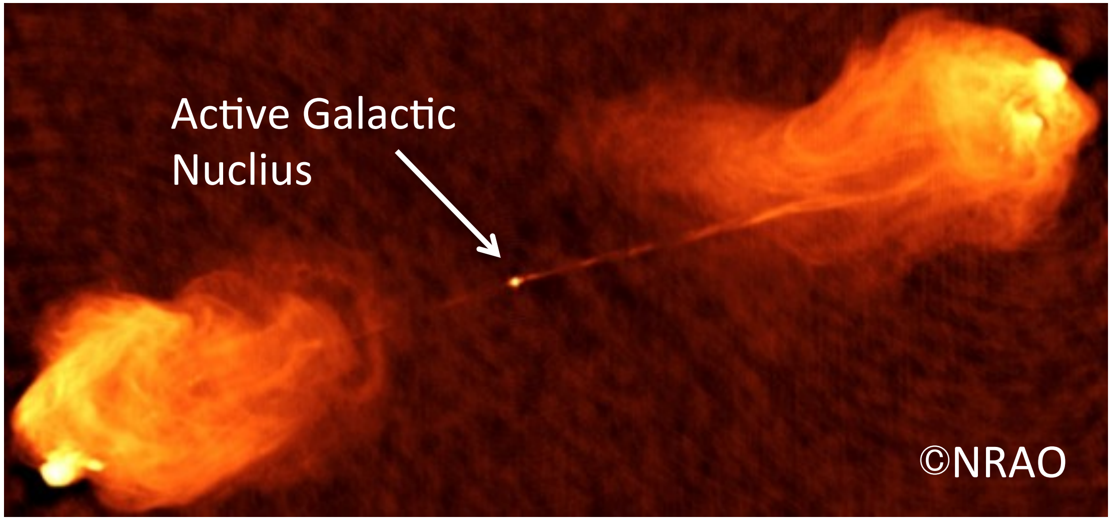
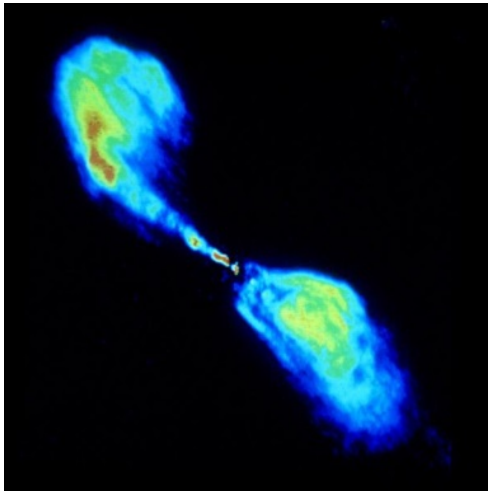
Levedev Physical Institute, Moscow, Mar 1, 2017

Outline

1. Introduction & issues
2. Kerr space-time
3. Field lines threading the equatorial plane in the ergosphere
4. Field lines threading the horizon
5. Conclusion

Black Hole (Relativistic) Jets

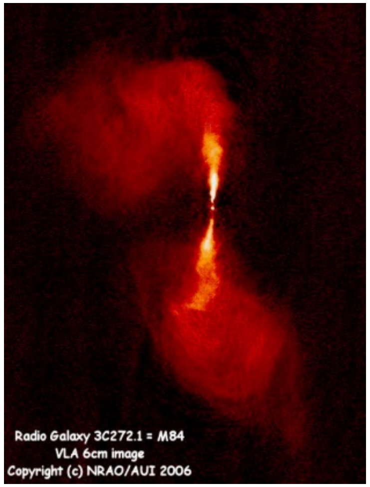




$$M_{\rm BH} \sim 10^7 - 10^9 \ M_\odot$$

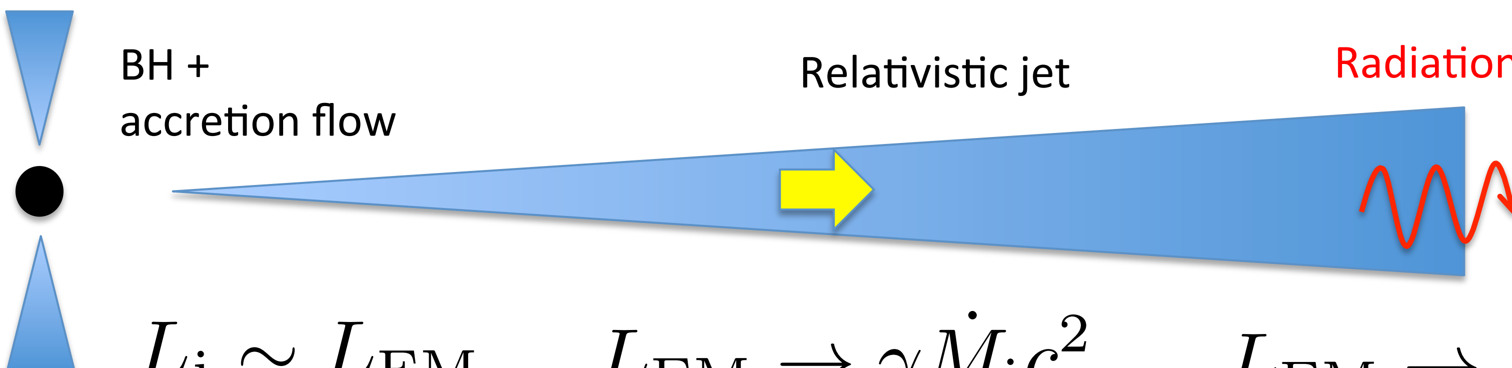
$$L_j \lesssim L_{\rm Edd} \simeq 10^{46} M_8 \ {\rm ergs}^{-1}$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = 10 - 100$$



Theoretical issues

$$\boxed{L_{\rm j} = L_{\rm th} + L_{\rm EM} + \gamma \dot{M}_{\rm j} c^2 = \text{const.}}$$



$L_{\rm j} \sim L_{\rm EM}$

or $L_{\rm th}$

$L_{\rm EM} \to \gamma \dot{M}_{\rm j} c^2$

$L_{\rm th} \to \gamma \dot{M}_{\rm j} c^2$

$L_{\rm EM} \to L_{\rm th}$

$\gamma \dot{M}_{\rm j} c^2 \to L_{\rm th}$

Driving mechanism

Acceleration/ collimation

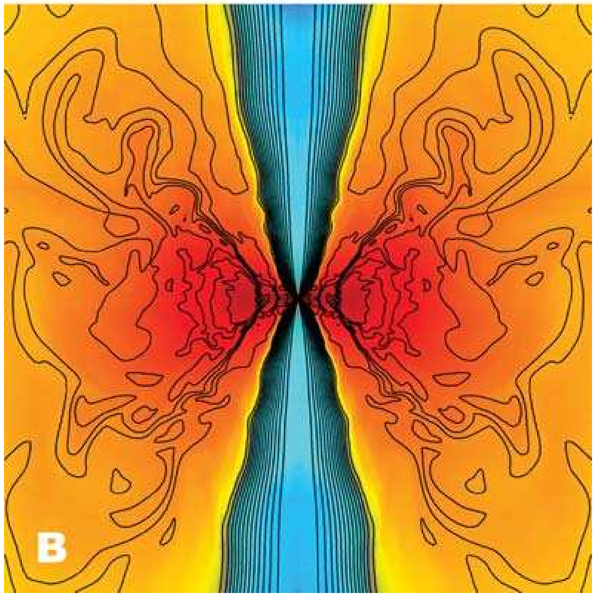
Dissipation mechanism

$L_{\rm j} \gg \dot{M}_{\rm j} c^2$

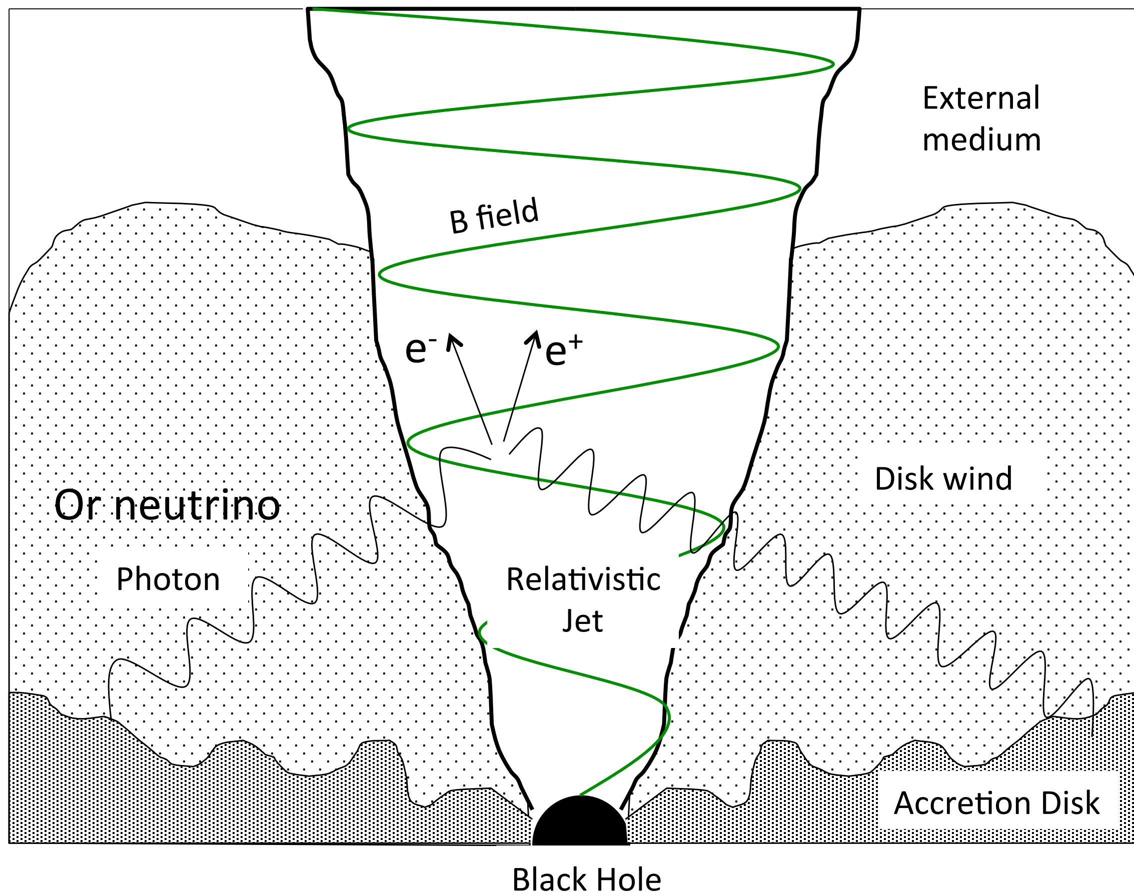
Stability

Mass loading mechanism

Electromagnetic jets



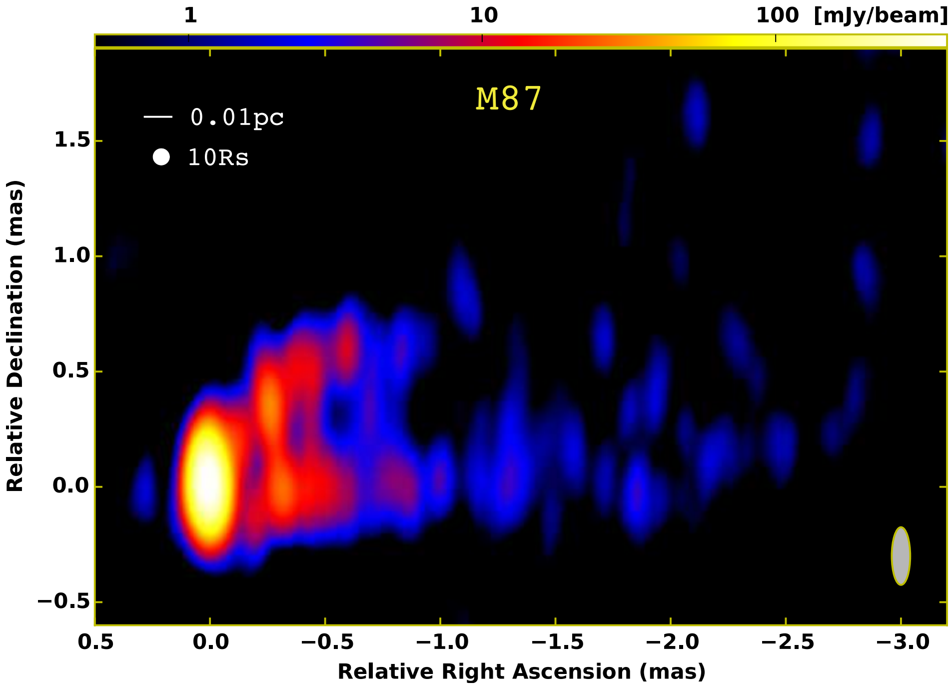
- GR MHD simulations (e.g. Komissarov 01; Koide+ 02; McKinney & Gammie 04; Tchekhovskoy+ 11)

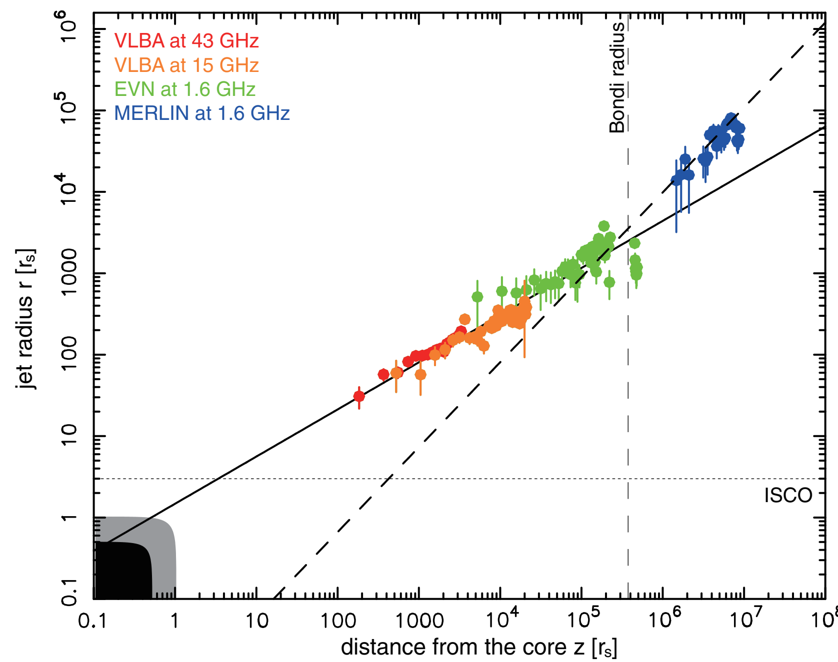


- Quasi-steady structure
- Extraction of BH energy
- $L_j \sim L_{EM}$ at the base
- Relativistic flow (γ > 10-100) around the axis
- Collimation by disk wind
- How to load mass ?

M87 jet

(Hada+ 2013; 2016)





The high-resolution radio imaging is being improved, revealing the structure and composition of M87 jet.

(Asada & Nakamura 12; Kino+15)

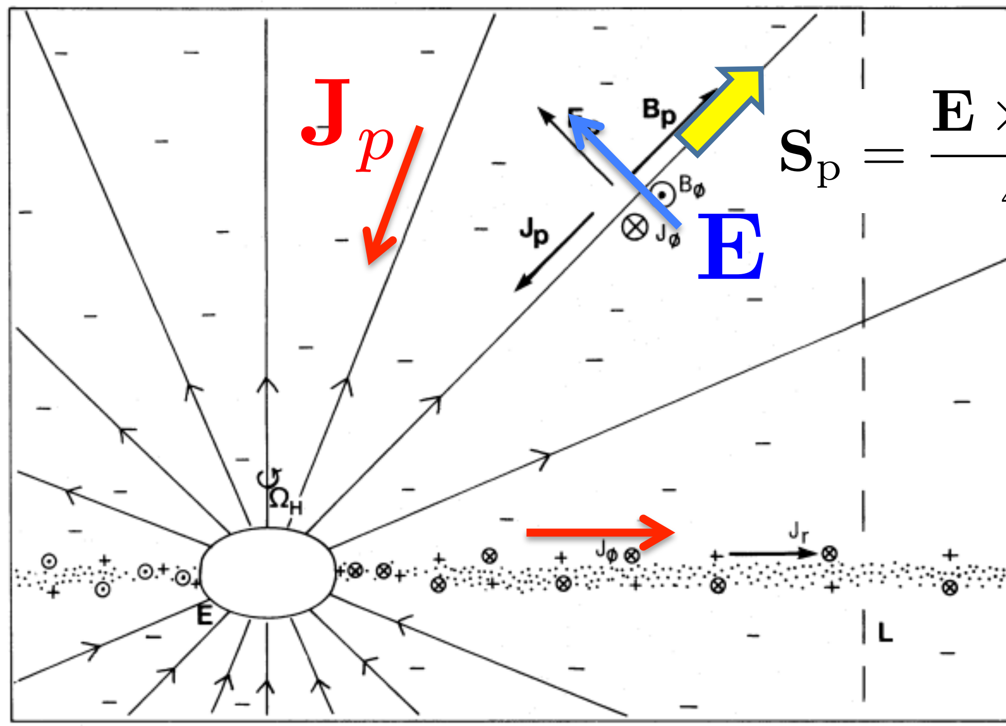
EHT+ALMA observation will be done in April, 2017.

Blandford & Znajek (1977)

- Slowly rotating Kerr BH

$$a = \frac{J}{Mr_g c} \ll 1$$

- Steady, axisymmetric
- Split-monopole B field
- Force-free approximation (Electromagnetically dominated)

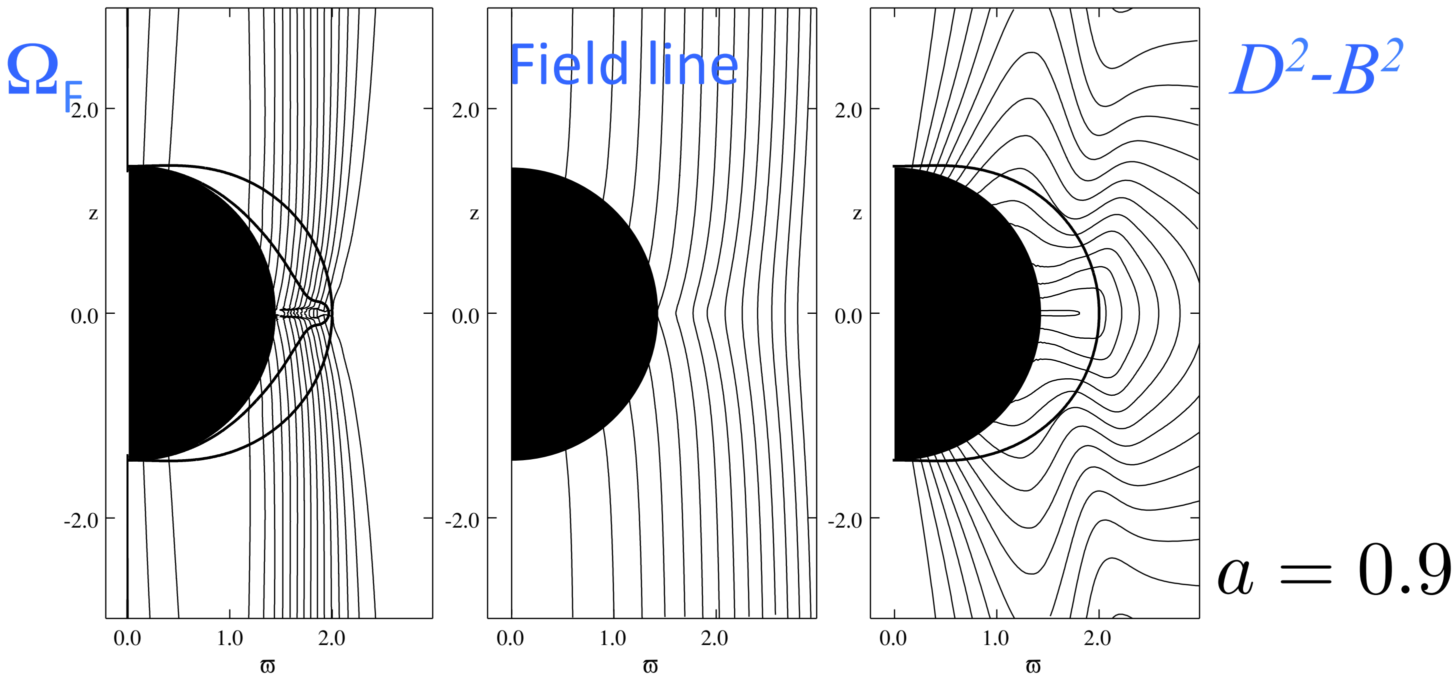


$$\mathbf{S}_\mathrm{p} = \frac{\mathbf{E} \times \mathbf{H}_\varphi}{4\pi}$$

(see also Beskin & Zheltoukhov 2013)

BZ process with large BH spin a

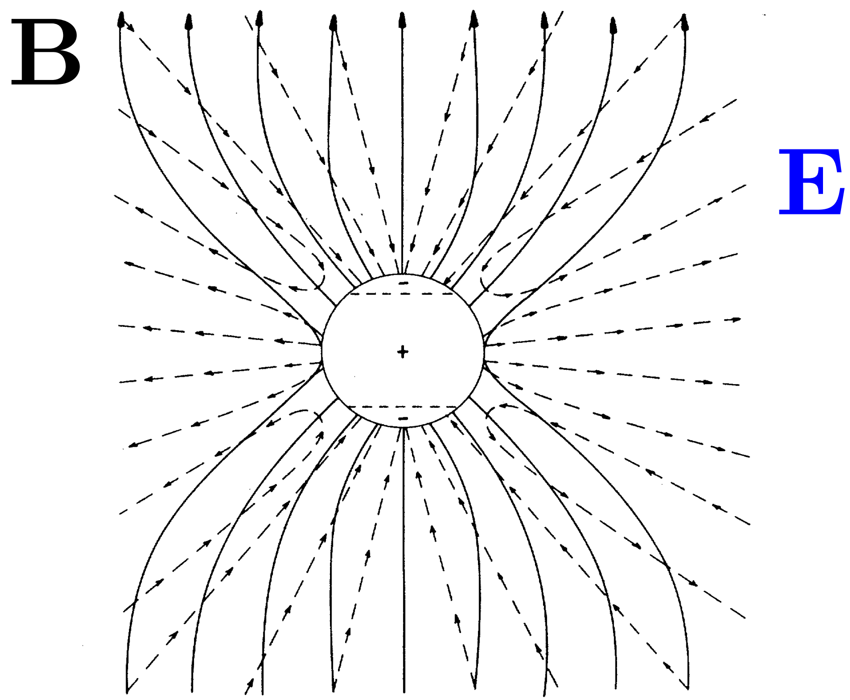
(Komissarov 2004)



- Many other FF/MHD numerical studies show BZ process works with large a. (e.g. Komissarov 01; Koide+ 02; McKinney & Gammie 04; Barkov & Komissarov 08; Tchekhovskoy+ 11; Ruiz+ 12; Contopoulos+ 13)

But the detailed mechanism of flux production is still debated

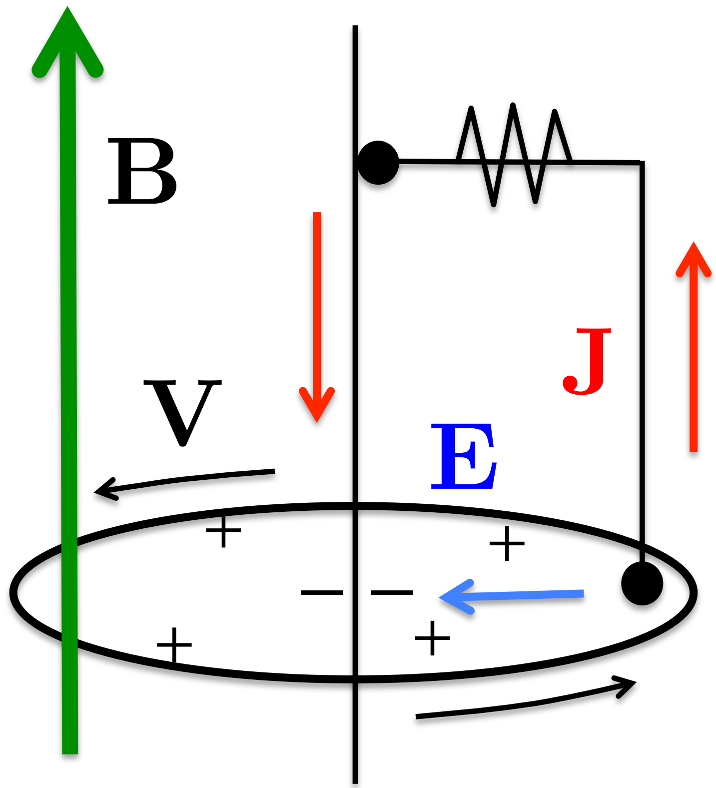
Vacuum Solution



(Wald 1974; Punsly & Coroniti 1989)

- Space-time rotation produces $\mathbb{E}$, but not B_ϕ
- B_ϕ is produced by J_p.

Unipolar induction process

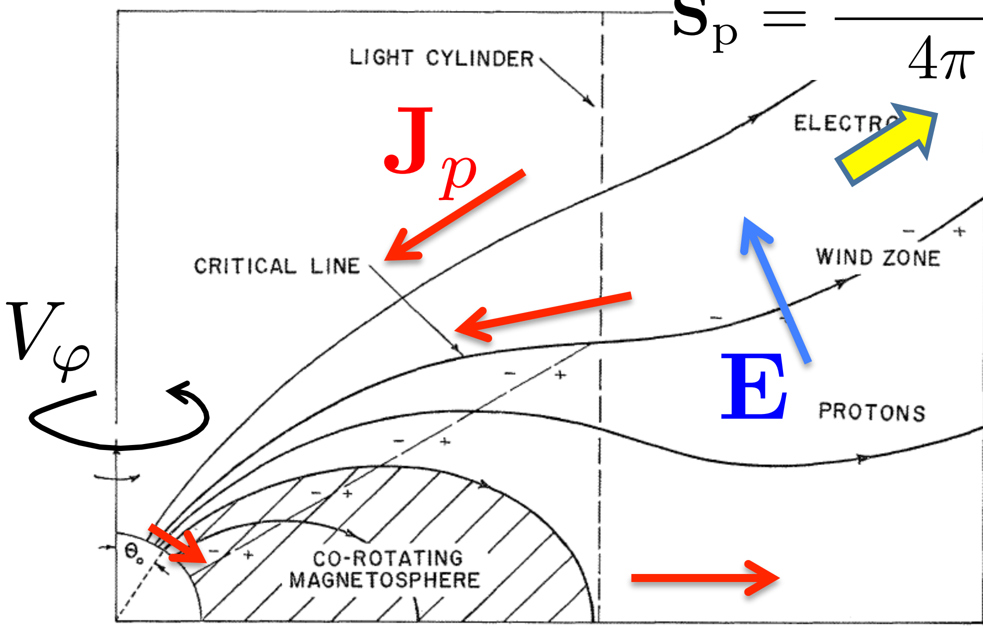


$$\boxed{\mathbf{E} = -\mathbf{V} \times \mathbf{B}}$$

Pulsar winds

(Goldreich & Julian 1969)

$$\mathbf{S}_\mathrm{p} = \frac{\mathbf{E} \times \mathbf{B}_\varphi}{4\pi}$$



$$\mathbf{E} = -\mathbf{V}_\varphi \times \mathbf{B}$$

Steady, axisymmetric

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla\phi$$

$$\Rightarrow E_\varphi = 0$$

Electric field screened by plasma

$$\mathbf{E} \cdot \mathbf{B} = 0$$

$$\Rightarrow \mathbf{E} = -\frac{r\Omega_\mathrm{F}}{c}\mathbf{e}_\varphi \times \mathbf{B}$$

Ω_F: constant along a field line

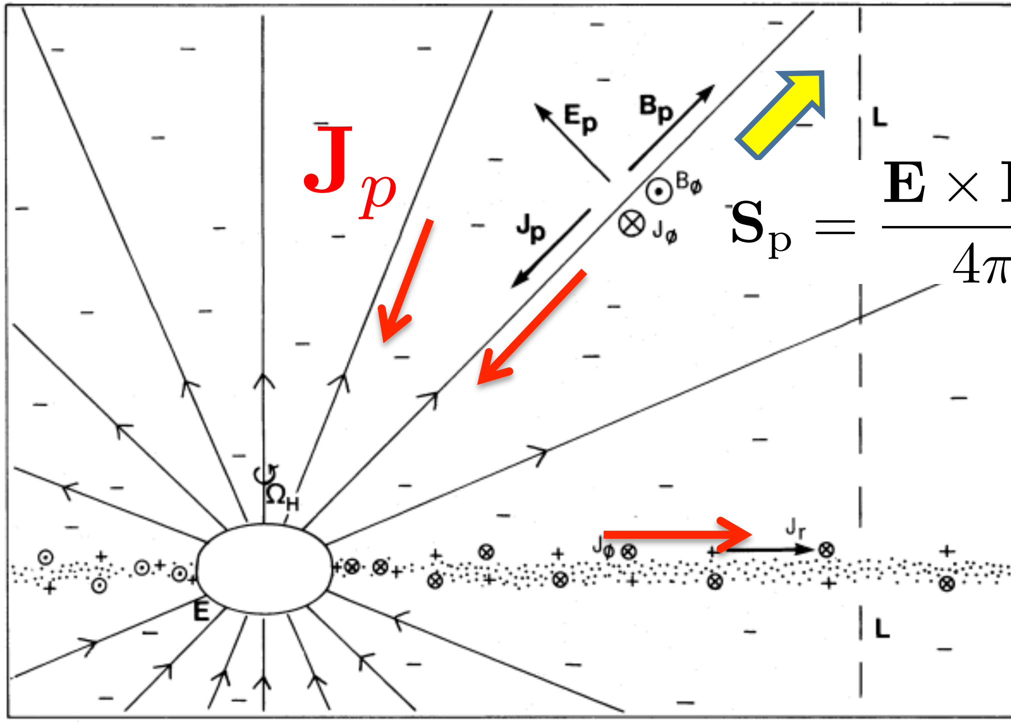
$$\Rightarrow \boxed{\Omega_\mathrm{F} = \Omega_\mathrm{pulsar}}$$

$$\boxed{\nabla \cdot \mathbf{S}_\mathrm{p} = -\mathbf{E} \cdot \mathbf{J}_\mathrm{p}}$$

Origin of electric potential & driving source of current are rotating star. Stellar rotation energy reduces as their feedbacks.

Blandford & Znajek (1977)

Condition at infinity $\boxed{H_\varphi = -2\pi\Omega_{\rm F} B^r \sqrt{\gamma}\sin\theta}$



$$\mathbf{S}_{\rm p} = \frac{\mathbf{E}\times\mathbf{H}_\varphi}{4\pi}$$

$\boxed{H_\varphi = 2\pi(\Omega_{\rm F}-\Omega_{\rm H})B^r\sqrt{\gamma}\sin\theta}$ At event horizon

In the MHD approximation, the conditions on the fast magnetosonic surfaces determine the solution (Beskin & Kuznetsova 00; Beskin 10)

- Kerr space-time
- Steady, axisymmetric
- Slowly rotating BH
- Split-monopole B field

$$B^r\sqrt{\gamma} = \text{const.}$$

- Force-free approximation (Electromagnetically dom.)

$$H_\varphi = \text{const.}$$

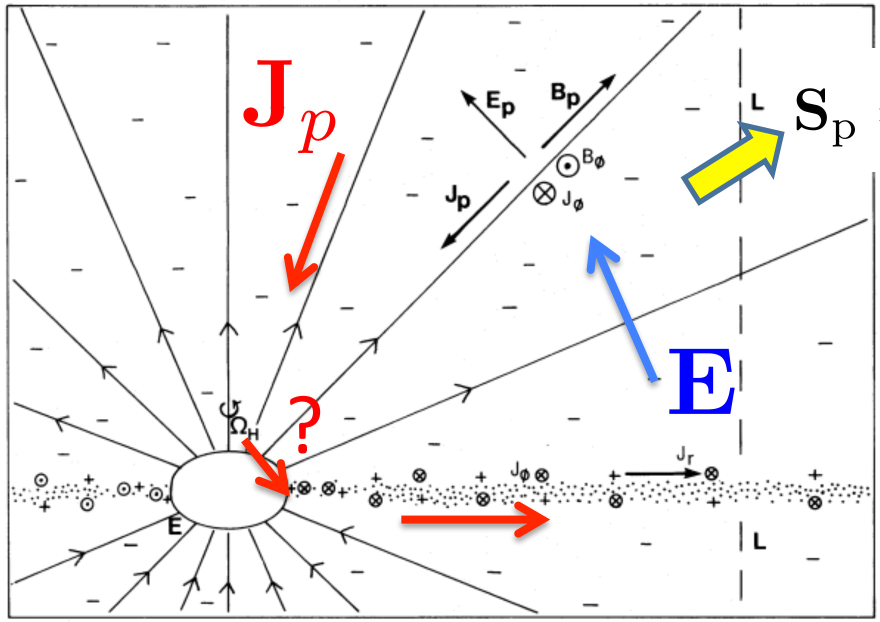
$$\mathbf{E} = -\Omega_{\rm F}\mathbf{e}_\varphi \times \mathbf{B}$$

$$\boxed{\nabla\cdot\mathbf{S}_{\rm p} = 0}$$

What drives current ?

How BH energy reduces ?

Membrane Paradigm ?

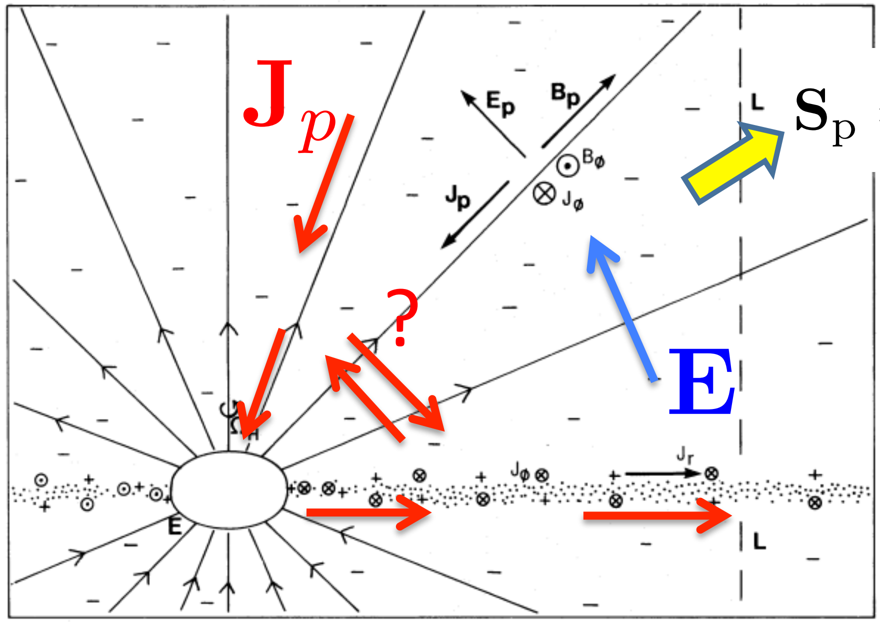


$$\mathbf{S}_{\rm p} = \frac{\mathbf{E} \times \mathbf{H}_{\varphi}}{4\pi}$$

- Horizon is assumed as a rotating conductor. Ohmic dissipation increases BH entropy (Thorne et al. 1986; Penna et al. 2013)
- But the horizon is causally disconnected (Punsly & Coroniti 1989)
- Current driving mechanism is unclear

- -> Mechanism producing the flux must work outside the horizon

Debates



$$\mathbf{S}_{\rm p} = \frac{\mathbf{E}\times\mathbf{H}_\varphi}{4\pi}$$

- Electric current is driven in a pair creation gap ?

(Okamoto 06; 09; 15)

- BH reduces its energy by negative electromagnetic energy inflow ?

(Lasota, Gourgoulhon, Abramowicz, Tchekhovskoy, & Narayan 2014; Koide & Baba 2014)

Kerr space-time

$$\boxed{ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(\beta^i dt + dx^i)(\beta^j dt + dx^j),}$$

Boyer-Lindquist coordinates

$$\alpha = \sqrt{\frac{\varrho^2\Delta}{\Sigma}}, \quad \boxed{\beta^\varphi = -\frac{2ar}{\Sigma}, \quad \equiv -\Omega}$$

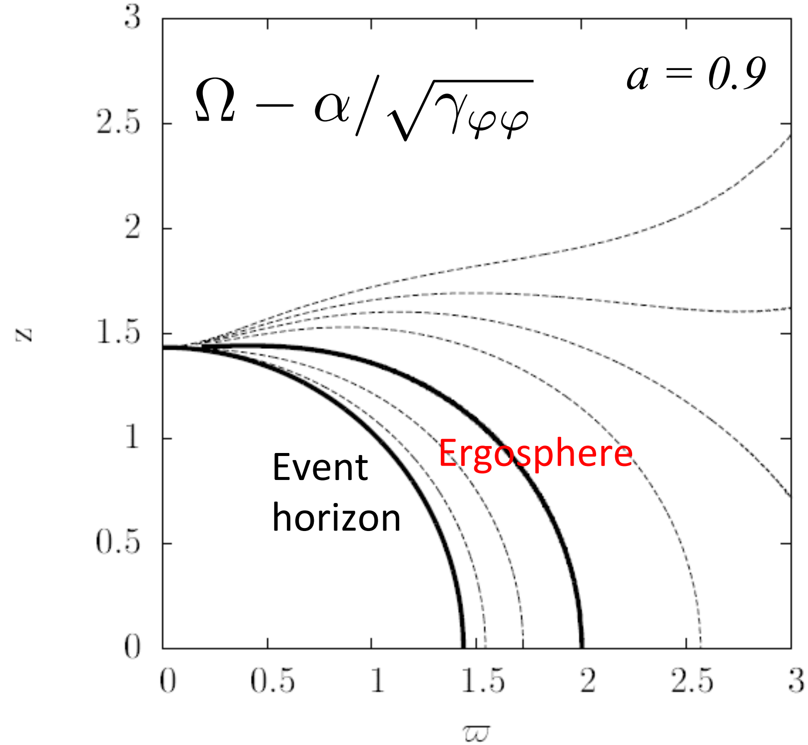
$$\gamma_{\varphi\varphi} = \frac{\Sigma}{\varrho^2}\sin^2\theta, \quad \gamma_{rr} = \frac{\varrho^2}{\Delta}, \quad \gamma_{\theta\theta} = \varrho^2,$$

$$\varrho^2 = r^2 + a^2\cos^2\theta, \quad \Delta = r^2 + a^2 - 2r,$$
$$\Sigma = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta,$$

$$r \to \infty : a \to 1, \Omega \to 0$$

$$r \to r_{\rm H} : a \to 0(\Delta \to 0)$$

Coordinate singularity



$$g_{tt} = -\alpha^2 + \gamma_{\varphi\varphi}\Omega^2 > 0$$

Kerr space-time

$$\boxed{\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \gamma_{ij}(\beta^i \mathrm{d}t + \mathrm{d}x^i)(\beta^j \mathrm{d}t + \mathrm{d}x^j),}$$

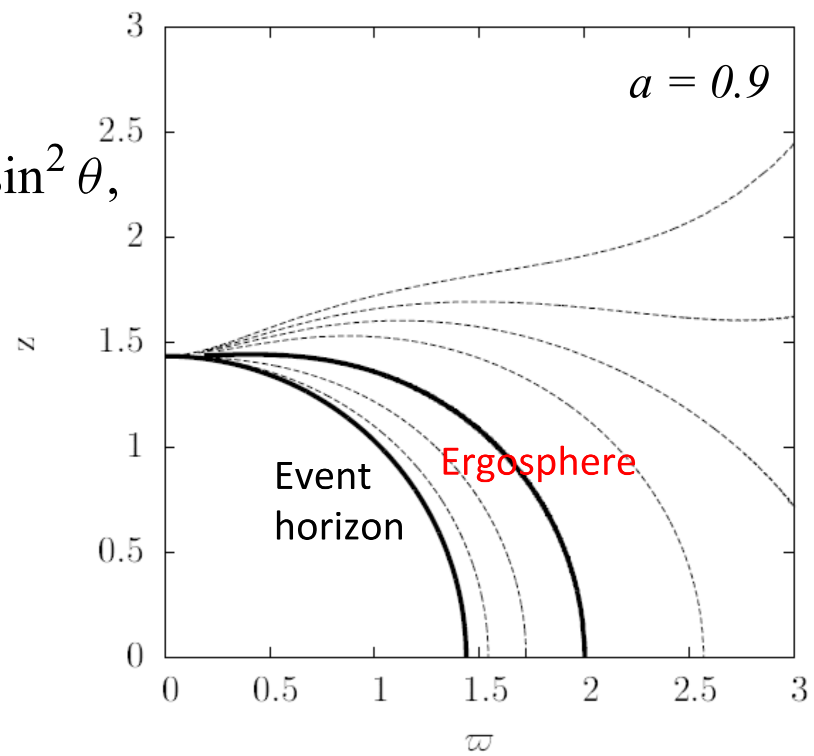
Kerr-Schild coordinates

$$\alpha = \frac{1}{\sqrt{1+z}}, \quad \beta^r = \frac{z}{1+z}, \quad \gamma_{r\varphi} = -a(1+z)\sin^2\theta,$$

$$\gamma_{\varphi\varphi} = \frac{\Sigma}{\varrho^2}\sin^2\theta, \quad \gamma_{rr} = 1+z, \quad \gamma_{\theta\theta} = \varrho^2,$$

$$z = 2r/\varrho^2$$

No coordinate singularity



Fiducial Observers (FIDOs)

$$\boxed{ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(\beta^i dt + dx^i)(\beta^j dt + dx^j),}$$

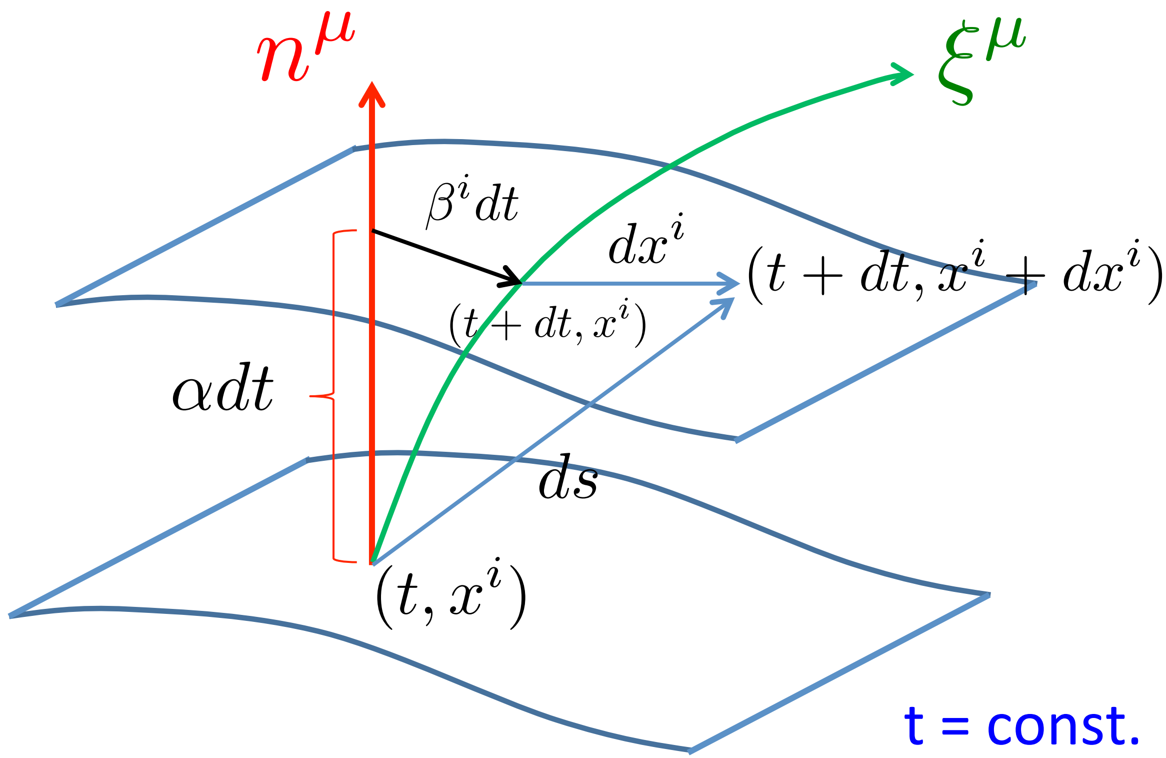
FIDOs:
Normal to t=const. surface

$$n^\mu = \left(\frac{1}{\alpha}, \frac{-\beta^i}{\alpha}\right),$$

$$n_\mu = g_{\mu\nu}n^\nu = (-\alpha, 0, 0, 0).$$

BL FIDOs rotate with $\Omega = -\beta^\phi$ in coordinate basis

Time-like Killing vector:
$dx^i = 0$



3+1 Electrodynamics

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \gamma_{ij}(\beta^i \mathrm{d}t + \mathrm{d}x^i)(\beta^j \mathrm{d}t + \mathrm{d}x^j),$$

$$E^\mu = \gamma^{\mu\nu} F_{\nu\alpha}\xi^\alpha, \quad H^\mu = -\gamma^{\mu\nu\,*}F_{\nu\alpha}\xi^\alpha$$ Fields in the coordinate basis

$$D^\mu = F^{\mu\nu} n_\nu, \quad B^\mu = -{}^*F^{\mu\nu} n_\nu$$ Fields as measured by FIDOs

$$\nabla \cdot \boldsymbol{B} = 0, \quad \partial_t \boldsymbol{B} + \nabla \times \boldsymbol{E} = 0,$$

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho, \quad -\partial_t \boldsymbol{D} + \nabla \times \boldsymbol{H} = 4\pi \boldsymbol{J},$$

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B},$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D},$$

Electromagnetic energy equation

$$\partial_t \underbrace{\left[\frac{1}{8\pi}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{B}\cdot\boldsymbol{H})\right]}_{\text{Energy density}} + \nabla \cdot \underbrace{\left(\frac{1}{4\pi}\boldsymbol{E}\times\boldsymbol{H}\right)}_{\text{Poynting flux}} = -\boldsymbol{E}\cdot\boldsymbol{J},$$

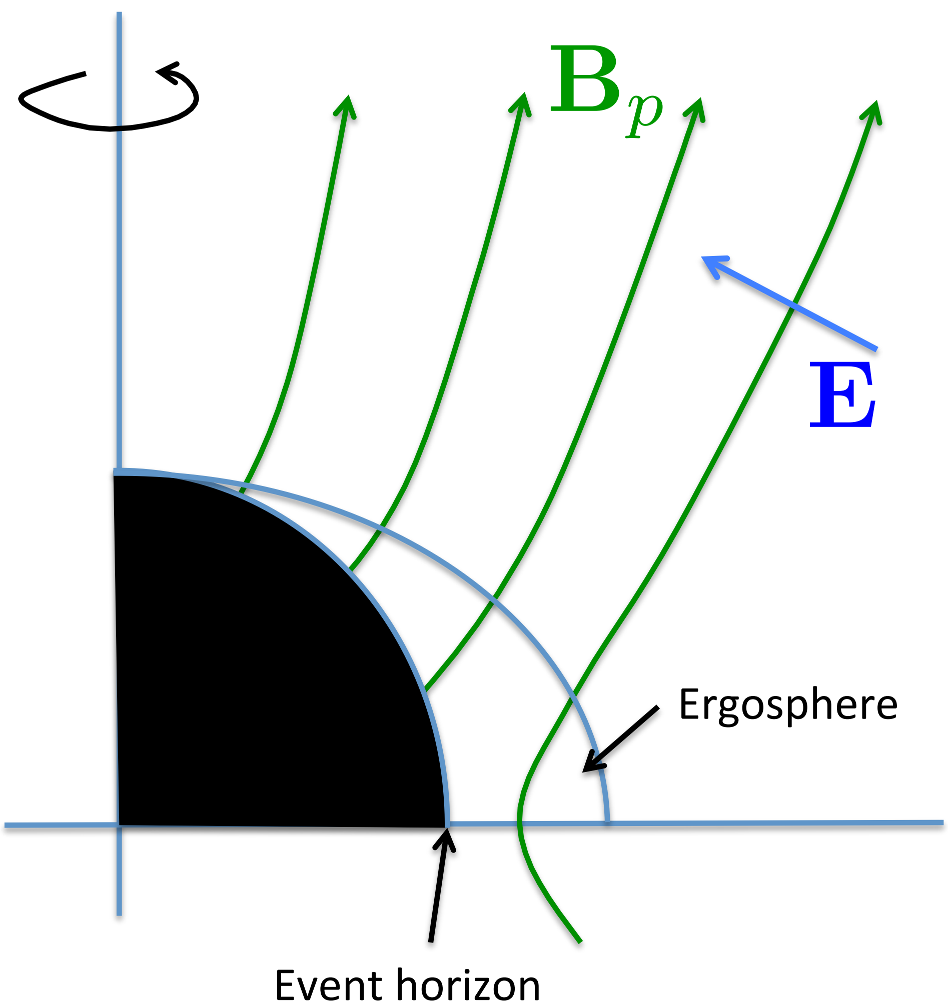
$$\frac{D\hat{u}_i}{d\hat{t}} = \frac{q}{m}(\hat{D}_i + \epsilon_{ijk}\hat{v}^j \hat{B}^k)$$

Particle EOM in FIDO's orthonormal basis

Current driving process & energy transport are clearly seen than in the Thorne formalism.

(Landau & Lifshitz 1975; Komissarov 2004)

General conditions of magnetosphere



- Kerr spacetime with arbitrary spin a (fixed)
- Axisymmetric
- Poloidal B field (with arbitrary shape) threading the ergosphere
- Plasma with sufficient number density

$$\mathbf{D} \cdot \mathbf{B} = 0$$

$$(\mathbf{E} \cdot \mathbf{B} = 0)$$

Steady axisymmetric field

$$\nabla \times \mathbf{E} = 0,$$
$$\mathbf{E} \cdot \mathbf{B} = 0$$

$$\boxed{\mathbf{E} = -\boldsymbol{\omega} \times \mathbf{B}, \quad \boldsymbol{\omega} = \Omega_{\mathrm{F}} \mathbf{m}.}$$

$$\nabla \cdot \left(\frac{-H_\varphi}{4\pi} \mathbf{B}_{\mathrm{p}} \right) = B^i \partial_i \left(\frac{-H_\varphi}{4\pi} \right) = -(\mathbf{J}_{\mathrm{p}} \times \mathbf{B}_{\mathrm{p}}) \cdot \mathbf{m},$$

$$\nabla \cdot \left(\Omega_{\mathrm{F}} \frac{-H_\varphi}{4\pi} \mathbf{B}_{\mathrm{p}} \right) = B^i \partial_i \left(\Omega_{\mathrm{F}} \frac{-H_\varphi}{4\pi} \right) = -\mathbf{E} \cdot \mathbf{J}_{\mathrm{p}},$$

$H_\varphi = {}^*F_{\mu\nu} \xi^\mu \chi^\nu$ and $\Omega_{\mathrm{F}} = -F_{t\theta} / F_{\varphi\theta}$

Same in BL & KS coordinates

$$\boldsymbol{E} = \alpha \boldsymbol{D} + \boldsymbol{\beta} \times \boldsymbol{B},$$

$$\boldsymbol{H} = \alpha \boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{D},$$

$$\mathbf{D} = \frac{1}{\alpha}(\Omega - \Omega_{\mathrm{F}})\mathbf{e}_{\varphi} \times \mathbf{B}$$

(Komissarov 2004)

BL coordinates

$$(B^2 - D^2)\alpha^2 = -B^2 f(\Omega_{\mathrm{F}}, r, \theta) + \frac{1}{\alpha^2}(\Omega_{\mathrm{F}} - \Omega)^2 H_{\varphi}^2,$$

$$f(\Omega_{\mathrm{F}}, r, \theta) \equiv (\xi + \Omega_{\mathrm{F}}\chi)^2 = -\alpha^2 + \gamma_{\varphi\varphi}(\Omega_{\mathrm{F}} - \Omega)^2.$$

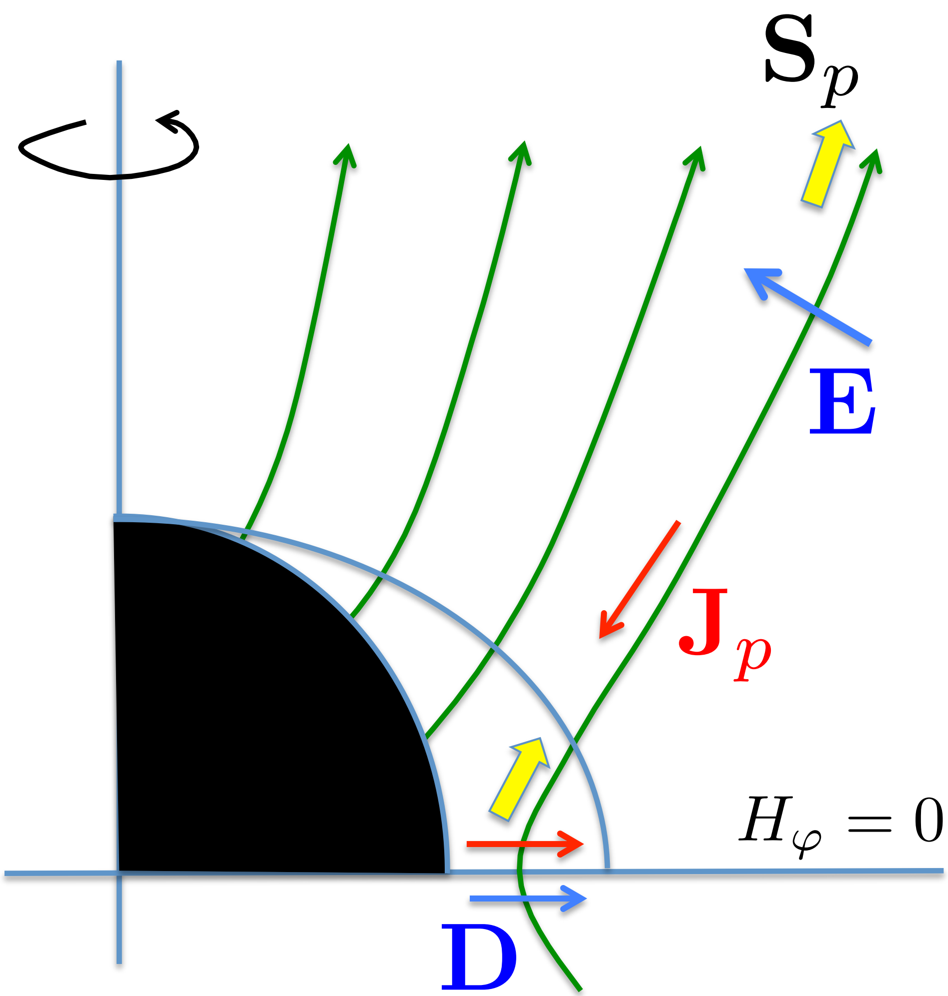
$f = 0$: light surfaces (same in BL & KS coordinates)

The inner light surface for $0 < \Omega_F < \Omega_H$

$$\Omega - \frac{\alpha}{\sqrt{\gamma_{\varphi\varphi}}} = \Omega_{\mathrm{F}} > 0$$

This is located in the ergosphere.

Steady state for field lines threading equatorial plane



- From the symmetry

$$H_\varphi = 0$$

- $D^2 > B^2$ is possible

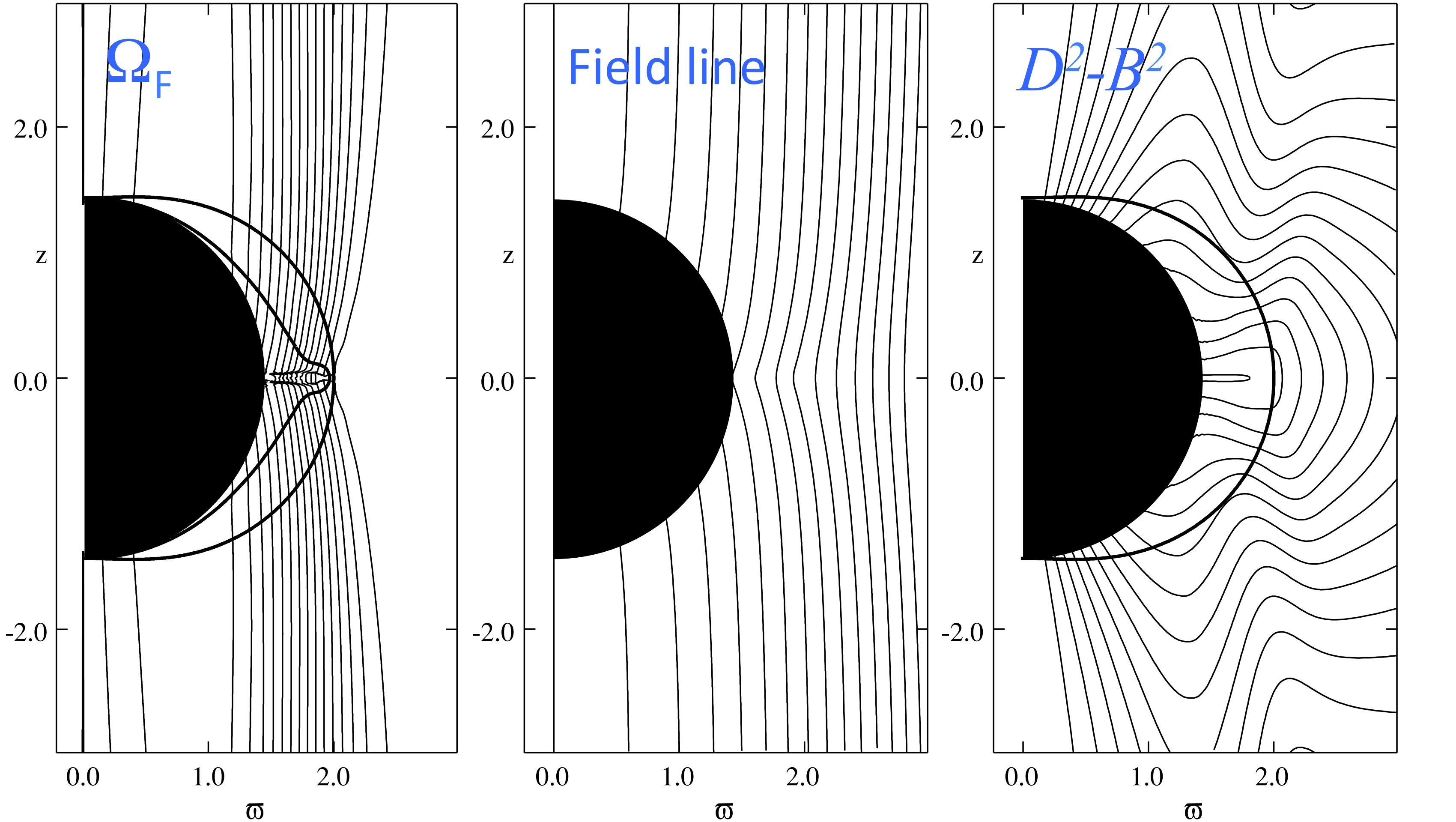
$$\nabla \cdot \mathbf{L}_p = -(\mathbf{J}_p \times \mathbf{B}_p) \cdot \mathbf{m}$$

$$\nabla \cdot \mathbf{S}_p = -\mathbf{E} \cdot \mathbf{J}_p$$

Similar to unipolar induction

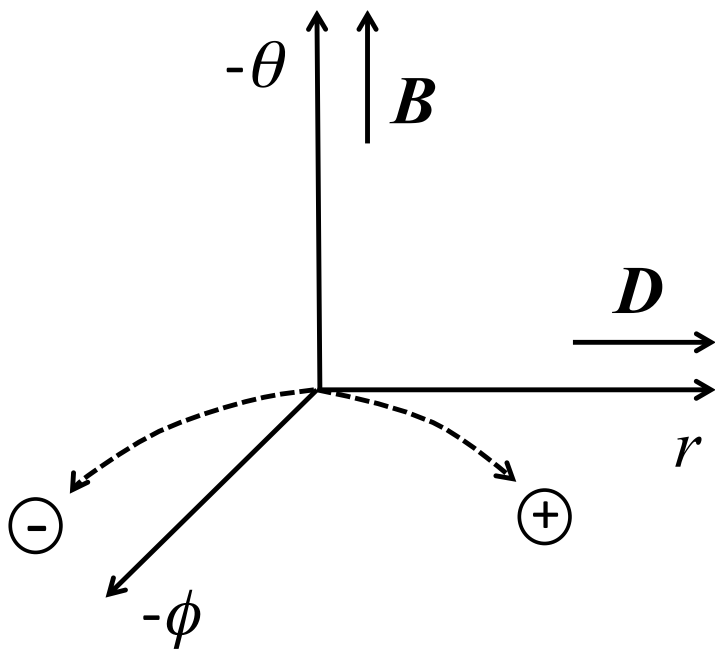
Resistive FF simulation results

(Komissarov 2004)

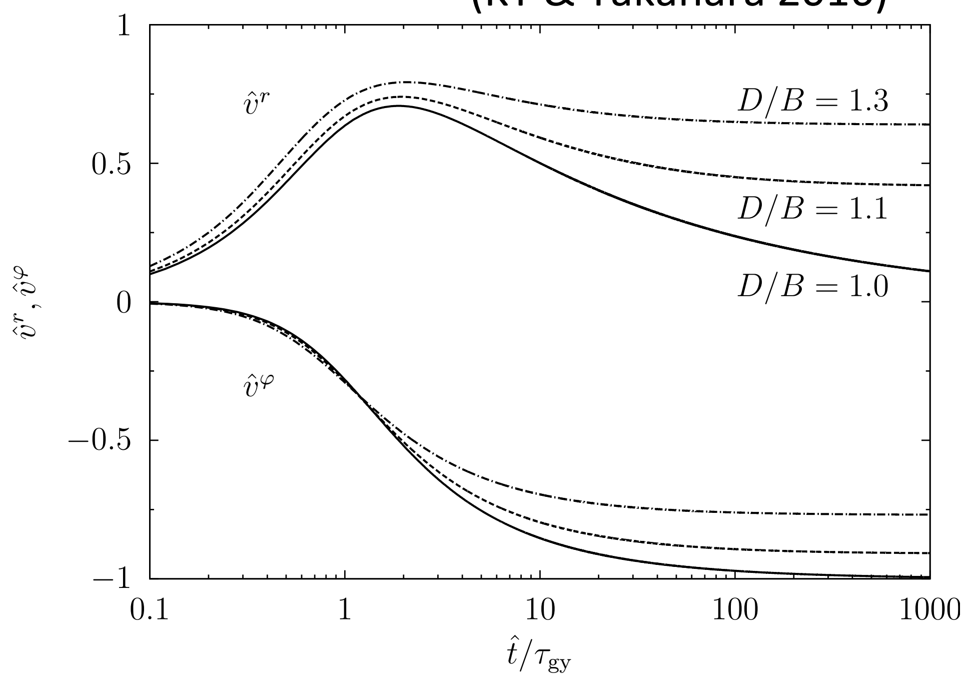


But note that few field lines are threading the equatorial plane in MHD simulations (Komissarov 2005).

Particles near equatorial plane



(KT & Takahara 2016)



$$\boxed{\hat{v}^\varphi \approx -1}$$

$$
\begin{aligned}
l_{\rm p} &= u_\mu \chi^\mu = \gamma_{\varphi\varphi}(v^\varphi - \Omega)u^t \\
&= \sqrt{\gamma_{\varphi\varphi}}\hat{v}^\varphi \hat{u}^t,
\end{aligned}
$$

$$
\begin{aligned}
e_{\rm p} &= -u_\mu \xi^\mu = [\alpha^2 + \gamma_{\varphi\varphi}\Omega(v^\varphi - \Omega)]u^t \\
&= (\alpha + \sqrt{\gamma_{\varphi\varphi}}\Omega\hat{v}^\varphi)\hat{u}^t,
\end{aligned}
$$

$\Rightarrow$ $l_{\rm p} < 0, \quad e_{\rm p} < 0$

Similar to mechanical Penrose process

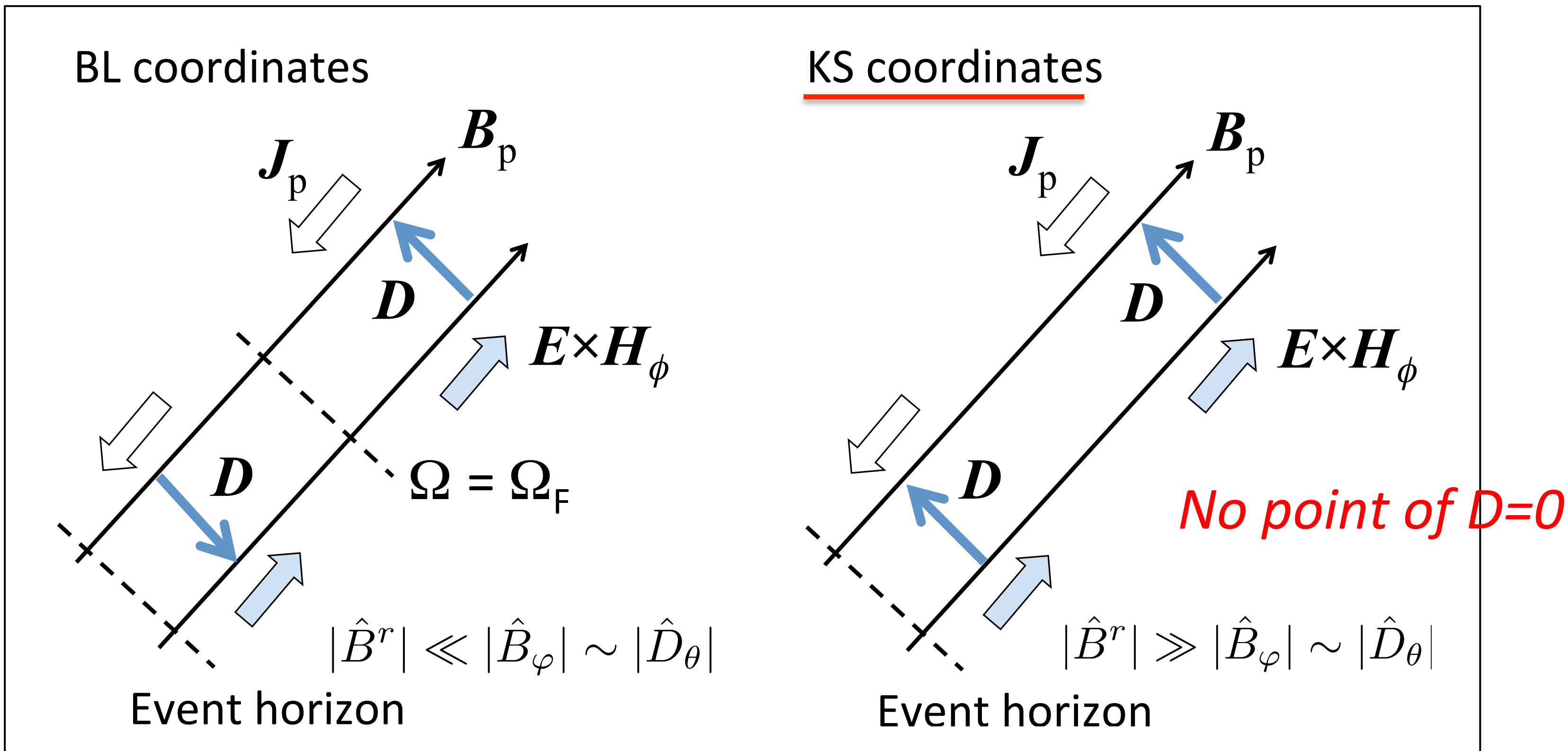
Steady state for field lines threading horizon

Condition on horizon

$$\boxed{H_\varphi \neq 0, \quad D^2 < B^2}$$

Force-free,
$\nabla \cdot \mathbf{S}_\mathrm{p} = 0$

<u>Structure of BZ solution</u>



(see Okamoto 06)

(KT & Takahara 2016)

Inflow of negative EM energy ?

$$l = \frac{1}{4\pi\alpha}\gamma_{\varphi\varphi}(\Omega_{\rm F} - \Omega)(B^\theta B_\theta + B^r B_r),$$
$$e = \frac{1}{8\pi\alpha}\left[\alpha^2 B^2 + \gamma_{\varphi\varphi}(\Omega_{\rm F}^2 - \Omega^2)(B^\theta B_\theta + B^r B_r)\right].$$

l < 0, *e* < 0

in BL coord.

(Lasota et al. 2014; Koide & Baba 2014)

$$\begin{aligned} 4\pi\alpha l &= \frac{\Sigma\sin^2\theta}{\varrho^2}(\Omega_{\rm F} - \Omega)B^\theta B_\theta - 2r\sin^2\theta B^r B^\varphi + \Omega_{\rm F}(\varrho^2 + 2r)\sin^2\theta (B^r)^2 \\ 8\pi\alpha e &= \left[\frac{\Sigma\sin^2\theta}{\varrho^2}(\Omega_{\rm F} + \Omega)(\Omega_{\rm F} - \Omega) + \frac{\varrho^2\Delta}{\Sigma}\right] B^\theta B_\theta + \Delta\sin^2\theta (B^\varphi)^2 \\ &\quad -2a\sin^2\theta B^r B^\varphi + \left[1 + \Omega_{\rm F}^2(\varrho^2 + 2r)\sin^2\theta\right](B^r)^2. \end{aligned}$$

l > 0, *e* > 0

in KS coord.

Concept of "flow of steady field" is ambiguous

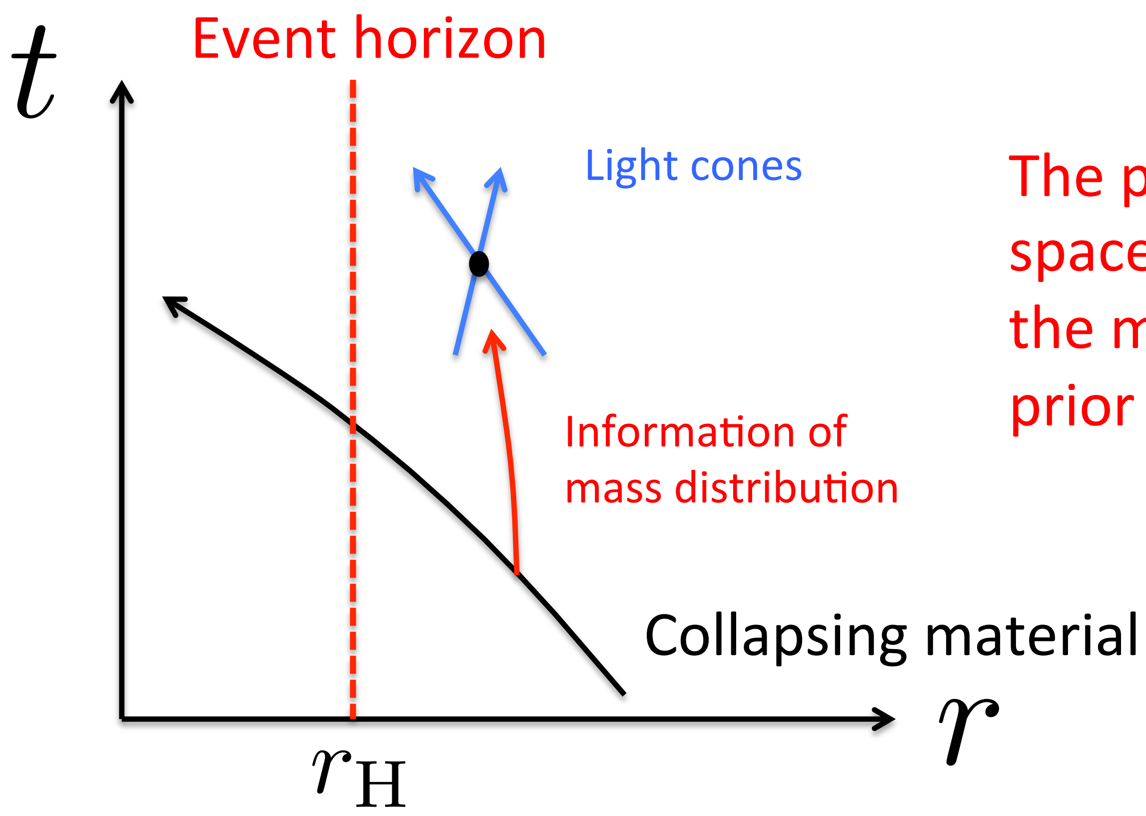
$$e = -\alpha T^t_t = T^\mu_\nu n_\mu \xi^\nu$$

$$S_{\rm p} = \frac{1}{4\pi} E H_\varphi \neq ev$$

(KT & Takahara 2016)

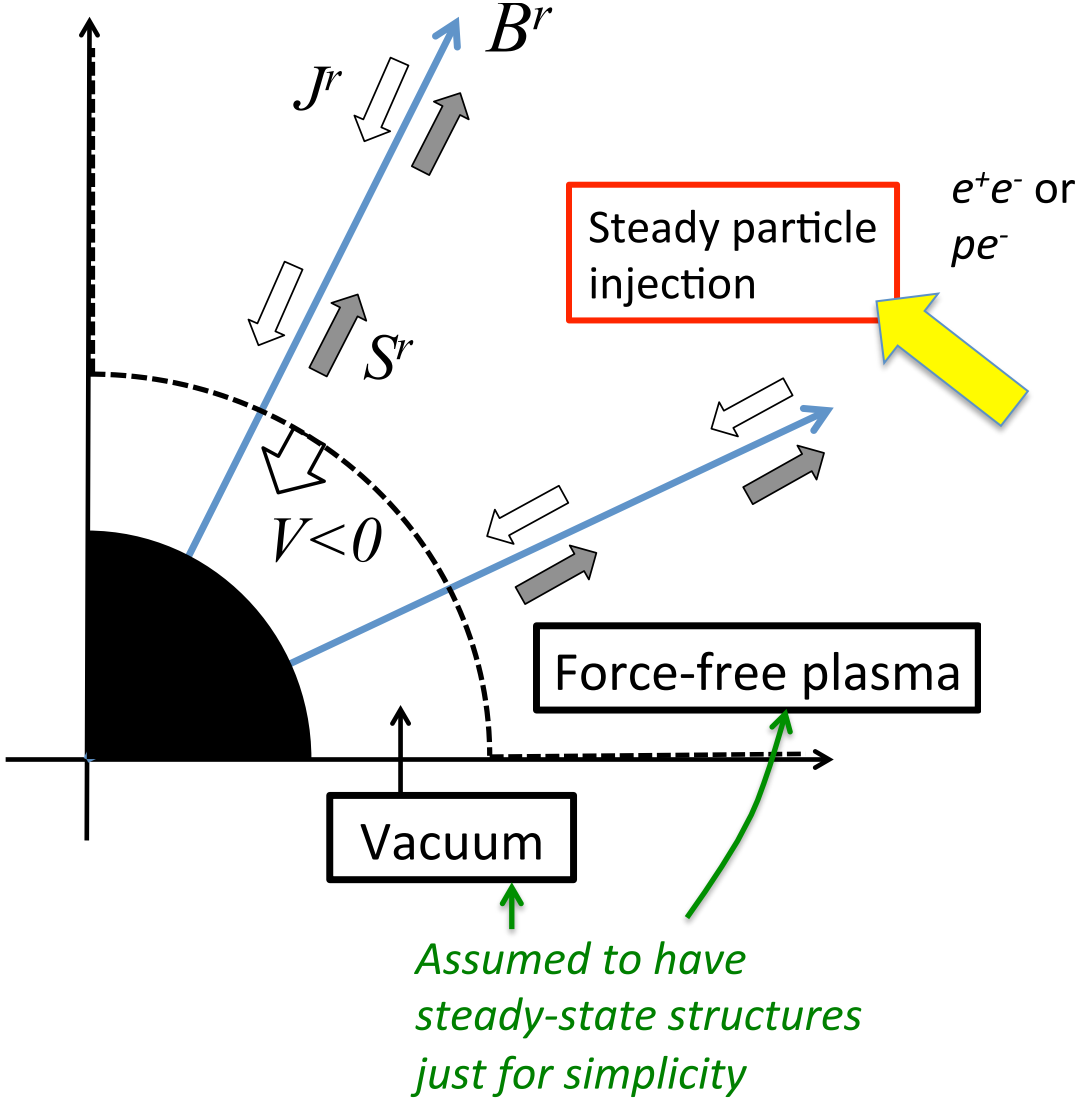
Origin of Schwarzschild spacetime

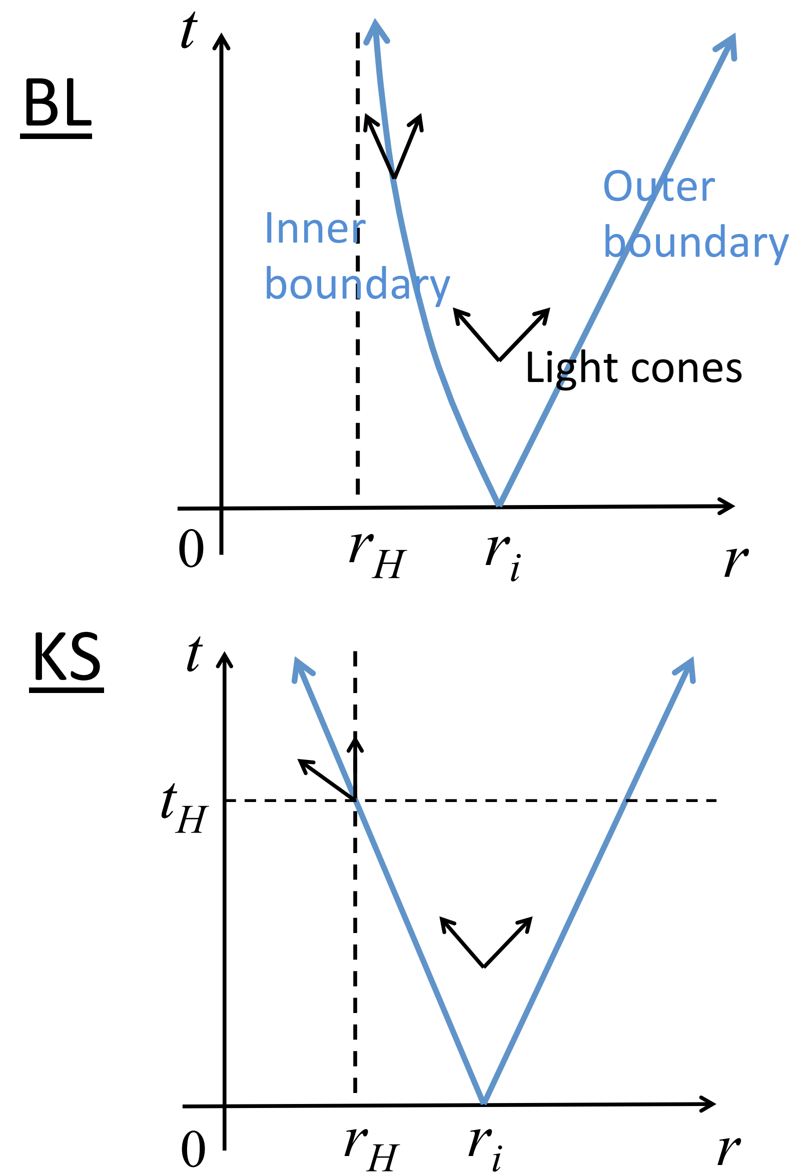
The source of the Schwarzschild gravitational field is the mass at the center, but this information does not escape to outside.



The physical origin of the space-time curvature is the mass distribution at prior times

Vacuum → Steady FF state





Junction condition at inner boundary (BL coordinates)

$$-\partial_t D^r + \frac{1}{\sqrt{\gamma}}\partial_\theta H_\varphi = 4\pi J^r, \quad \Rightarrow \quad \eta^r = \left.\frac{-D^r_{\rm vac}}{4\pi}\right|_{R=0} V,$$

$$-\partial_t D^\theta - \frac{1}{\sqrt{\gamma}}\partial_r H_\varphi = 4\pi J^\theta, \quad \Rightarrow \quad V = \frac{1}{\sqrt{\gamma}} \left.\frac{H^{\rm ff}_\varphi + 4\pi\sqrt{\gamma}\eta^\theta}{D^\theta_{\rm ff} - D^\theta_{\rm vac}}\right|_{R=0}.$$

$$\partial_t B^\varphi + \frac{1}{\sqrt{\gamma}}(\partial_r E_\theta - \partial_\theta E_r) = 0, \quad \Rightarrow \quad V = \frac{1}{\sqrt{\gamma}} \left.\frac{E^{\rm ff}_\theta - E^{\rm vac}_\theta}{B^\varphi_{\rm ff}}\right|_{R=0},$$

$$\Downarrow$$

$$\boxed{ds^2 = -\alpha^2 dt^2 + \gamma_{rr} dr^2}$$

If $\eta^\theta = 0$, it would be null

$$V = \frac{\pm\alpha}{\sqrt{\gamma_{rr}}}\sqrt{1 + \frac{4\pi\sqrt{\gamma}\eta^\theta}{H^{\rm ff}_\varphi}}. \quad \Rightarrow \quad \eta^\theta > 0,$$

Current crosses field lines !

Same conclusion in KS coordinates

Sources of current & fluxes

$$H_\varphi^{\rm ff} = \left[\sqrt{\gamma}(D^\theta_{\rm ff} - D^\theta_{\rm vac})V - 4\pi\sqrt{\gamma}\eta^\theta\right]_{R=0}$$

Displacement current & Cross-field current on the inner boundary

$$L^r_{\rm ff} = \left[V(l_{\rm ff} - l_{\rm vac}) + \sqrt{\gamma}\eta^\theta B^r\right]_{R=0}.$$

$$S^r_{\rm ff} = \left[V(e_{\rm ff} - e_{\rm vac}) - E_r\eta^r - E_\theta\eta^\theta\right]_{R=0}.$$

Energy conversion, work of cross-field current, Ohmic dissipation on the inner boundary

$$\nabla \cdot \mathbf{S}_p = -\partial_t e - \mathbf{E}\cdot\mathbf{J}_p$$

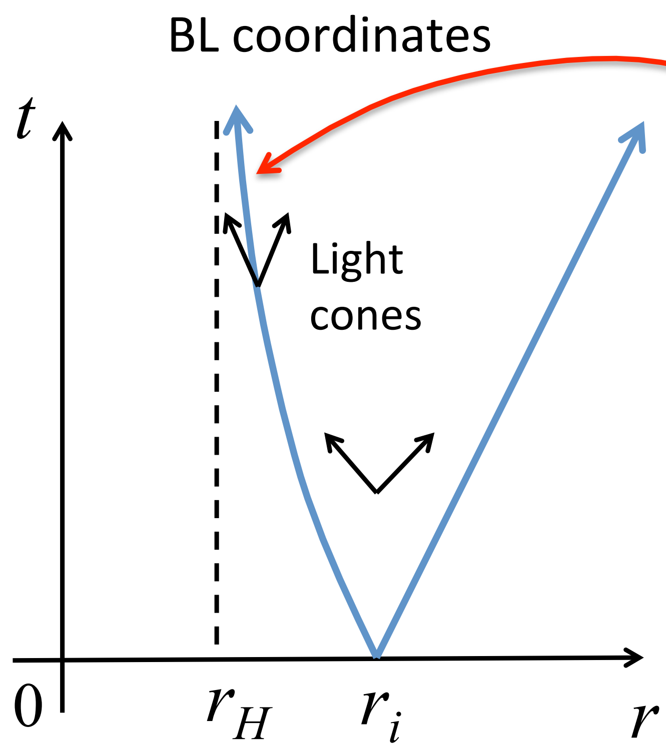
Steady State

$$\nabla \cdot \mathbf{L}_p = -\partial_t l - (\mathbf{J}_p \times \mathbf{B}_p) \cdot \mathbf{m}$$

$$\nabla \cdot \mathbf{S}_p = -\partial_t e - \mathbf{E} \cdot \mathbf{J}_p$$

$$\nabla \cdot \mathbf{L}_\mathrm{p} = 0$$

$$\nabla \cdot \mathbf{S}_\mathrm{p} = 0$$



The boundary (AM/energy source) does not affect the exterior

- Driving source of current is not required in steady state (no resistivity is assumed)

- BH loses energy directly by outward Poynting flux (unlike mechanical Penrose process)

Condition at the horizon

$$H_\varphi = -\alpha\sqrt{\frac{\gamma_{\varphi\varphi}}{\gamma_{\theta\theta}}}D_\theta$$

BL coordinates

- Ohm's law for the current flowing on the membrane (Thorne et al. 1986 "Membrane Paradigm")

- Rather, it may be interpreted as displacement current (see also Punsly 2008)

$$H_\varphi^{\mathrm{ff}} = \sqrt{\gamma}(D_{\mathrm{ff}}^\theta - D_{\mathrm{vac}}^\theta)V - 4\pi\sqrt{\gamma}\eta^\theta$$

$$V = \frac{\pm\alpha}{\sqrt{\gamma_{rr}}}\sqrt{1 + \frac{4\pi\sqrt{\gamma}\eta^\theta}{H_\varphi^{\mathrm{ff}}}}.$$

$$\eta^\theta \to 0$$

$$\alpha D_{\mathrm{vac}}^\theta \to 0$$

Conclusion

- The current driving (S_p production) mechanism in BZ process can be discussed only in the time-dependent state towards steady state
- In the steady state, S_p needs no electromagnetic source. The steady currents can keep flowing in the ideal MHD condition. No gap is needed. The BH rotational energy is reduced directly by S_p without being mediated by the negative energies.
- Our argument is based on some assumptions. Detailed plasma simulations are needed to validate it

MHD model

Energy flux density

Bernoulli constant

$$S_{\mathrm{p}} = 4\pi\rho c^2 \Gamma v_{\mathrm{p}} \mathcal{E} > 0 \quad \text{for} \quad v_{\mathrm{p}} < 0, \quad \mathcal{E} < 0$$

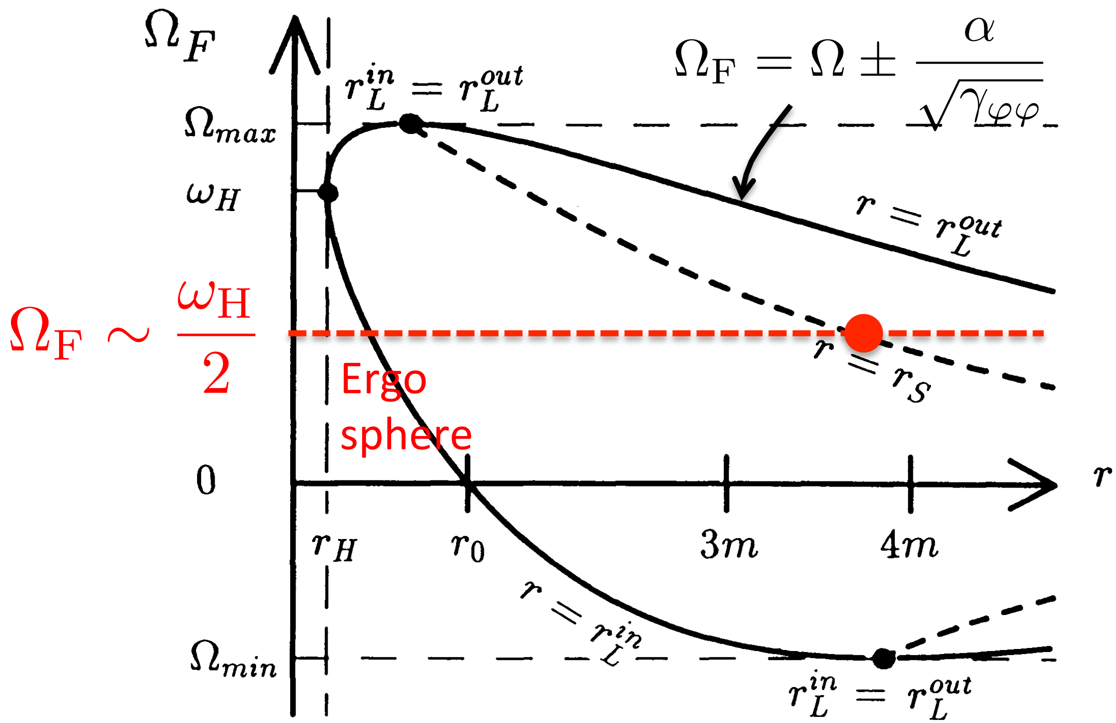


FIG. 1.—Positions of the light surfaces $r = r_L^{in}$, r_L^{out} (*solid lines*) and the separation point $r = r_S$ (*broken lines*) for a monopole geometry in the equatorial plane with $a = 0.8m$. These points are determined by Ω_F.

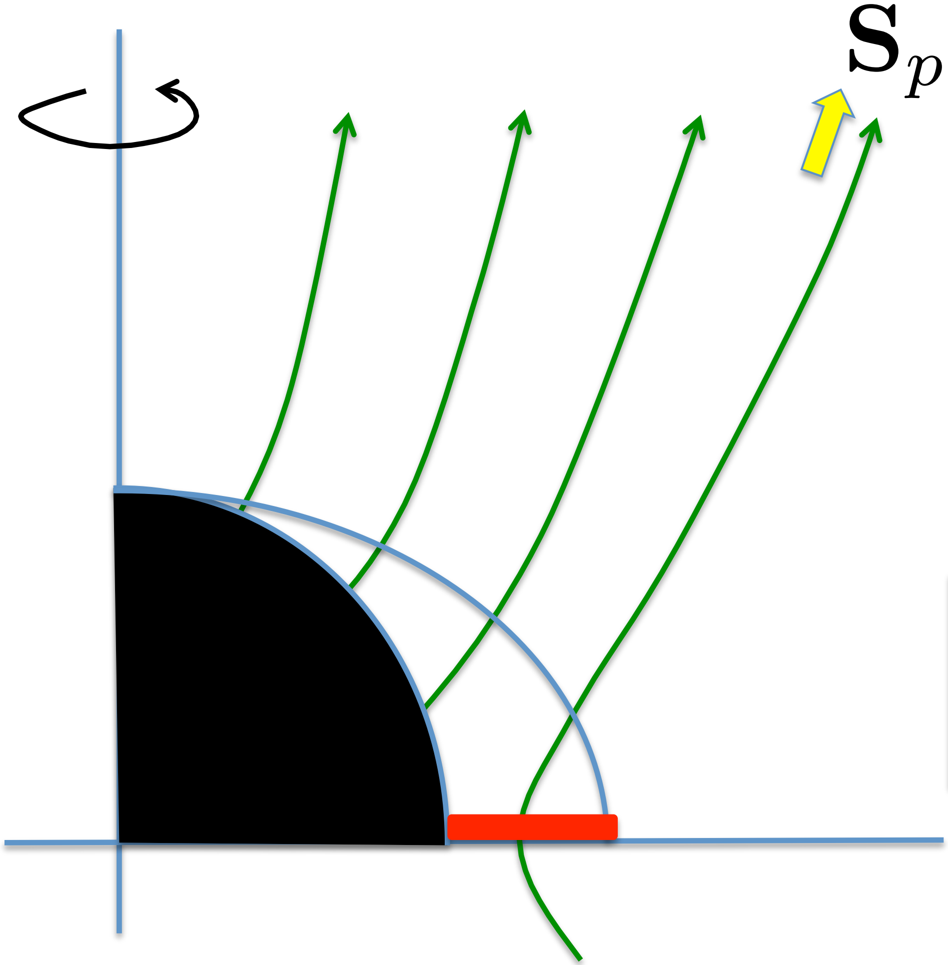
(Takahashi et al. 1990)

Separation surface may be located outside the ergosphere.

(Komissarov 2009)

- Cross-field (inertial drift) currents cannot produce all of S_{p}
- MHD simulations show the steady state without negative particle energy (Komissarov 2005)

Negative-energy particle inflow



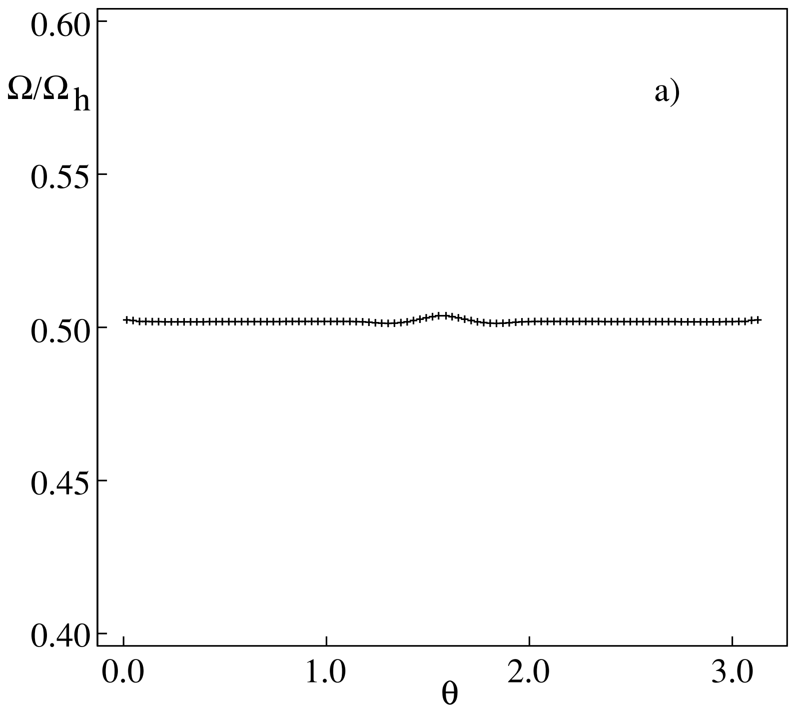
$$\boxed{-U_t < 0, \quad U^r < 0}$$

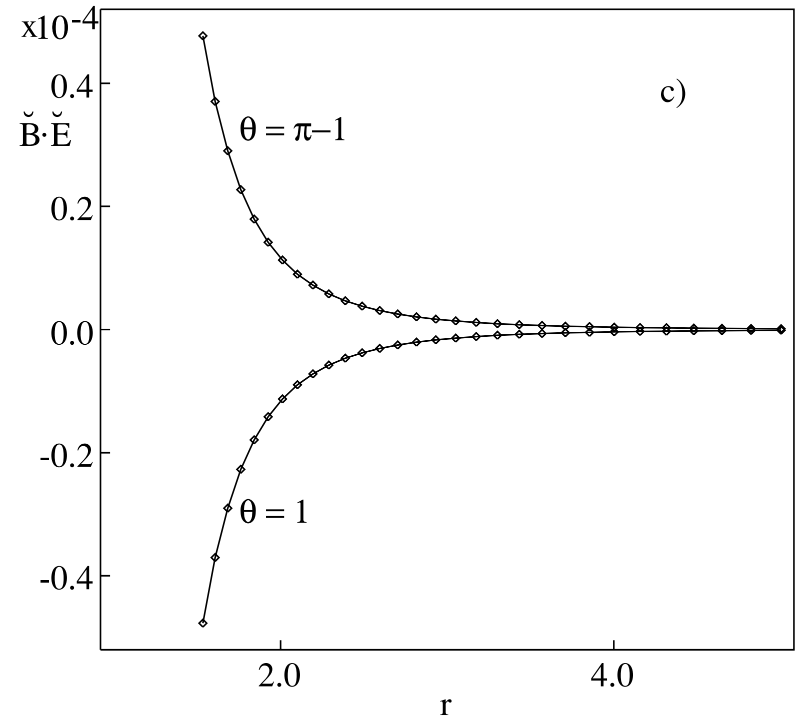
$$\partial_r \sqrt{\gamma}(-\alpha \rho_{\mathrm{m}} U_t U^r) = \mathbf{E} \cdot \mathbf{J}_{\mathrm{p}} < 0$$

FF simulation results

<u>Monopole solution with a = 0.1</u>

(Komissarov 2004)





$$\mathbf{D} \cdot \mathbf{B} \neq 0$$

We consider that a small field-aligned electric field may appear in numerical simulations and in reality with small resistivity